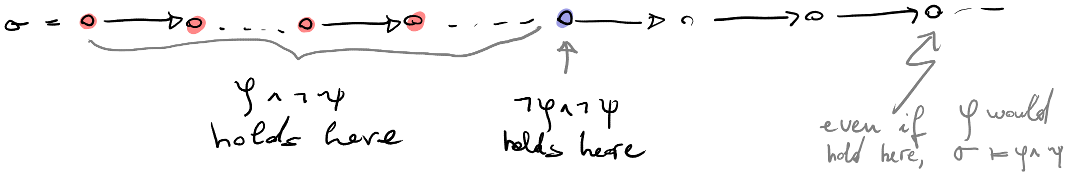
Duality in LTL

As said, the De Morgan laws still hold for propositional logic connectives of LTL.
What about $\circ\_$ and $\_\mathcal{U}\_$?

- Next is self-dual: $\quad \neg\circ\varphi \equiv \circ\neg\varphi$
- For $\_\mathcal{U}\_$ note that $\quad \neg(\varphi\,\mathcal{U}\,\psi) \equiv ((\varphi\wedge\neg\psi)\,\mathcal{U}\,(\neg\varphi\wedge\neg\psi)) \vee \underbrace{\neg(\text{true}\,\mathcal{U}\,\neg(\varphi\wedge\neg\psi))}_{\substack{\neg\Diamond\neg(\varphi\wedge\neg\psi) \\ \square(\varphi\wedge\neg\psi)}}$

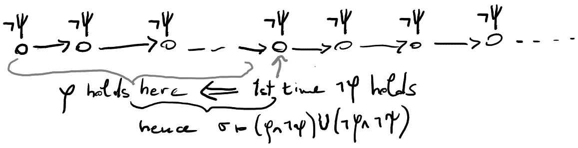
Case $\Leftarrow$

1. clearly for any word σ s.t. $\sigma \models \square(\varphi\wedge\neg\psi)$, we have $\sigma \not\models \varphi\,\mathcal{U}\,\psi$
2. assume $\sigma \models (\varphi\wedge\neg\psi)\,\mathcal{U}\,(\neg\varphi\wedge\neg\psi)$, then we have



Case $\Rightarrow$

1. If $\sigma \models \square\neg\psi$ then
   - (1.1) if $\sigma \models \square\varphi$ then $\sigma \models \square(\varphi\wedge\neg\psi)$
   - (1.2) if $\sigma \models \Diamond\neg\varphi$ then



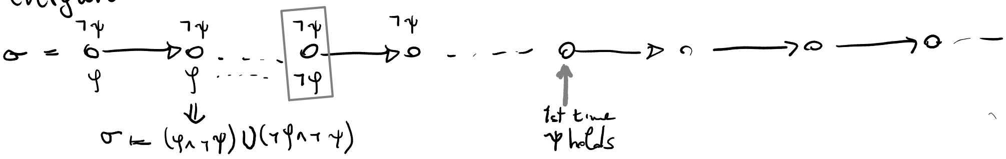
2. If $\sigma \not\models \square\neg\psi$ then

Ex: why? { & there is ¬φ somewhere!

$\Downarrow$

If $\neg\varphi$ everywhere here

else



So let

$$\varphi\,\mathcal{W}\,\psi \triangleq \varphi\,\mathcal{U}\,\psi \vee \neg(\text{true}\,\mathcal{U}\,\neg\varphi)$$

then

$$\neg(\varphi\,\mathcal{U}\,\psi) \equiv (\varphi\wedge\neg\psi)\,\mathcal{W}\,(\neg\varphi\wedge\neg\psi) \qquad \& \qquad \neg(\varphi\,\mathcal{W}\,\psi) \equiv (\varphi\wedge\neg\psi)\,\mathcal{U}\,(\neg\varphi\wedge\neg\psi)$$