

Introduction to Model Checking

<https://clegra.github.io/mc.html>

Lecture 1

Clemens Grabmayer

<https://clegra.github.io>

Emilio Tuosto

<https://cs.gssi.it/emilio.tuosto/>

Department of Computer Science



January 26, 2026

Organization

Lectures (Clemens 4 / Emilio 3)

- ▶ this week:
 - ▶ Monday, 10.30–12.30, room B, Aurora
 - ▶ Tuesday, 14.30–16.30, P-2.6, Zenith
 - ▶ Thursday and Friday, 14.30–16.30, conference room -1, Zenith

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stable for later: <https://clegra.github.io/mc/25-26/mc.html>

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Exam (more later)

- ▶ options:
 - ▶ small verification project (of an algorithm, e.g. in Maude)
 - ▶ presentation about a paper
 - ▶ written exam?

Topics of the course

- ▶ modeling systems by [labeled transition systems \(LTSs\)](#)
- ▶ [safety](#), [liveness](#), and [fairness](#) properties

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- ▶ modeling systems by **labeled transition systems (LTSs)**
- ▶ **safety**, **liveness**, and **fairness** properties
- ▶ **Linear Temporal Logic (LTL)**
 - ▶ model checking formulas
 - ▶ express properties by **Büchi automata**
 - ▶ model check LTSs and properties via **product automata**
- ▶ **Computation Tree Logic (CTL)**

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- ▶ **Maude** examples **model-checker**

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- ▶ **Maude** examples **model-checker**
- ▶ (**partial** model checking)
 - ▶ (partially known systems (state properties/states/transitions))

Model Checking

... is an effective automatable technique:

- ▶ *to expose potential software design errors;*
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 - ▶ embedded systems, software engin., hardware design, explainable AI
- ▶ supports partial verification (of system parts)
- ▶ provides diagnostic information for debugging
- ▶ has sound mathematical underpinning (logic and process theory)

Model Checking

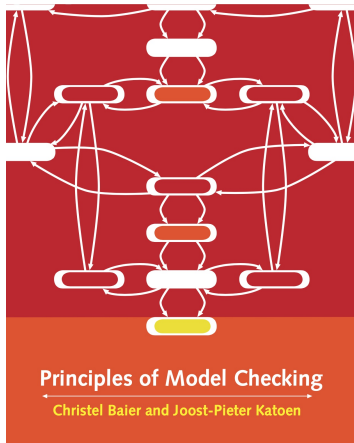
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Course Goals are introduction to:

- ▶ **Theory:**
 - ▶ modeling systems by labeled transition systems,
 - ▶ expressing properties by temporal-logic formulas
 - ▶ model-checking algorithms
- ▶ **Practice related:** see Maude examples

Book



- ▶ pdf online available (see [1])
- ▶ we study chapters 1–3, 5, 6

Lectures

1. Introduction (preview extended, LTSs)
2. Modeling by labeled transition systems
 - ▶ executions; traces; non-determinism; examples
3. Linear-Time Behaviour and Properties
 - ▶ invariant, safety, and liveness (and fairness) properties
4. Linear Temporal Logic (LTL)
 - ▶ syntax and semantics; interpretation of LTSs; examples
5. LTL (continued)
 - ▶ model checking of LTL formulas, and fairness in LTL
 - ▶ Maude examples
6. Computation Tree Logic (CTL)
 - ▶ syntax and semantics, examples
7. Extensions of CTL, and Outlook
 - ▶ expressibility differences with LTL
 - ▶ model checking formulas in CTL
 - ▶ (μ -calculus | partial model-checking | Maude examples)

Software correctness

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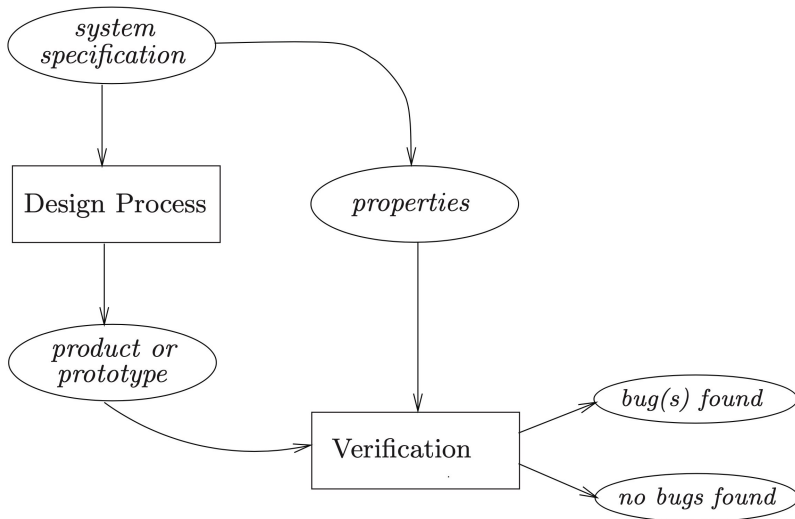
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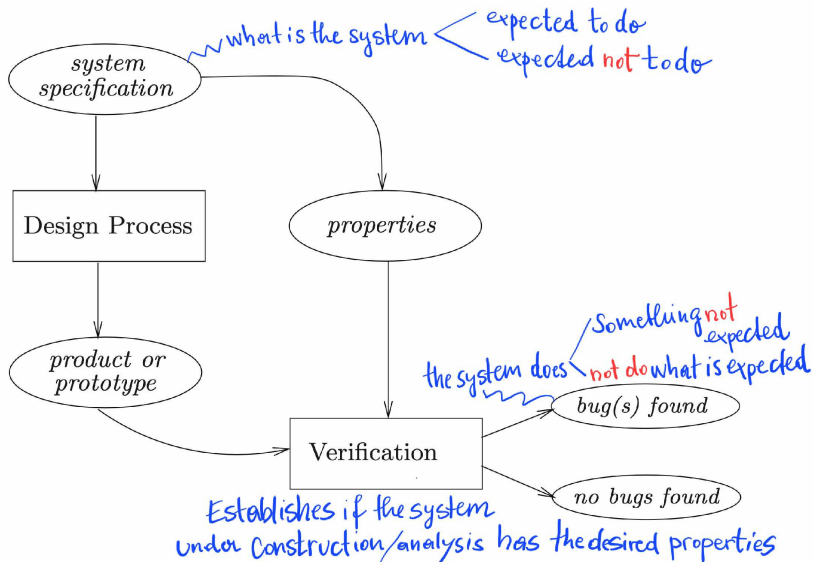
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Hard-/Software Verification (traditionally)



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Software verification methods

► Peer review

- $\left\{ \begin{array}{l} + \text{ rather useful: between 31\% and 93\% (median 60\%)} \\ - \text{ hard to catch subtle errors (concurrency and algorithmic defects)} \end{array} \right.$

Software verification methods

► Peer review

- + rather useful: between 31% and 93% (median 60%)
- hard to catch subtle errors (concurrency and algorithmic defects)

► Software testing

- + applicable to all sorts of software
- + test generation can (partly) be automated
- exhaustive testing of all execution paths is infeasible
- When do we stop?

Software verification methods

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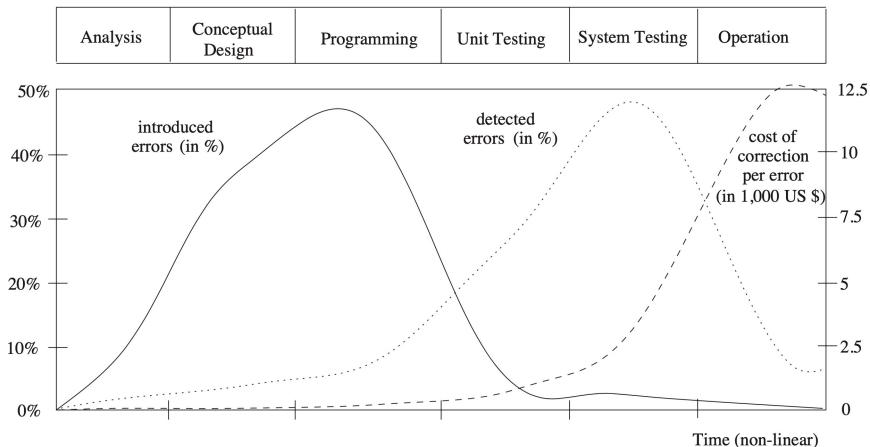
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► Catching software errors: **the sooner the better.**

Software lifecycle, error introduction/detection, repair costs

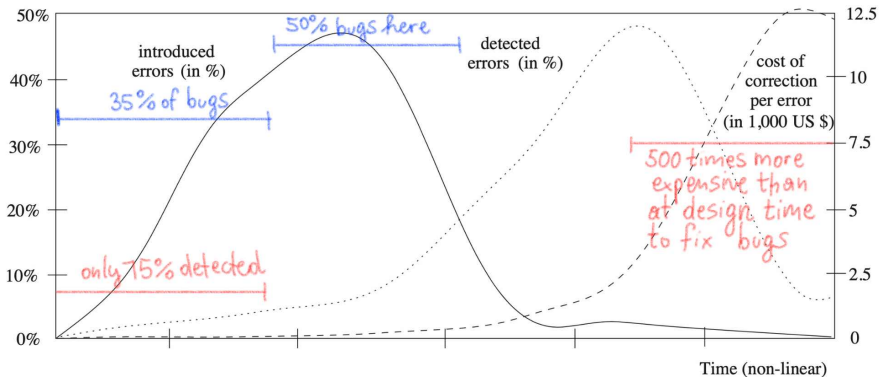


Liggesmeyer et al: "Qualitätssicherung technischer Systeme ..." [4, 1998]

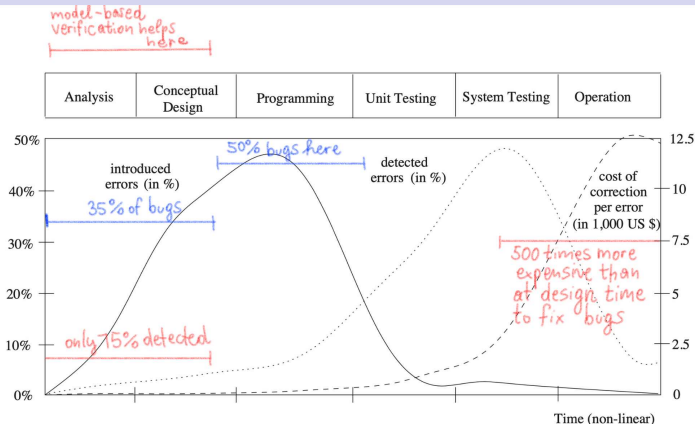
Software lifecycle, error introduction/detection, repair costs

model-based
verification helps
here

Analysis	Conceptual Design	Programming	Unit Testing	System Testing	Operation
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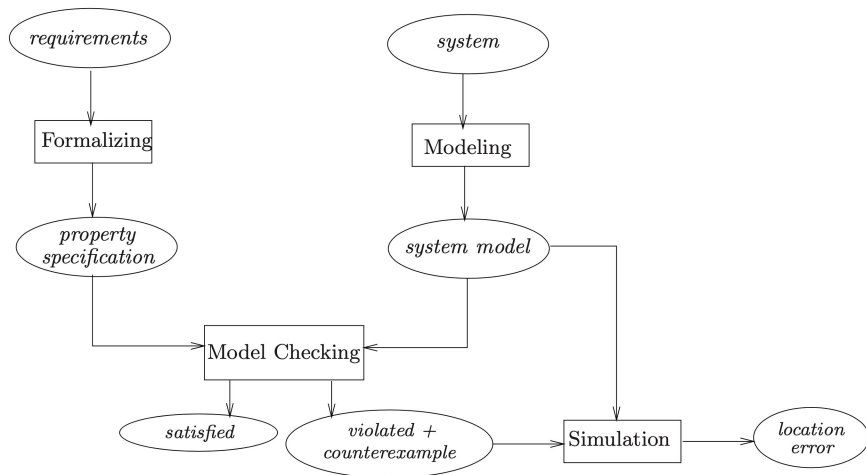


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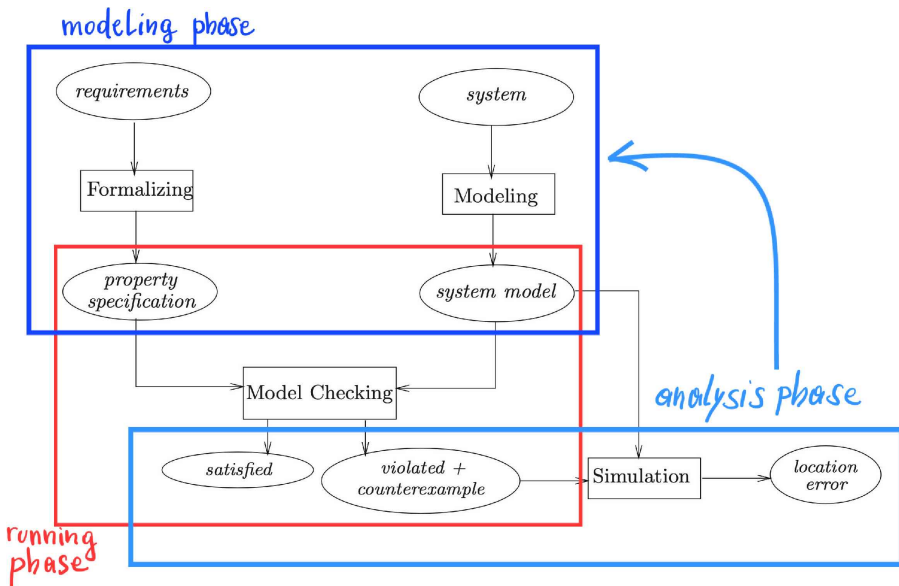


“In software and hardware design of complex systems, more time and effort are spent on verification than on construction. **Techniques are sought to reduce and ease the verification efforts while increasing their coverage.** Formal methods offer a large potential to obtain an early integration of verification in the design process, to provide more effective verification techniques, and to reduce the verification time.”

Model checking



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Glancing at temporal logic

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- ▶ stems from philosophy: modal logic to reason about time
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Modality $\Box\phi \hat{=}$ for all time moments, ϕ holds.

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Then we have:

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Example

Modality $\Box \phi \hat{=}$ for all time moments, ϕ holds.

Then we have:

$\Box(\neg(a \wedge b)) \hat{=}$ for all time moments,
events a and b do not occur at the same time,

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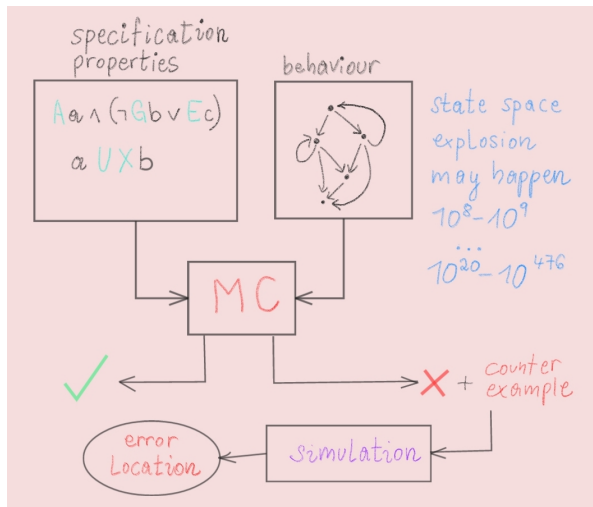
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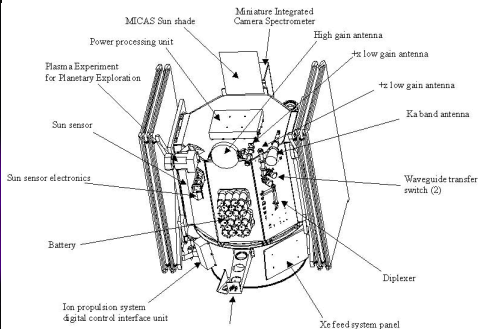
Then we have:

$$\begin{aligned} \Box(\neg(a \wedge b)) &\hat{=} \text{for all time moments,} \\ &\quad \text{events } a \text{ and } b \text{ do not occur at the same time,} \\ &= \text{it will never happen that} \\ &\quad \text{events } a \text{ and } b \text{ occur at the same time.} \end{aligned}$$

Glancing at temporal logic (its use for model checking)



Deep Space 1 (NASA)



- ▶ Flyby of asteroid 9969 Braille (1999)
- ▶ Entered the coma of Comet Borrelly (2001)
- ▶ Model checking discovered **5 concurrency errors**

Example (program concurrency/non-determinism)

Programs **Inc**, **Dec**, and **Reset** cooperate, and use a shared variable x :

proc Inc

```
while  $0 \leq x \leq 200$ 
do
  if  $x < 200$ 
  then  $x := x + 1$ 
  fi
od
```

proc Dec

```
while  $0 \leq x \leq 200$ 
do
  if  $x > 0$ 
  then  $x := x - 1$ 
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proc Reset

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while  $0 \leq x \leq 200$ 
do
  if  $x = 200$ 
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Question: When started on $x = 0$, do the counter programs run forever?

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Is $0 \leq x \leq 200$ always guaranteed?

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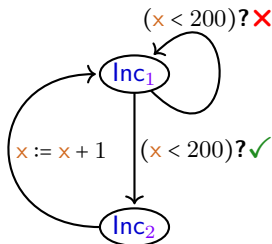
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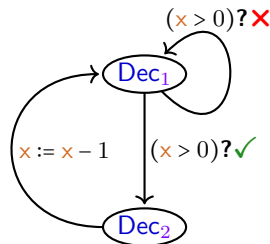


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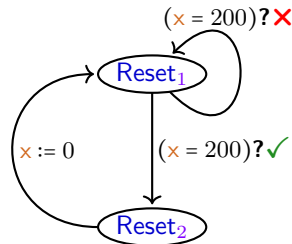


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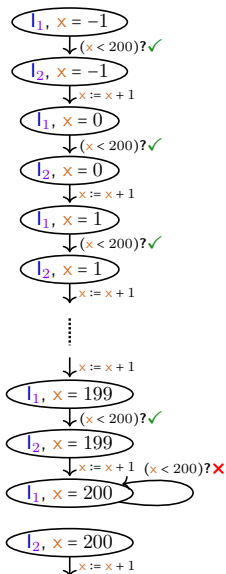
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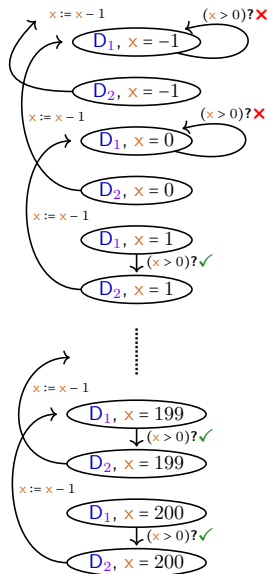


Program graphs (PG)

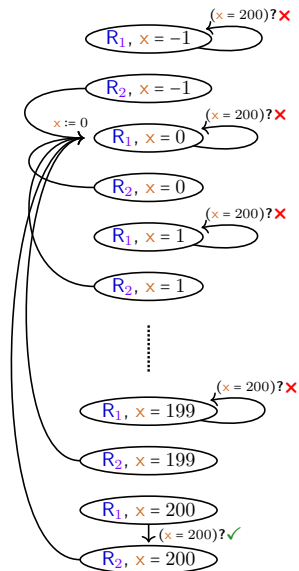
Modeling (by **labeled transition systems**, with state space explosion)



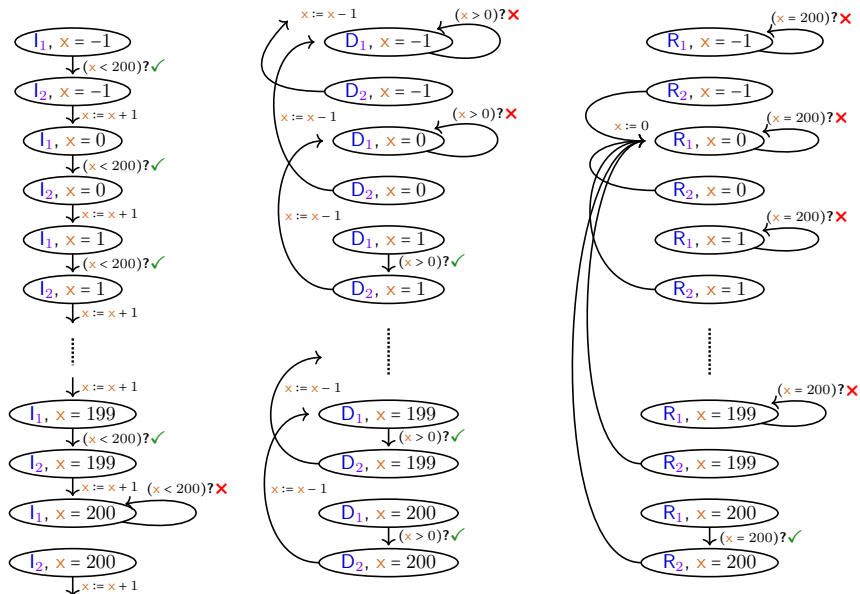
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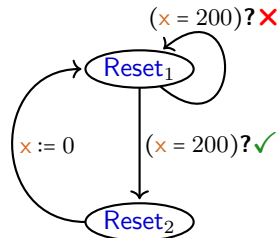
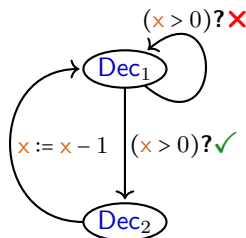
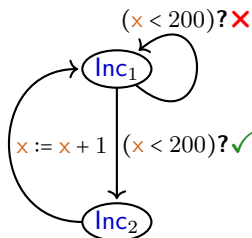
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Formalizing properties (in temporal logic)



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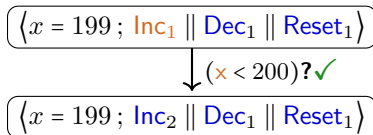
Counterexample (offending execution trace)

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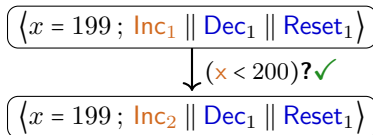
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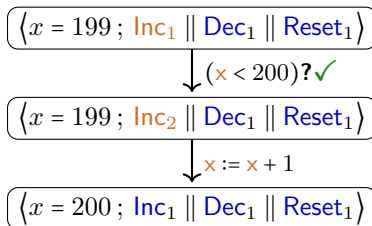
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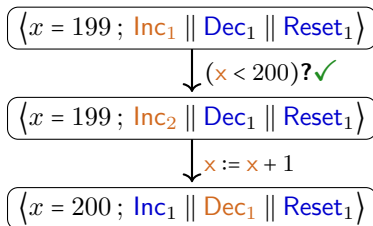
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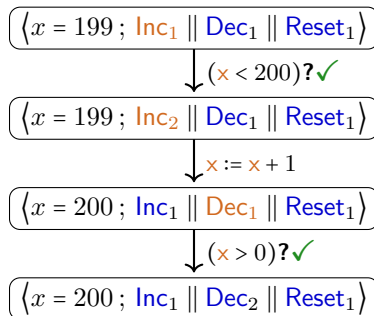
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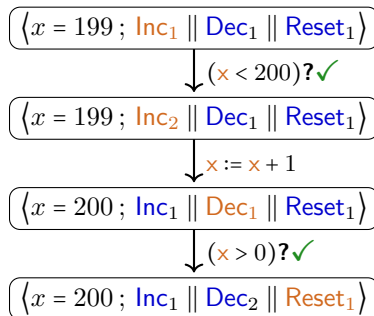
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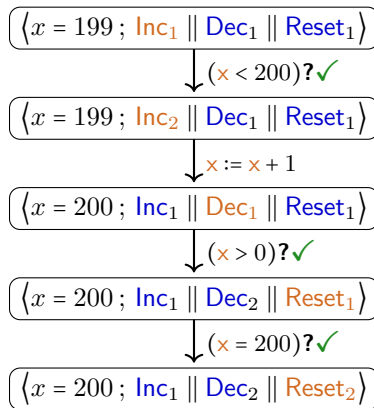
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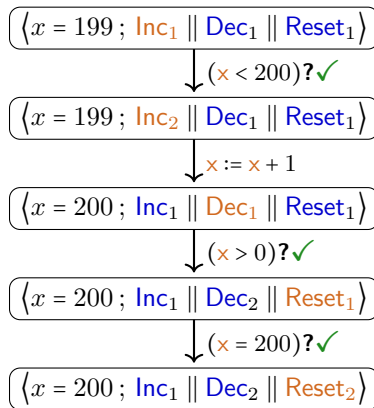
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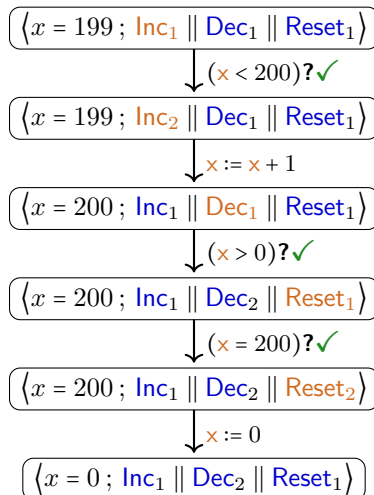
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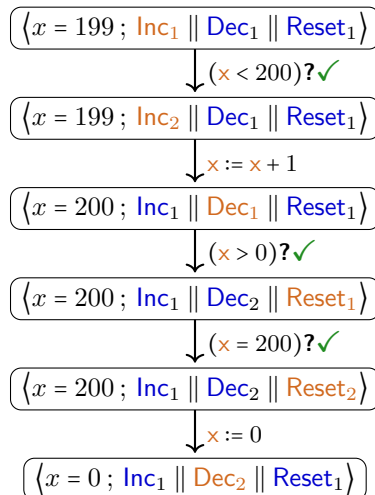
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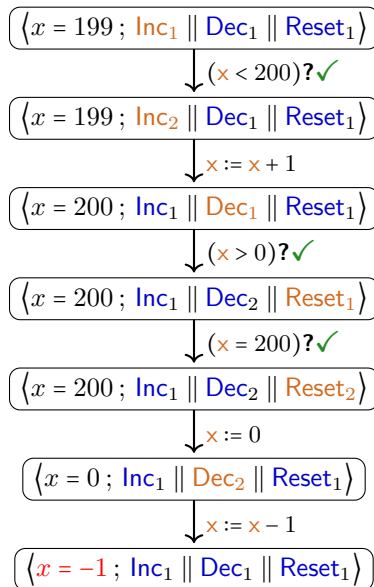
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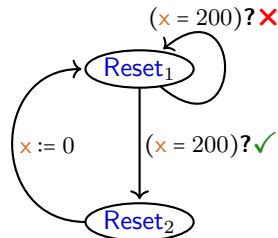
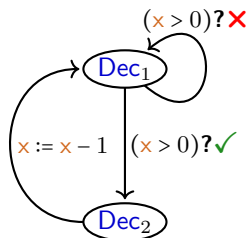
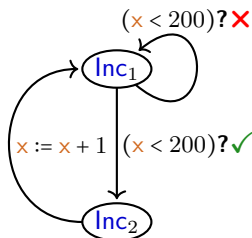
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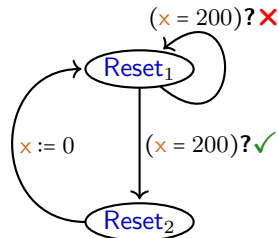
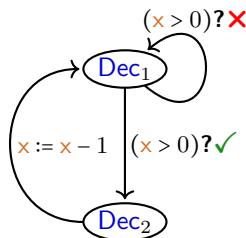
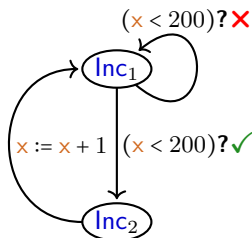
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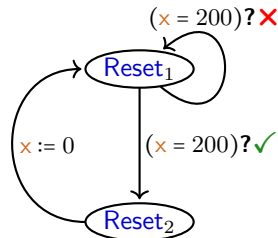
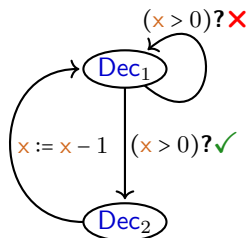
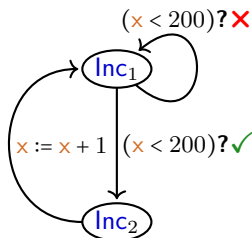
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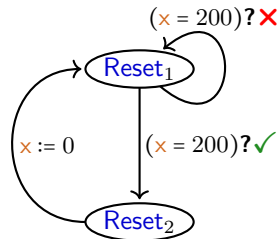
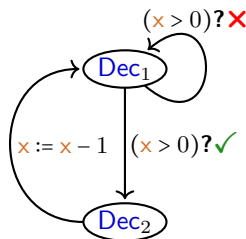
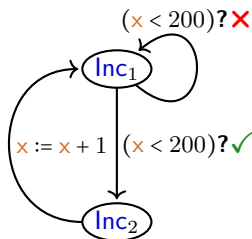


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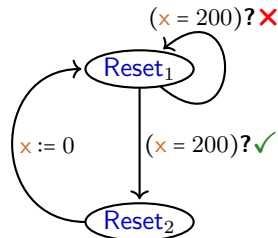
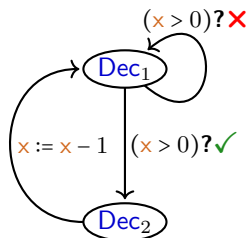
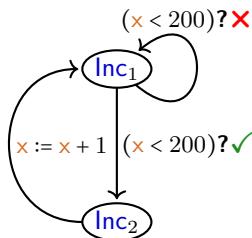


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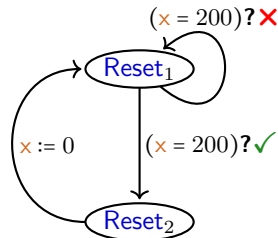
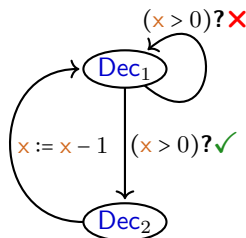
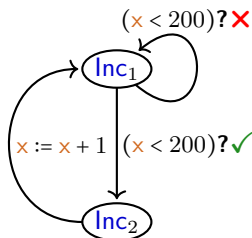
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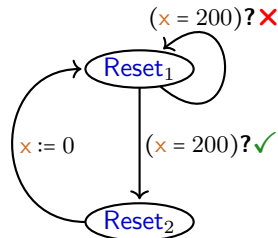
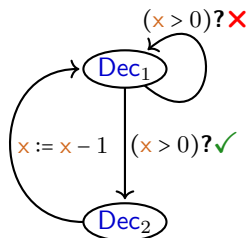
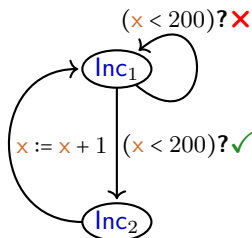
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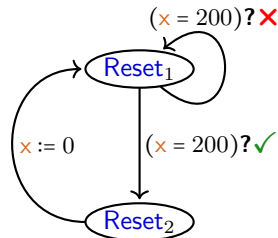
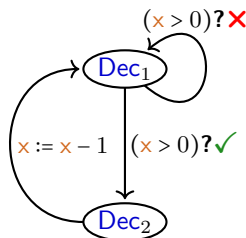
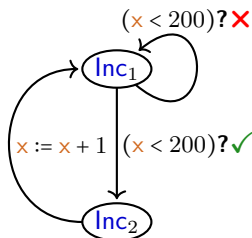
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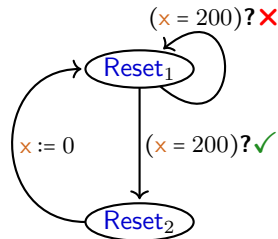
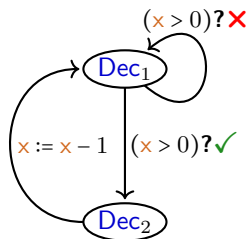
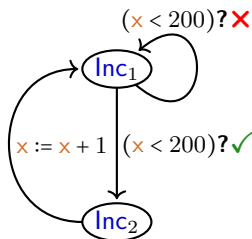
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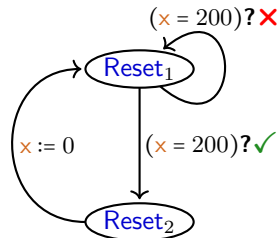
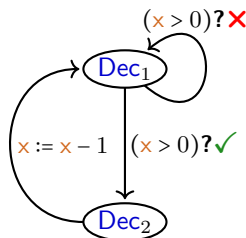
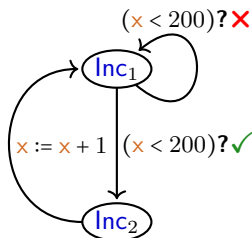
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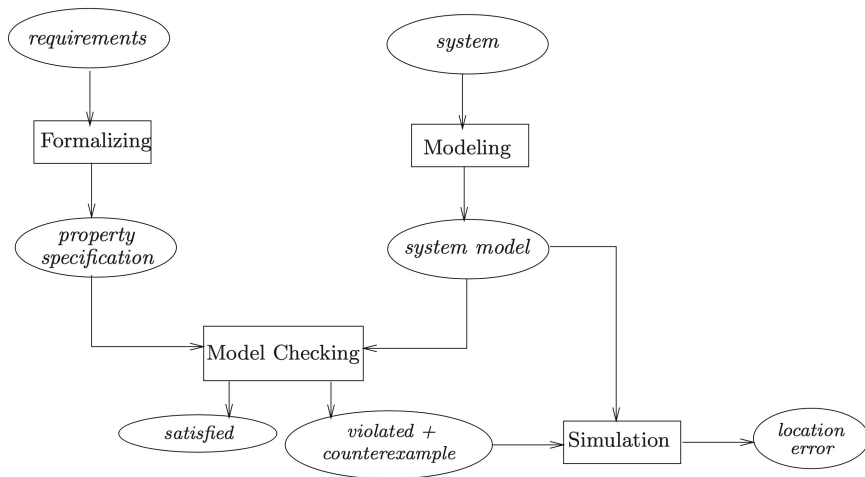
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Model checking: validation



Any [such] verification is only as good as the model of the system.

Modeling: Verification versus Validation

VERIFICATION

Are we building the **thing** right?



Does the **design** satisfy the
expected properties?

VALIDATION

Are we building the **right thing**?



Is the **design** faithfully capturing
the requirements?

Model-checking: strengths and weaknesses

Strengths:

- ▶ it is a **general** technique
- ▶ supports **partial** verification, checking properties individually

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Origins of model checking

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See Baier and Katoen [1] for more references.

Lectures

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2. Modeling by labeled transition systems
 - ▶ executions; traces; non-determinism; examples
3. Linear-Time Behaviour and Properties
 - ▶ invariant, safety, and liveness (and fairness) properties
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5. LTL (continued)
 - ▶ model checking of LTL formulas, and fairness in LTL
 - ▶ Maude examples
6. Computation Tree Logic (CTL)
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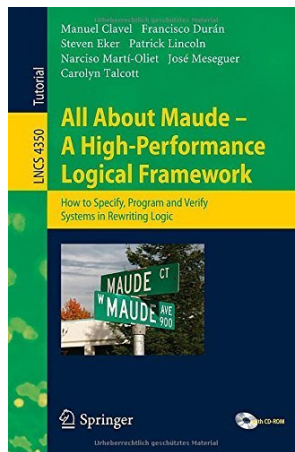
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Book Maude



- Maude documentation page:
<https://maude.cs.illinois.edu/documentation>

Maude code (idea)

```
cr1 [Inc1a]    :  Inc1 x => Inc2 x   if x < 200
r1  [Inc2]     :  Inc2 x => Inc1 (x + 1)
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cr1 [Dec1a]      :  Dec1 x => Dec2 x  if 0 < x
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cr1 [Reset1a]    :  Reset1 x => Reset2 x   if x = 200
r1  [Reset2]     :  Reset2 x => Reset1 0
cr1 [Reset1b]    :  Reset1 x => Reset1 x   if not(x = 200)
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eq initial = { Inc1 Dec1 Reset1 0 }

ceq (S1 S2 S3 x |= counterge0) = true   if (0 = x \/ 0 < x)
ceq (S1 S2 S3 x |= counterlt0) = true   if (x < 0)
ceq (S1 S2 S3 x |= counterle200) = true if (x < 200 \/ x = 200)

```

Maude output (simplified)

```
Maude> red modelCheck(initial, <> counterlt0)
reduce in COUNTERS-CHECK : modelCheck(initial, <> counterlt0)
result ModelCheckResult:
result Bool : true
```

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```
Maude> red modelCheck(initial, [] (counterge0 /\ counterle200))
reduce in COUNTERS-CHECK :
      modelCheck(initial, [] (counterge0 /\ counterle200))
result ModelCheckResult:
counterexample({Inc1 Dec1 Reset1 199}
               {Inc2 Dec1 Reset1 199}
               {Inc1 Dec1 Reset1 200}
               {Inc1 Dec2 Reset1 200}
               {Inc1 Dec2 Reset2 200}
               {Inc1 Dec2 Reset1 0}
               {Inc1 Dec1 Reset1 -1})
```

Lectures (tentative)

1. Introduction (preview extended)
2. Modeling by labeled transition systems
 - ▶ executions; traces; non-determinism; examples
3. Linear-Time Behaviour and Properties
 - ▶ invariant, safety, and liveness (and fairness) properties
4. Linear Temporal Logic (LTL)
 - ▶ syntax and semantics; interpretation of LTSs; examples
5. LTL (continued)
 - ▶ fairness in LTL
 - ▶ model checking of LTL formulas
6. Computation Tree Logic (CTL)
 - ▶ syntax and semantics, examples
7. Extensions of CTL, and Outlook
 - ▶ expressibility differences with LTL
 - ▶ model checking formulas in CTL
 - ▶ Maude examples | partial model-checking | μ -calculus

Exam options

Together we decide on a topic.

[Study an article, or a book chapter.](#)

or

[Dive deeper into a proof from the lecture.](#)

or

[Model a basic algorithm, and check basic properties.](#)

or

[Develop an idea that motivates you.](#)

or

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You give a 25-minute presentation about what you found.

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Exam examples

Presentations about articles:

- ▶ *Progress, Justness, and Fairness* (2019) by R. van Glabbeek, P. Höfner
- ▶ *Comparison of Model Checking Tools for Information Systems* (2010) by M. Frappier et al. (focusing on [SPIN](#) and [NuSMV](#))

References I



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Principles of Model Checking.

MIT Press, 2008.

Available at:

https://is.ifmo.ru/books/_principles_of_model_checking.pdf.



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References II



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Lecture Notes in Computer Science. Springer Berlin Heidelberg, 2007.



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