$\boxed{\text{Release Operator}}$

$\varphi_1 R \varphi_2 := \neg((\neg\varphi_1) U (\neg\varphi_2))$ defined from U

We prove its semantics, for all paths $\pi = \pi[0]\pi[1]\pi[2]\ldots$

$$\pi \models \varphi_1 R \varphi_2 \Leftrightarrow$$

$$\Leftrightarrow \pi \models \neg((\neg\varphi_1) U (\neg\varphi_2))$$

$$\Leftrightarrow \pi \not\models (\neg\varphi_1) U (\neg\varphi_2)$$

$$\Leftrightarrow \text{NOT}(\pi \models (\neg\varphi_1) U (\neg\varphi_2))$$

$$\Leftrightarrow \text{NOT } \exists i \geq 0 : (\underbrace{\pi^{\geq i} \models \neg\varphi_2}_{\pi^{\geq i} \not\models \varphi_2} \text{ AND } \forall 0 \leq j < i : \underbrace{\pi^{\geq j} \models \neg\varphi_1}_{\pi^{\geq j} \not\models \varphi_1})$$

$$\Leftrightarrow \forall i \geq 0 : (\pi^{\geq i} \models \varphi_2 \text{ OR } \textbf{NOT } \forall 0 \leq j < i : \pi^{\geq j} \not\models \varphi_1)$$

$$\Leftrightarrow \forall i \geq 0 : ((\forall 0 \leq j < i : \pi^{\geq j} \not\models \varphi_1) \rightarrow \pi^{\geq i} \models \varphi_2)$$

φ_2 must hold for as long as φ_1 is false and also for the first time step in which φ_1 is true

$$\Leftrightarrow \forall i \geq 0 : ((\pi^{\geq i} \models \varphi_1 \text{ AND } \forall 0 \leq j < i : \pi^{\geq j} \not\models \varphi_1) \Rightarrow \pi^{\geq j} \models \varphi_2)$$

$$\text{AND } (\underbrace{\text{NOT}(\exists i \geq 0 : \pi^{\geq i} \models \varphi_1)}_{\forall i \geq 0 : \pi^{\geq i} \not\models \varphi_1} \Rightarrow \forall i \geq 0 : \pi^{\geq i} \models \varphi_2)$$