$\varphi_1 \equiv \varphi_2 \iff \mathrm{Words}(\varphi_1) = \mathrm{Words}(\varphi_2)$

$\Updownarrow$

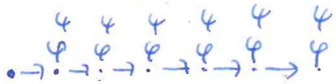
$\forall \sigma \in (2^{AP})^\omega$

$(\sigma \models \varphi_1 \iff \sigma \models \varphi_2)$

$\mathrm{Words}(\varphi) := \{\sigma \in (2^{AP})^\omega \mid \sigma \models \varphi\}$

$\boxed{1}$ $\Box(\varphi \wedge \psi) \overset{?}{\equiv} \Box\varphi \wedge \Box\psi$ $(\checkmark)$

$\Rightarrow$:



$\Box(\varphi\wedge\psi)$ $\qquad \varphi\wedge\psi$

$\Box\varphi$ φ $\qquad \varphi$

$\Box\psi$ ψ $\qquad \psi$

$\Leftarrow$:

$\Box\varphi \wedge \Box\psi$

$\Box\varphi$

$\Box\psi$

φ, ψ

$\sigma \models \Box(\varphi\wedge\psi) \iff$ for all $i \geq 0$: $\sigma^{\geq i} \models \varphi \wedge \psi$

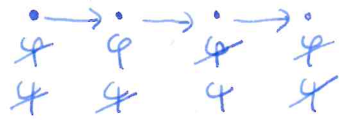
$\iff$ for all $i \geq 0$: $\sigma^{\geq i} \models \varphi$ and $\sigma^{\geq i} \models \psi$

$\iff$ for all $i \geq 0$: $\sigma^{\geq i} \models \varphi$
and
for all $i \geq 0$: $\sigma^{\geq i} \models \psi$

$\iff (\sigma \models \Box\varphi)$ and $(\sigma \models \psi)$

$\iff \sigma \models \Box\varphi \wedge \Box\psi$

$\boxed{2}$ $\Diamond(\varphi \wedge \psi) \overset{?}{\equiv} \Diamond\varphi \wedge \Diamond\psi$ $(\times)$



$\Diamond\varphi$

$\Diamond\psi$

$\cancel{\Diamond(\varphi\wedge\psi)}$

i.p. $\Diamond\varphi \wedge \Diamond\psi \rightarrow \Diamond(\varphi\wedge\psi)$ is not valid.

$\boxed{3}$ $\bigcirc\Diamond\varphi \overset{?}{\equiv} \Diamond\bigcirc\varphi$ $(\checkmark)$

$\sigma \models \bigcirc\Diamond\varphi$ iff $\sigma^{\geq 1} \models \Diamond\varphi$

iff $\sigma^{\geq i} \models \varphi$ for some $i \geq 1$

iff $\sigma^{\geq i+1} \models \varphi$ for some $i \geq 0$

iff $\sigma^{\geq i} \models \bigcirc\varphi$ for some $i \geq 0$

iff $\sigma \models \Diamond\bigcirc\varphi$

$\boxed{\text{Extending Thor's solution}}$

$\underbrace{(crit_1 \vee \neg crit_2)}_{\top} \wedge (\neg crit_2 \vee \neg crit_1) \equiv \neg crit_1 \vee \neg crit_2$

$\equiv$

$(crit_1 \wedge \neg crit_2) \vee \neg crit_1$

$\equiv$

$(crit_1 \wedge \neg crit_2) \vee (\neg crit_1 \wedge \underbrace{(crit_2 \vee \neg crit_2)}_{\top})$

$\equiv$

$(crit_1 \wedge \neg crit_2) \vee (\neg crit_1 \wedge crit_2) \vee \boxed{(\neg crit_1 \wedge \neg crit_2)}$

$\neg crit_1 \vee \neg crit_2 \equiv \neg(crit_1 \wedge crit_2)$

<u>Exercise.</u> (a) define "infinitely often" φ

(b) define "eventually forever" φ

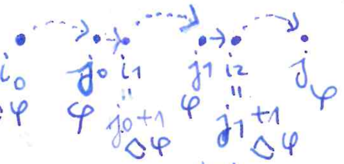
(a) $\varphi_1 := \Box\Diamond\varphi$. For all traces/words $\sigma \in (2^{AP})^\omega$:

$\sigma \models \overbrace{\Box\Diamond\varphi}^{\varphi_1}$ iff for all $i \geq 0$: $\sigma^{\geq i} \models \Diamond\varphi$

iff for all $i \geq 0$ there is $j \geq i$ such that $\sigma^{\geq j} \models \varphi$

iff $\forall i \geq 0\ \exists j \geq i : \sigma^{\geq j} \models \varphi$

iff $\underbrace{\overset{\infty}{\exists} j \geq 0}_{\text{for infinitely many}} : \sigma^{\geq j} \models \varphi$



(b) $\varphi_2 := \Diamond\Box\varphi$. For all traces/words $\sigma \in (2^{AP})^\omega$:

$\sigma \models \varphi_2$ iff $\sigma \models \Diamond\Box\varphi$

iff there exists $i \geq 0$ such that $\sigma^{\geq i} \models \Box\varphi$

iff there exists $i \geq 0$ such that for all $j \geq i$ $\sigma^{\geq j} \models \varphi$

iff $\underbrace{\overset{\infty}{\forall} j \geq 0}_{\text{for } \underline{\text{all but}} \text{ finitely many}} : \sigma^{\geq j} \models \varphi$.

= eventually forever

<u>Relevance for fairness notions:</u>

	LTL-formula description
execution $\varrho: s_0 \xrightarrow{\alpha_1} s_1 \xrightarrow{\alpha_2} s_2 \xrightarrow{\alpha_3} s_3 \xrightarrow{\alpha_4} \dots$ $A \subseteq Act$, A set of actions "of interest"	
ϱ is <u>unconditionally fair</u>: $\Leftrightarrow \overset{\infty}{\exists} j \geq 0.\ \alpha_j \in A$ $\sim$	$\Diamond\Box\, taken(A)$
ϱ is <u>strongly A-fair</u>: $\Leftrightarrow \overset{\infty}{\exists} j \geq 0: A \cap Act(s_j) \neq \emptyset$ $\Rightarrow \overset{\infty}{\exists} j \geq 0: \alpha_j \in A$ $\sim$	$\Diamond\Box\, enabled(A)$ $\Rightarrow \Diamond\Box\, taken(A)$
ϱ is weakly A-fair: $\Leftrightarrow \overset{\infty}{\forall} j \geq 0: A \cap Act(s_j) \neq \emptyset$ $\Rightarrow \overset{\infty}{\exists} j \geq 0: \alpha_j \in A$ $\sim$	$\Box\Diamond\, enabled(A)$ $\Rightarrow \Diamond\Box\, taken(A)$