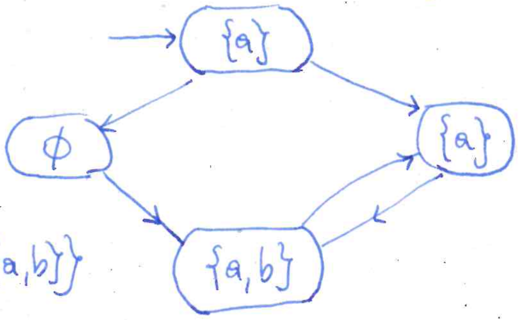
Exercise 3.1 Give the traces on the set of atomic propositions $\{a,b\}$ of the following transition system:



$AP = \{a,b\}$

$2^{AP} = \{\phi, \{a\}, \{b\}, \{a,b\}\}$

$\{a\}\ \phi\ \{a,b\}\ \{a\}\ \{a,b\}\ \{a\} \ldots$

$\{a\}\ \{a\}\ \{a,b\}\ \{a\}\ \{a,b\}\ \{a\} \ldots$

Exercise 3.5 $AP = \{x=0, x>1\}$

Formulate as LT-properties and determine whether they are invariant / safety / liveness properties:

(a) false $\quad P_1 = \{\} = \phi = \{A_0A_1A_2\ldots \in (2^{AP})^\omega \,/\, A_i \models false\}$

invariant ~~[illegible]~~, safety (bad prefixes: $\{x=0\}, \{x>1\}, \phi, \{x=0, x>1\}$) $= 2^{AP}$

~~liveness~~

(b) initially x is equal to zero

$P_2 = \{\, \{x=0\} A_1 A_2 A_3 \ldots \in 2^{AP} \,/\, A_i \subseteq AP \text{ for all } i \geq 1 \}$

~~invariance~~, safety (bad prefix: $\{x>1\}$), ~~liveness~~

(c) initially x differs from zero

$P_3 = \{\, \{x>1\} A_1 A_2 A_3 \ldots \in 2^{AP} \,/\, A_i \subseteq AP \text{ for all } i \geq 1 \}$

~~invariance~~, safety (bad prefix: $\{x=0\}$), ~~liveness~~

(d) initially x is equal to zero, but at some point exceeds one

$P_4 = \{\, \{x=0\} A_1 A_2 A_3 \ldots \in 2^{AP} \,/\, A_i \subseteq AP \text{ for all } i \geq 1, \ \exists j \geq 0.\ A_j = \{x>1\} \}$

~~invariance~~, ~~safety~~, ~~liveness~~, intersection of safety and liveness

$$P_4 = \underbrace{\{\, \{x=0\} A_1 A_2 A_3 \ldots \in 2^{AP} \,/\, A_i \subseteq AP \text{ for all } i \geq 1 \}}_{\text{safety prop}} \cap$$

$$\cap \underbrace{\{A_0 A_1 A_2 \ldots \,/\, A_i \subseteq AP \text{ for all } i \geq 1, \exists j \geq 0 : A_j = \{x>1\}\}}_{\text{liveness prop}}$$

(e) x exceeds one only finitely often

$P_5 = \{A_0 A_1 A_2 \ldots \in 2^{AP} \,/\, \overset{\infty}{\forall} j\ \ A_j = \{x=0\} \}$

$\neg \overset{\infty}{\exists} j.\ A_j = \{x=0\}$

~~invariance~~, ~~safety~~, liveness (because $pref(P_5) = (2^{AP})^*$).

(f) x exceeds one infinitely often

$$P_6 = \{A_0A_1A_2\ldots \in (2^{AP})^\omega \mid \overset{\infty}{\exists} j\geq 0:\ A_j = \{x>1\}\}$$

$$\exists j_0 \geq 0\ \forall j \geq j_0 : A_j = \{x>1\}$$

~~invariance~~, ~~safety~~, liveness since $pref(P_6) = (2^{AP})^*$

(g) the value of x alternates between zero and two:

$$P_7 \approx \{A_0A_1A_2\ldots \in (2^{AP})^\omega \mid A_i \subseteq AP \text{ for all } i\geq 0,\ (A_i = \{x=0\} \Leftrightarrow A_{i+1} = \{x>1\}) \text{ for all } i\geq 0\}$$

because $\{x=2\}, \{x\neq 2\} \notin AP$

~~invariance~~, safety (bad prefixes: e.g. $\{x=0\}\{x=0\}$, $\{x>1\}\{x>1\}$,
~~liveness~~ in general: $(\{x=0\}\{x>1\})^*\{x=0\}\{x=0\}$
$(\{x=0\}\{x>1\})^+\{x>1\}$
and the same with roles inversed

(h) true

$$P_8 = (2^{AP})^\omega = \{A_0A_1A_2A_3\ldots \in (2^{AP})^\omega \mid A_i \models true\}$$

invariant, safety, liveness
(see above $BP(P_8) = \phi$ $Pref((2^{AP})^\omega) = (2^{AP})^*$

$\boxed{\text{Exercise 3.6}}$ Consider $AP = \{a, b\}$. Formulate as AP-properties and characterize as invariance, safety, or liveness properties:

(a) a should never occur:

$$P_1 = \{A_0A_1A_2\ldots \in (2^{AP})^\omega \mid \overbrace{\underbrace{a\notin A_i}_{A_i \in \{\phi, \{b\}\}}}^{A_i \models \neg a} \text{ for all } i\geq 0\}$$

invariant, safety, ~~liveness~~

(b) a should occur exactly once

$$P_2 = \{A_0A_1A_2\ldots \in (2^{AP})^\omega \mid \exists! j\geq 0: a\in A_j\}$$

~~invariant~~, ~~safety~~, ~~liveness~~

$$= \underbrace{\{\ldots \mid |\{j\geq 0 \mid a\in A_j\}| \leq 1\}}_{\text{safety}} \cap \underbrace{\{\ldots \mid |\{j\geq 0 \mid a\in A_j\}| \geq 1\}}_{\text{liveness}}$$

$$BP = \{A_0\ldots A_nA_{n+1} \in (2^{AP})^\omega \mid |\{j\in\{0,\ldots,n\} \mid a\in A_j\}| = 1,\ a\in A_{n+1}\}$$

(c) a and b alternate infinitely often, $AP = \{a, b\}$

first attempt $P_{30} = \{A_0 A_1 A_2 \ldots \in (2^{AP})^\omega \mid \forall i \geq 0 (A_i = \{a\} \Rightarrow \exists j > i.\ A_j = \{b\})$ and $\forall i \geq 0 (A_i = \{b\} \Rightarrow \exists j > i\ A_j = \{a\})\}$

$= \{A_0 A_1 A_2 \ldots \in (2^{AP})^\omega \mid (\overset{\infty}{\exists} i \geq 0.\ A_i = \{a\}) \,\&\, (\overset{\infty}{\exists} i \geq 0.\ A_i = \{b\})\}$

~~invariant~~, ~~safety~~, liveness

$P_3 = \{A_0 A_1 A_2 \ldots \in (2^{AP})^\omega \mid \forall i \geq 0 (A_i = \{a\} \text{ or } A_i = \{b\})$ and $\forall i \geq 0 (A_i = \{a\} \Rightarrow \exists j > i.\ A_j = \{b\})$ and $\forall i \geq 0 (A_i = \{b\} \Rightarrow \exists j > i.\ A_j = \{a\})\}$

~~invariant~~, ~~safety~~, ~~liveness~~

$= \underbrace{\{A_0 A_1 A_2 \ldots \in (2^{AP})^\omega \mid \forall i \geq 0 (A_i = \{a\} \text{ or } A_i = \{b\})\}}_{\text{safety}} \cap \underbrace{P_{30}}_{\text{liveness}}$

(d) (every) a should eventually be followed by b

$P_4 = \{A_0 A_1 A_2 \ldots \in (2^{AP})^\omega \mid \forall i \geq 0 (a \in A_i \Rightarrow \exists j > i.\ b \in A_j)\}$

liveness, ~~invariant~~, ~~safety~~