Structure-Constrained Process Graphs for the Process Interpretation of Regular Expressions

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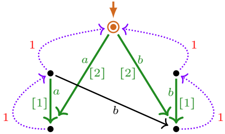

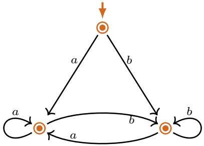

TERMGRAPH 2020

July 5, 2020

Overview

- process semantics of regular expressions
- graph property: loop existence and elimination (LEE)
  - LEE-witnesses: structure-constrained process graphs
    (inspired by higher-order term graphs)

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- process semantics of regular expressions
- graph property: loop existence and elimination (LEE)
  - LEE-witnesses: structure-constrained process graphs
    (inspired by higher-order term graphs)
- LEE fails for process semantics in general
- recover LEE for variant process interpretation
  - use 1-transitions (similar to ‘silent steps’ in NFAs)
- outlook: use result for tackling open problems

Process semantics of star expressions *(Milner,1984)*

$\mathbf{0} \xrightarrow{\mathcal{C}(\cdot)}$ deadlock δ, no termination

$\mathbf{1} \xrightarrow{\mathcal{C}(\cdot)}$ empty process ϵ, then terminate

$a \xrightarrow{\mathcal{C}(\cdot)}$ atomic action a, then terminate

$e + f \xrightarrow{\mathcal{C}(\cdot)}$ alternative composition of $\mathcal{C}(e)$ and $\mathcal{C}(f)$

$e \cdot f \xrightarrow{\mathcal{C}(\cdot)}$ sequential composition of $\mathcal{C}(e)$ and $\mathcal{C}(f)$

$e^{*} \xrightarrow{\mathcal{C}(\cdot)}$ unbounded iteration of $\mathcal{C}(e)$, option to terminate

Process chart semantics $\mathcal{C}(\cdot)$

Definition (Transition system specification (TSS) for star expressions)

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'} \ (i \in \{1,2\})$$

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$$\frac{}{\mathbf{1}{\downarrow}} \qquad \frac{e_i{\downarrow}}{(e_1 + e_2){\downarrow}}\ (i \in \{1,2\}) \qquad \frac{e_1{\downarrow} \quad e_2{\downarrow}}{(e_1 \cdot e_2){\downarrow}} \qquad \frac{}{(e^*){\downarrow}}$$

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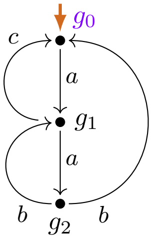
Definition

The (process) chart interpretation $\mathcal{C}(e)$ of a star expression e:

$\mathcal{C}(e)$:= labeled transition system generated by e with the rules above.

Process chart semantics (examples | non-examples)

$\mathcal{C}(g_0)$



$$g_0 = ((1 \cdot a) \cdot \overbrace{(c \cdot a + a \cdot (b + b \cdot a))^*}^{=:\, g}) \cdot 0 \,,$$

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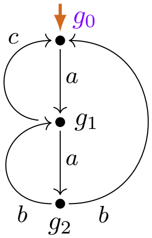

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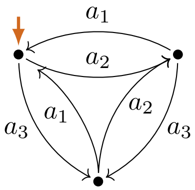

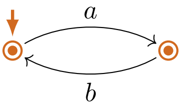

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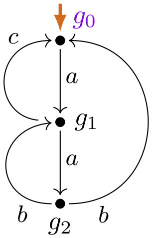

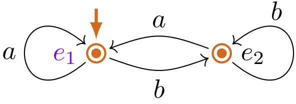

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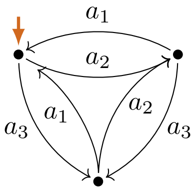

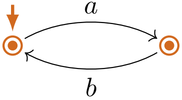

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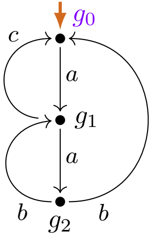

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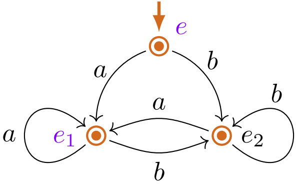

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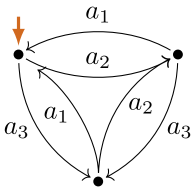

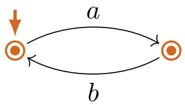

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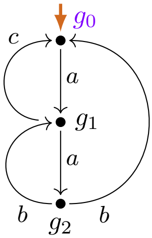

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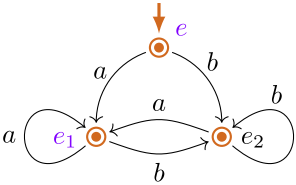

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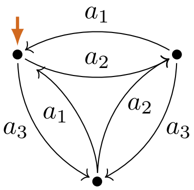

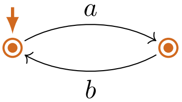

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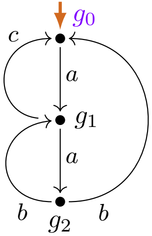

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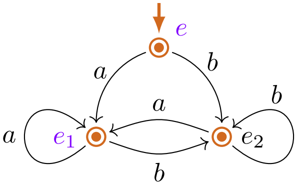

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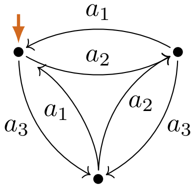

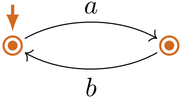

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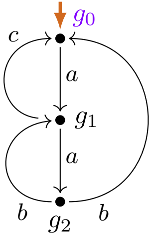

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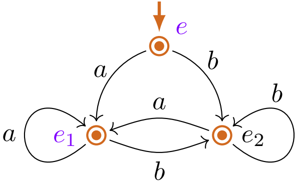

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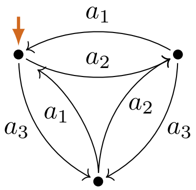

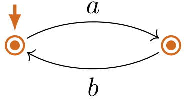

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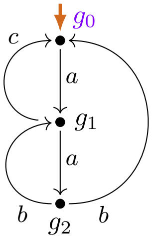
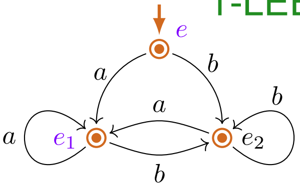


$\mathcal{C}(e)$ ~~LEE~~ 1-LEE



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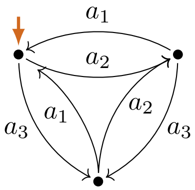

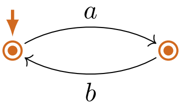

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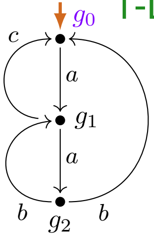

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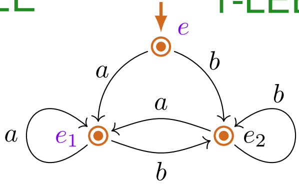

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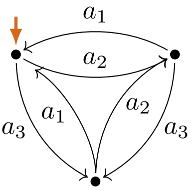

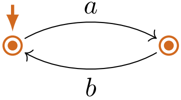

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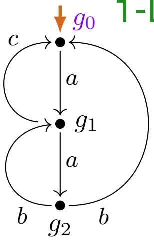

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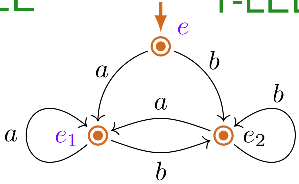

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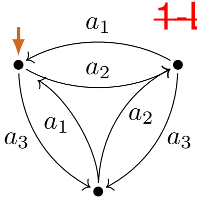

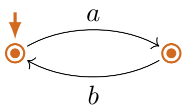

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Loop charts (interpretations of innermost iterations)

Definition

A chart is a loop chart if:

(L1) There is an infinite path from the start vertex.

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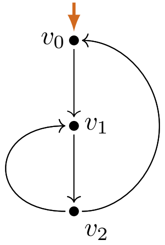

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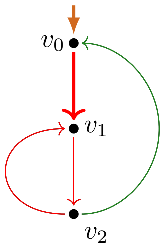

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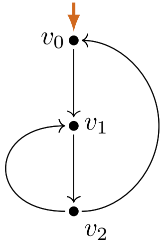

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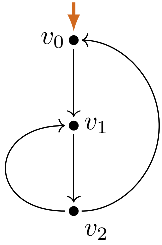

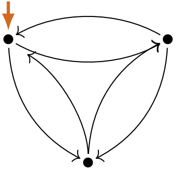

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Loop charts (interpretations of innermost iterations)

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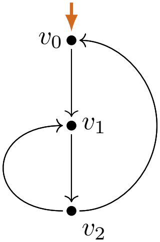

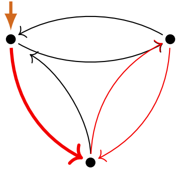

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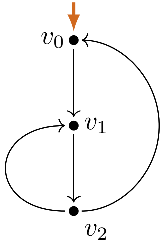

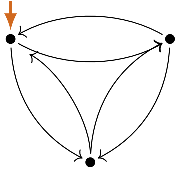

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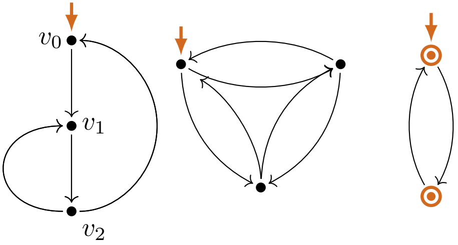

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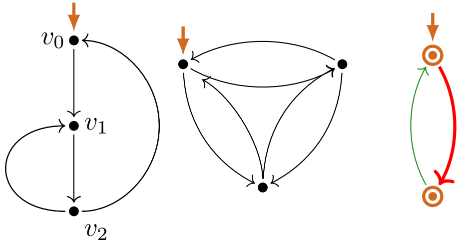

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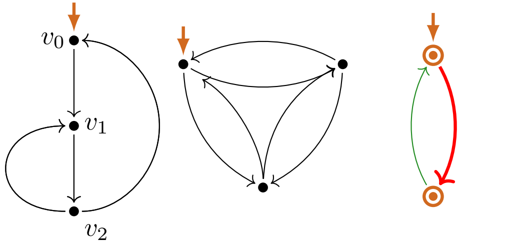

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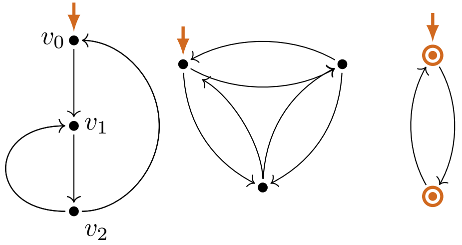

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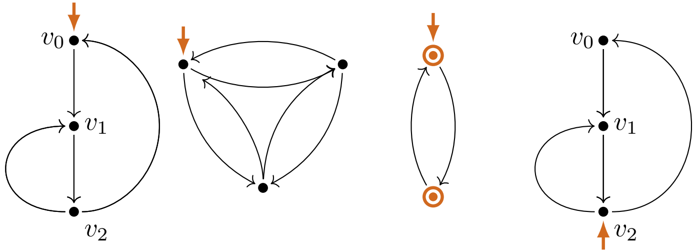

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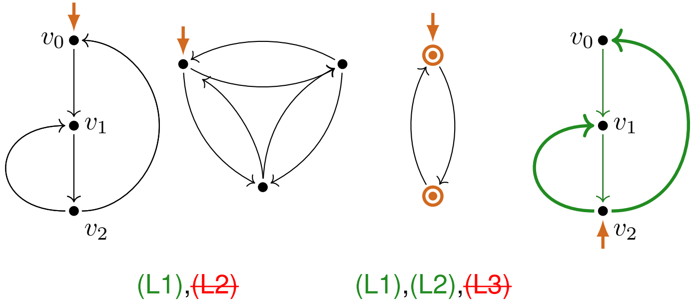

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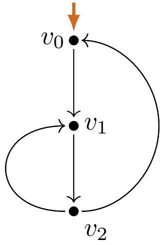

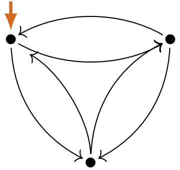

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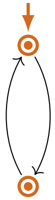
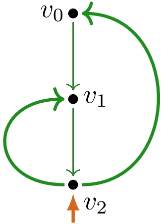


(L1),(L2),~~(L3)~~



loop chart

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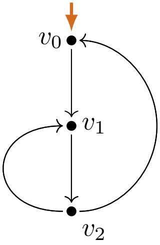
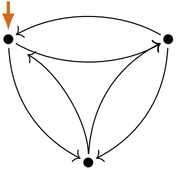
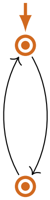
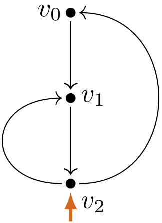




(L1),~~(L2)~~



(L1),(L2),~~(L3)~~



loop chart

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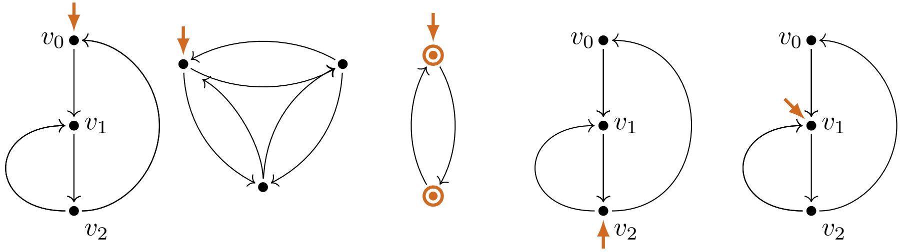

(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart

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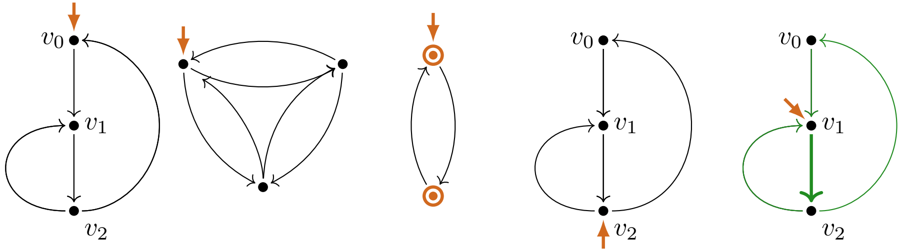

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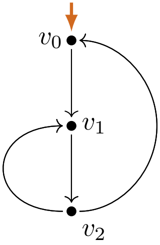
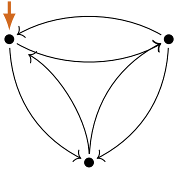
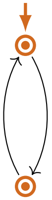
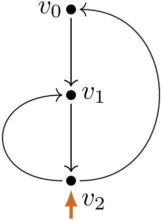




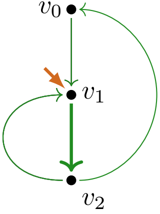
(L1),~~(L2)~~



(L1),(L2),~~(L3)~~



loop chart



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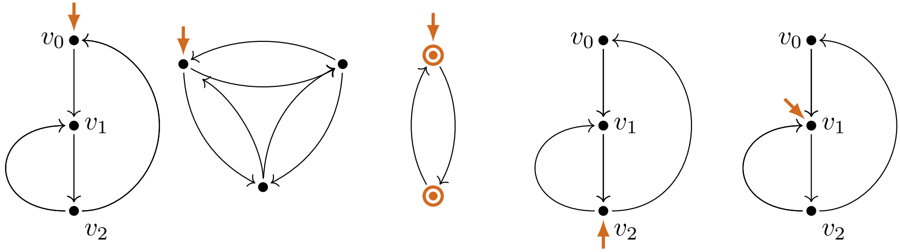

(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart loop chart

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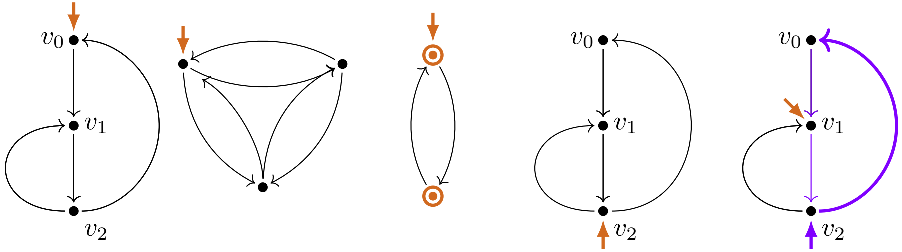

(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart loop subchart

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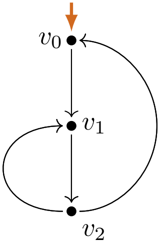

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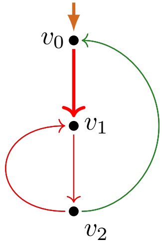

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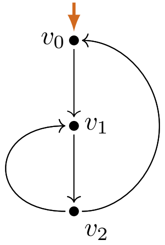

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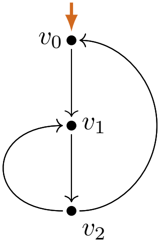

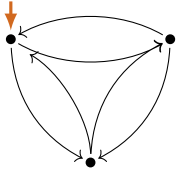

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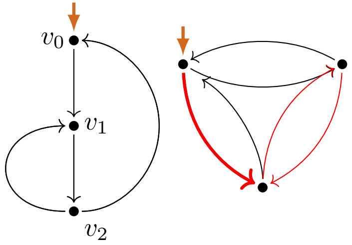

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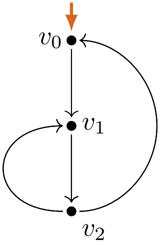

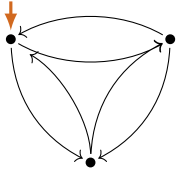

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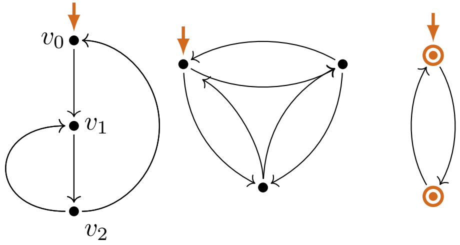

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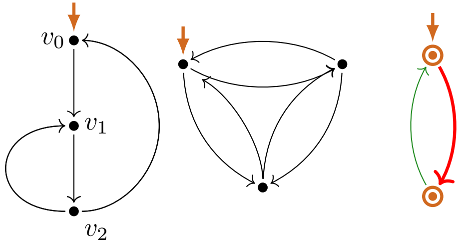

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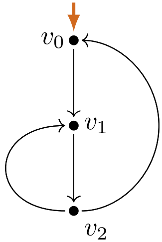

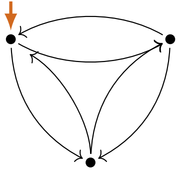

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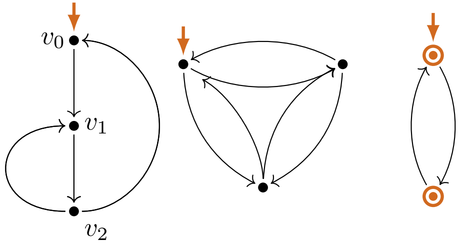

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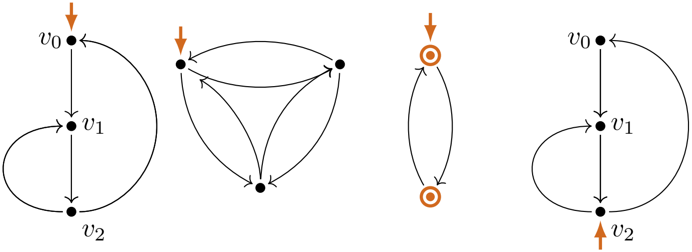

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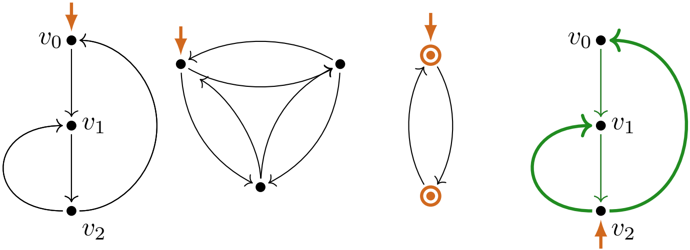

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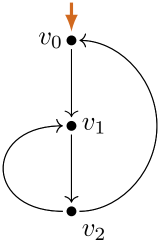
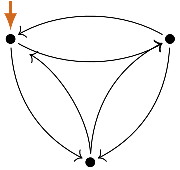
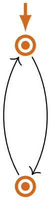
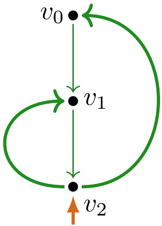




(L1),~~(L2)~~



(L1),(L2),~~(L3)~~



loop chart

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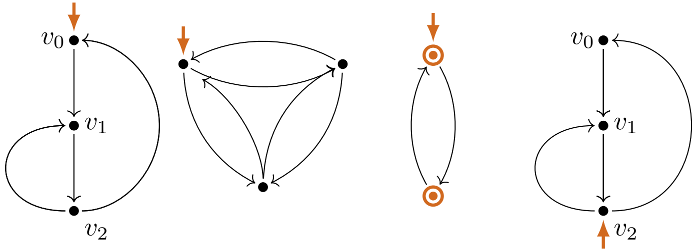

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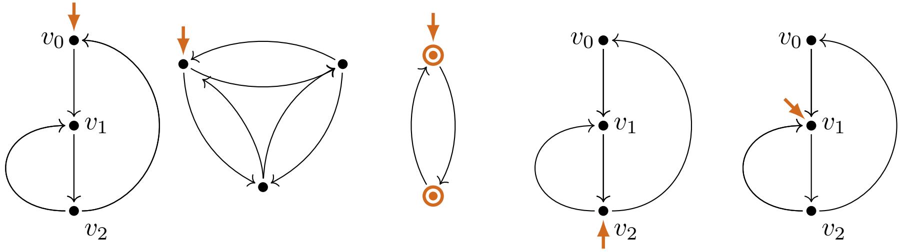


(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart

Loop charts (interpretations of innermost iterations)

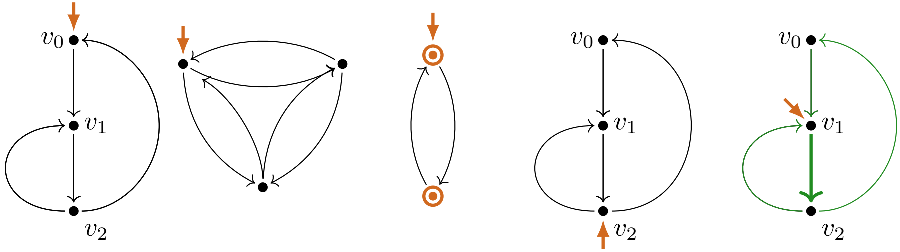
Definition

A chart is a loop chart if:

(L1) There is an infinite path from the start vertex.

(L2) Every infinite path from the start vertex returns to it.

(L3) Termination is only possible at the start vertex.



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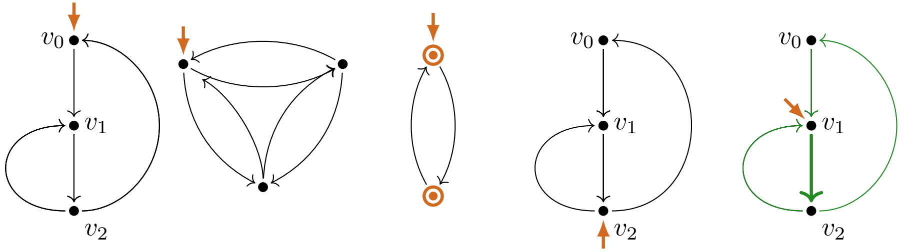

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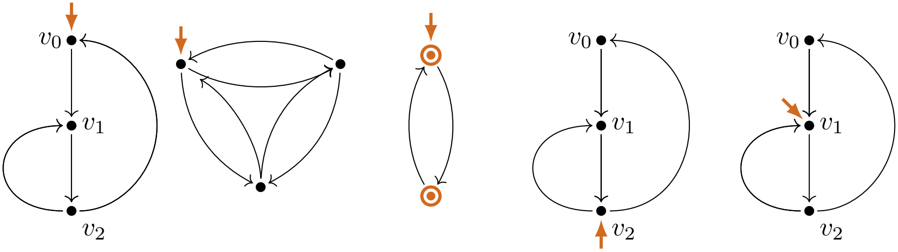

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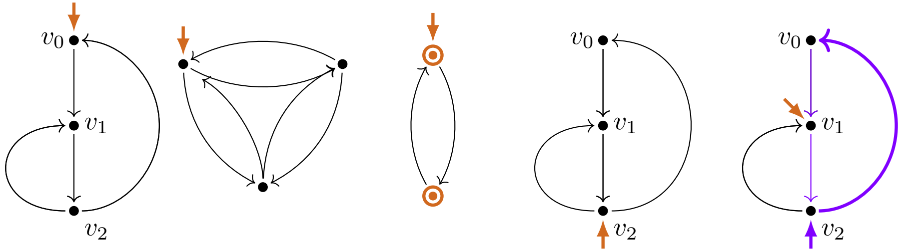
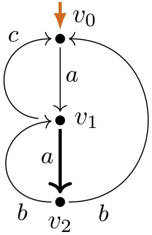


(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart loop subchart

Loop existence and elimination



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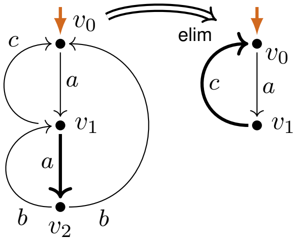

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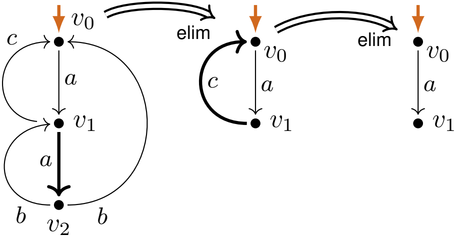

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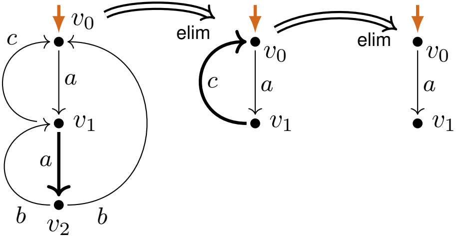

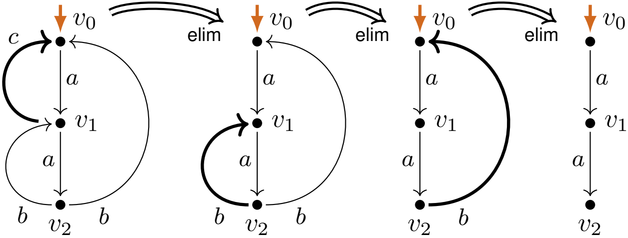

LEE

Definition

A chart $\mathcal{C}$ satisfies LEE (*loop existence and elimination*) if:

$$\exists \mathcal{C}_0 \left(\mathcal{C} \Longrightarrow^{*}_{\text{elim}} \mathcal{C}_0 \not\Longrightarrow_{\text{elim}} \right.$$
$$\left. \wedge\ \mathcal{C}_0 \text{ permits no infinite path} \right).$$

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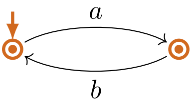

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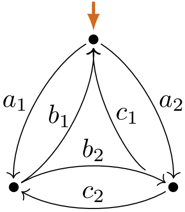
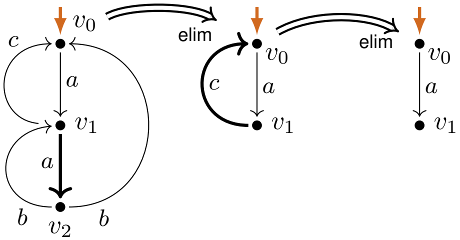


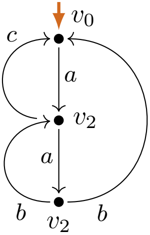
~~LEE~~

LEE

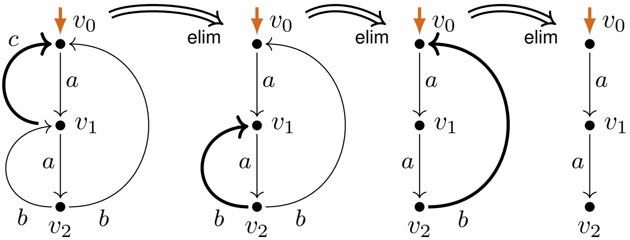


v_0
c
a
v_1
a
b
v_2
b
elim
v_0
c
a
v_1
elim
v_0
a
v_1

LEE

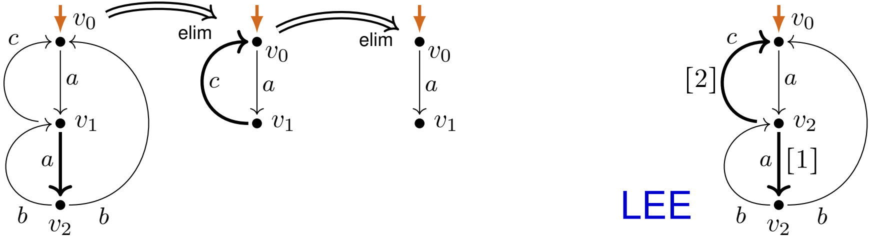


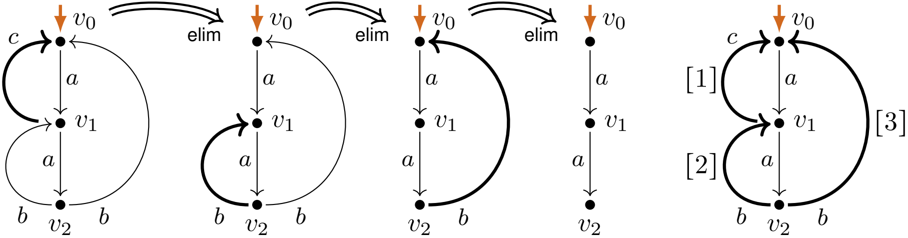
v_0
c
a
v_2
a
b
v_2
b



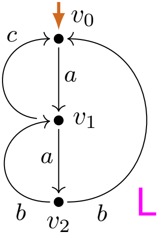
v_0
c
a
v_1
a
b
v_2
b
elim
v_0
a
v_1
a
b
v_2
b
elim
v_0
a
v_1
a
v_2
b
elim
v_0
a
v_1
a
v_2

LEE



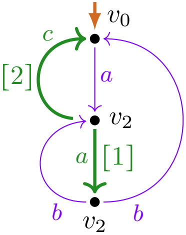


LEE witness and LEE-charts

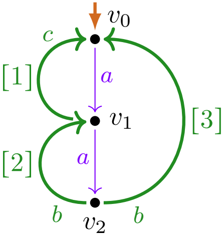


LEE-chart

LEE-witness



LEE-witness



Properties of LEE-charts

Theorem (G/Fokkink, LICS 2020)

A chart is expressible by a 1-free star expression modulo bisimilarity
if and only if
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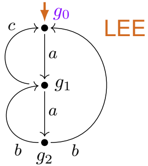
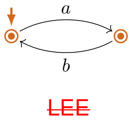
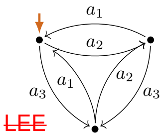
Hence expressible | not expressible by 1-free star expressions:

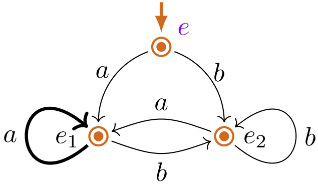






Failure of LEE in general (example)

$\mathcal{C}((a^* \cdot b^*)^*)$



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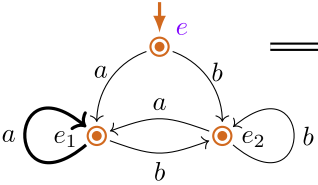

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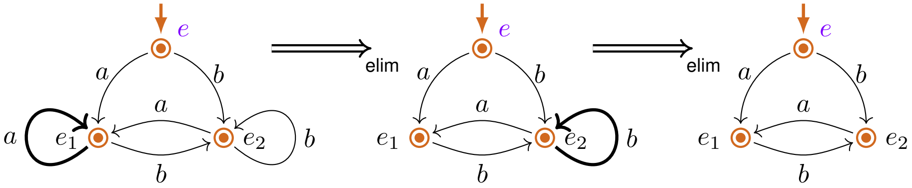

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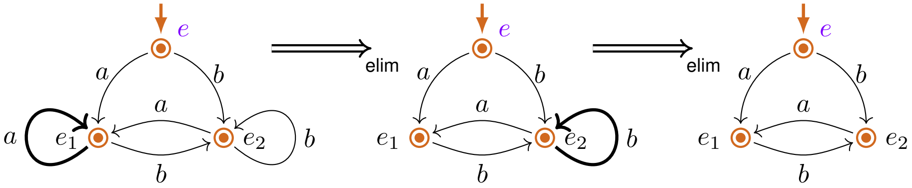

~~LEE~~

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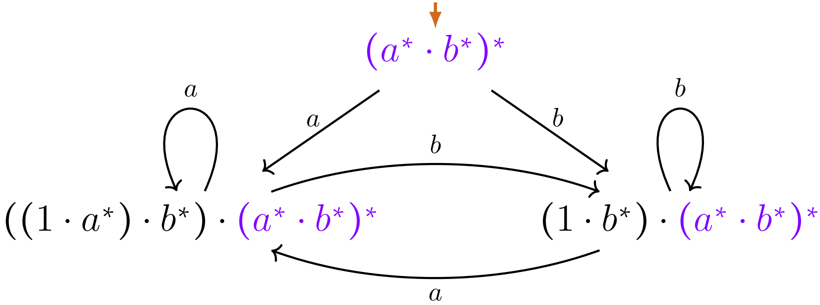
Analysis of failure of LEE (example)



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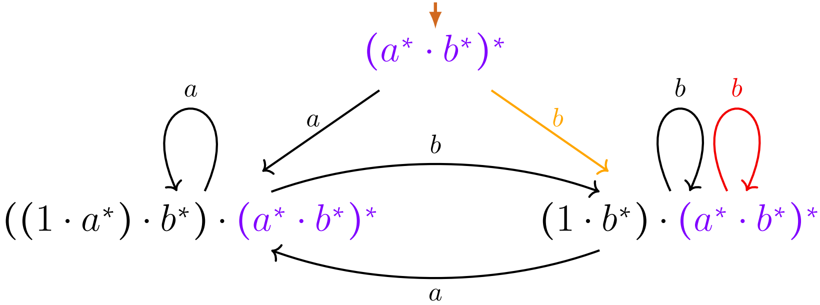

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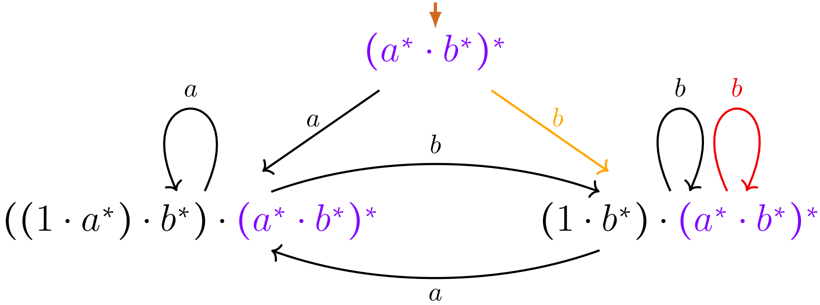

$$\frac{(1 \cdot b^*){\downarrow} \qquad (a^* \cdot b^*)^* \xrightarrow{b} (1 \cdot b^*) \cdot (a^* \cdot b^*)^*}{(1 \cdot b^*) \cdot (a^* \cdot b^*)^* \xrightarrow{b} (1 \cdot b^*) \cdot (a^* \cdot b^*)^*}$$

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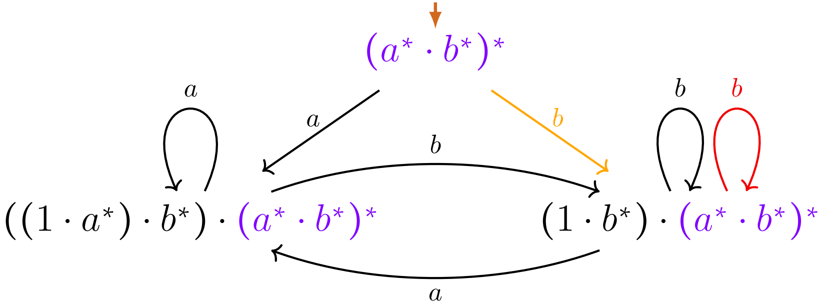

reason: iteration is bypassed from inside

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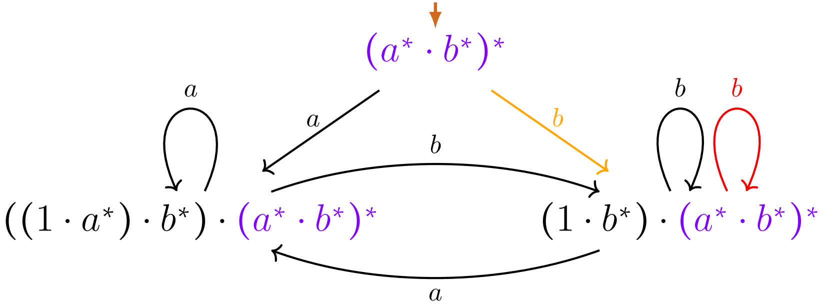

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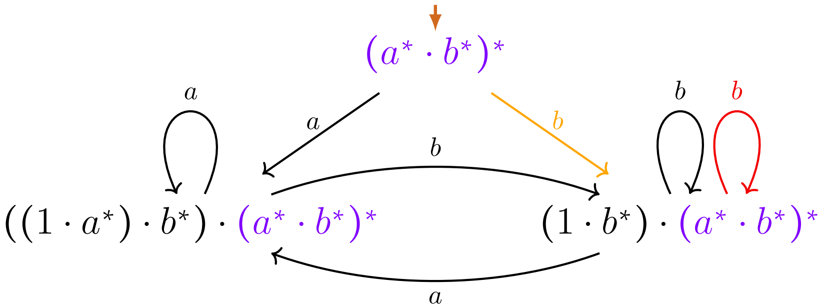

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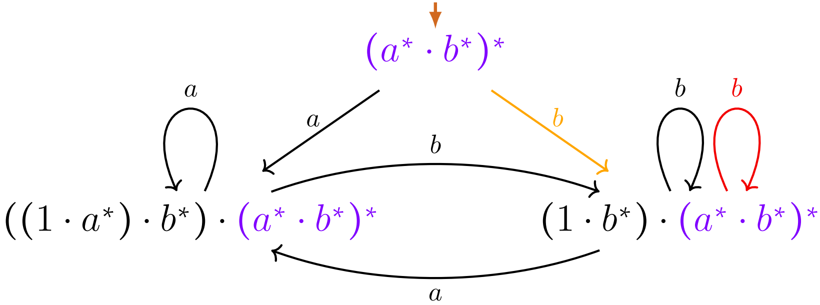

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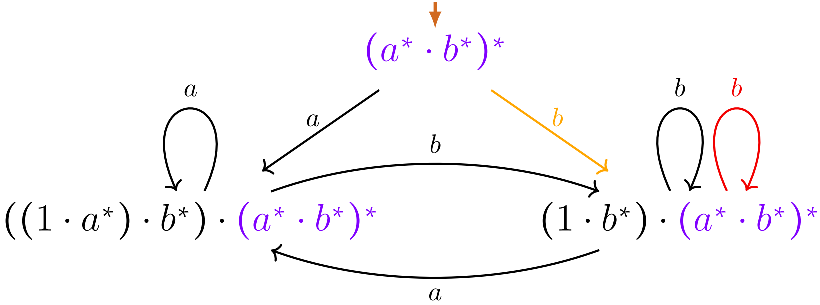

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we would like: $(1 \cdot b^*) \circledast (a^* \cdot b^*)^* \xrightarrow{1} (a^* \cdot b^*)^* \xrightarrow{b} (1 \cdot b^*) \cdot (a^* \cdot b^*)^*$

Stacked star expressions

Definition (Stacked star expressions)

$$E \;::=\; e \mid E \cdot e \mid E \circledast e^{*} \qquad \text{(where } e \text{ star expression)}\,.$$

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$$\frac{}{\mathbf{1}{\downarrow}} \qquad \frac{e_i{\downarrow}}{(e_1 + e_2){\downarrow}}\,(i \in \{1,2\}) \qquad \frac{e_1{\downarrow} \quad e_2{\downarrow}}{(e_1 \cdot e_2){\downarrow}} \qquad \frac{}{(e^{*}){\downarrow}}$$

$$\frac{e_1{\downarrow}}{e_1 \circledast e_2^{*} \xrightarrow{\mathbf{1}} e_2^{*}}$$

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$$\frac{e \xrightarrow{a} E'}{e^{*} \xrightarrow{a} E' \circledast e^{*}}$$

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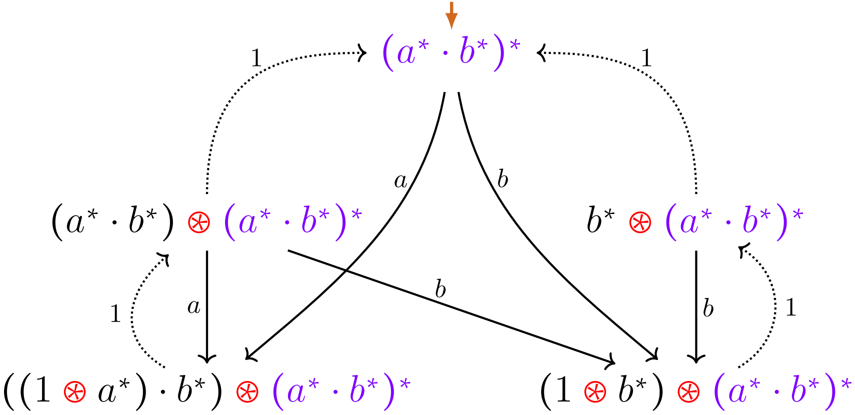
$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} E_i'}{e_1 + e_2 \xrightarrow{a} E_i'}\,(i \in \{1,2\})$$

$$\frac{E_1 \xrightarrow{a} E_1'}{E_1 \cdot e_2 \xrightarrow{a} E_1' \cdot e_2} \qquad \frac{e_1{\downarrow} \quad e_2 \xrightarrow{a} E_2'}{e_1 \cdot e_2 \xrightarrow{a} E_2'} \qquad \frac{e \xrightarrow{a} E'}{e^{*} \xrightarrow{a} E' \circledast e^{*}}$$

$$\frac{E_1 \xrightarrow{a} E_1'}{E_1 \circledast e_2^{*} \xrightarrow{a} E_1' \circledast e_2^{*}} \qquad \frac{e_1{\downarrow}}{e_1 \circledast e_2^{*} \xrightarrow{1} e_2^{*}}$$

1-chart interpretation (example)



1-Charts and induced charts

Definition

$\xrightarrow{1} \cdot \cdots \cdot \xrightarrow{1} \cdot \xrightarrow{a} \quad \widehat{=} \quad \xrightarrow{(a]}$ induced a-transitions, for $a \in A$

$\xrightarrow{1} \cdot \cdots \cdot \xrightarrow{1} \cdot \downarrow \quad \widehat{=} \quad \downarrow^{(1)}$ induced termination.

1-Charts and induced charts

Definition

$v_1 \xrightarrow{1} \cdot \;\cdots\; \cdot \xrightarrow{1} \cdot \xrightarrow{a} v_2 \quad \widehat{=} \quad v_1 \xrightarrow{(a]} v_2$ induced a-transitions, for $a \in A$

$v \xrightarrow{1} \cdot \;\cdots\; \cdot \xrightarrow{1} \cdot \downarrow \quad \widehat{=} \quad v \downarrow^{(1)}$ induced termination.

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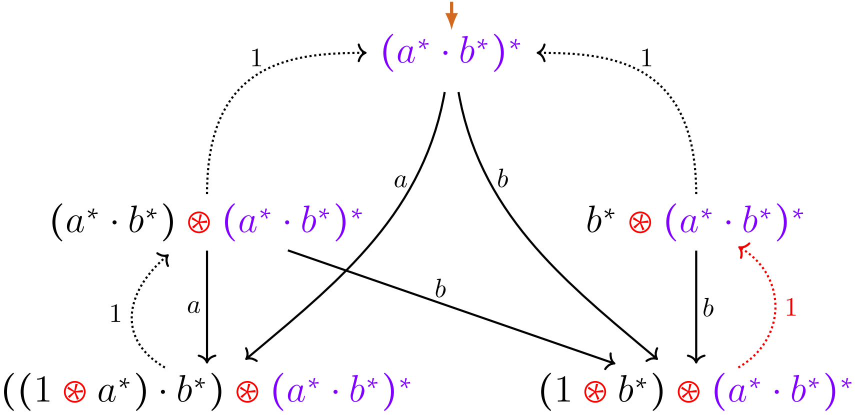
Definition

The induced chart of a 1-chart $\underline{\mathcal{C}} = \langle V, A, 1, v_s, \rightarrow, \downarrow \rangle$ is:

$$\mathcal{I}(\underline{\mathcal{C}}) = \langle V, A, v_s, \xrightarrow{(\cdot]}, \downarrow^{(1)} \rangle .$$

1-chart interpretation (example)



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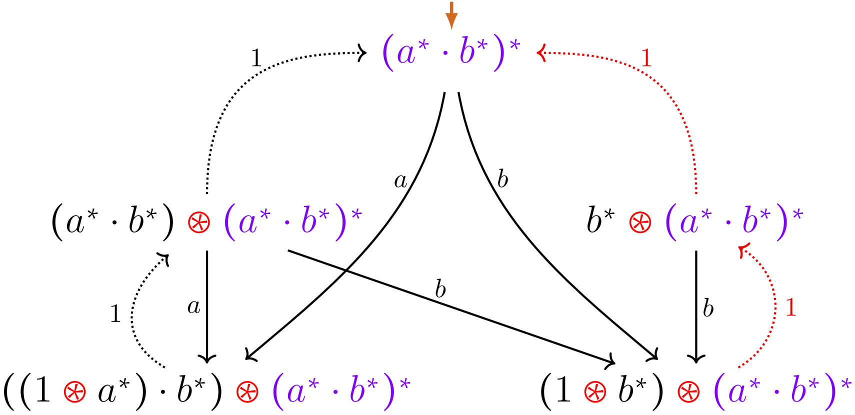

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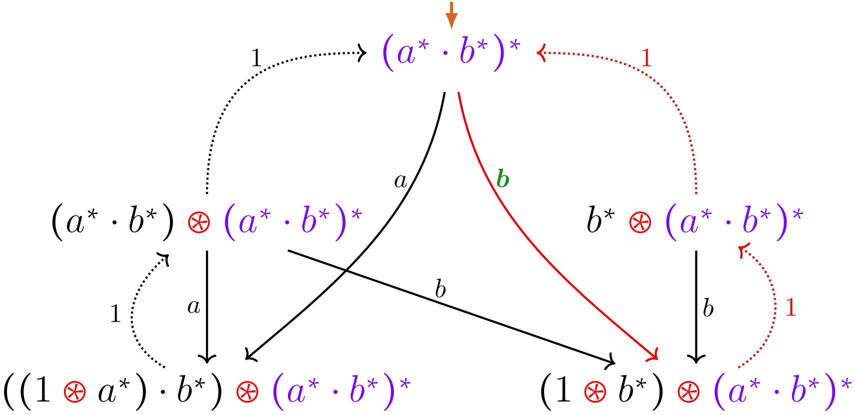

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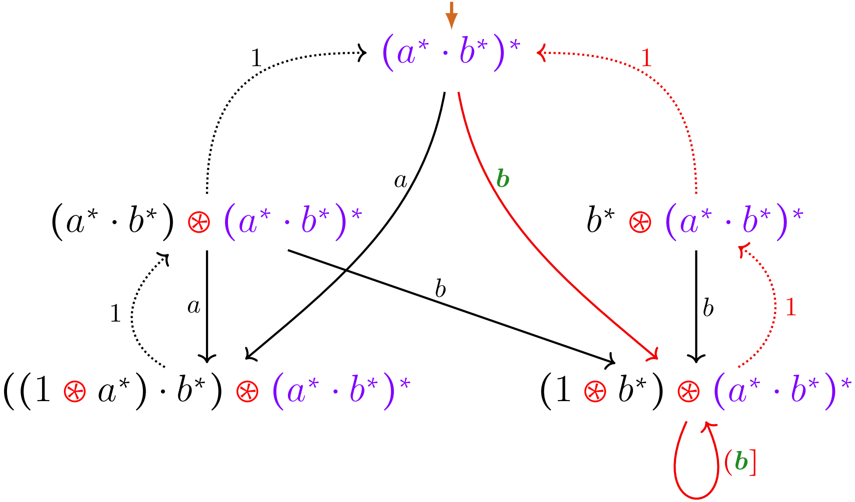
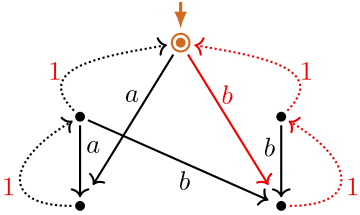


1-chart interpretation vs. chart interpretation (example)

$\underline{\mathcal{C}}((a^* \cdot b^*)^*)$



1-chart interpretation

1-chart interpretation vs. chart interpretation (example)

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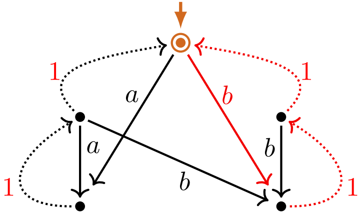
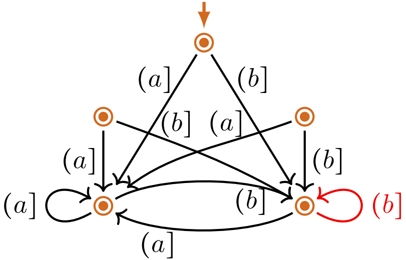


1-chart interpretation

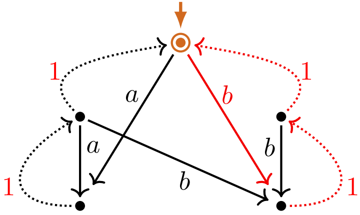
$\mathcal{I}(\underline{\mathcal{C}}(e))$



induced chart

1-chart interpretation vs. chart interpretation (example)

$\underline{\mathcal{C}}((a^* \cdot b^*)^*)$



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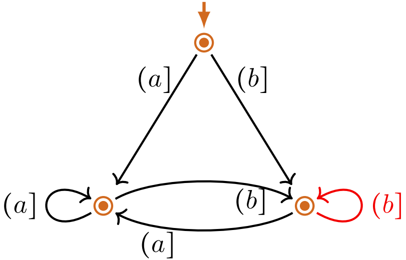
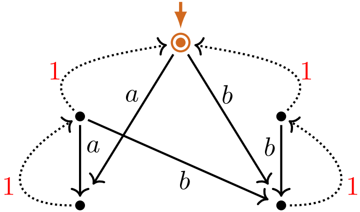


induced chart

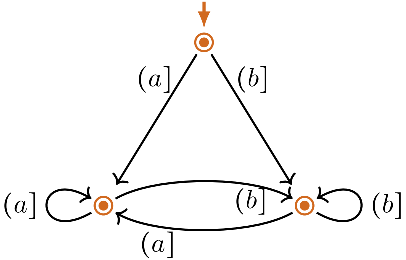
1-chart interpretation vs. chart interpretation (example)

$\underline{\mathcal{C}}((a^* \cdot b^*)^*)$



1-chart interpretation

$\mathcal{I}(\underline{\mathcal{C}}(e))$

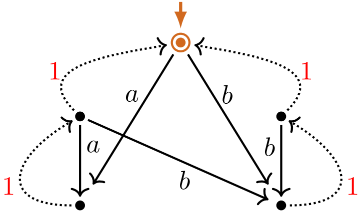


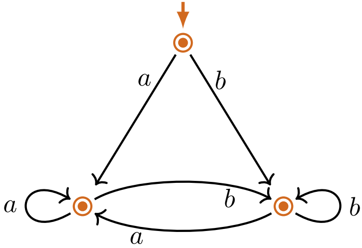
induced chart

1-chart interpretation vs. chart interpretation (example)

$\underline{\mathcal{C}}((a^* \cdot b^*)^*)$

$\mathcal{I}(\underline{\mathcal{C}}(e)) = \mathcal{C}(e)$



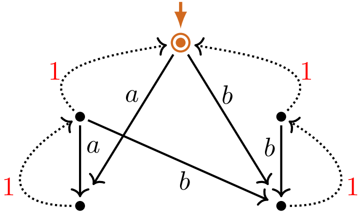


1-chart interpretation

induced chart

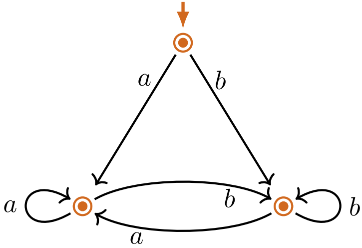
1-chart interpretation vs. chart interpretation (example)

$\underline{\mathcal{C}}((a^* \cdot b^*)^*)$



1-chart interpretation

$\mathcal{I}(\underline{\mathcal{C}}(e)) = \mathcal{C}(e)$

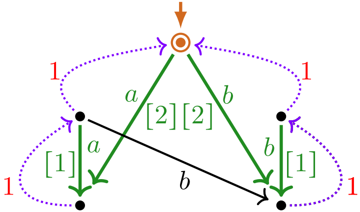


induced chart

LEE

1-chart interpretation vs. chart interpretation (example)

$\widehat{\underline{\mathcal{C}}(e)}$

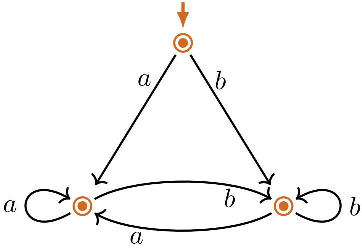


1-chart interpretation

LEE-witness

LEE

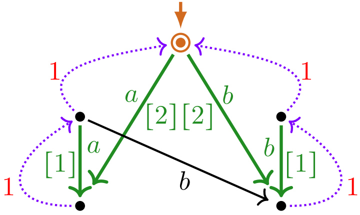
$\mathcal{I}(\underline{\mathcal{C}}(e)) = \mathcal{C}(e)$



induced chart

1-chart interpretation vs. chart interpretation (example)

$\widehat{\underline{\mathcal{C}}(e)}$



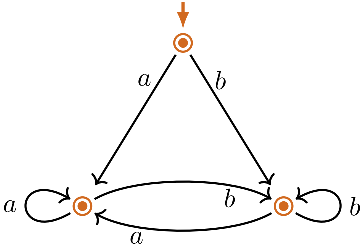
1-chart interpretation

LEE-witness

LEE-1-chart

LEE

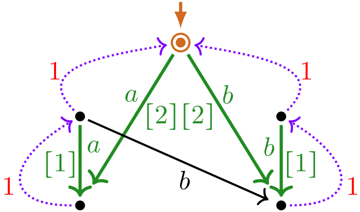
$\mathcal{I}(\underline{\mathcal{C}}(e)) = \mathcal{C}(e)$



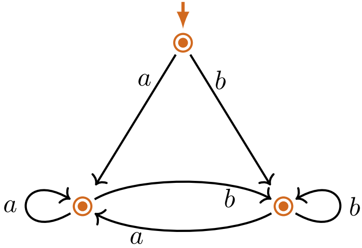
induced chart

1-chart interpretation vs. chart interpretation (example)

$\widehat{\underline{\mathcal{C}}(e)}$



$\mathcal{I}(\underline{\mathcal{C}}(e)) = \mathcal{C}(e)$



1-chart interpretation

LEE-witness

LEE-1-chart

LEE

chart interpretation

(= induced chart of
1-chart interpretation)

1-LEE

Properties of the variant process chart semantics

Theorem

The 1-chart interpretation $\underline{\mathcal{C}}(e)$ *of a star expression* e *satisfies:*

1. $\underline{\mathcal{C}}(e)$ *is a dag of proper transitions with* 1*-transition backbindings,*
2. $\mathcal{I}(\underline{\mathcal{C}}(e)) \rightarrow \mathcal{C}(e)$ *for all star expressions* e*:*
   there is a functional bisimulation from the induced chart $\mathcal{I}(\underline{\mathcal{C}}(e))$ *of* $\underline{\mathcal{C}}(e)$ *to the chart interpretation* $\mathcal{C}(e)$*.*
3. $\underline{\mathcal{C}}(e)$ *satisfies* LEE*.*

Outlook: Milner's questions

Q1. *Recognition: How to recognize charts that are expressible by a star expression modulo bisimilarity?*

- definability by well-behaved specifications *(Baeten/Corradini, 2005)*
  that is decidable (super–exponentially) *(Baeten/Corradini/G, 2007)*
- expressibility by 1-free star expr's: bisimulation collapse is LEE-chart
  that is polynomially decidable *(G, Fokkink, 2020)*
- efficiently recognizable also in the general case?

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- efficiently recognizable also in the general case?

Q2. *Complete proof system:* *Is Milner's system Mil complete for* $\underline{\leftrightarrow}_{P}$ *?*

- Yes, when restricted to 0,1-free star expr's. *(Fokkink, Zantema, 1994)*
- Yes, when adapted to 1-free star expressions. *(G, Fokkink, 2020)*
  - LEE-charts are preserved under bisimulation collapse.
- general case?
  - Results here facilitate use of LEE-1-charts.

Summary

- loop existence and elimination (LEE)
  - holds for process interpretations of 1-free star expressions
- LEE fails for process semantics in general
  - loop structure is lost, because
    bypassing of iterations e^* from inside is permitted by TSS-rules

Summary

- loop existence and elimination (LEE)
  - holds for process interpretations of 1-free star expressions
- LEE fails for process semantics in general
  - loop structure is lost, because
    bypassing of iterations e^* from inside is permitted by TSS-rules
- recover LEE for variant process interpretation
  - loop structure is regained by: additional symbol $\circledast$ and TSS-rules
    that force 1-transitions from iteration bodies $e' \circledast e^*$ back to e^*
  - functionally bisimilar to process interpretation
  - LEE holds for image
- basis for:
  - tackling Milner's axiomatization and recognition problems

Resources

- LICS article, and report version
  - CG & Wan Fokkink: A Complete Proof System for 1-Free Regular Expressions Modulo Bisimilarity, LICS 2020, report: arXiv:2004.12740, May 2020.
- extended abstract
  - CG: Structure-Constrained Process Graphs for the Process Interpretation of Regular Expressions, TERMGRAPH 2020, July 5, 2020. http://www.termgraph.org.uk/2020/.

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I want to thank:

- Luca Aceto (comments)
- Wan Fokkink (idea LEE-witnesses from TSSs)

Thank you for your attention!

LEE-witnesses for 1-chart interpretations

Definition (TSS for labels for LEE-witnesses)

$$\frac{}{a \xrightarrow{a}_{\mathsf{bo}} \mathbf{1}} \qquad \frac{e_i \xrightarrow{a}_{l} E_i'}{e_1 + e_2 \xrightarrow{a}_{\mathsf{bo}} E_i'} \; (i \in \{1,2\})$$

$$\frac{e \xrightarrow{a}_{l} E' \quad (e \text{ not normed})}{e^{*} \xrightarrow{a}_{\mathsf{bo}} E' \circledast e^{*}} \qquad \frac{e \xrightarrow{a}_{l} E' \quad (e \text{ normed})}{e^{*} \xrightarrow{a}_{[|e^{*}|_{*}]} E' \circledast e^{*}}$$

LEE-witnesses for 1-chart interpretations

Definition (TSS for labels for LEE-witnesses)

$$\frac{}{a \xrightarrow{a}_{\mathbf{bo}} \mathbf{1}} \qquad \frac{e_i \xrightarrow{a}_{l} E'_i}{e_1 + e_2 \xrightarrow{a}_{\mathbf{bo}} E'_i} \; (i \in \{1,2\})$$

$$\frac{E_1 \xrightarrow{a}_{l} E'_1}{E_1 \cdot e_2 \xrightarrow{a}_{l} E'_1 \cdot e_2} \qquad \frac{e_1{\downarrow} \qquad e_2 \xrightarrow{a}_{l} E'_2}{e_1 \cdot e_2 \xrightarrow{a}_{\mathbf{bo}} E'_2}$$

$$\frac{E_1 \xrightarrow{a}_{l} E'_1}{E_1 \circledast e_2^{*} \xrightarrow{a}_{l} E'_1 \circledast e_2^{*}} \qquad \frac{e_1{\downarrow}}{e_1 \circledast e_2^{*} \xrightarrow{1}_{\mathbf{bo}} e_2^{*}}$$

$$\frac{e \xrightarrow{a}_{l} E' \qquad (e \text{ not normed})}{e^{*} \xrightarrow{a}_{\mathbf{bo}} E' \circledast e^{*}} \qquad \frac{e \xrightarrow{a}_{l} E' \qquad (e \text{ normed})}{e^{*} \xrightarrow{a}_{[|e^{*}|_{*}]} E' \circledast e^{*}}$$