Modeling Terms by Graphs with Structure Constraints

(Two Illustrations)

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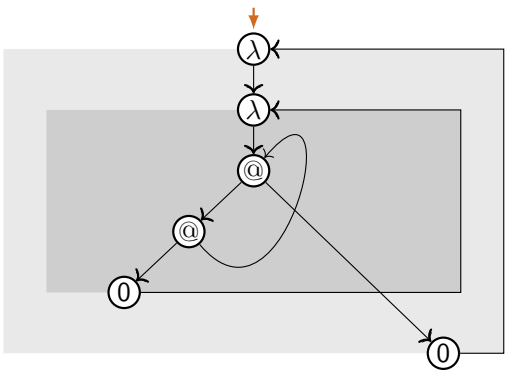
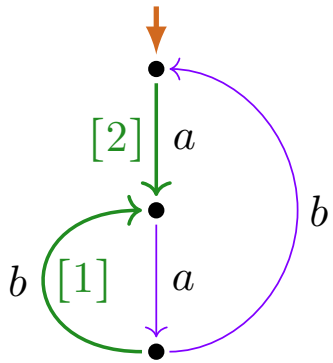
The Netherlands



Computer Science Seminar

GSSI

August 30, 2018



Overview

1. Maximal sharing of functional programs
   - higher-order λ-term graphs
2. Process interpretation of regular expressions
   - LEE-witnesses: graph labelings based on a loop–condition LEE

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   - from terms in the λ-calculus with letrec to:
     - higher-order λ-term graphs
     - first-order λ-term graphs
     - λ-NFAs, and λ-DFAs
   - minimization / readback / efficiency / Haskell implementation
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   - Milner's questions, known results
   - structure-constrained process graphs:
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     - preservation under bisimulation collapse
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- Comparison desiderata

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λ-calculus with letrec under unfolding semantics

Not available: term graph interpretation that is studied under $\underline{\leftrightarrow}$

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Regular expressions under process semantics (bisimilarity $\underline{\leftrightarrow}$)

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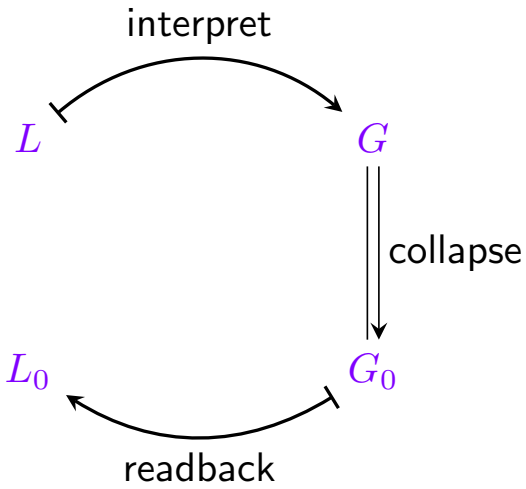
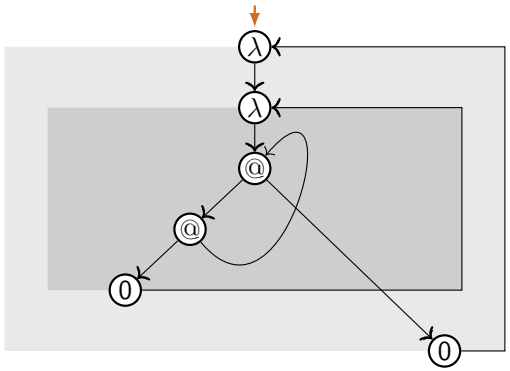
Desired: reason with graphs that are $[\![\cdot]\!]_{\boldsymbol{P}}$-expressible modulo $\underline{\leftrightarrow}$ (at least with 'sufficiently many')

understand incompleteness by a structural graph property

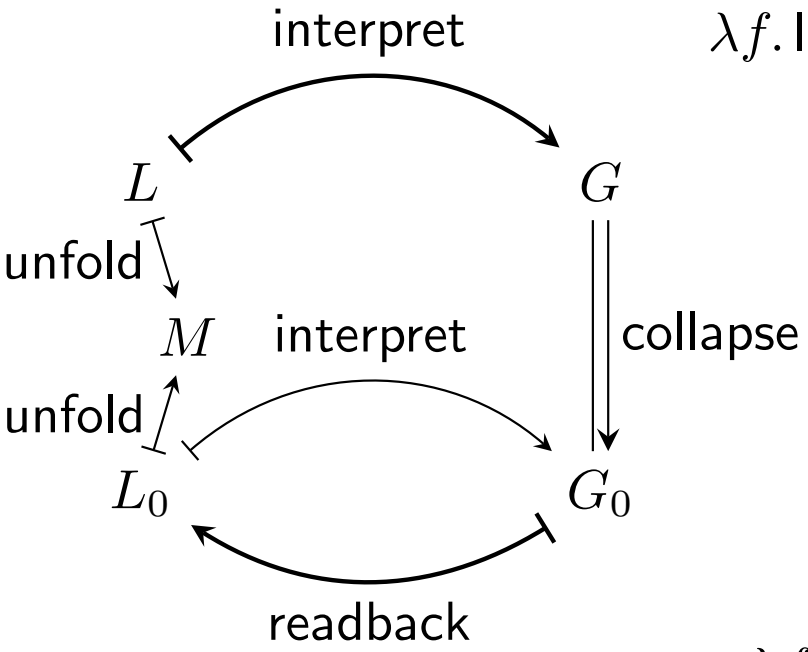
Maximal sharing of functional programs

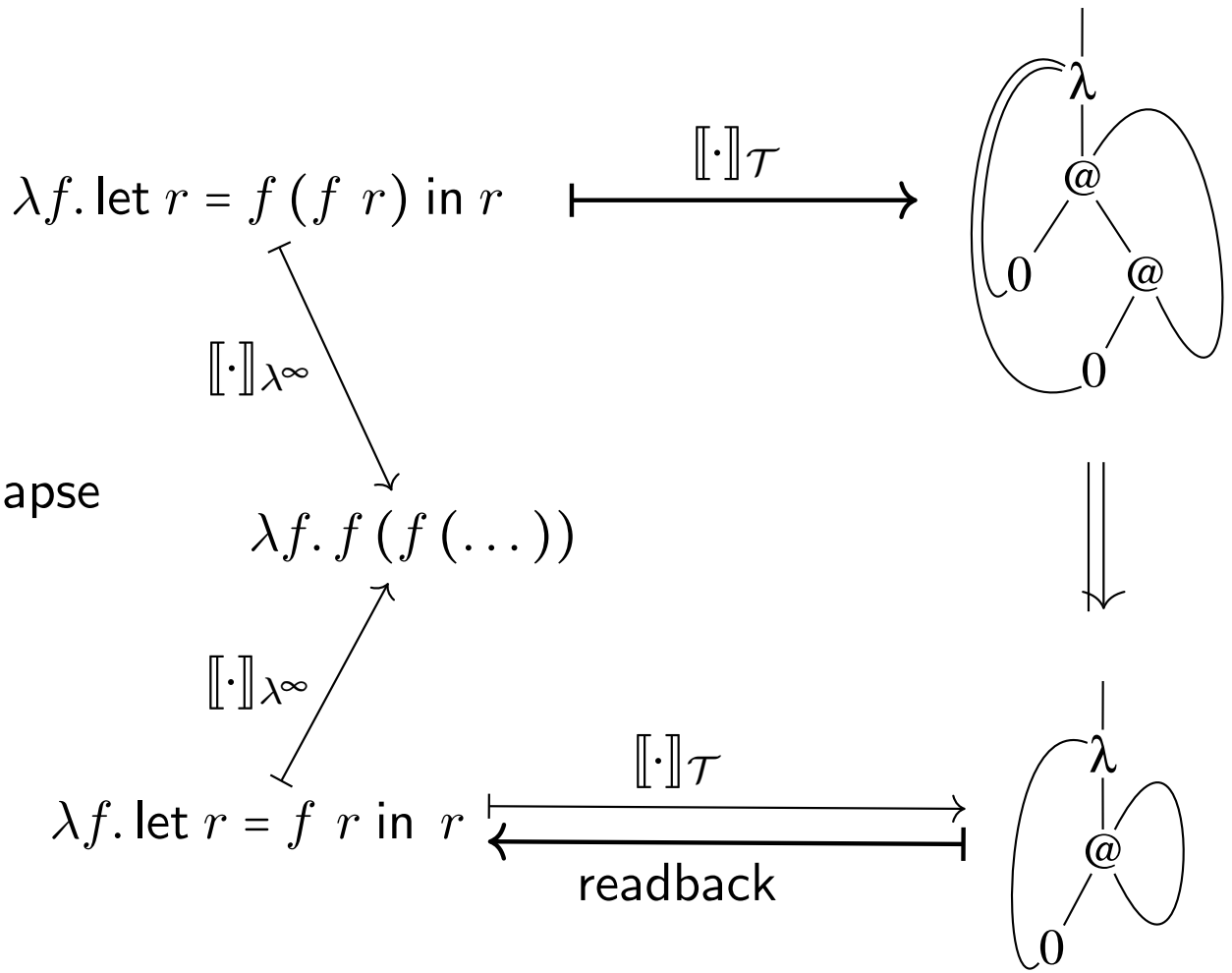
(joint work with Jan Rochel)





maximal sharing: example (fix)





maximal sharing: the method

$$L \overset{[\![\cdot]\!]_{\mathcal{H}}}{\longmapsto} \mathcal{G}$$

1. term graph interpretation $[\![\cdot]\!]$.
   of λ_{letrec}-term L as:
   a. higher-order term graph
      $\mathcal{G} = [\![L]\!]_{\mathcal{H}}$

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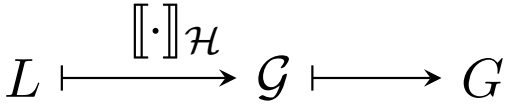

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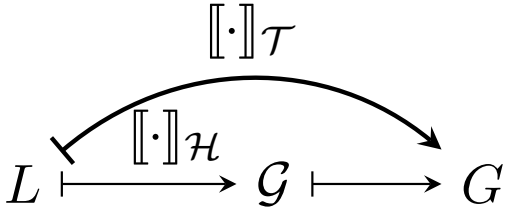

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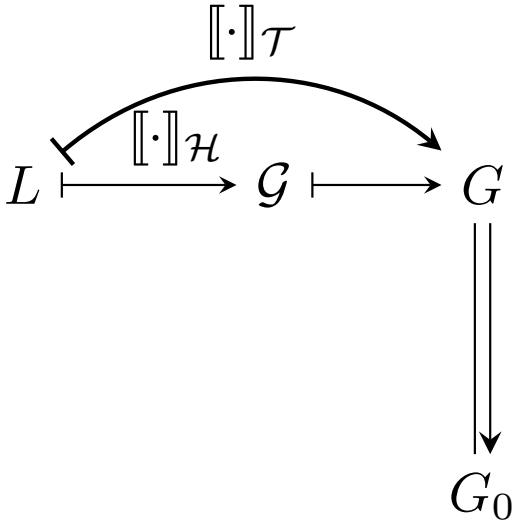

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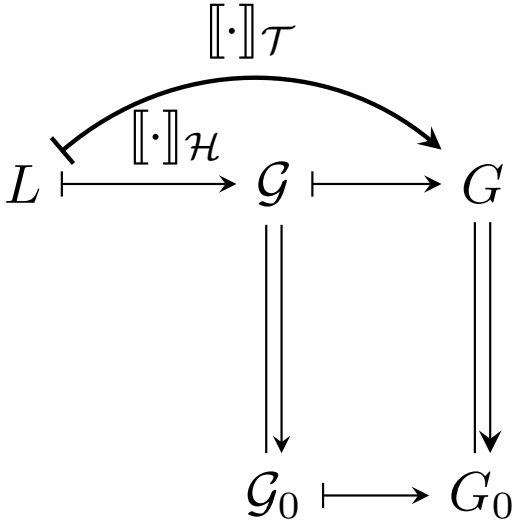

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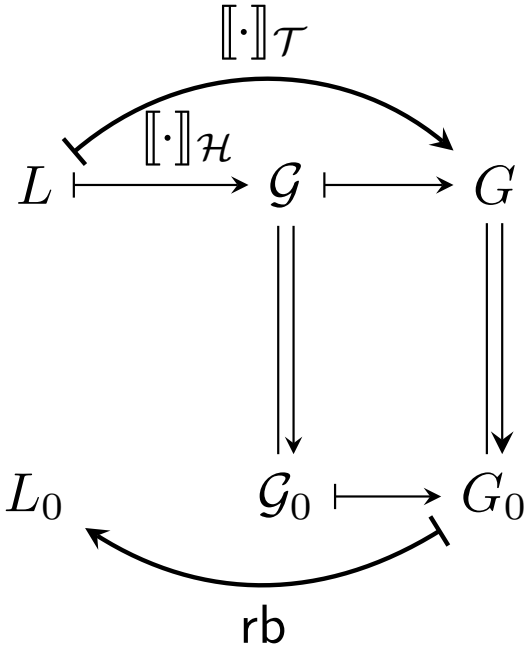

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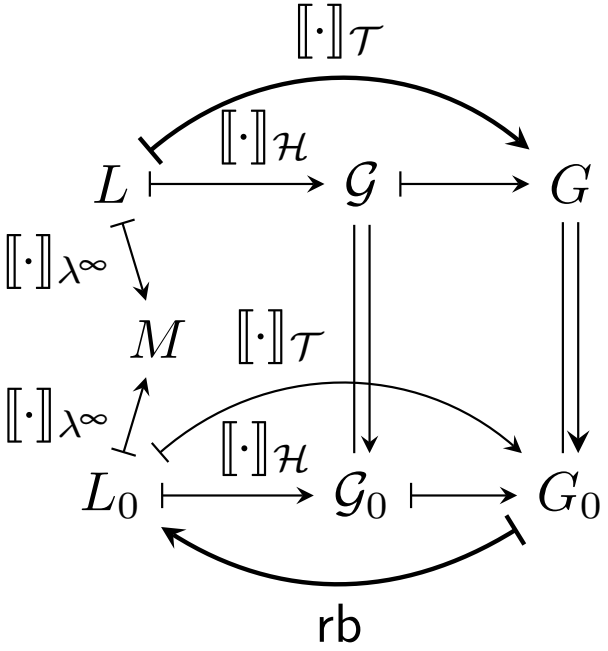

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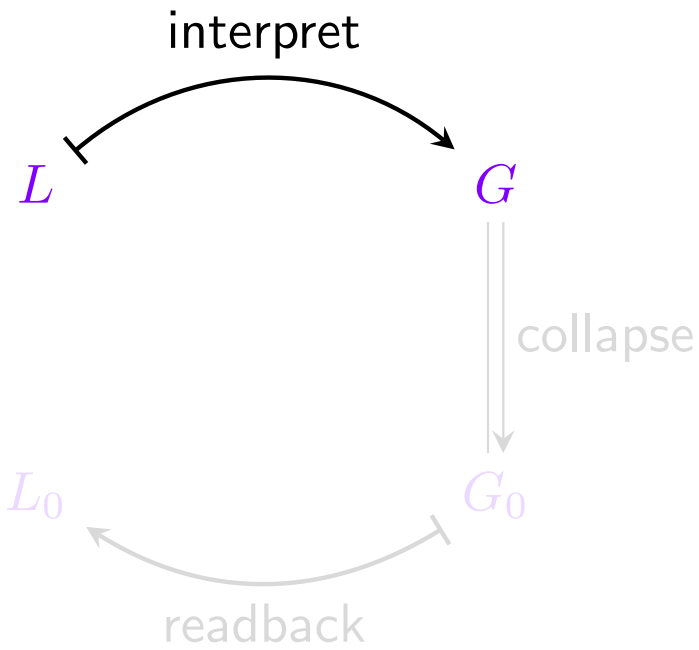
interpretation



running example

instead of:

$\lambda f.\,\mathsf{let}\ r = f\ (f\ r)\ \mathsf{in}\ r$ $\longmapsto_{\text{max-sharing}}$ $\lambda f.\,\mathsf{let}\ r = f\ r\ \mathsf{in}\ r$

we use:

$\lambda x.\,\lambda f.\,\mathsf{let}\ r = f\ (f\ r\ x)\ x\ \mathsf{in}\ r$ $\longmapsto_{\text{max-sharing}}$ $\lambda x.\,\lambda f.\,\mathsf{let}\ r = f\ r\ x\ \mathsf{in}\ r$

L $\longmapsto_{\text{max-sharing}}$ L_0

graph interpretation (example 1)

$L_0 = \lambda x.\, \lambda f.\, \mathsf{let}\ r = f\ r\ x\ \mathsf{in}\ r$

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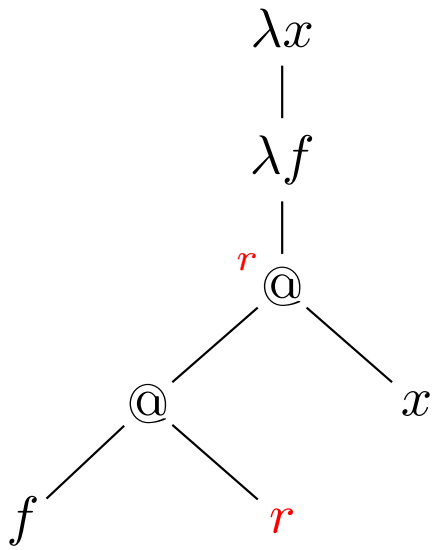

syntax tree

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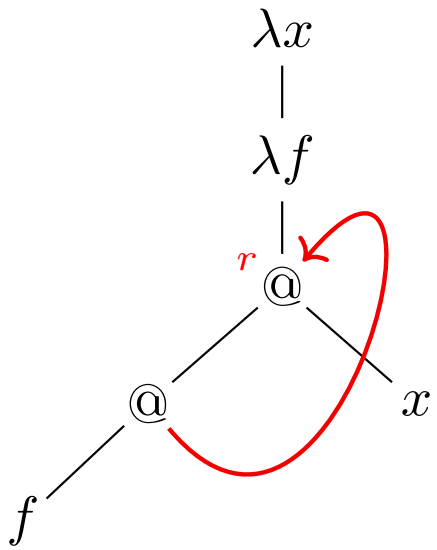

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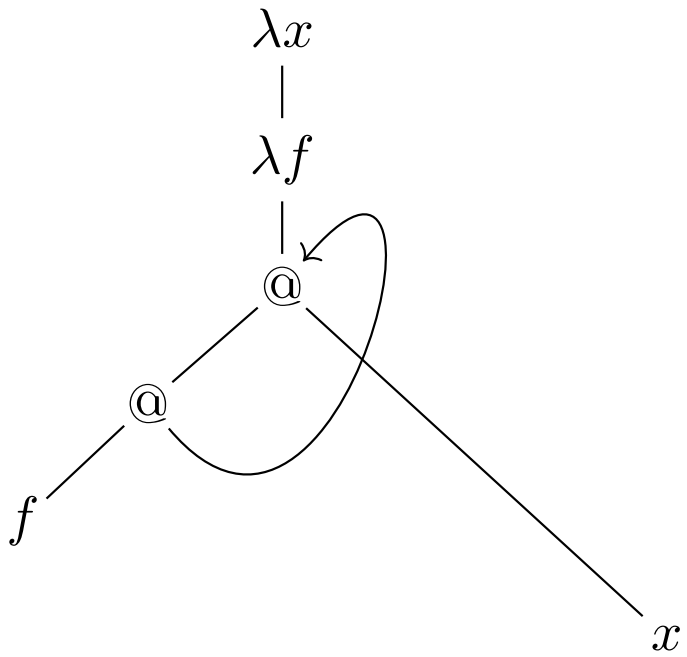

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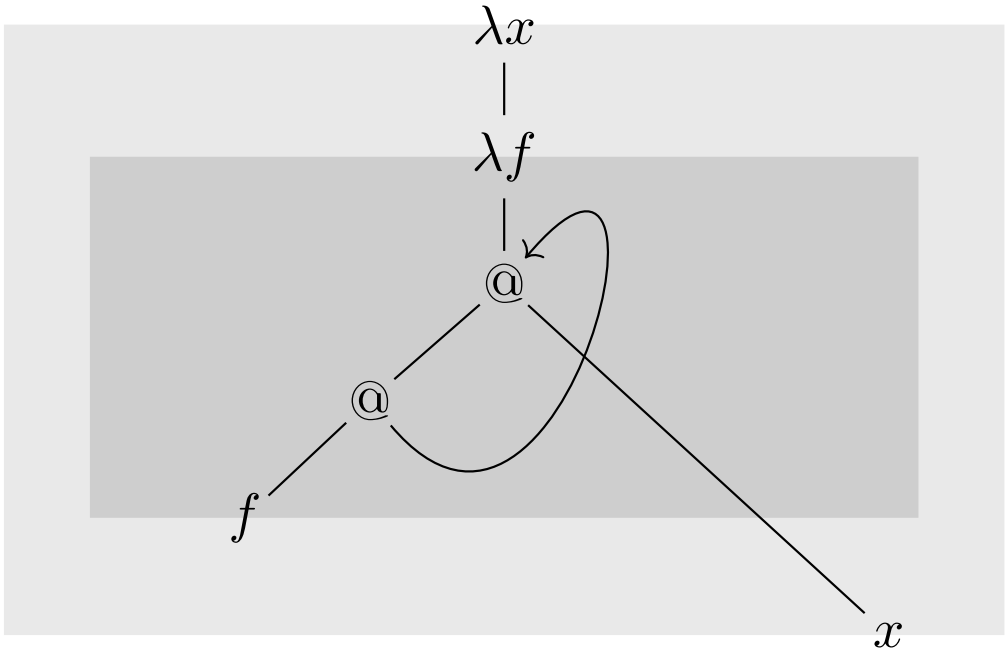

syntax tree (+ recursive backlink, + scopes)

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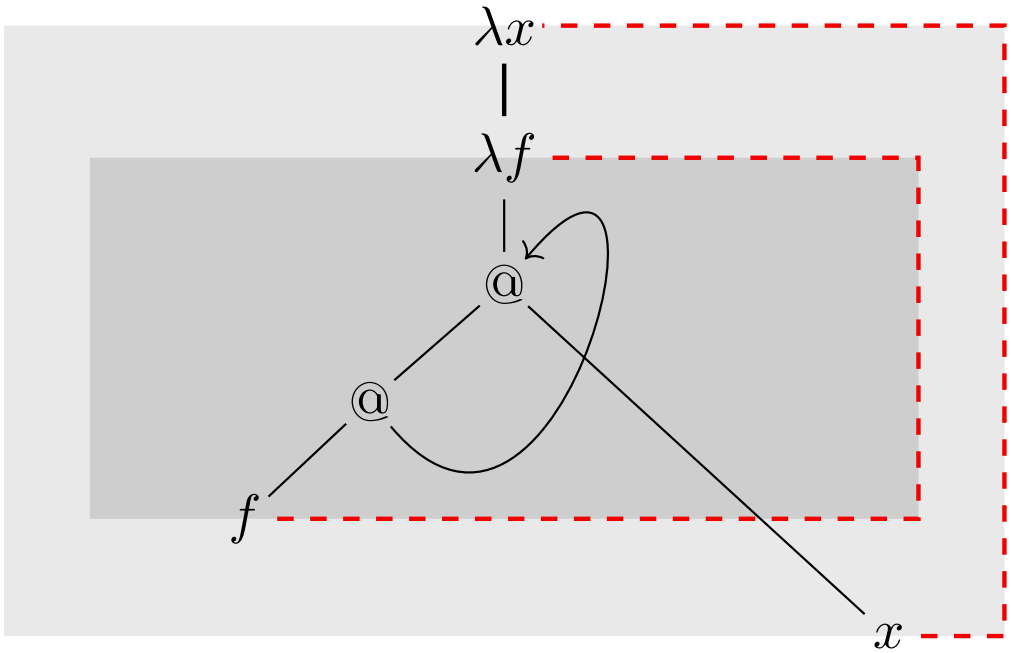

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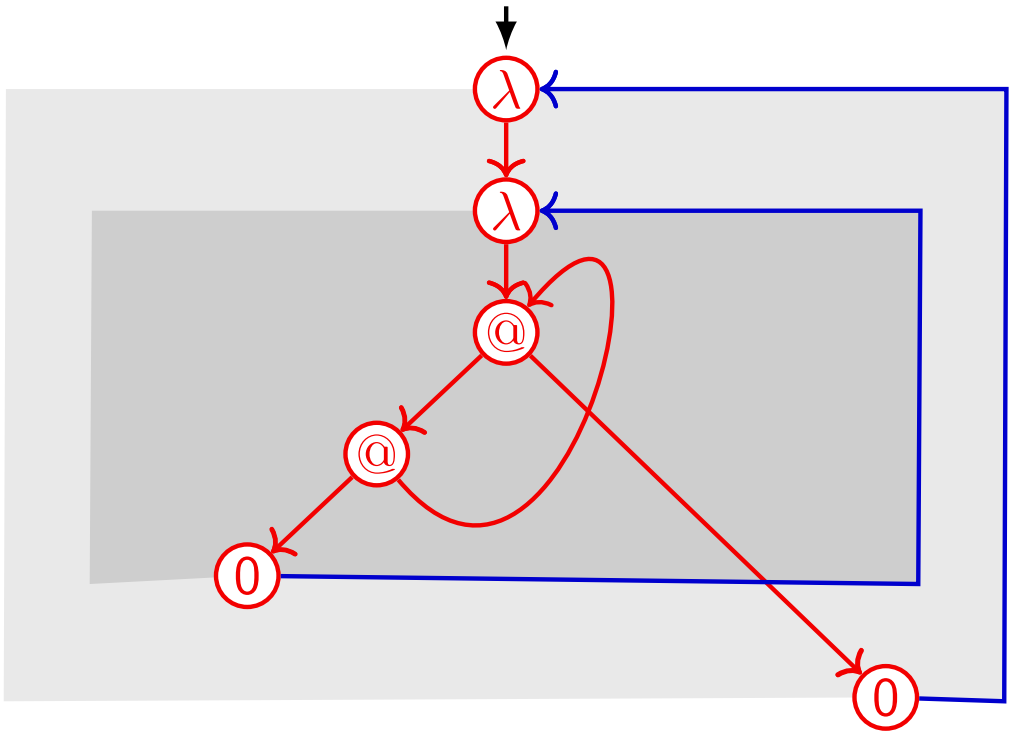

first-order term graph with binding backlinks (+ scope sets)

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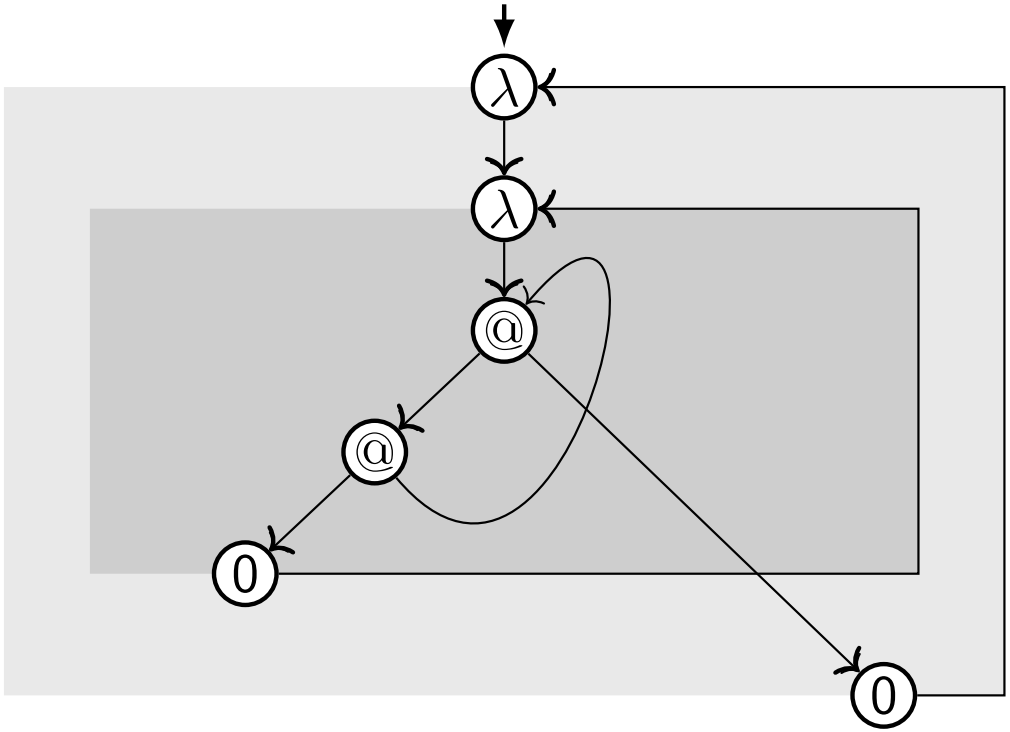

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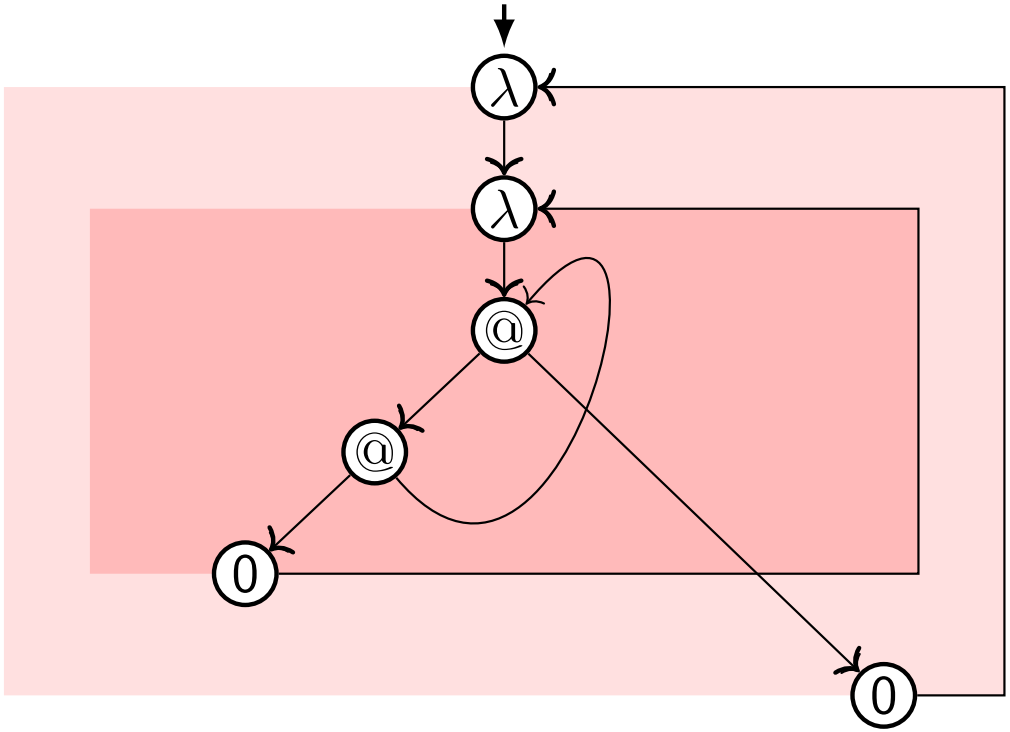

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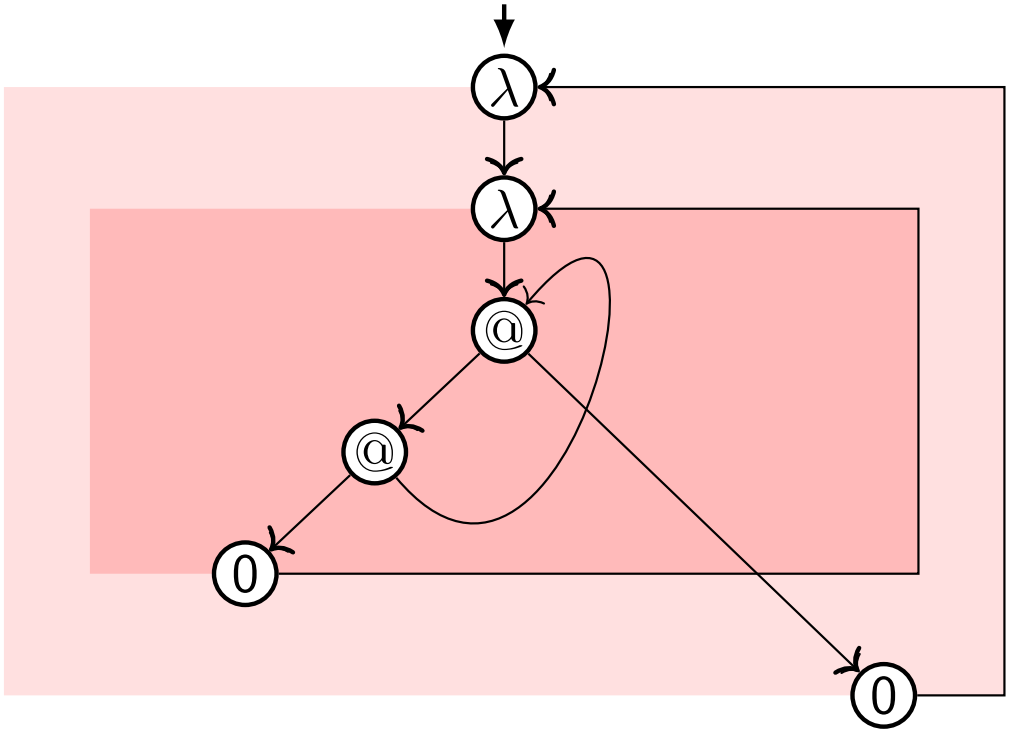

higher-order term graph (with scope sets, Blom [2003])

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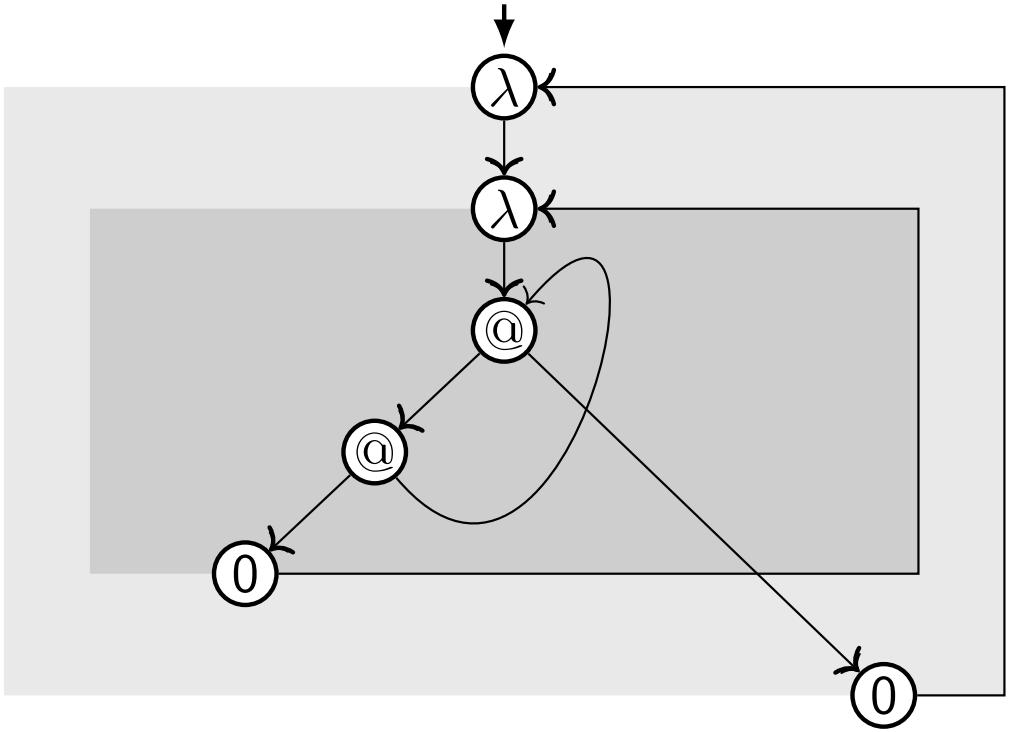

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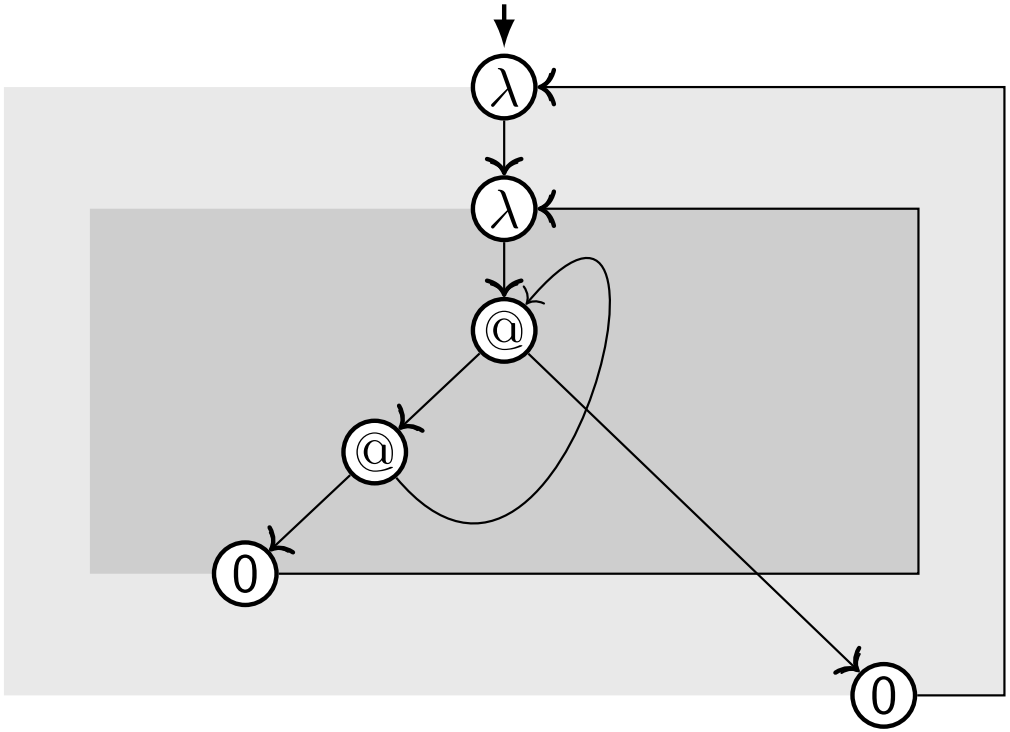

first-order term graph with binding backlinks (+ scope sets)

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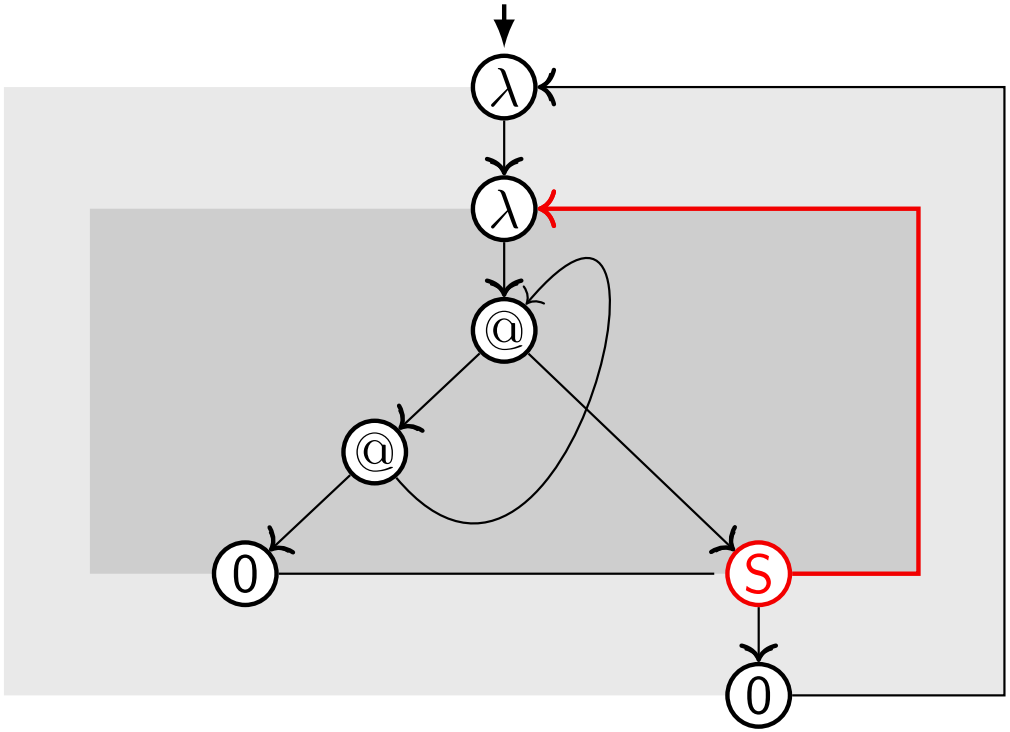

first-order term graph with scope vertices with backlinks (+ scope sets)

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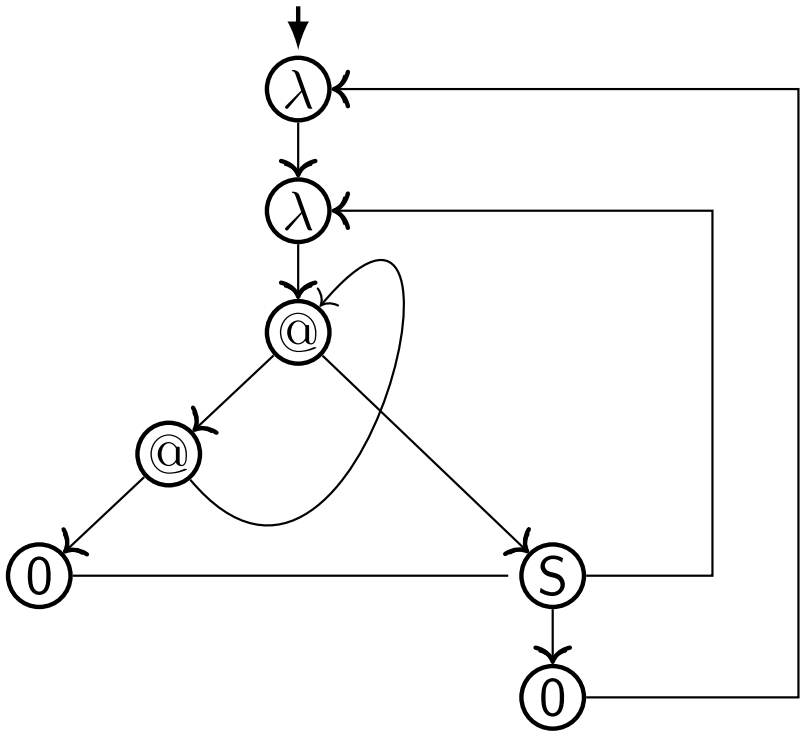

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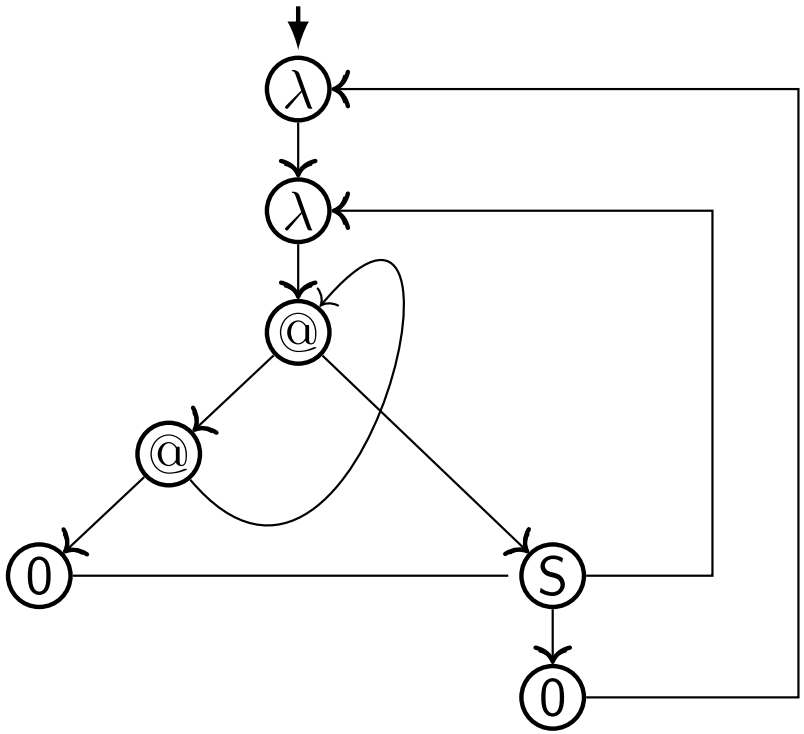

λ-term-graph $[\![L_0]\!]_{\mathcal{T}}$

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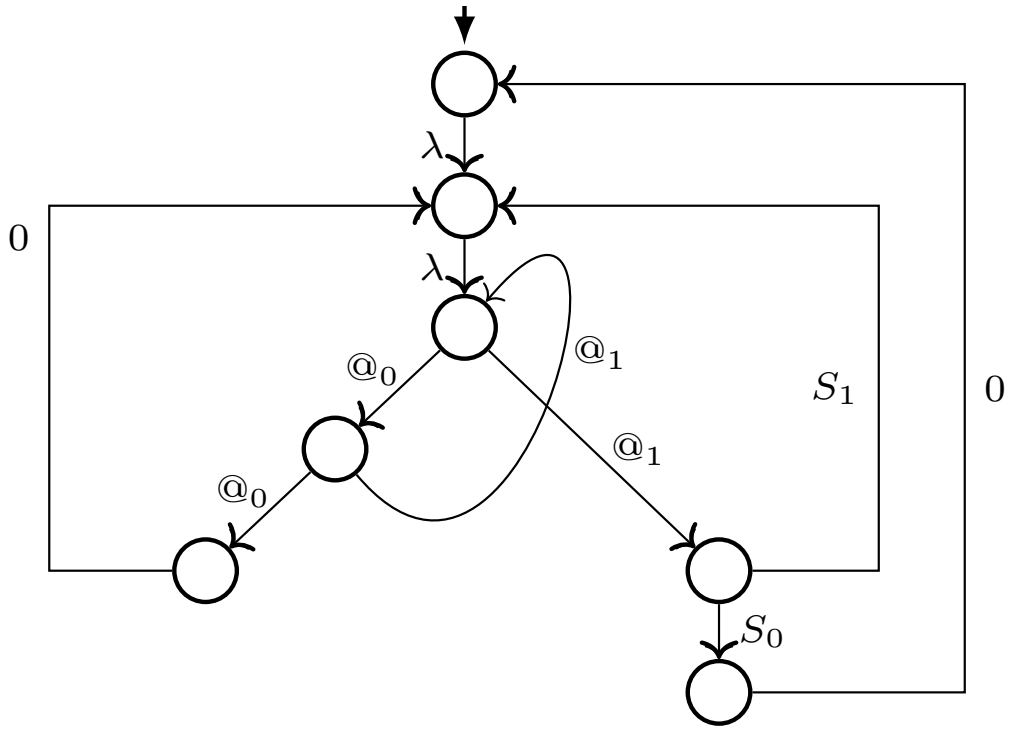

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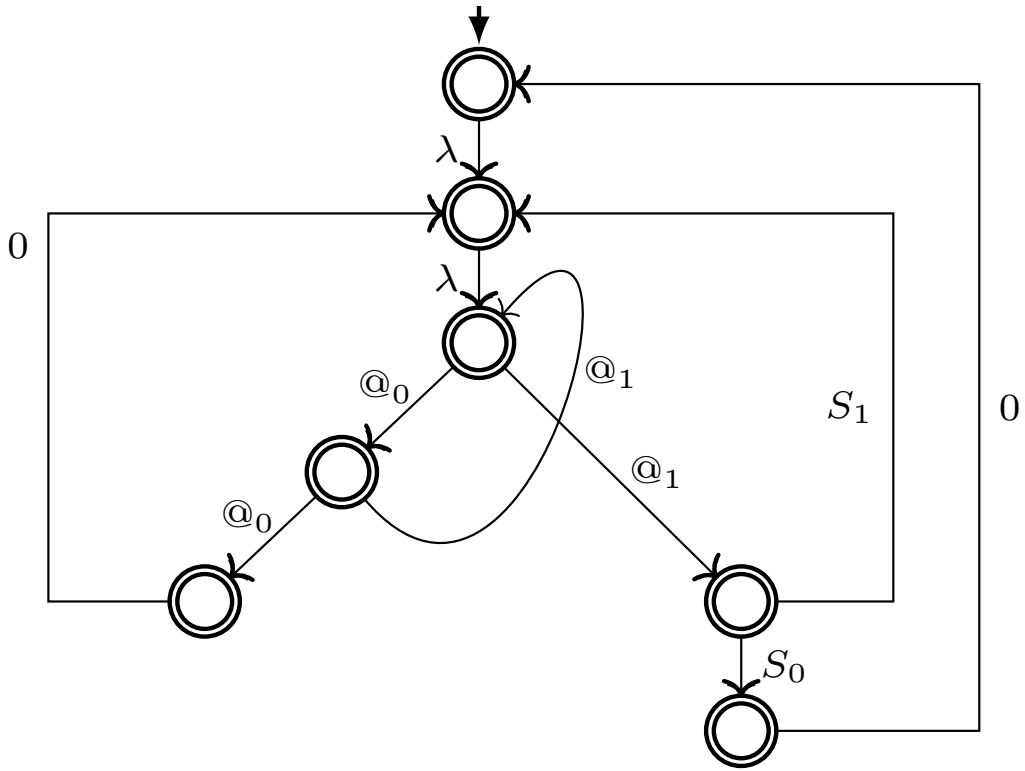

λ-NFA

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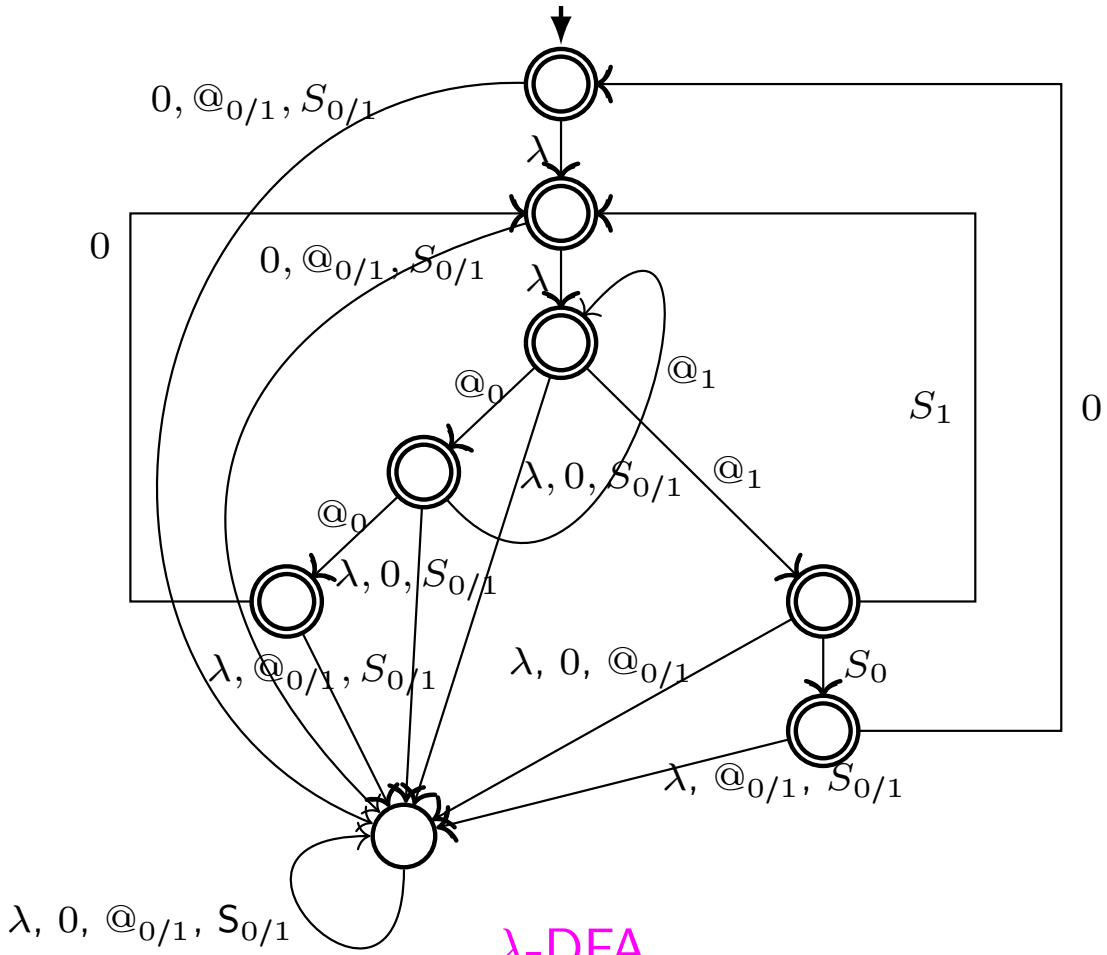

λ-DFA

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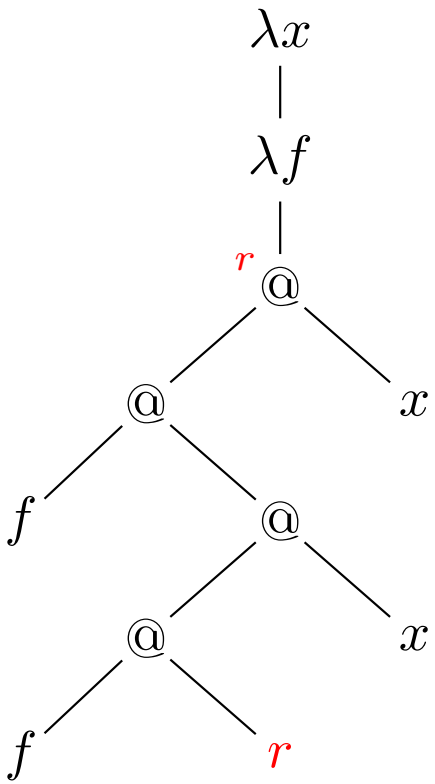
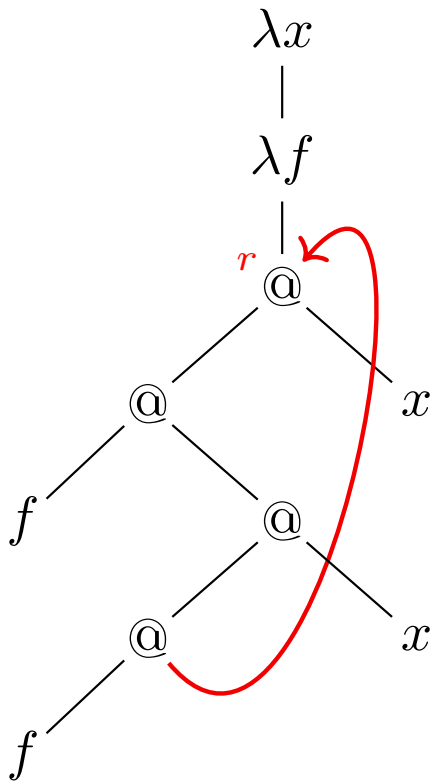


syntax tree

graph interpretation (example 2)

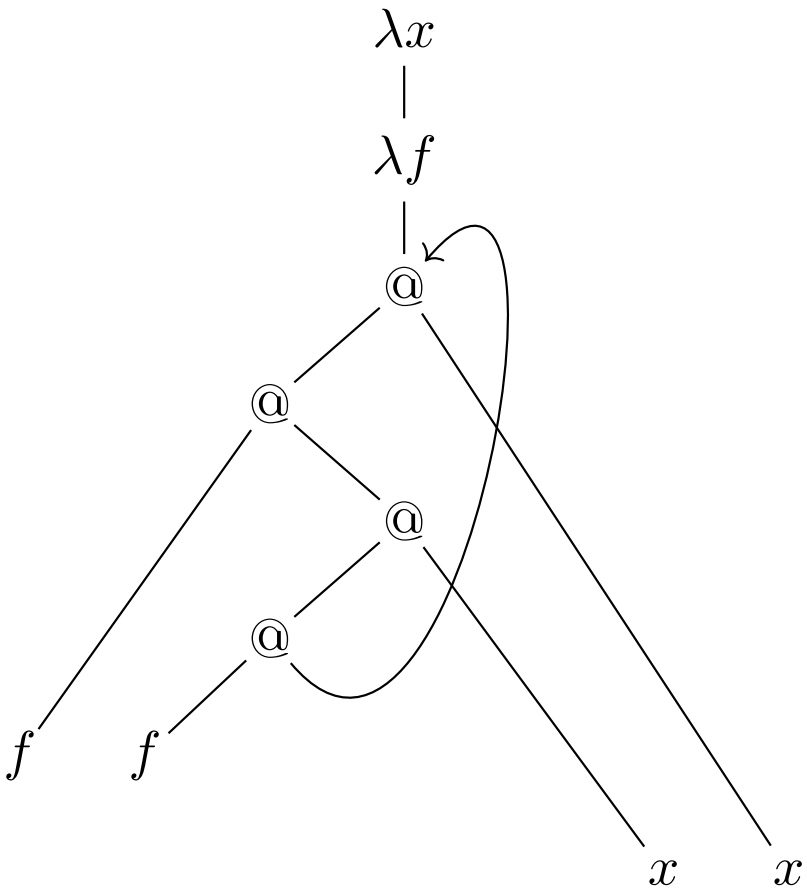
$L = \lambda x.\, \lambda f.\, \text{let } r = f\ (f\ r\ x)\ x \text{ in } r$



syntax tree (+ recursive backlink)

graph interpretation (example 2)

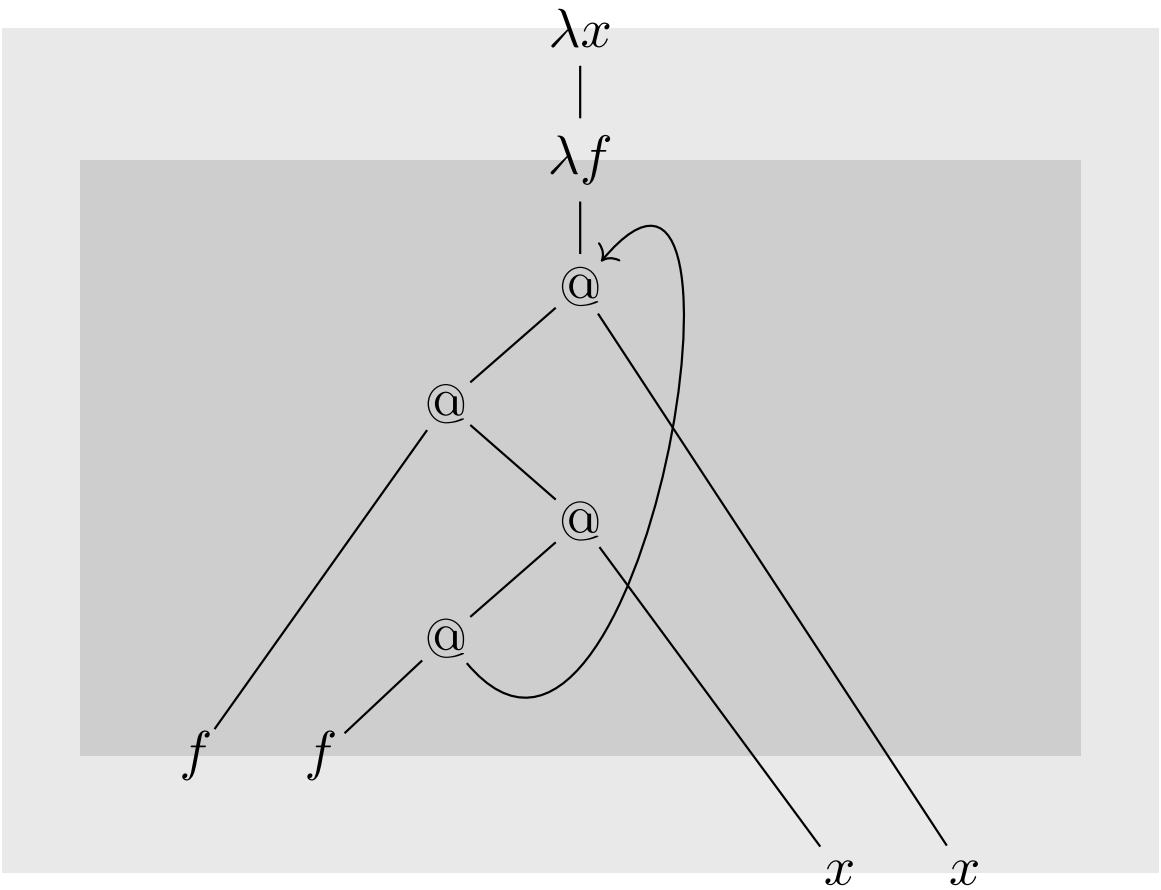
$L = \lambda x.\, \lambda f.\, \mathsf{let}\ r = f\ (f\ r\ x)\ x\ \mathsf{in}\ r$



syntax tree (+ recursive backlink)

graph interpretation (example 2)

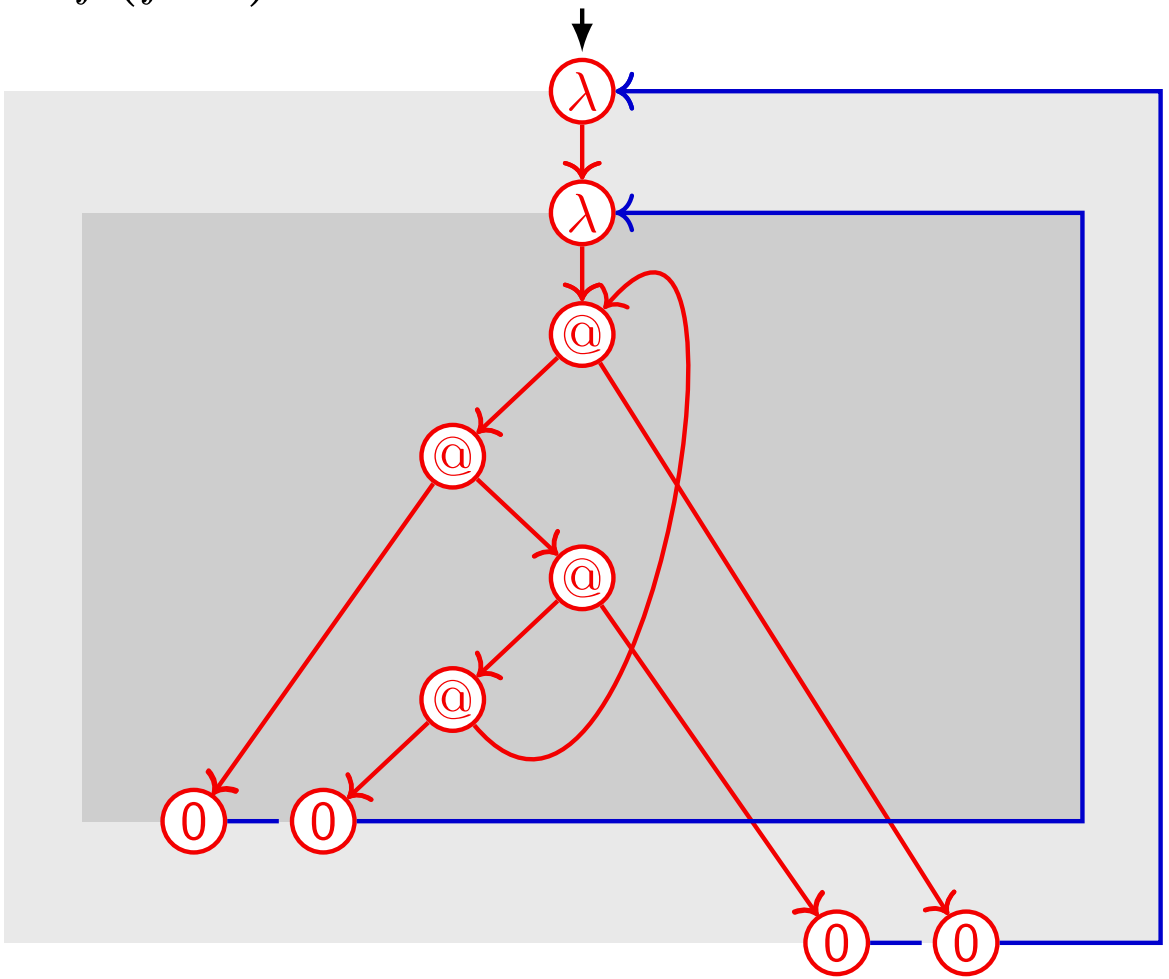
$L = \lambda x.\, \lambda f.\, \text{let } r = f\ (f\ r\ x)\ x \text{ in } r$



syntax tree (+ recursive backlink, + scopes)

graph interpretation (example 2)

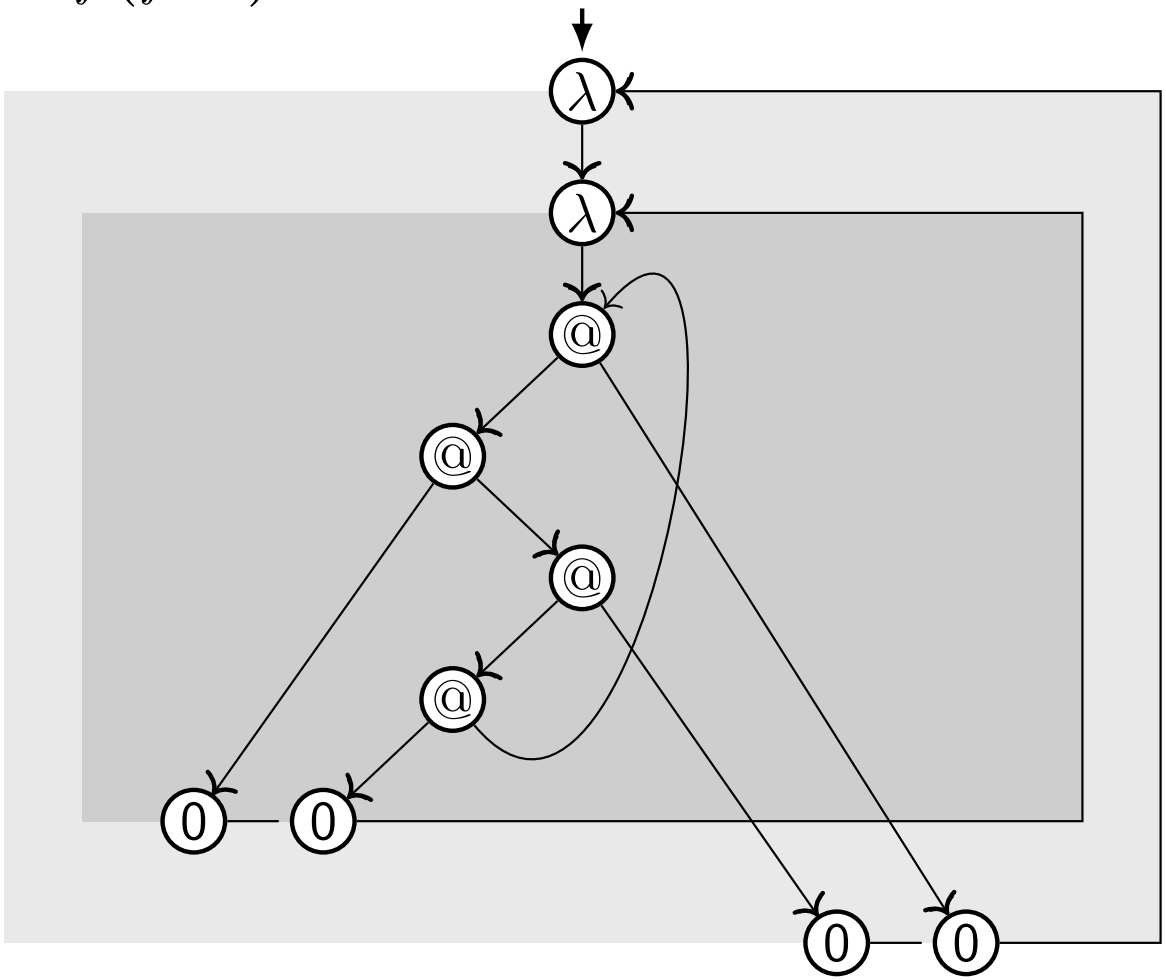
$L = \lambda x.\, \lambda f.\, \text{let } r = f\,(f\,r\,x)\,x \text{ in } r$



first-order term graph with binding backlinks (+ scope sets)

graph interpretation (example 2)

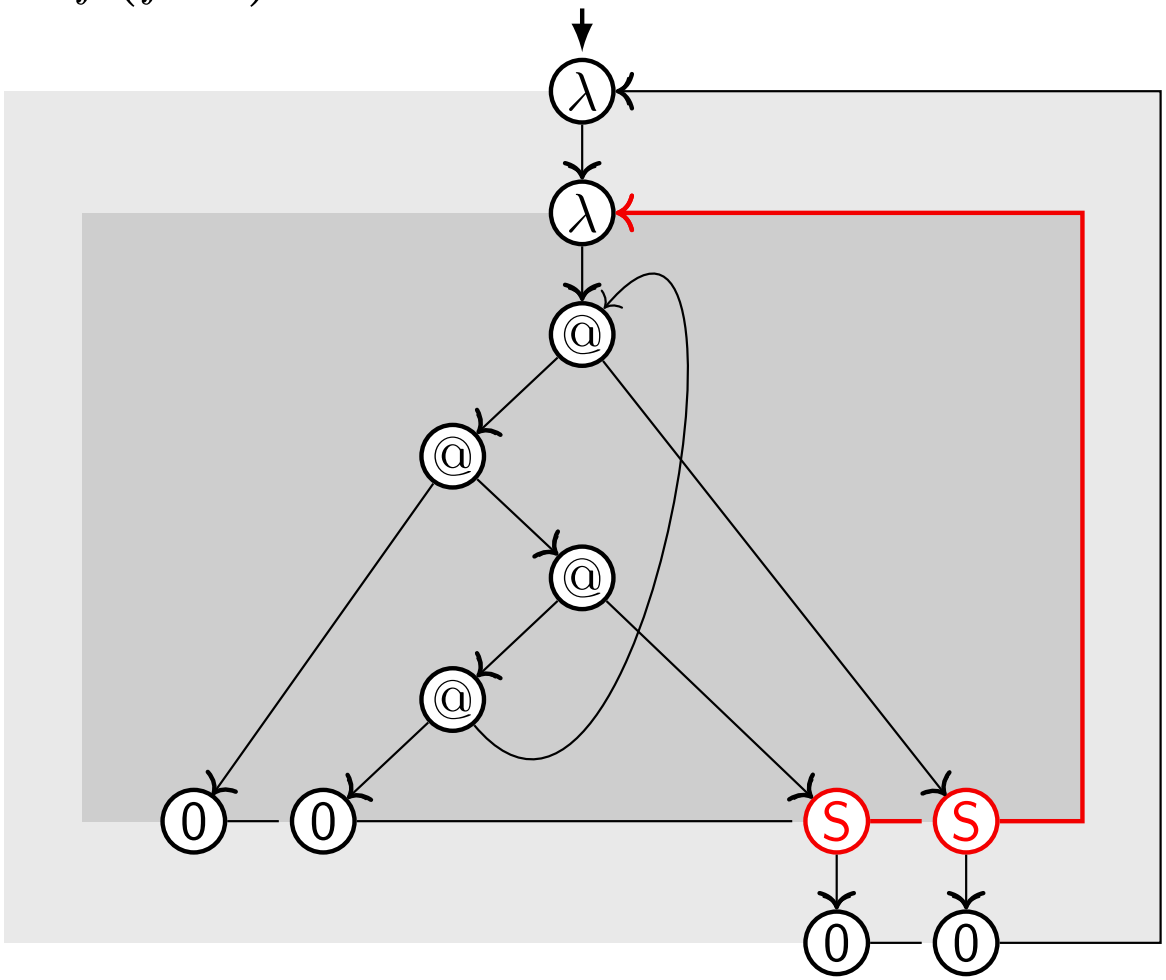
$L = \lambda x.\, \lambda f.\, \text{let } r = f\,(f\,r\,x)\,x \text{ in } r$



λ-higher-order-term-graph $[\![L]\!]_{\mathcal{H}}$

graph interpretation (example 2)

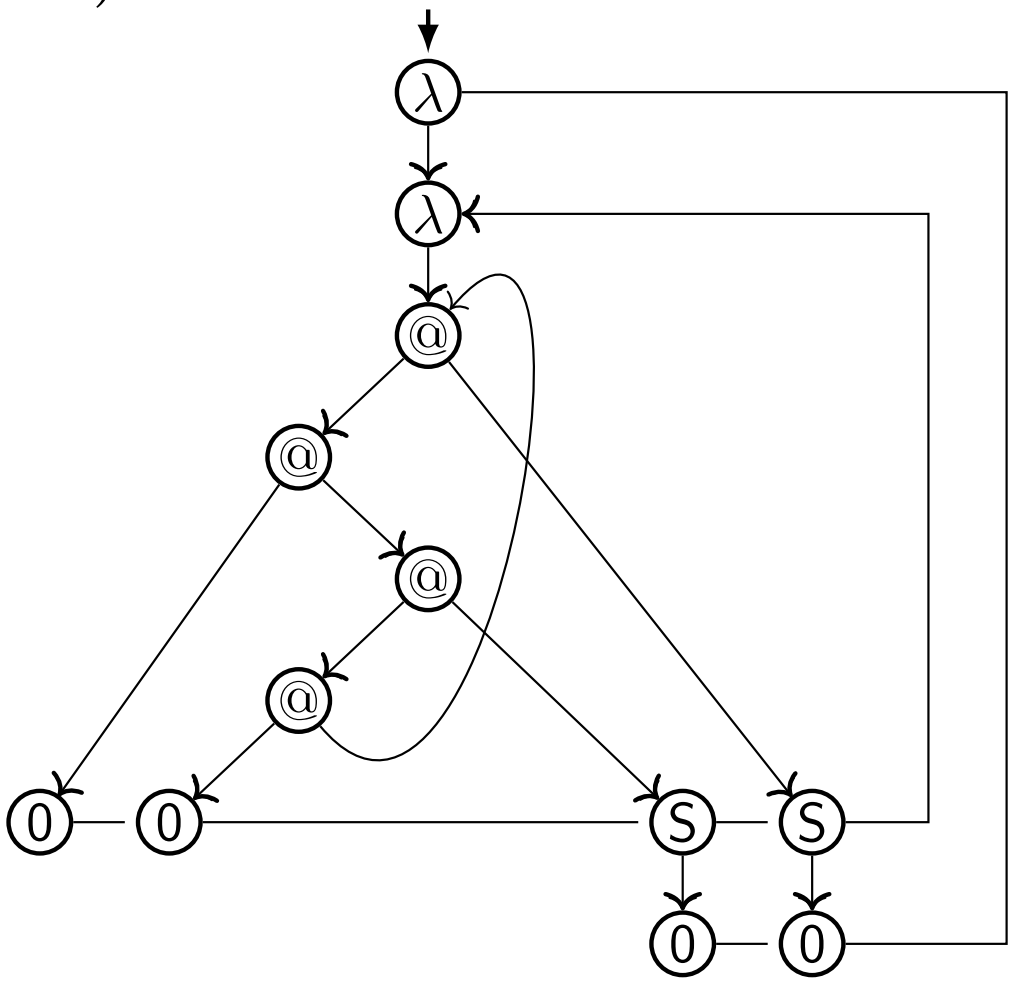
$L = \lambda x.\, \lambda f.\, \text{let } r = f\ (f\ r\ x)\ x \text{ in } r$



first-order term graph with scope vertices with backlinks (+ scope sets)

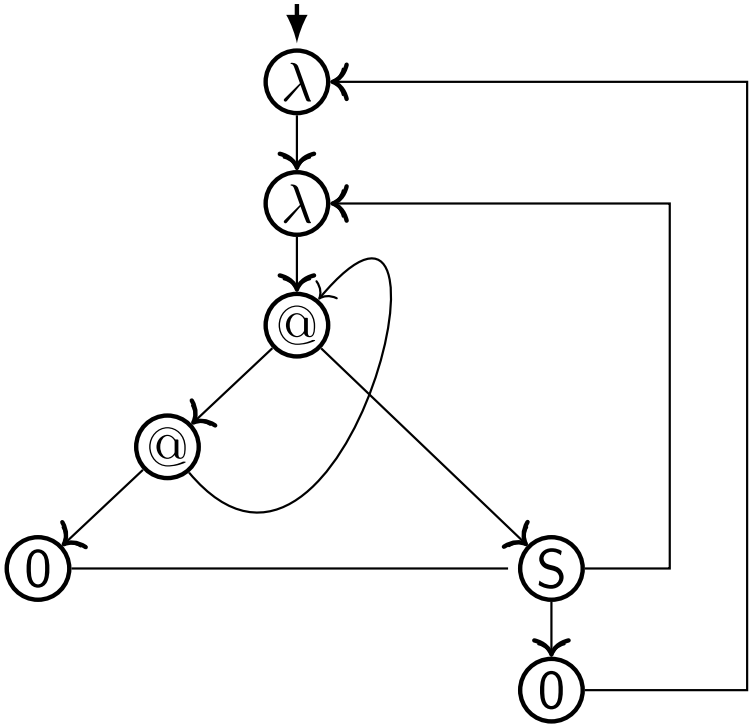
graph interpretation (example 2)

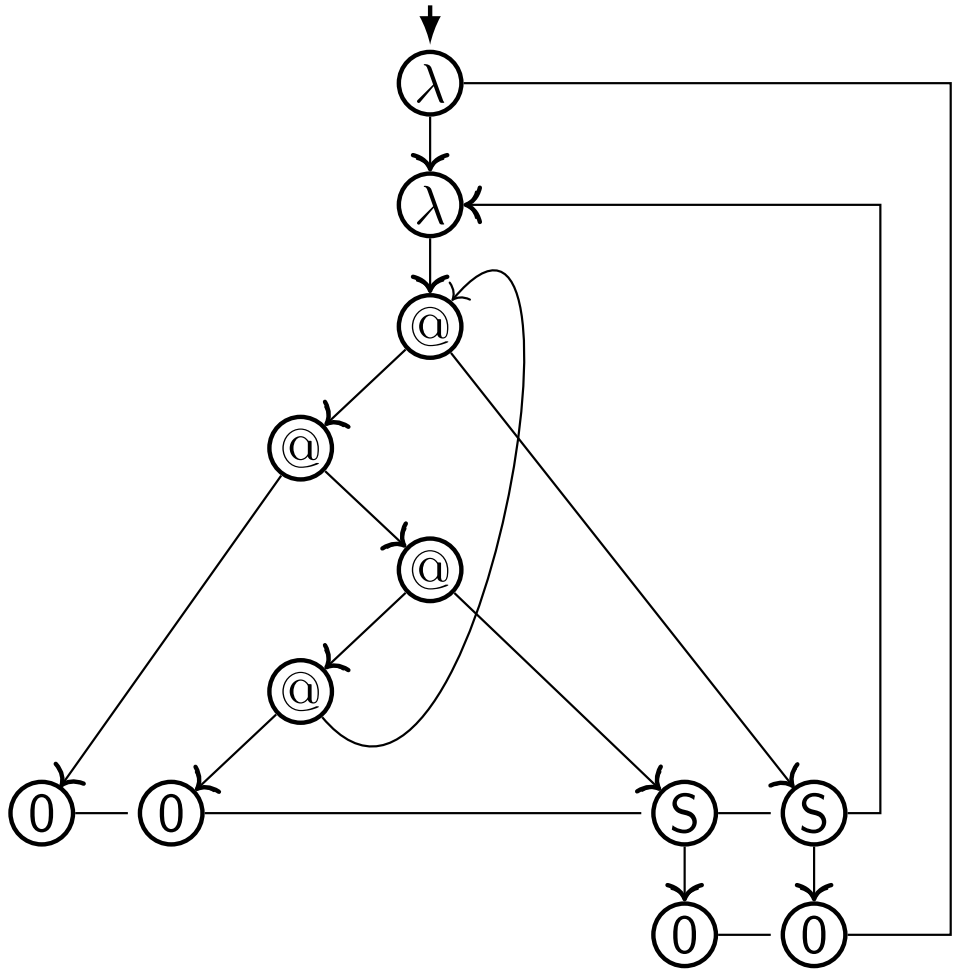
$L = \lambda x.\, \lambda f.\, \mathsf{let}\ r = f\ (f\ r\ x)\ x\ \mathsf{in}\ r$



λ-term-graph $[\![L]\!]_{\mathcal{T}}$

graph interpretation (examples 1 and 2)





$[\![L_0]\!]\tau$

$[\![L]\!]\tau$

interpretation $[\![\cdot]\!]_{\mathcal{T}}$: properties (cont.)

interpretation λ_{letrec}-term $L \longmapsto$ λ-term-graph $[\![L]\!]_{\mathcal{T}}$

- defined by induction on structure of L
- similar analysis as fully-lazy lambda-lifting
- yields eager-scope λ-term-graphs: $\sim$ minimal scopes

Theorem

For λ_{letrec}-terms L_1 and L_2 it holds: Equality of infinite unfolding coincides with bisimilarity of λ-term-graph interpretations:

$$[\![L_1]\!]_{\lambda^\infty} = [\![L_2]\!]_{\lambda^\infty} \iff [\![L_1]\!]_{\mathcal{T}} \leftrightarrows [\![L_2]\!]_{\mathcal{T}}$$

interpretation $[\![\cdot]\!]_{\mathcal{T}}$: properties (cont.)

interpretation $\boldsymbol{\lambda}_{\text{letrec}}$-term $L \longmapsto$ λ-term-graph $[\![L]\!]_{\mathcal{T}}$

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For $\boldsymbol{\lambda}_{\text{letrec}}$-terms L_1 and L_2 it holds: Equality of infinite unfolding coincides with bisimilarity of λ-term-graph interpretations:

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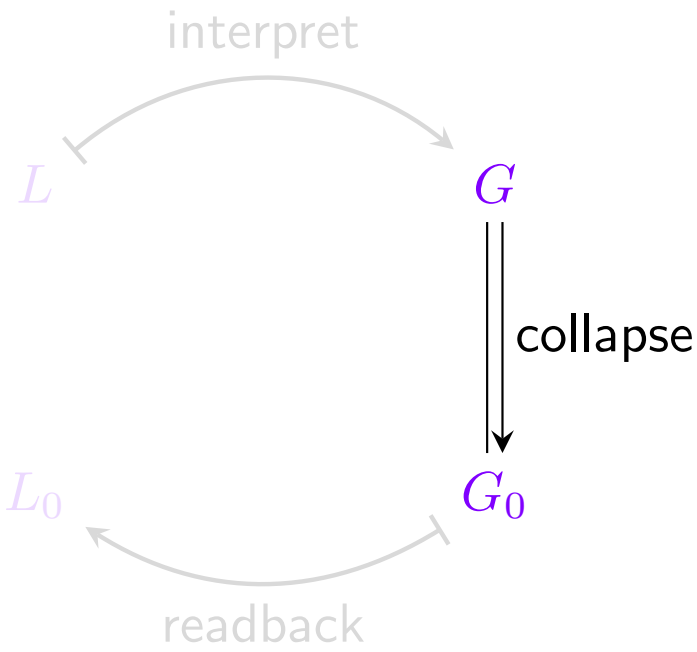
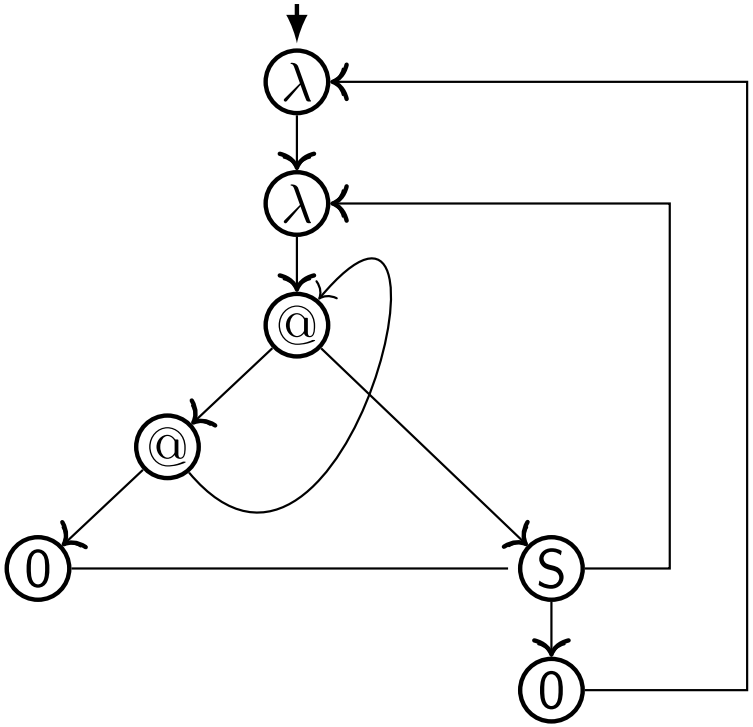
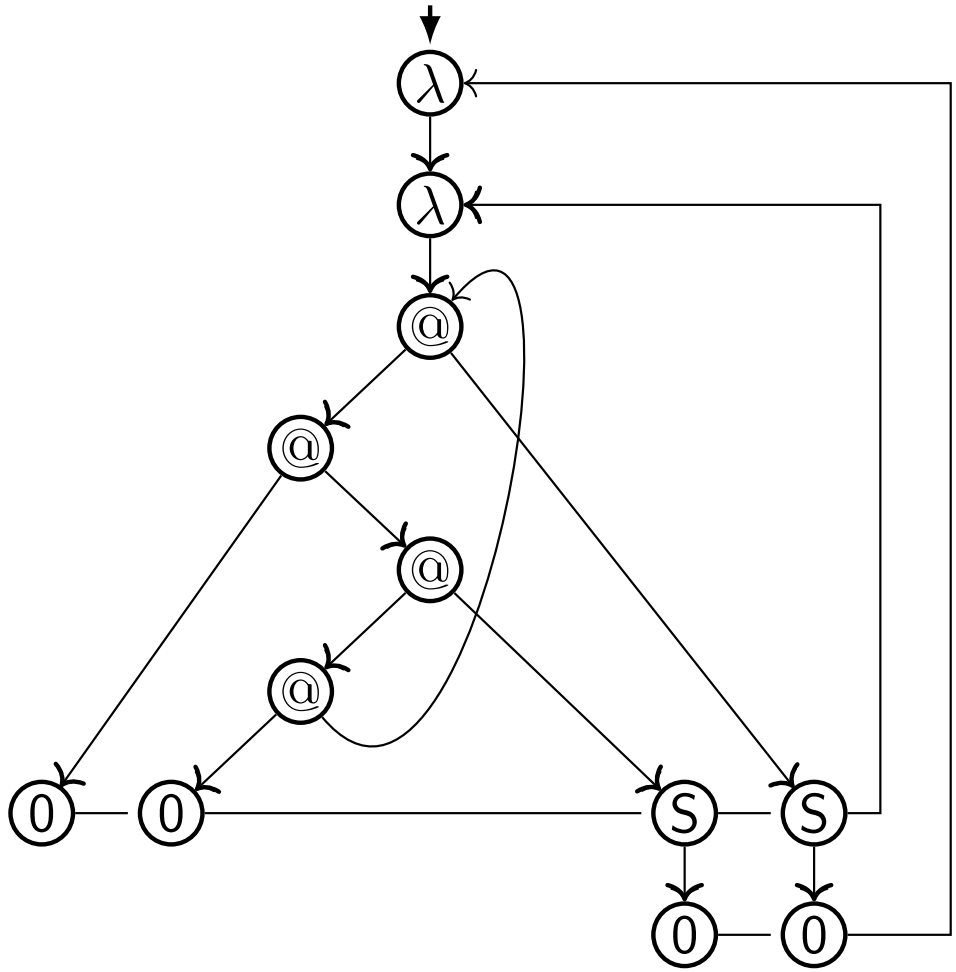
collapse



bisimulation check between λ-term-graphs

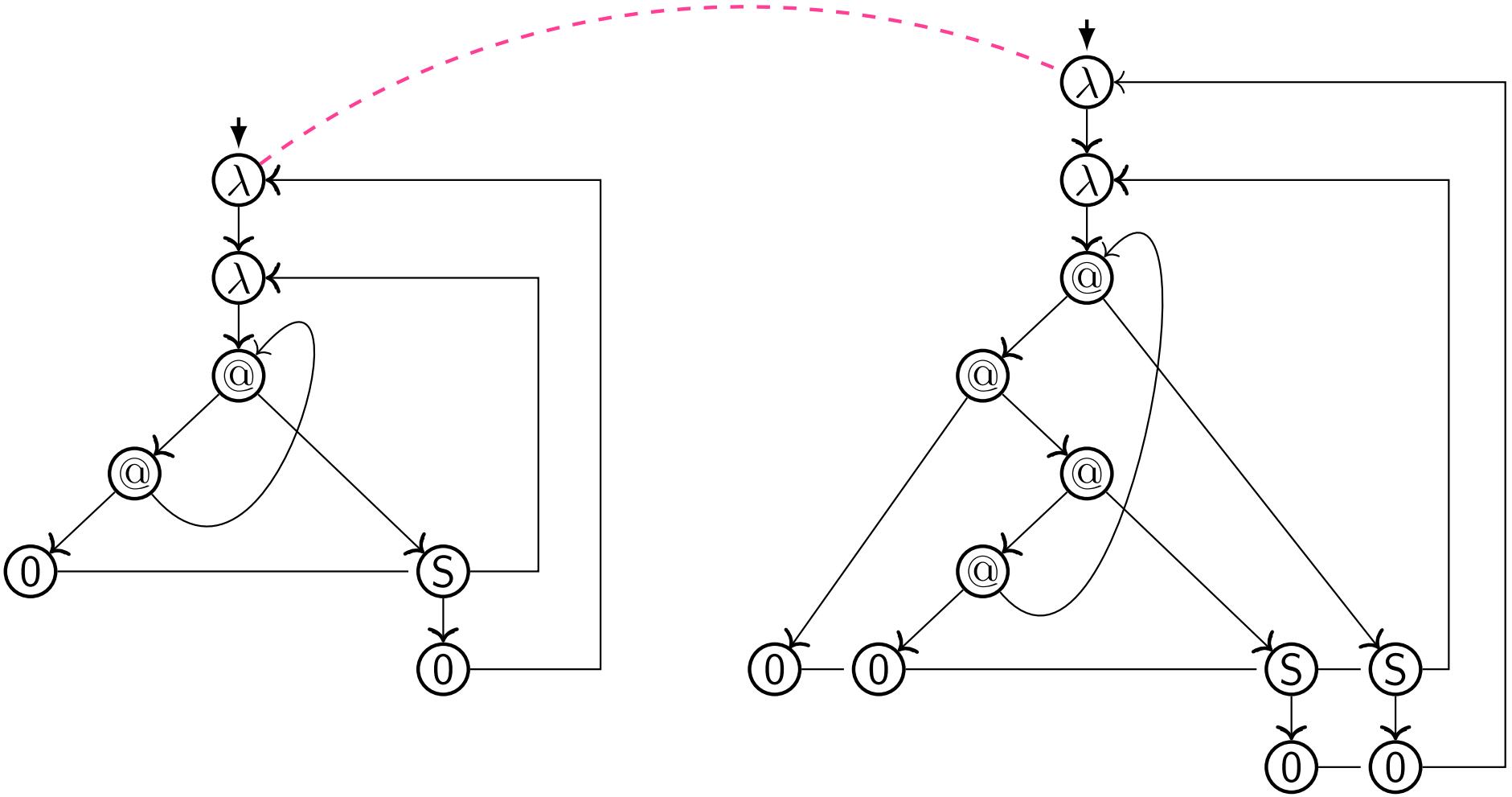




$[\![L_0]\!]\tau$

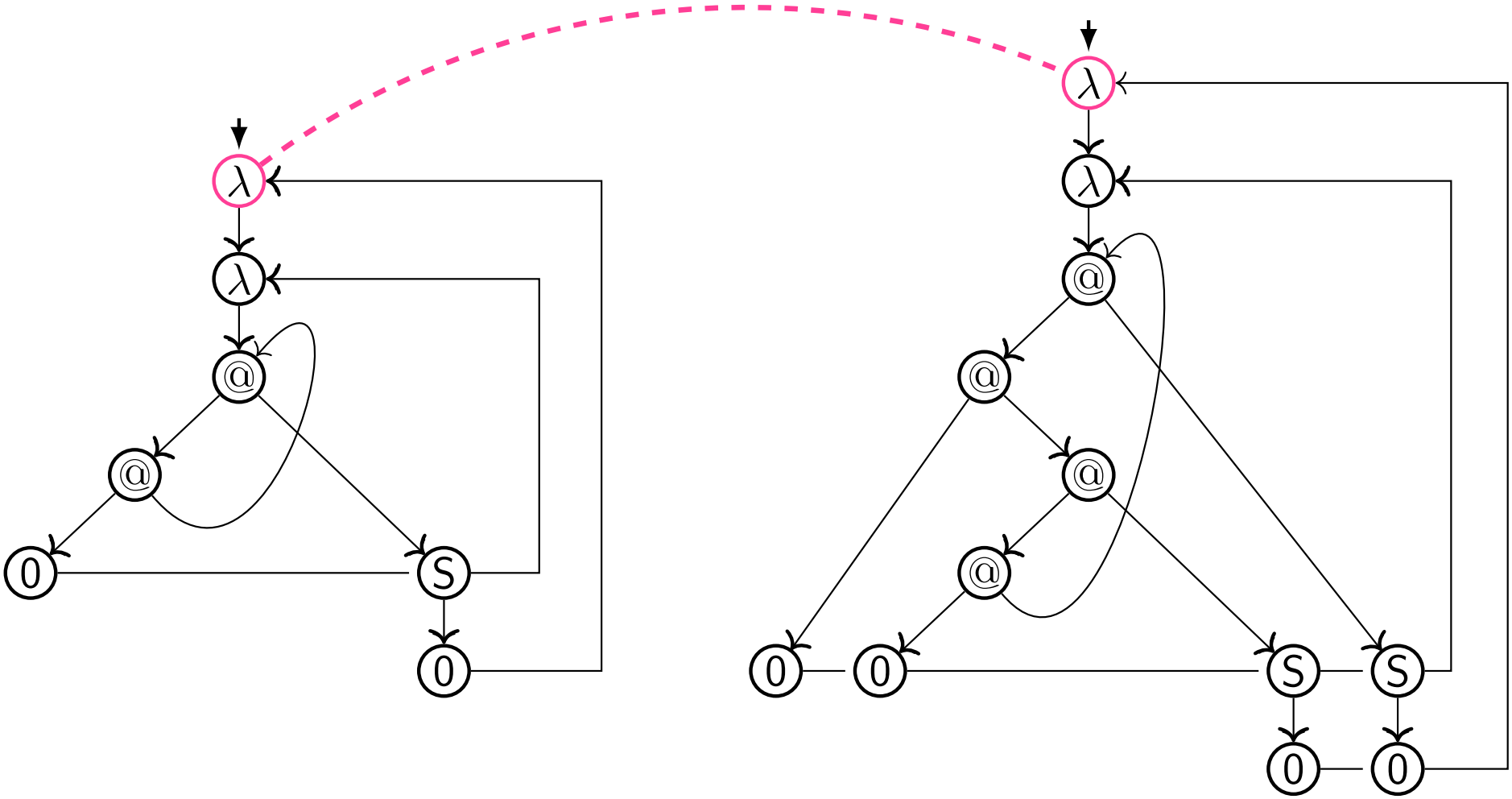
$[\![L]\!]\tau$

bisimulation check between λ-term-graphs



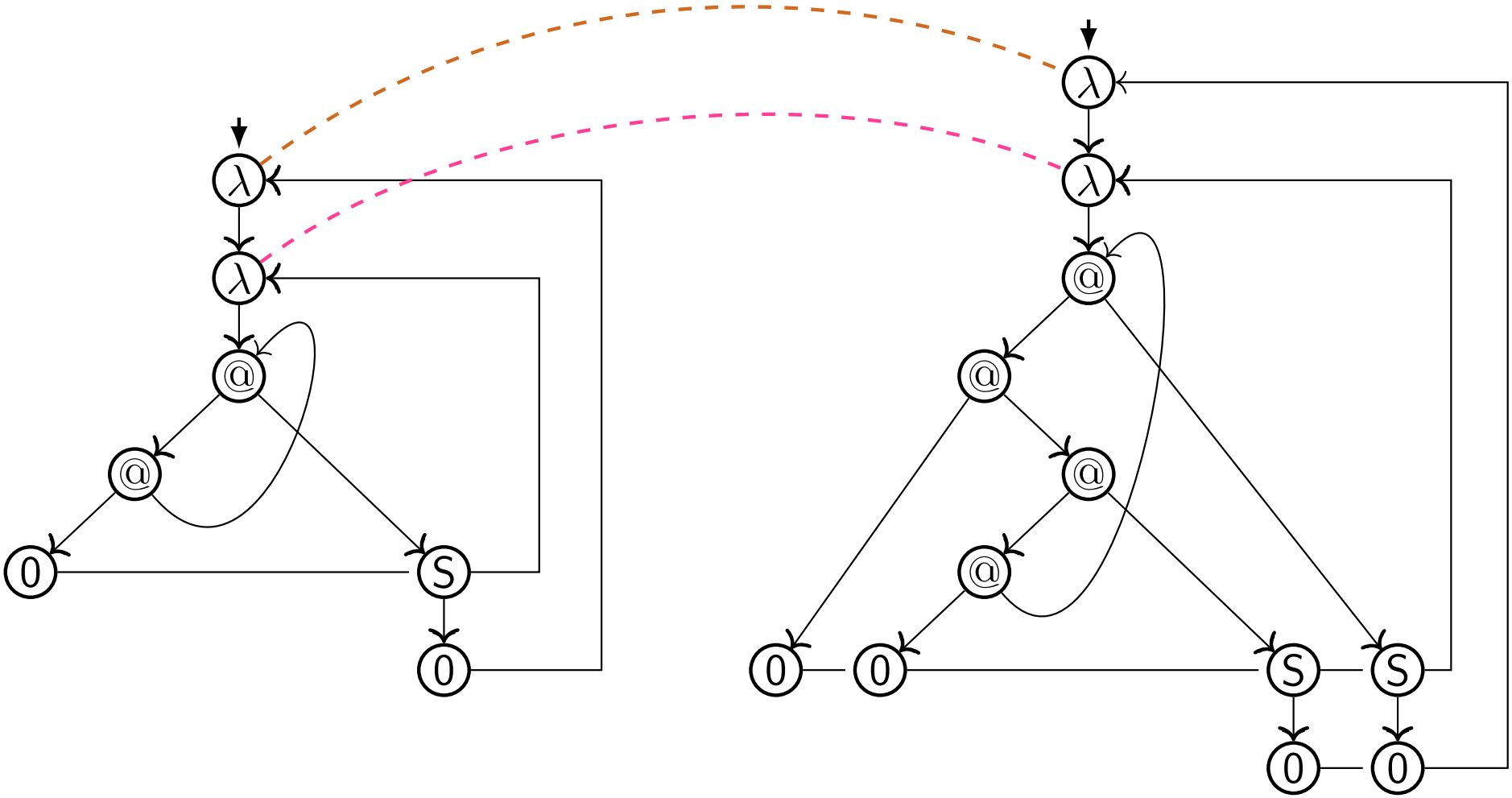
$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



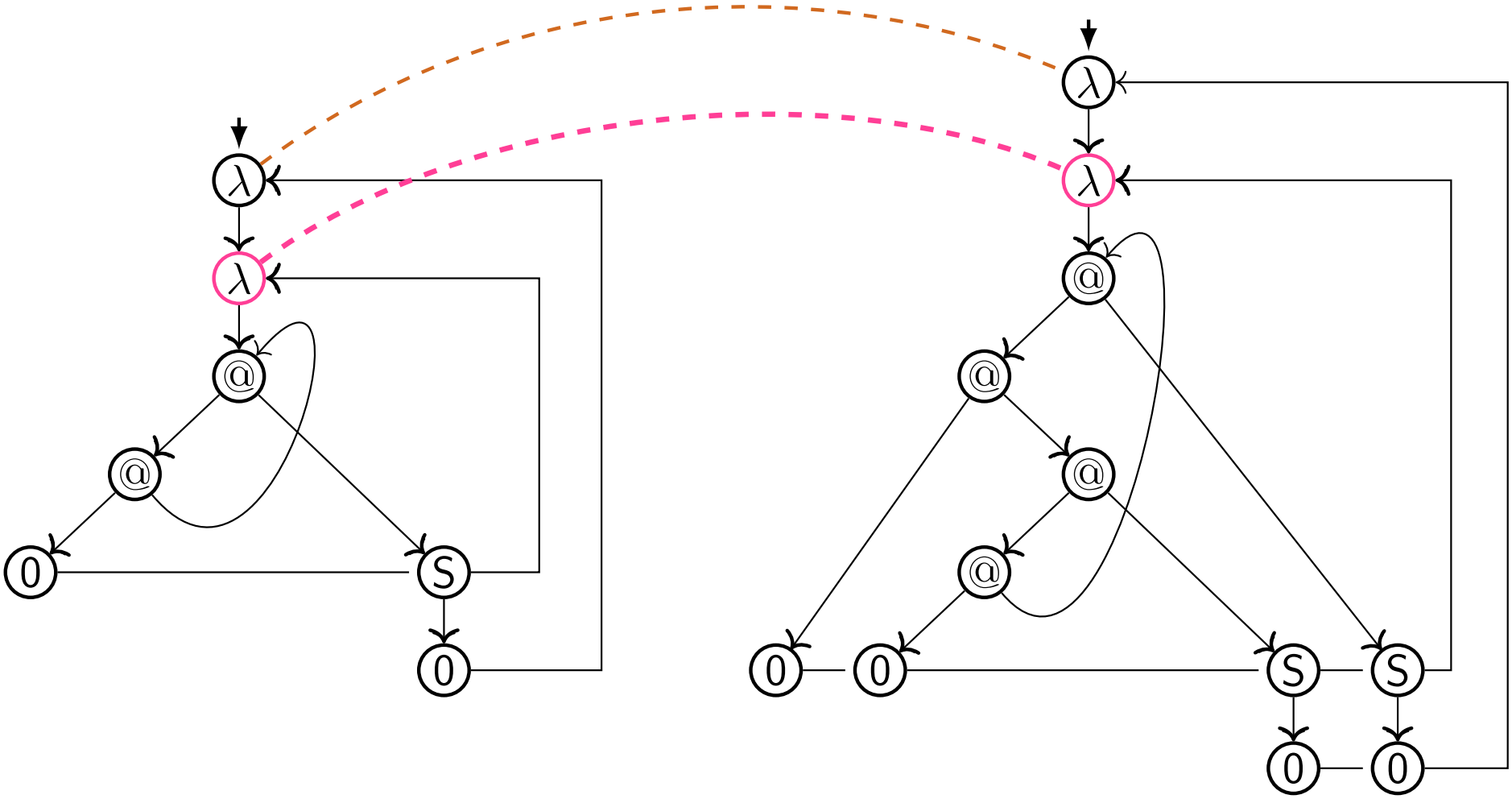
$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



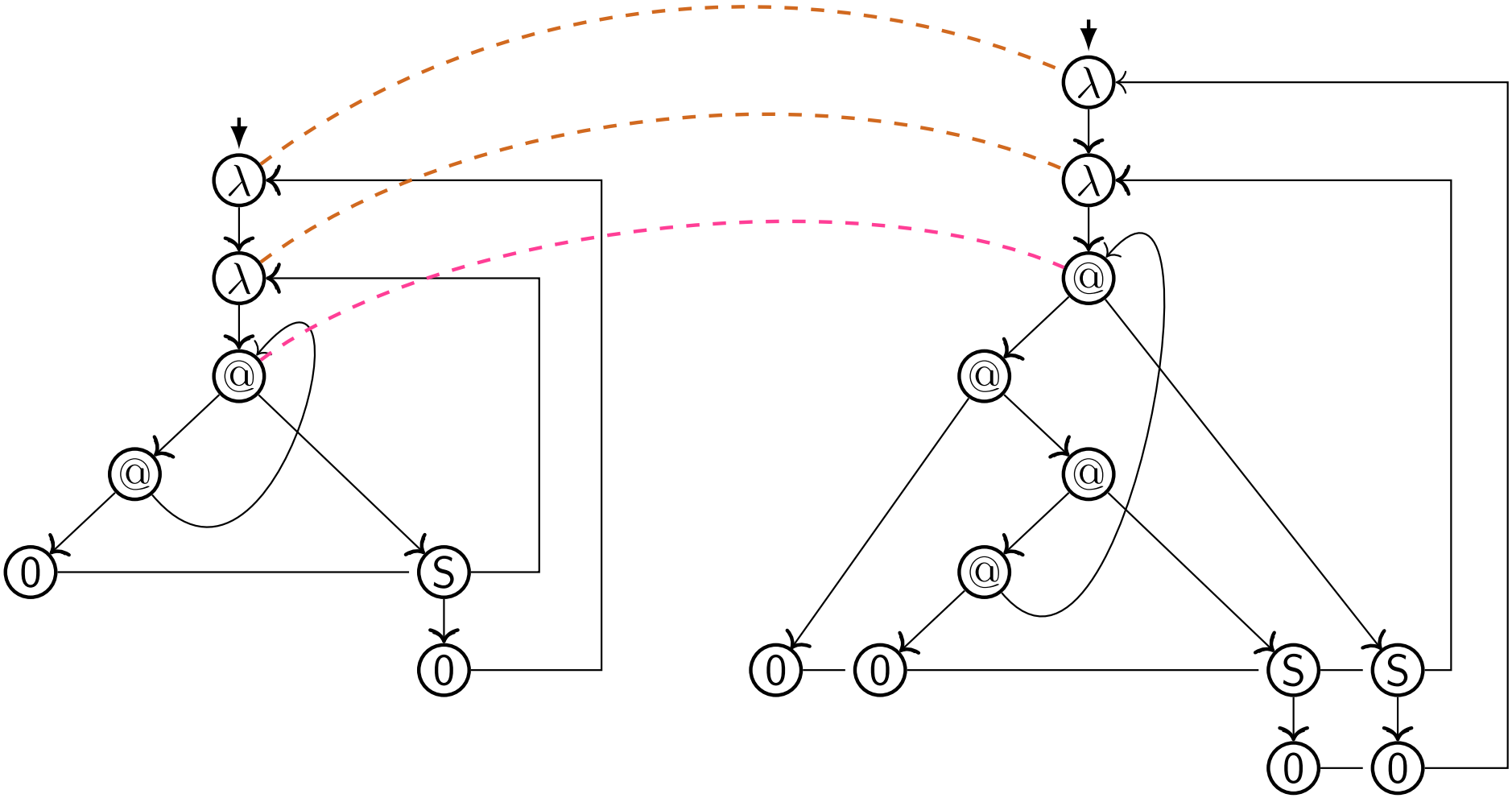
$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



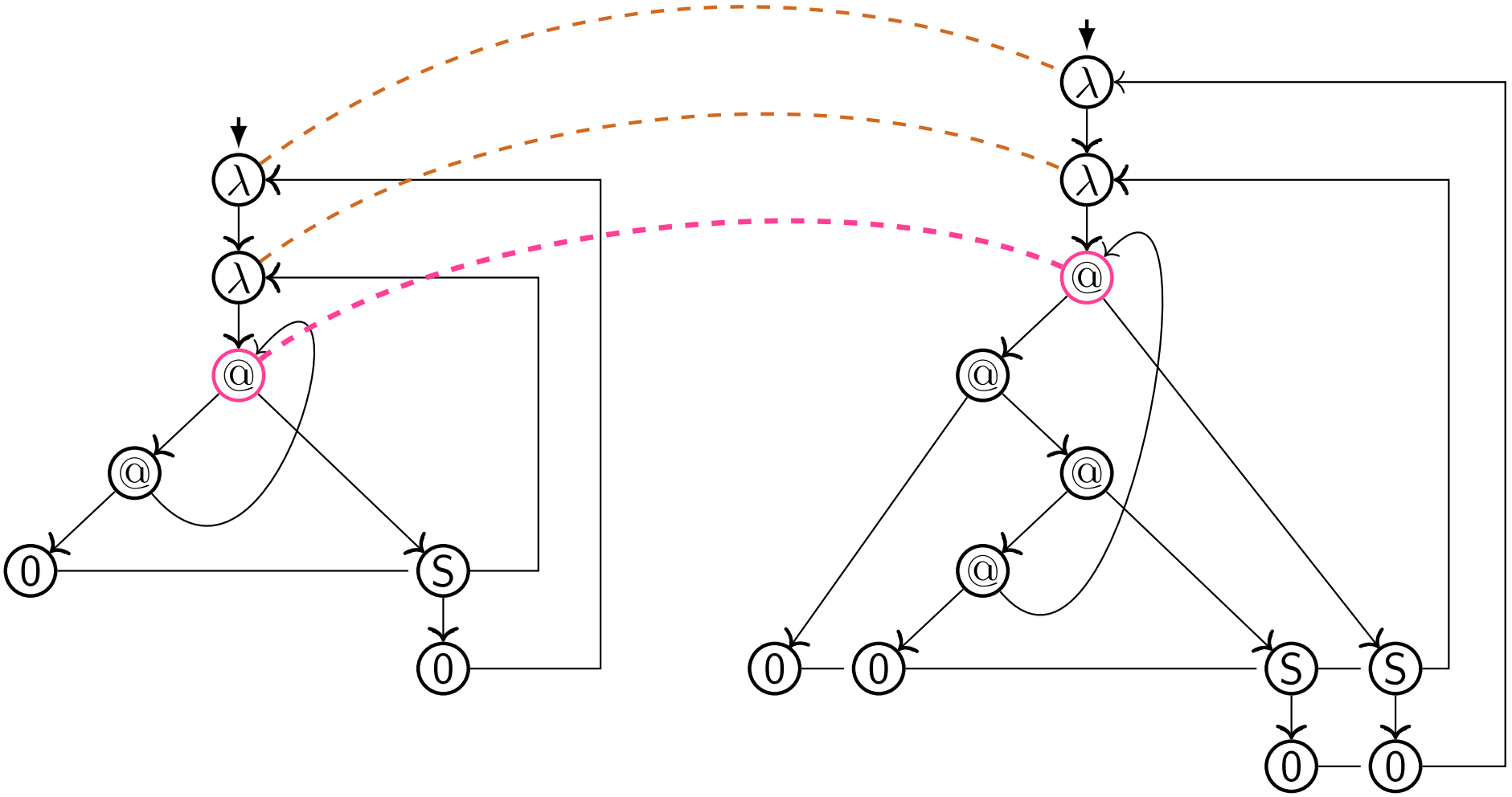
$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



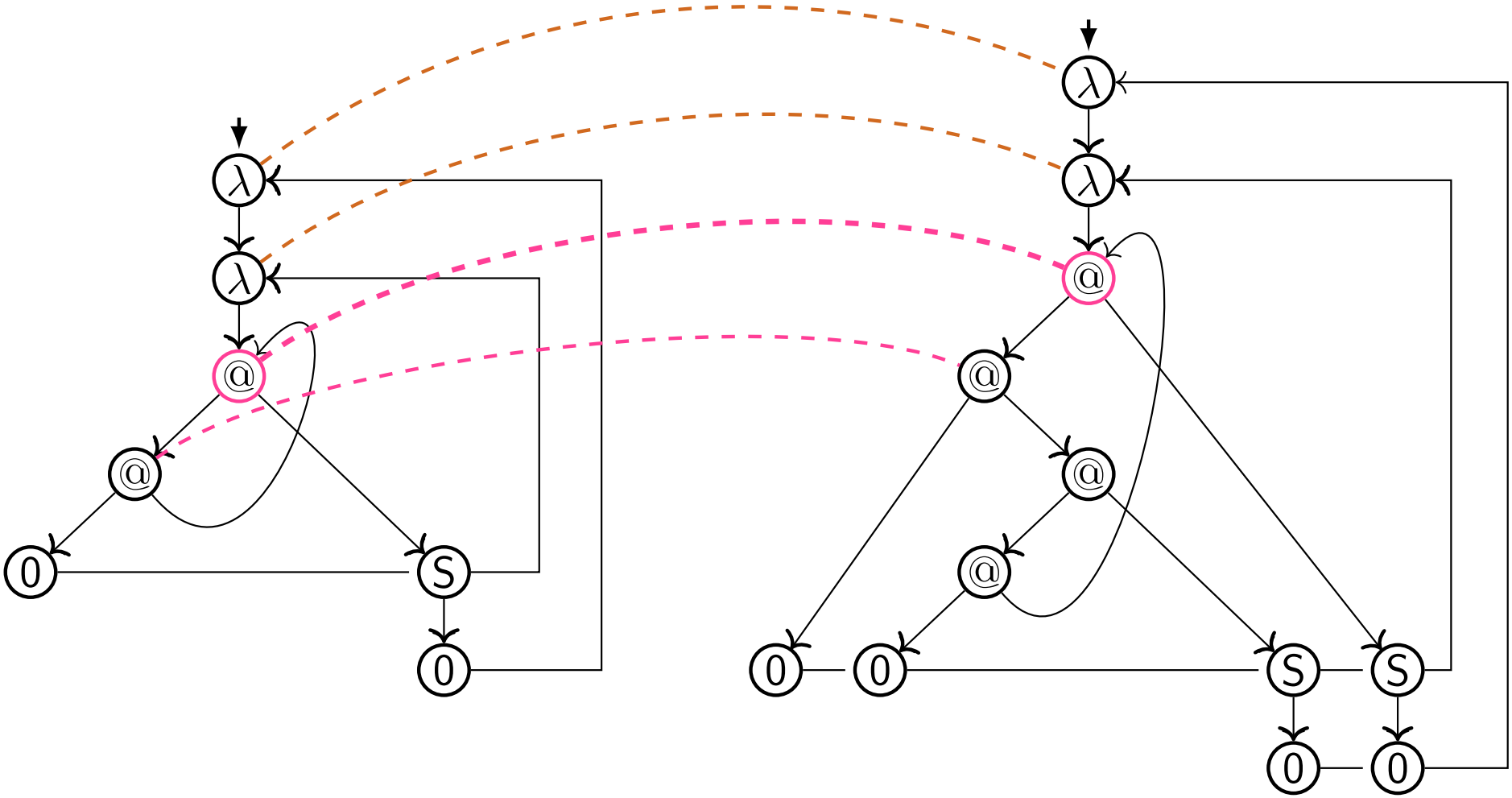
$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



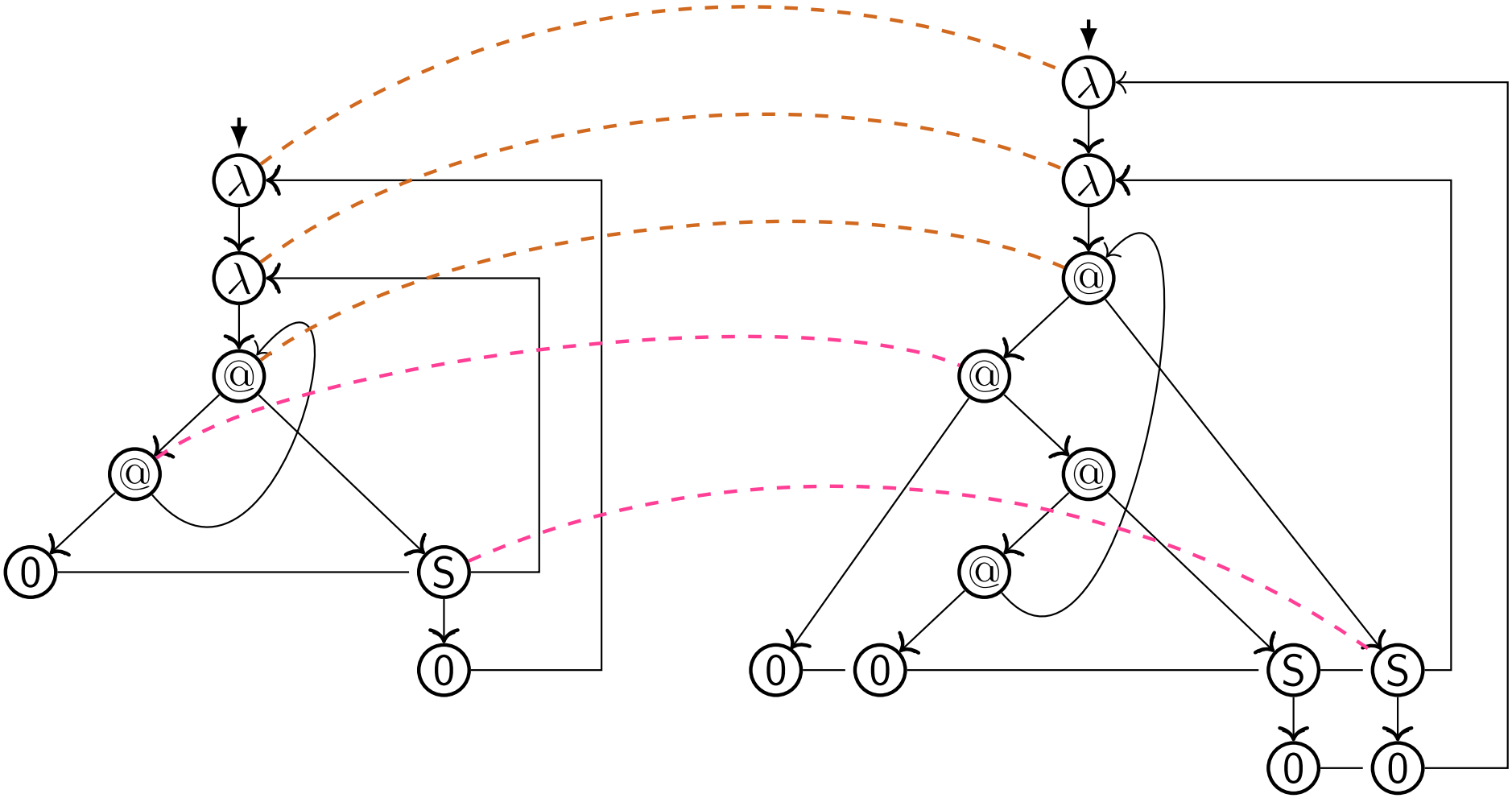
$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



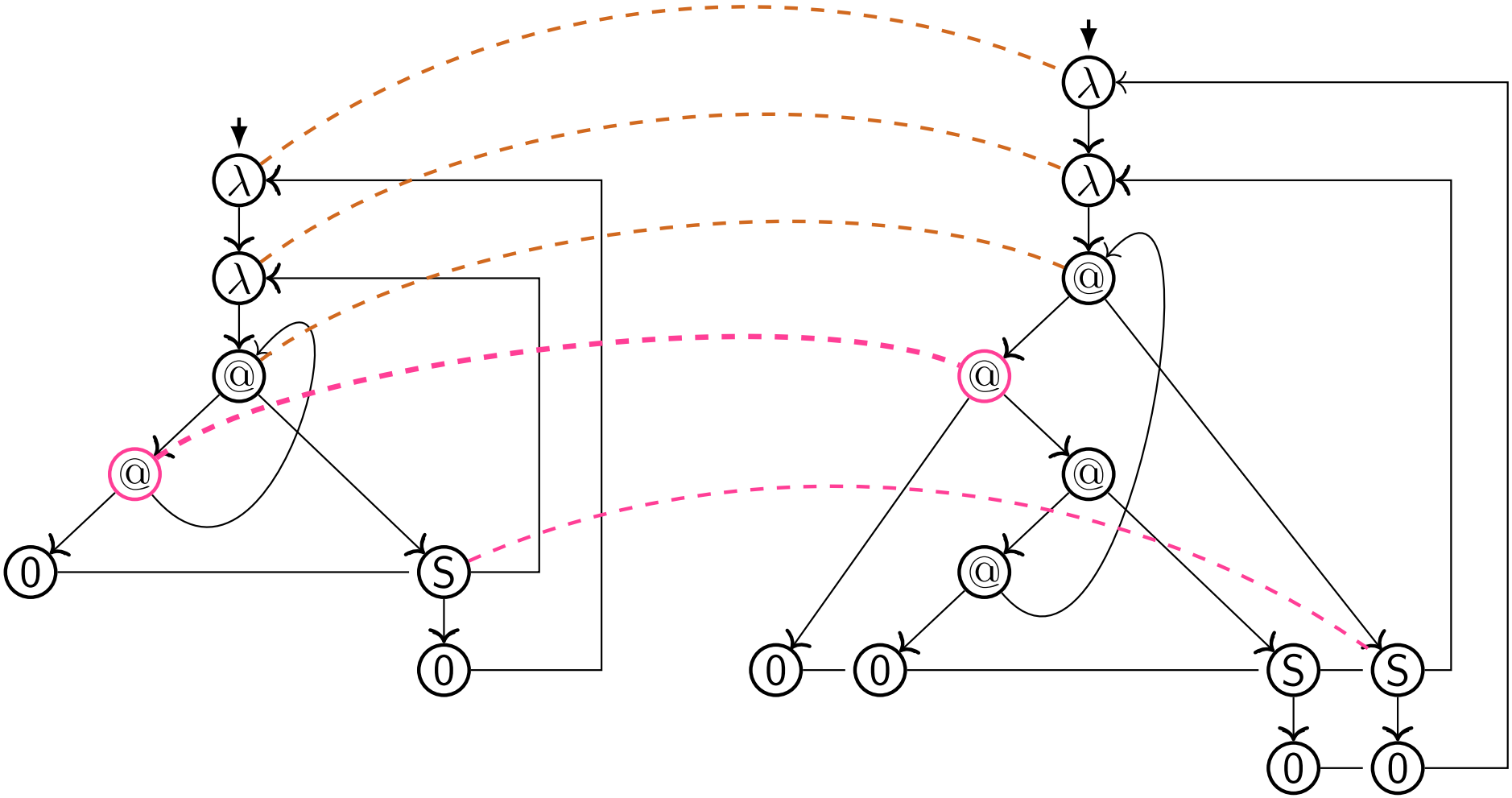
$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



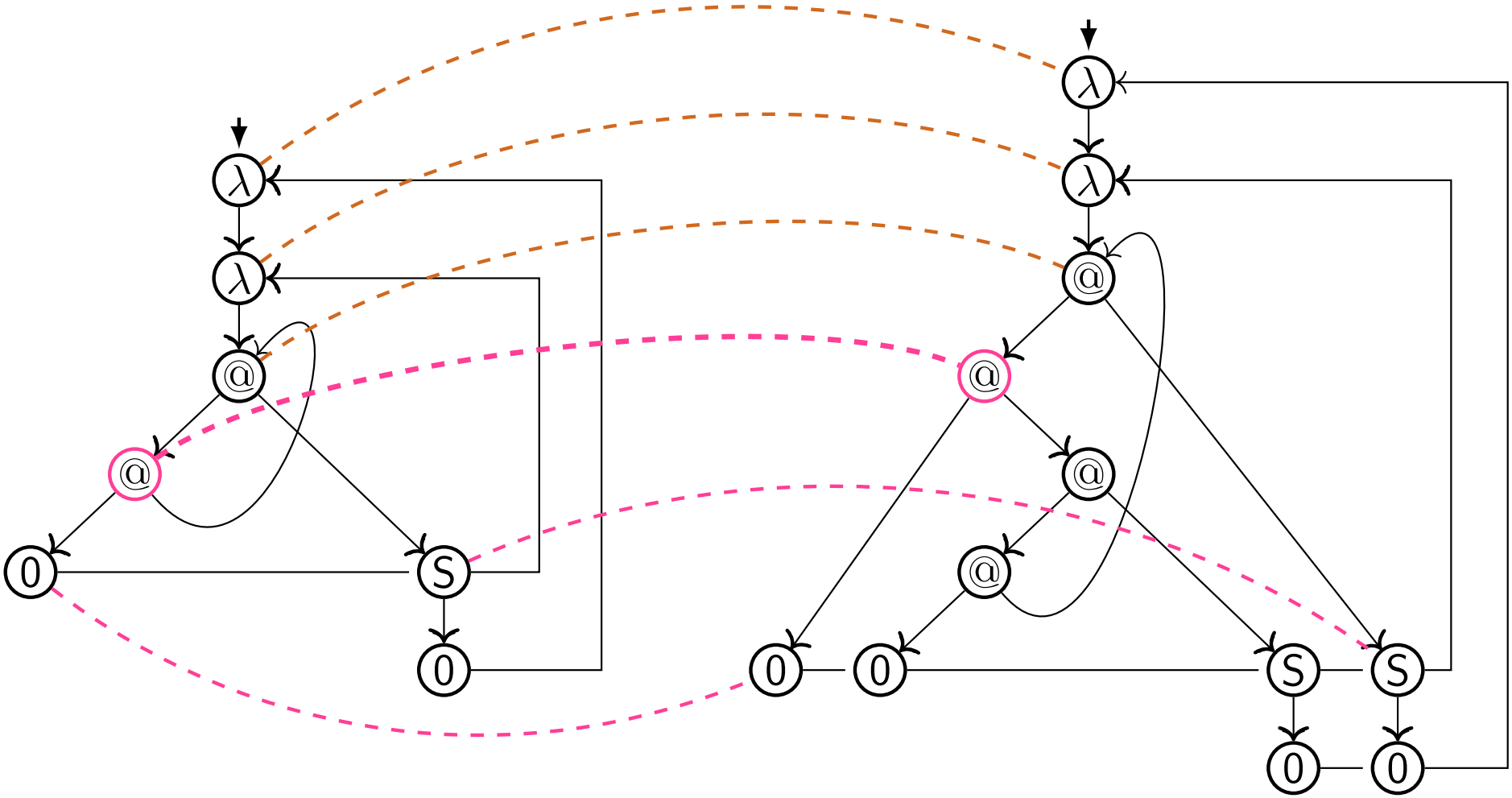
$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



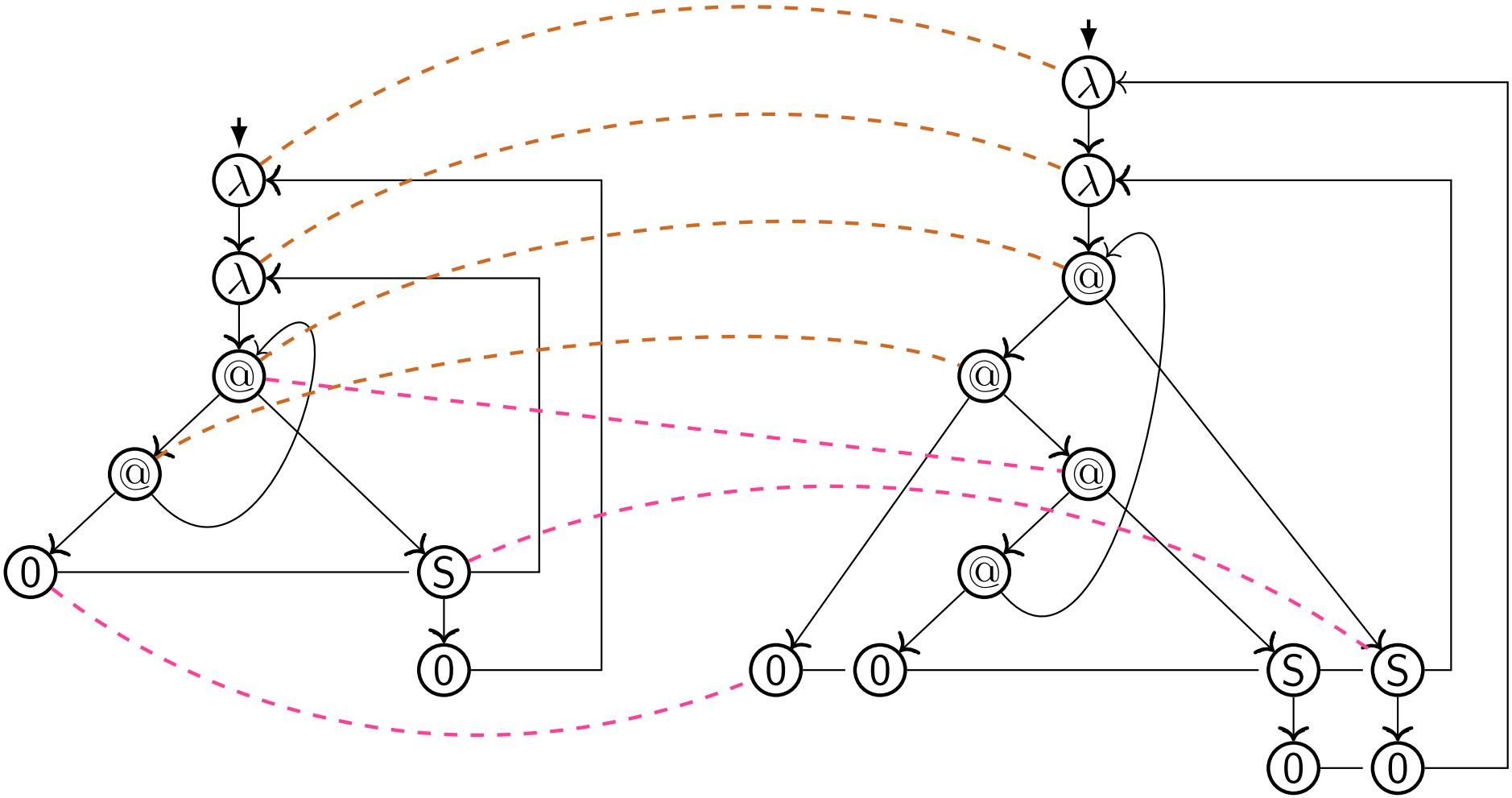
$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



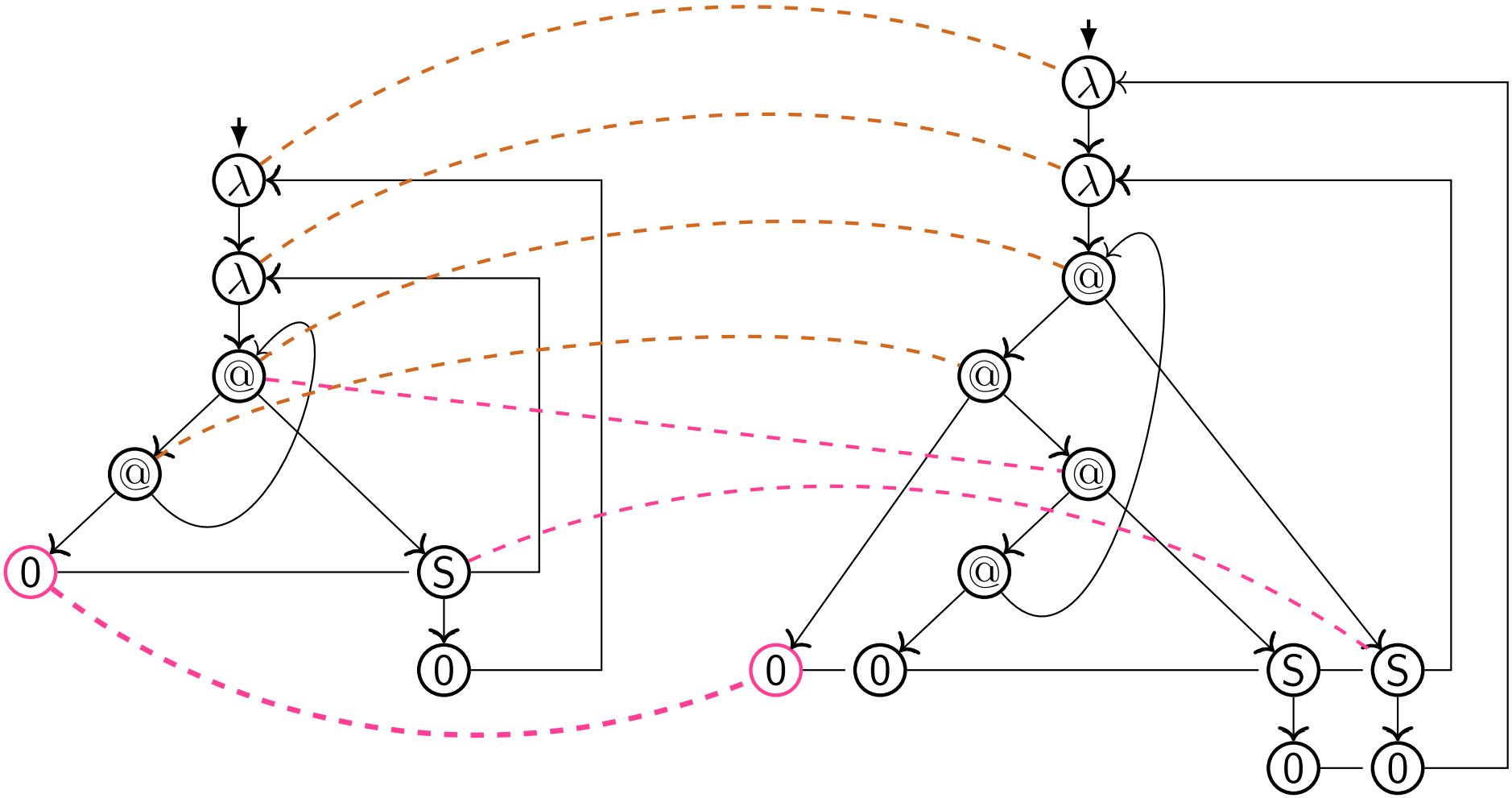
$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



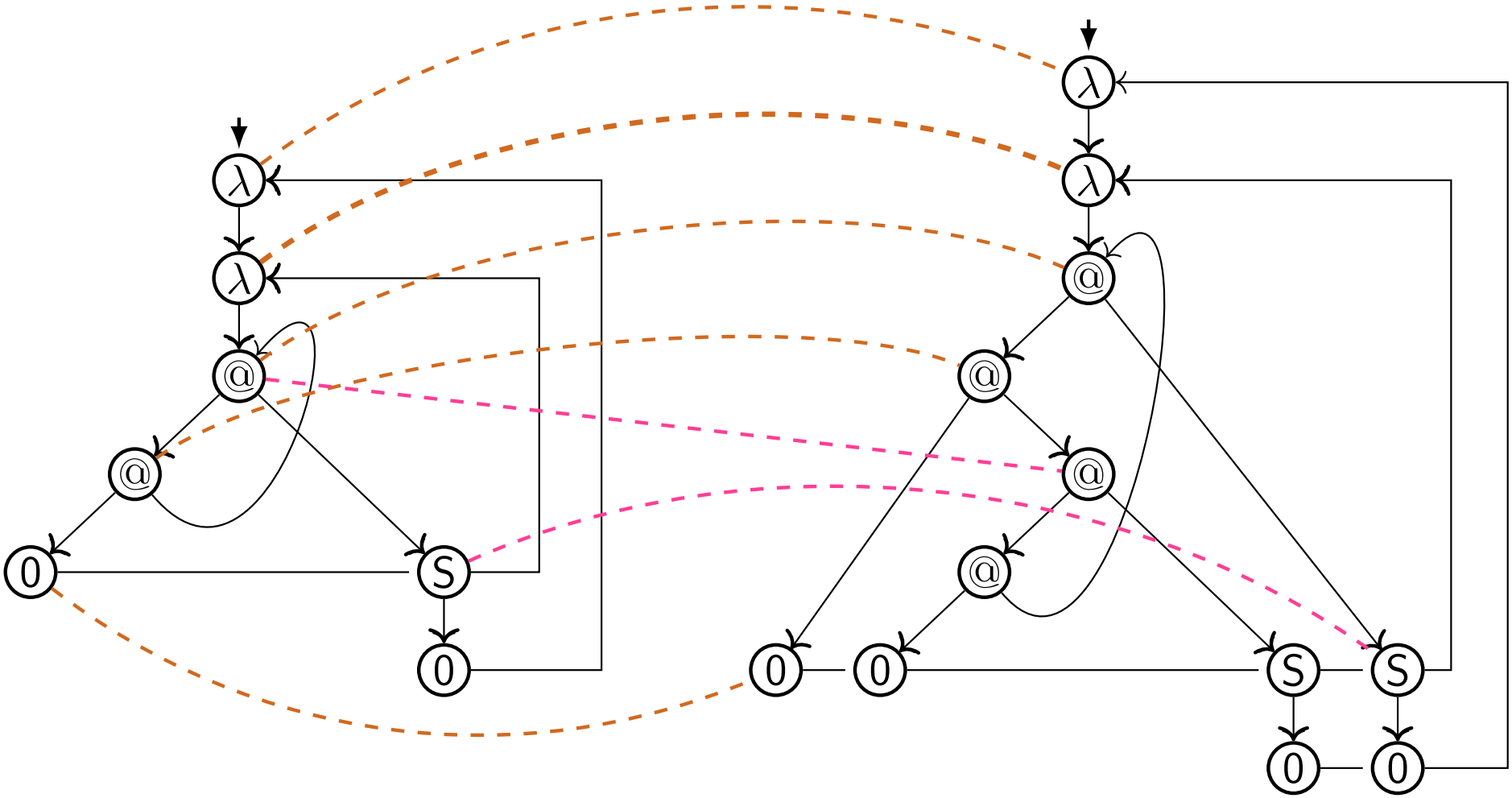
$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



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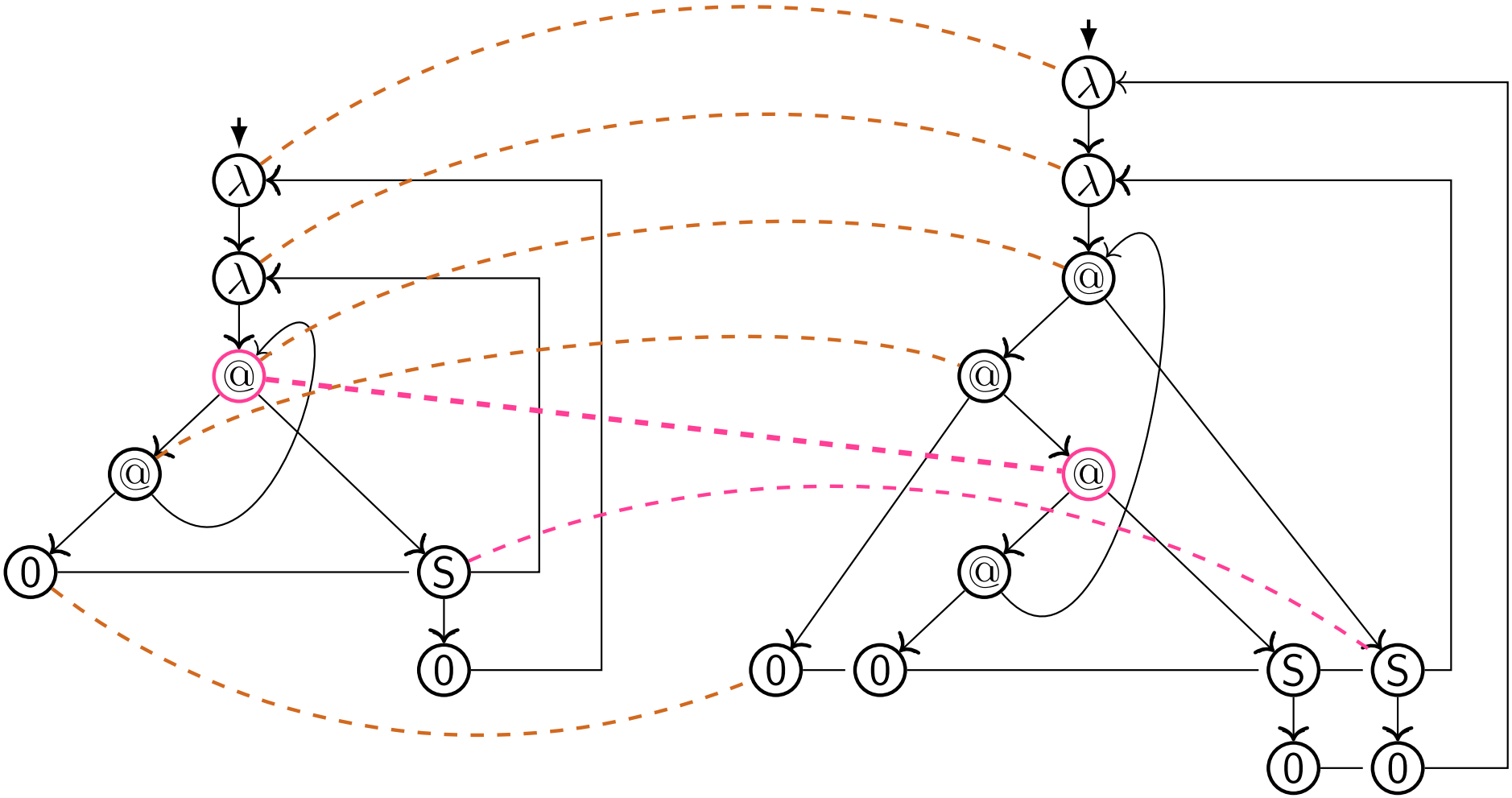
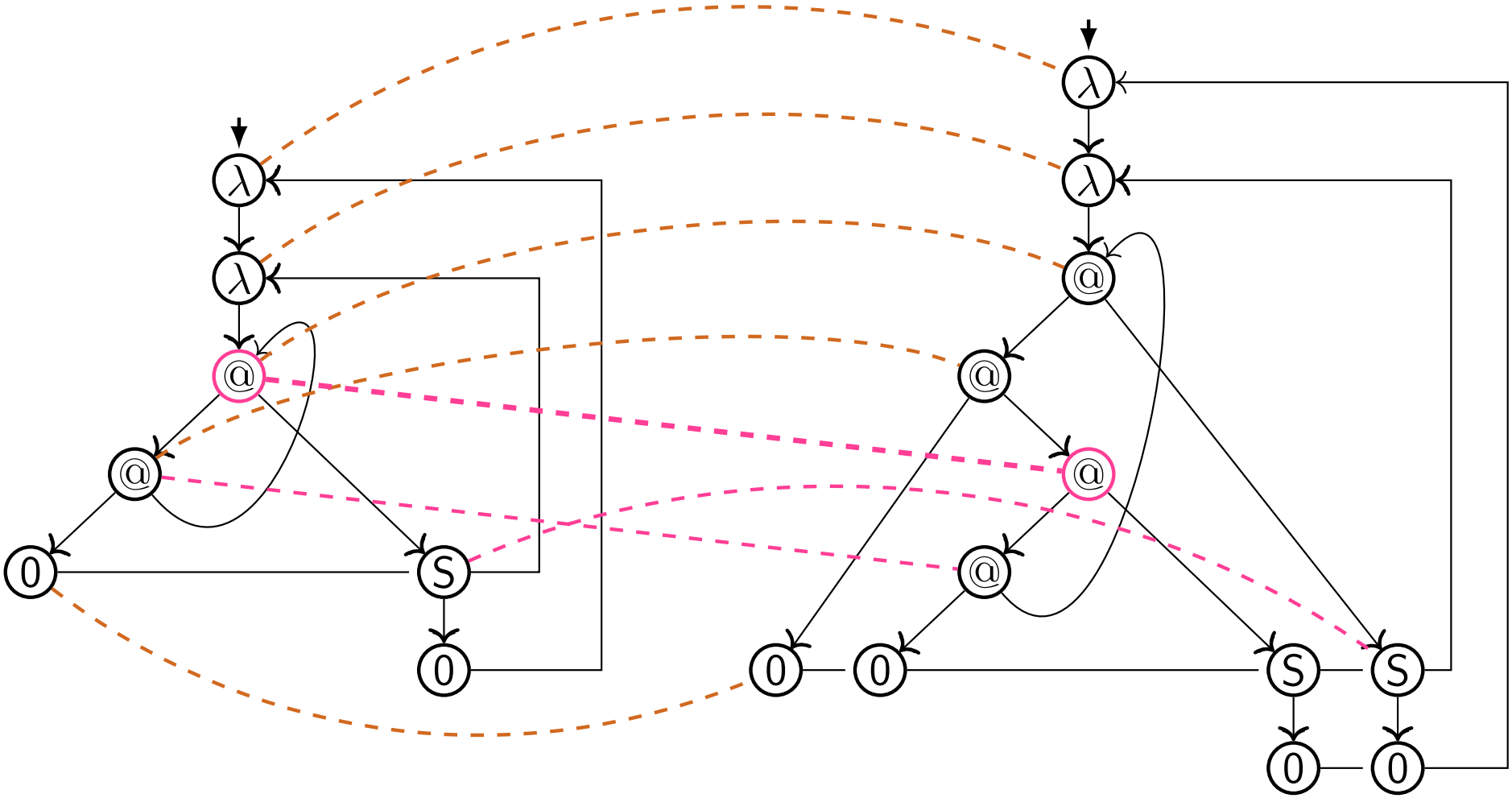


$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



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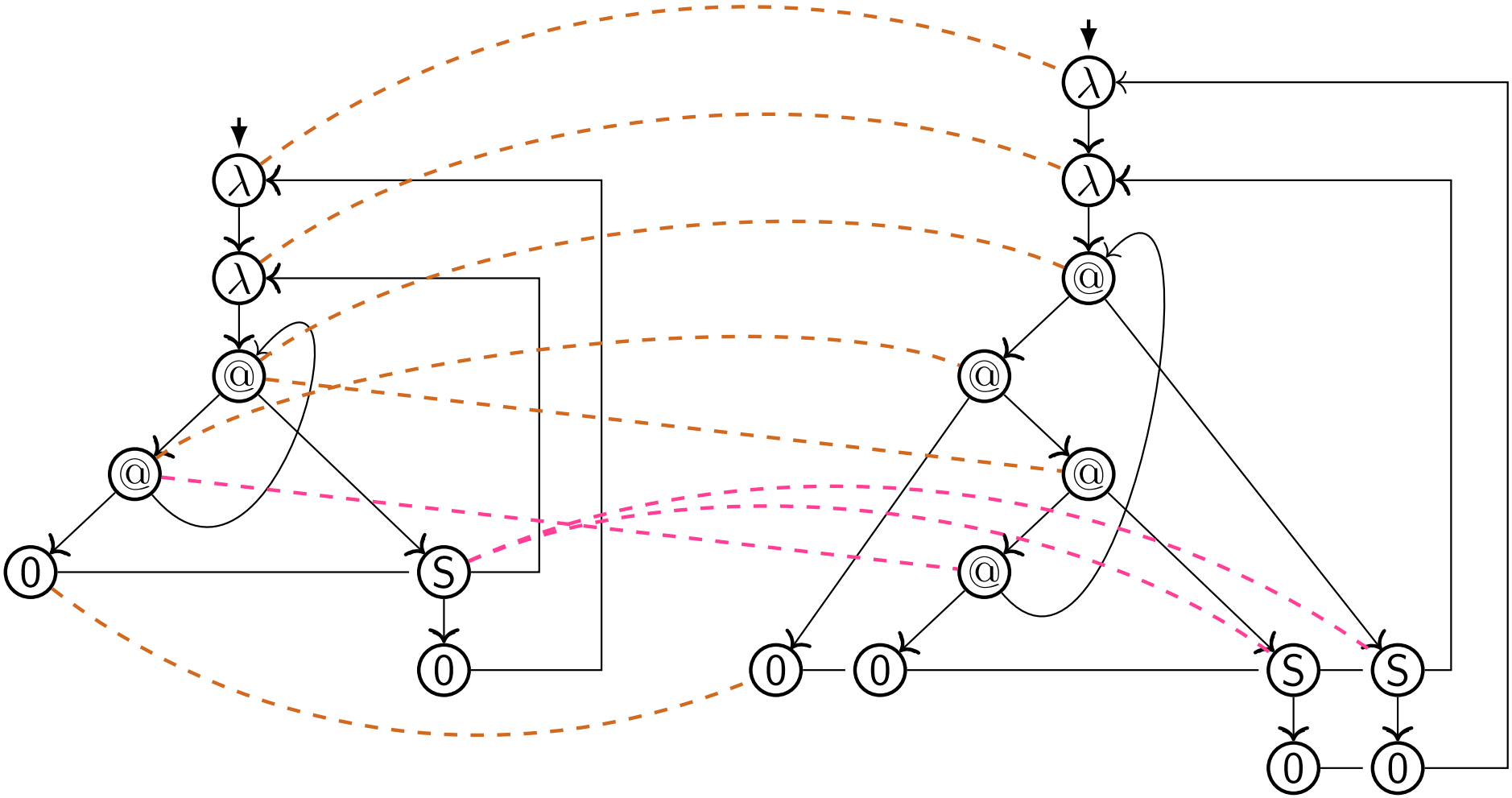
bisimulation check between λ-term-graphs



$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

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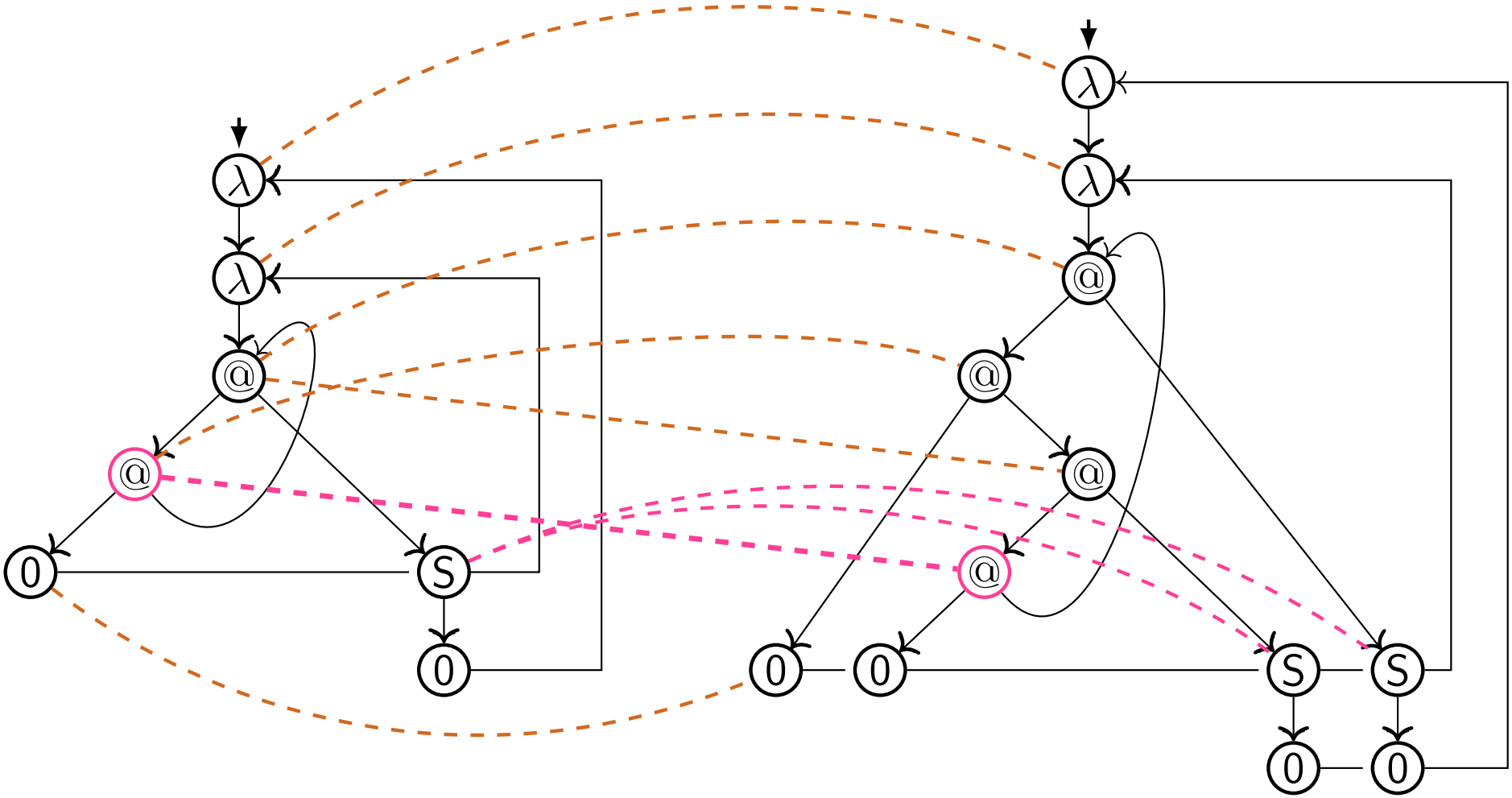

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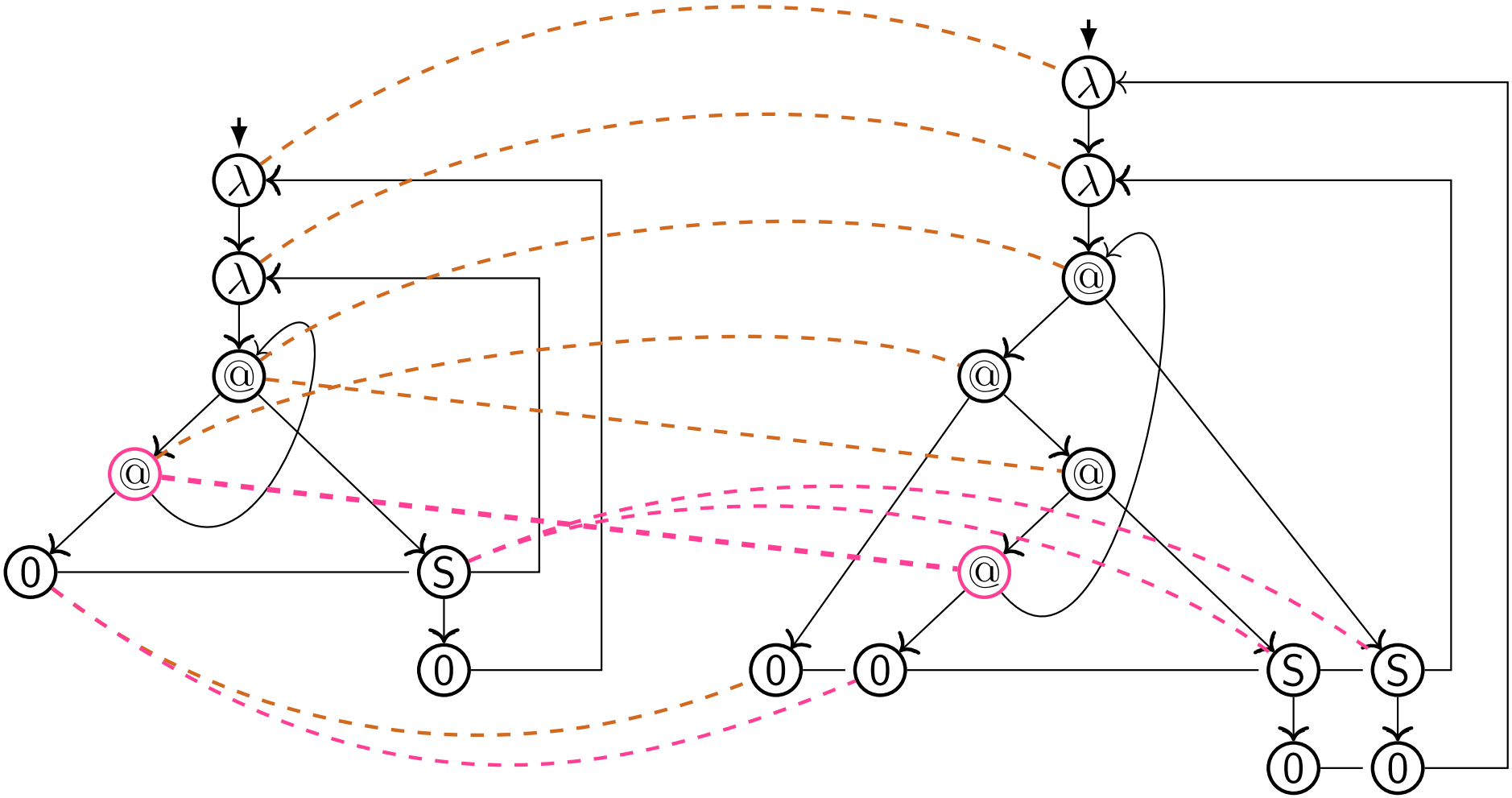
bisimulation check between λ-term-graphs



$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

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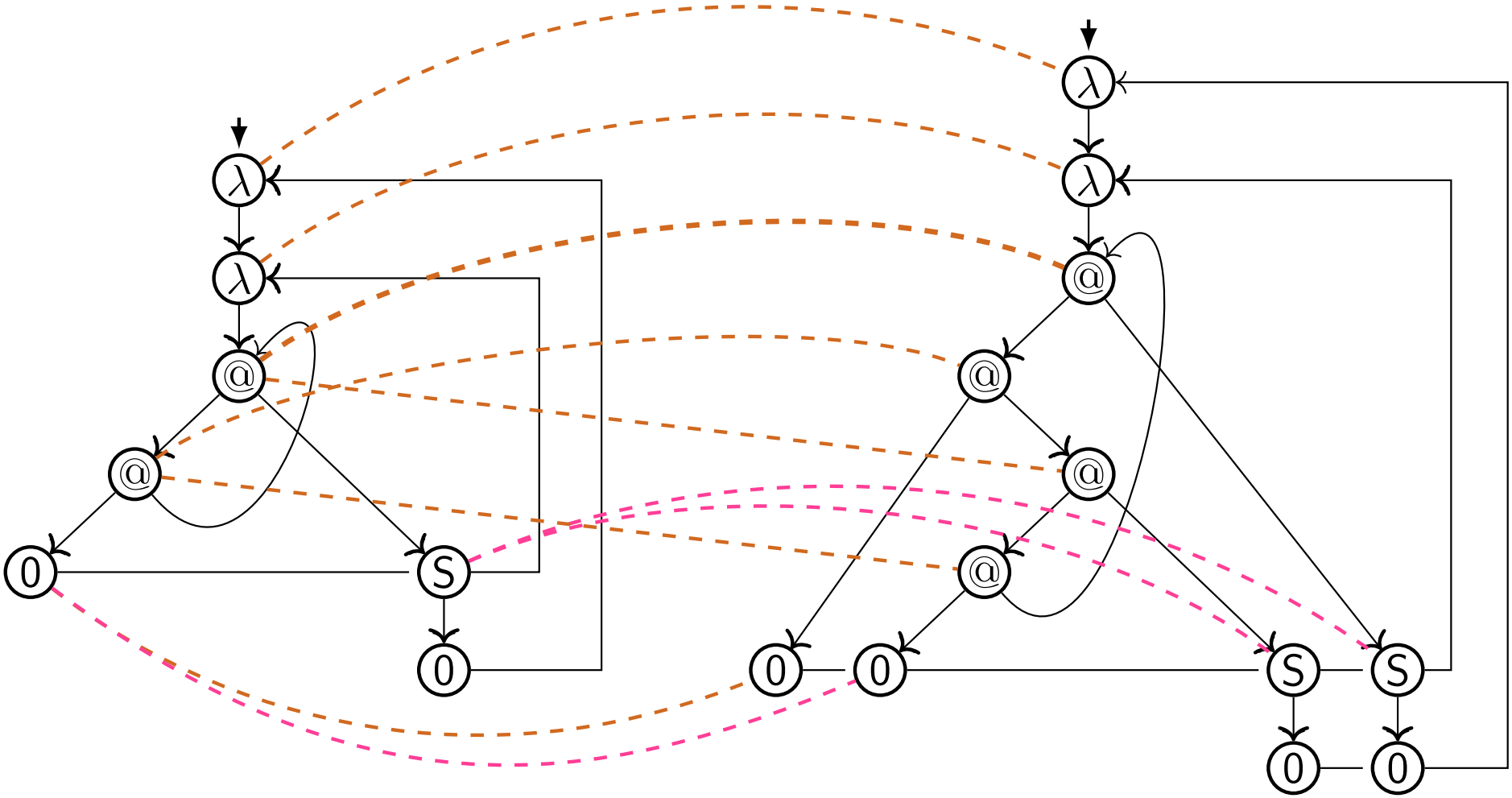

$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

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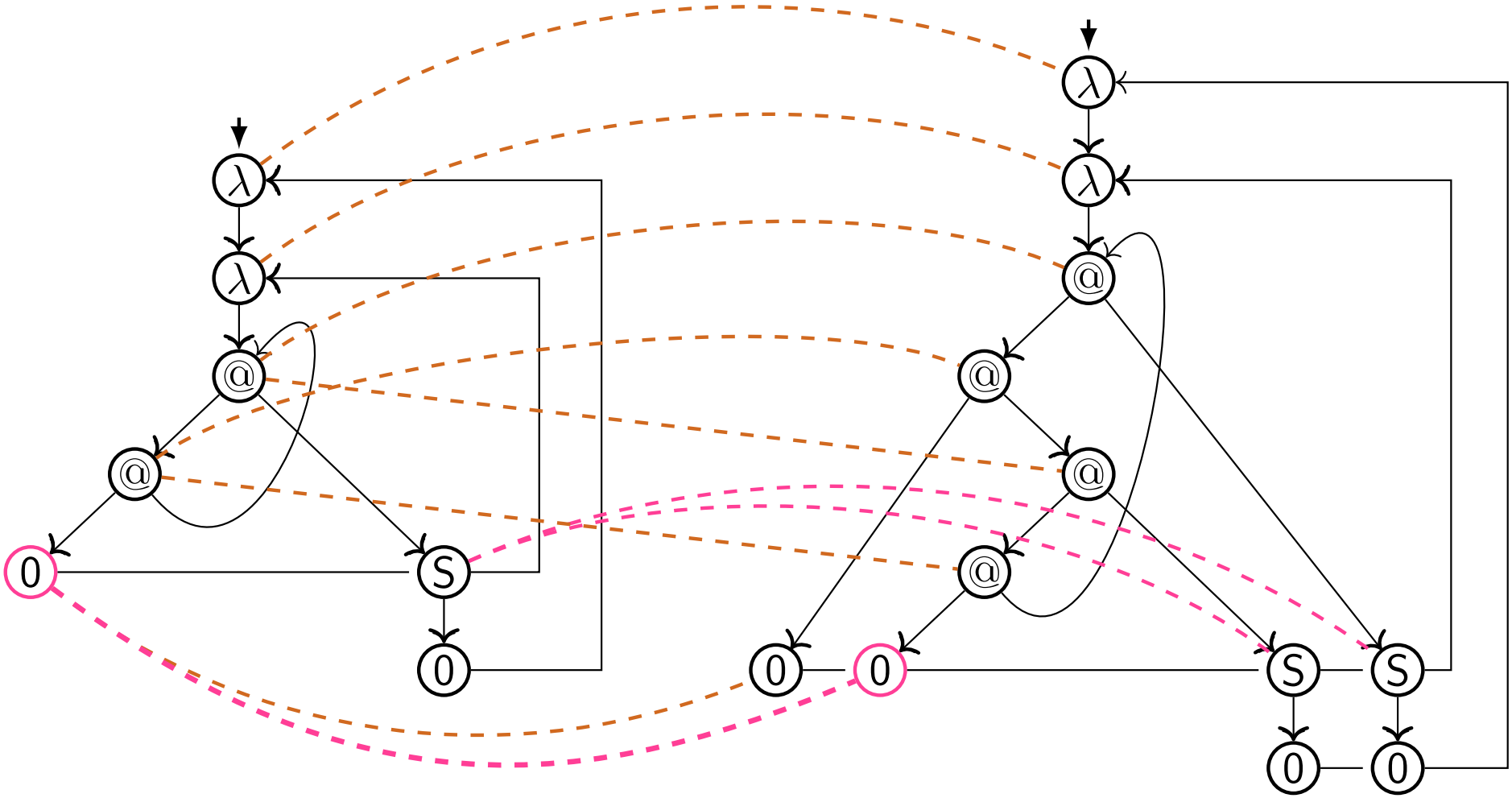
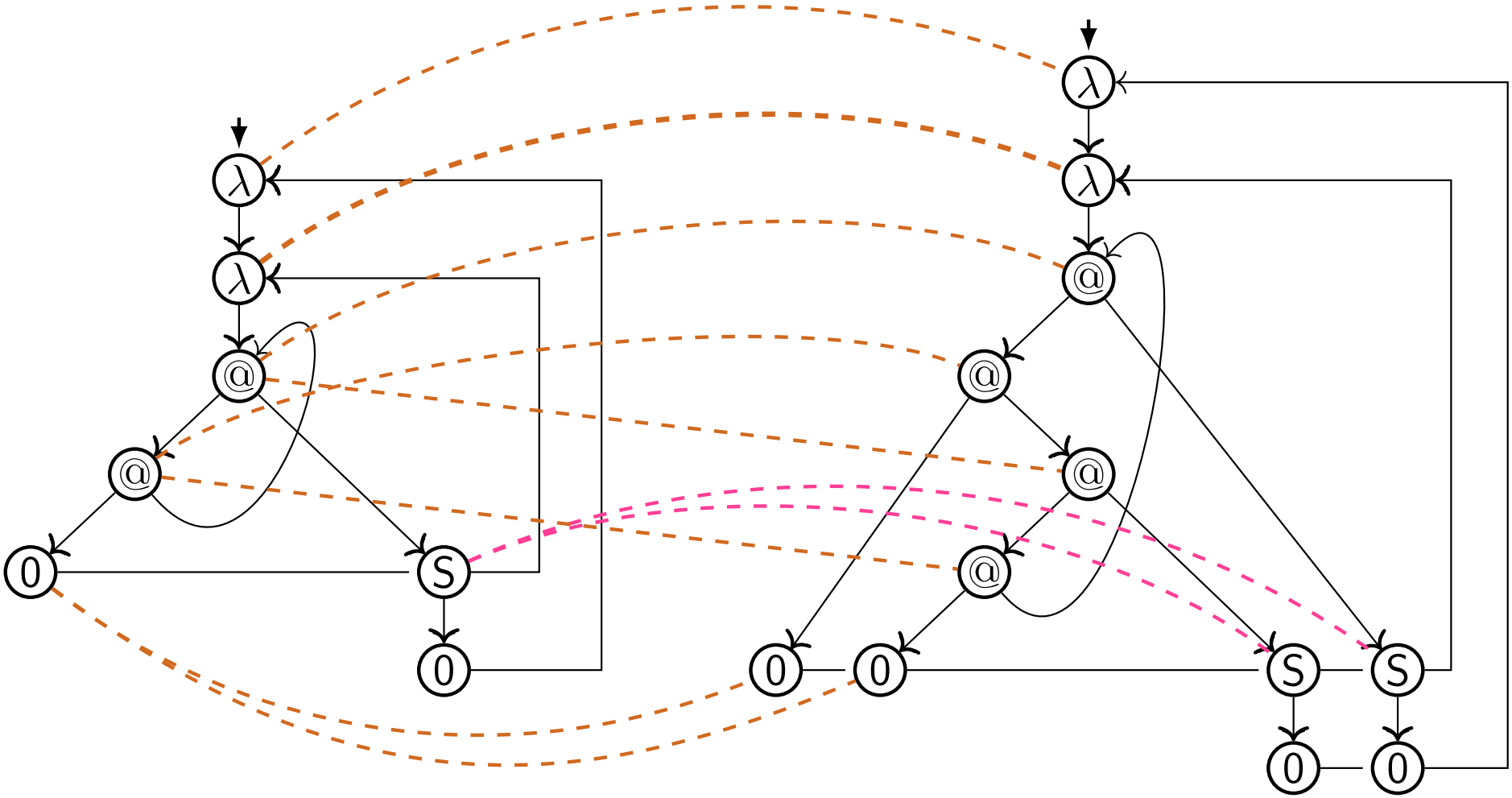


$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



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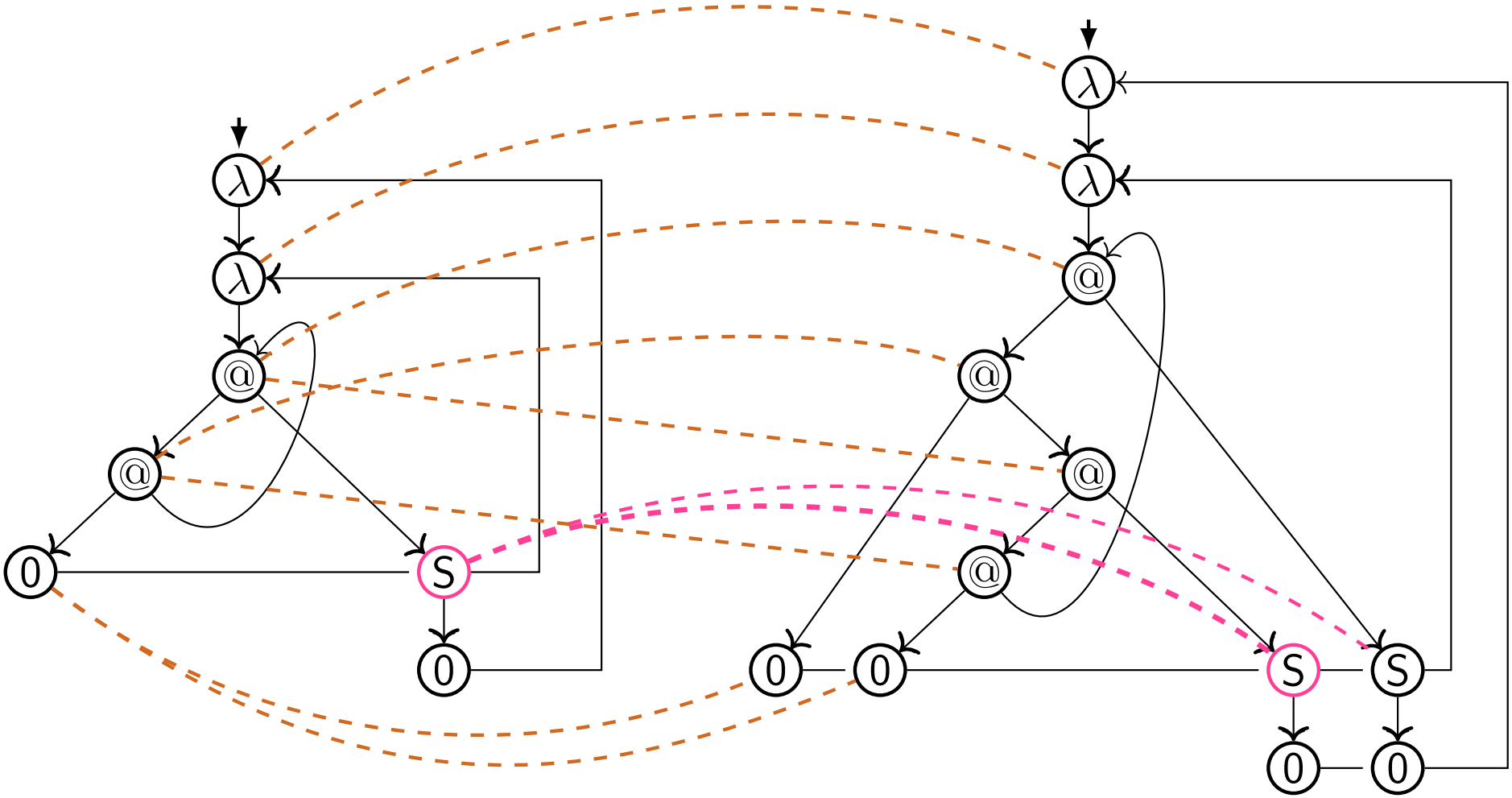
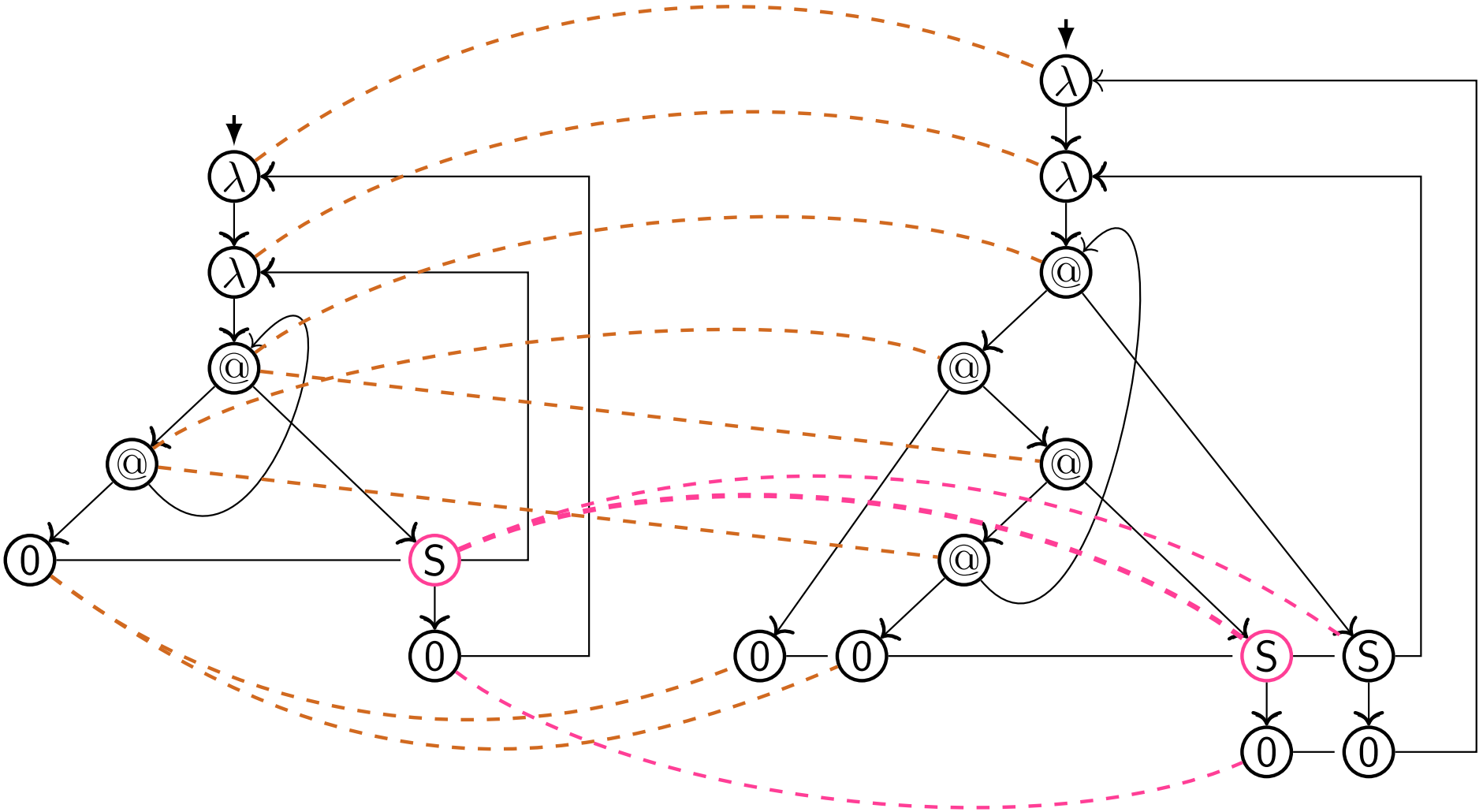


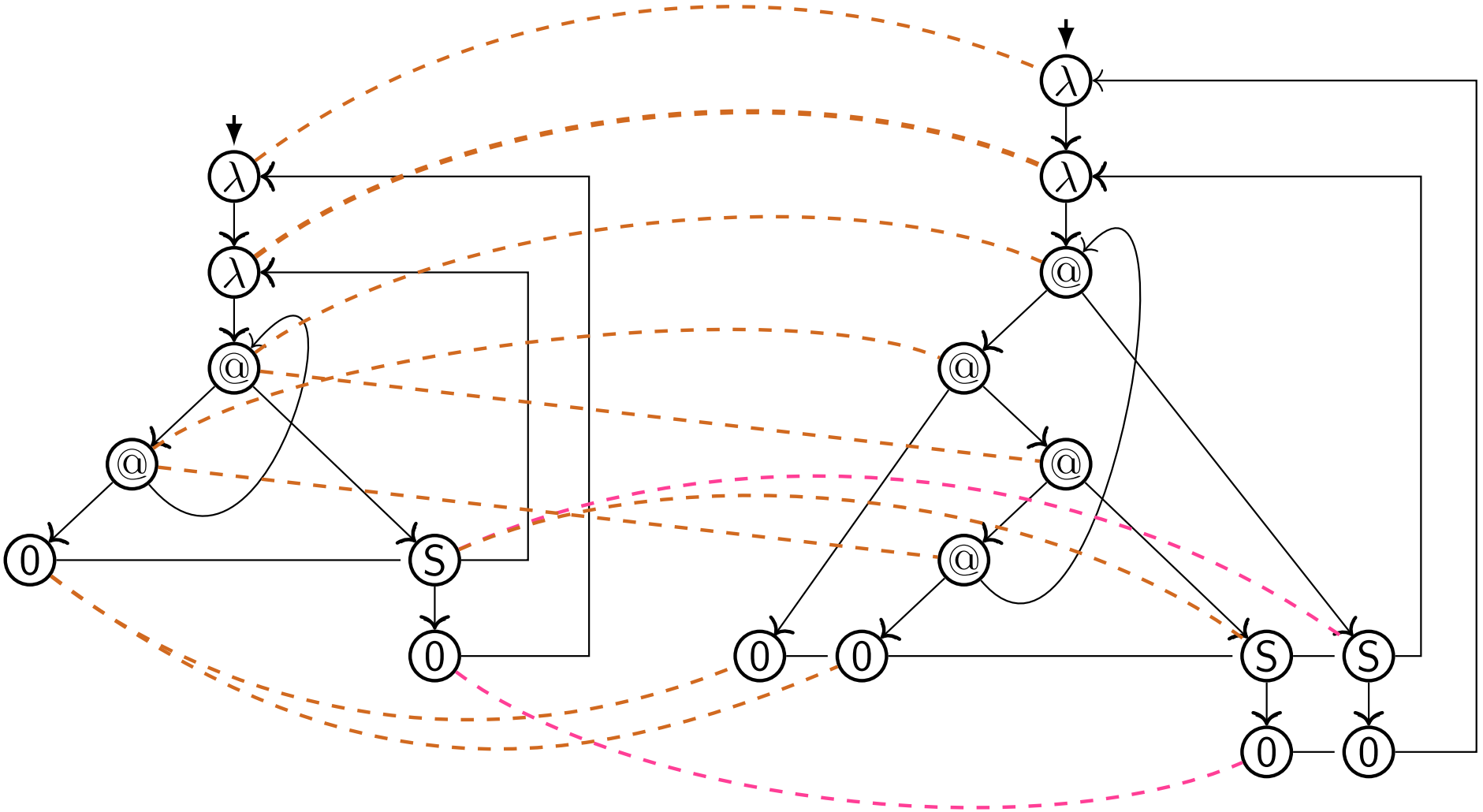
$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



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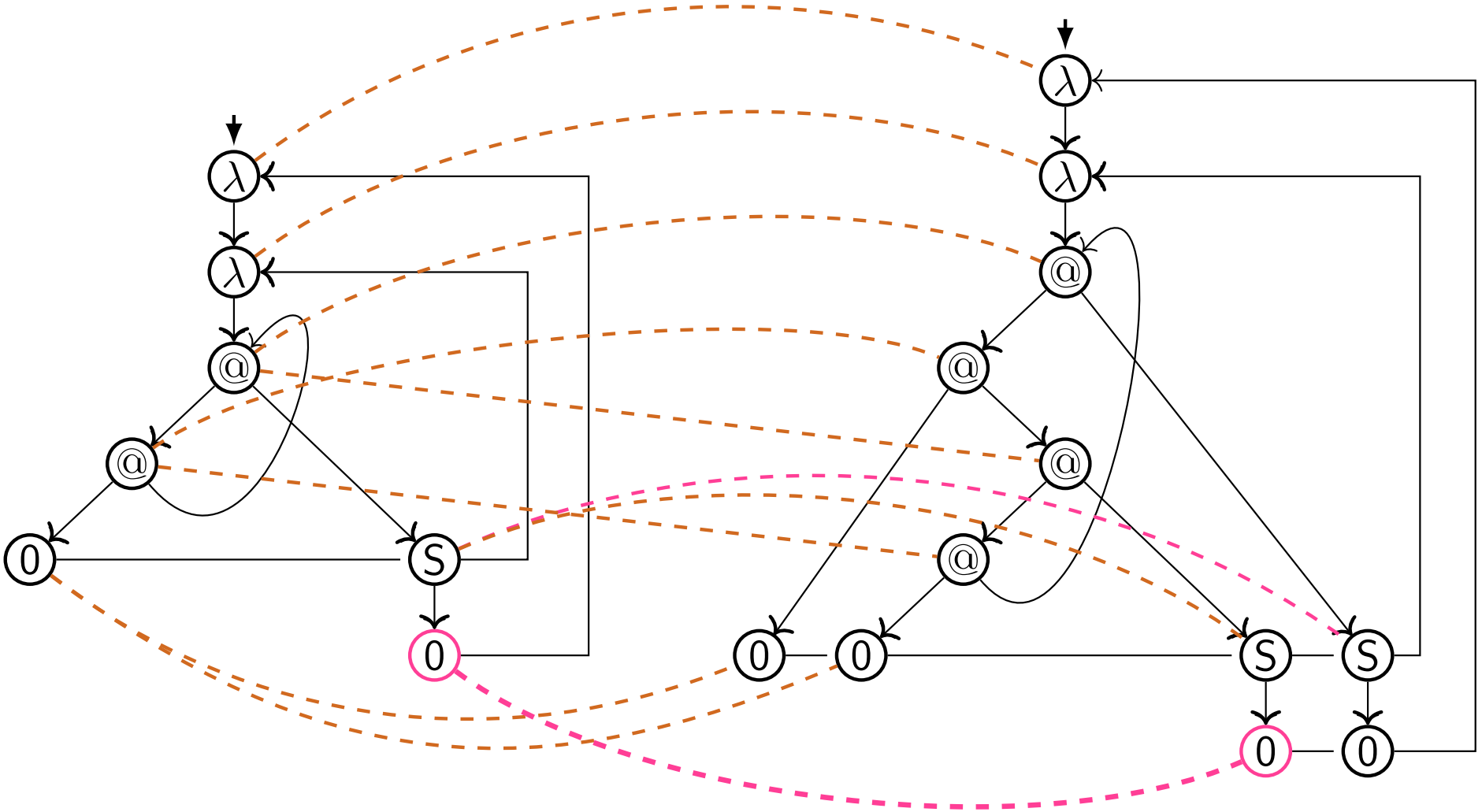
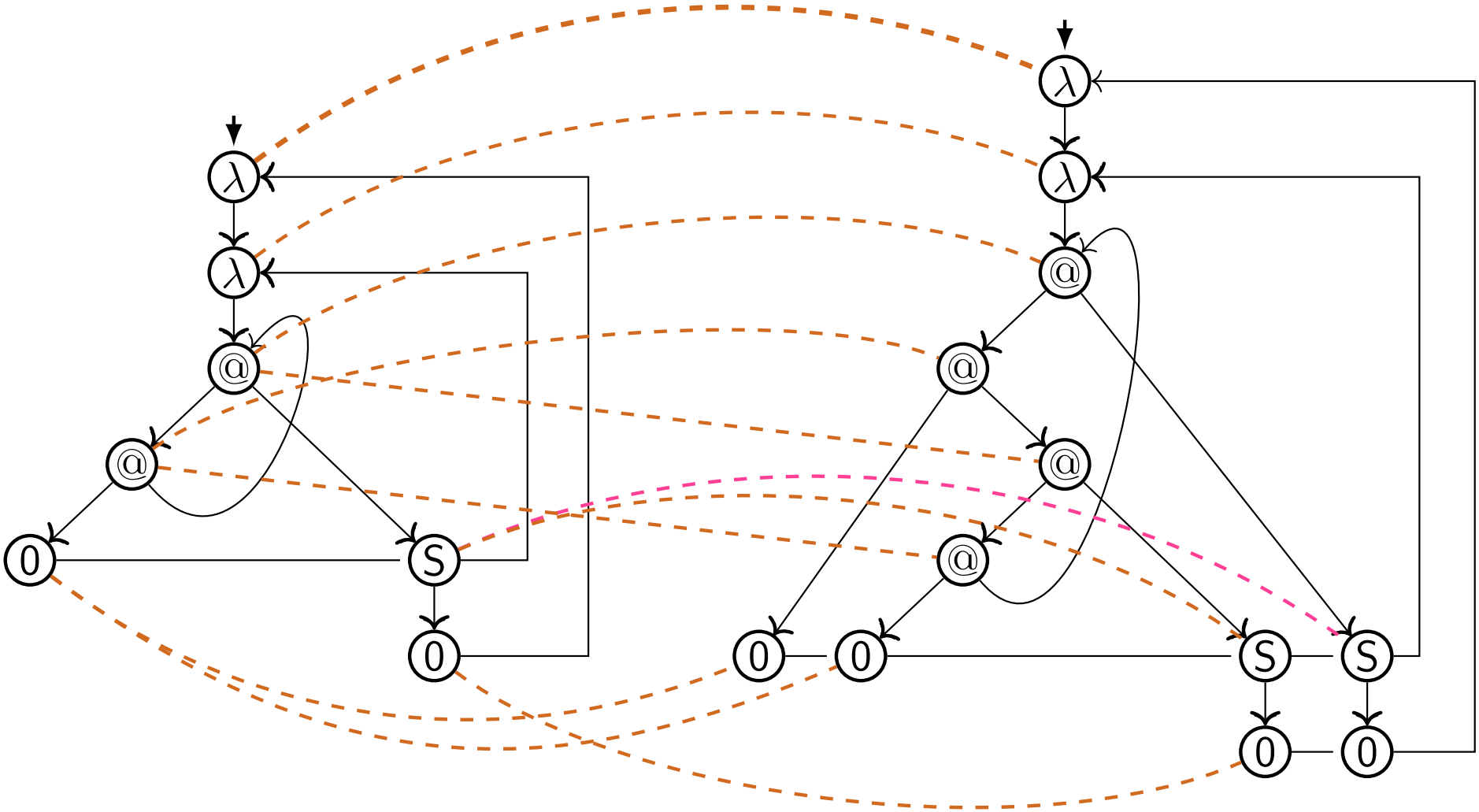
bisimulation check between λ-term-graphs



$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs



$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

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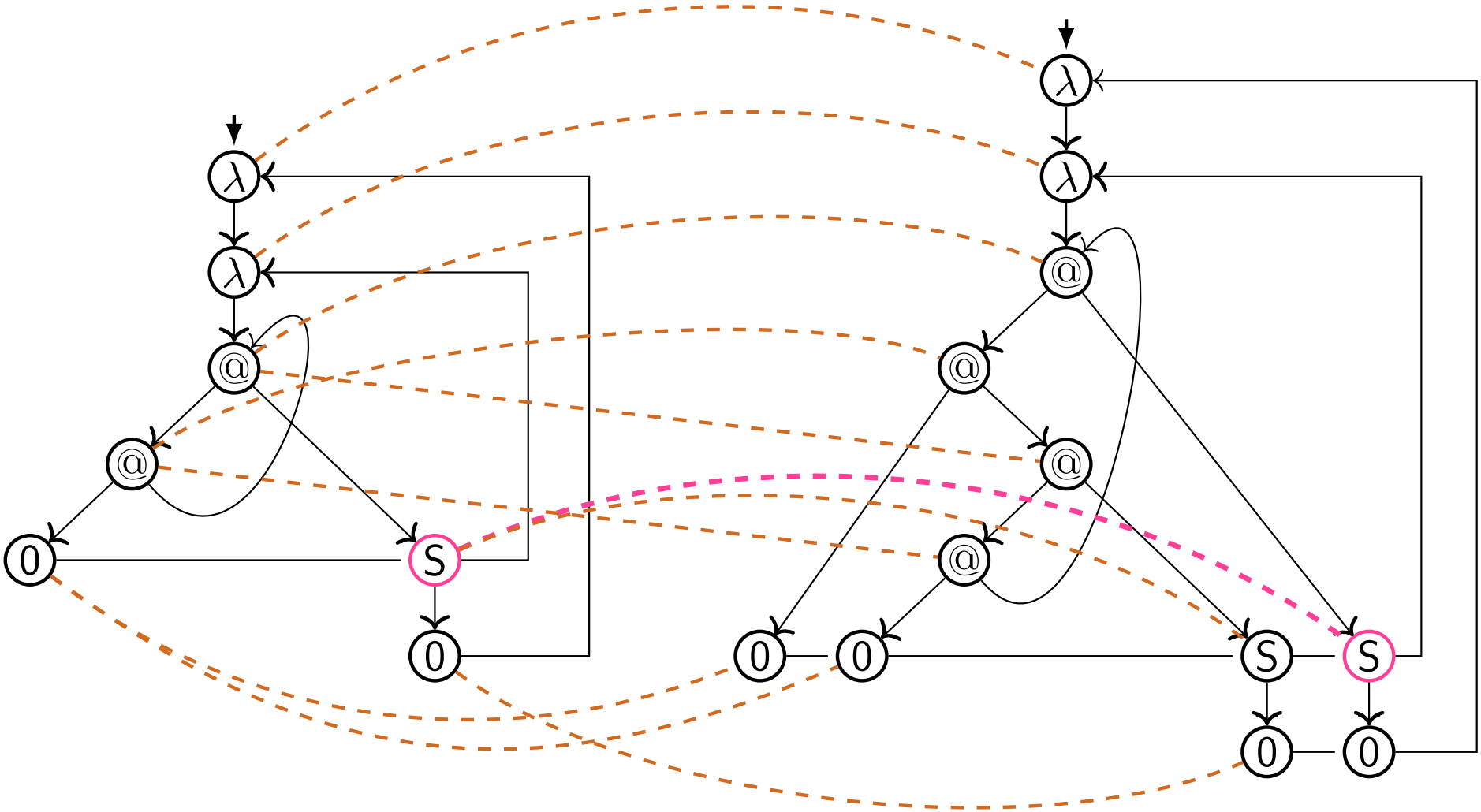

$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

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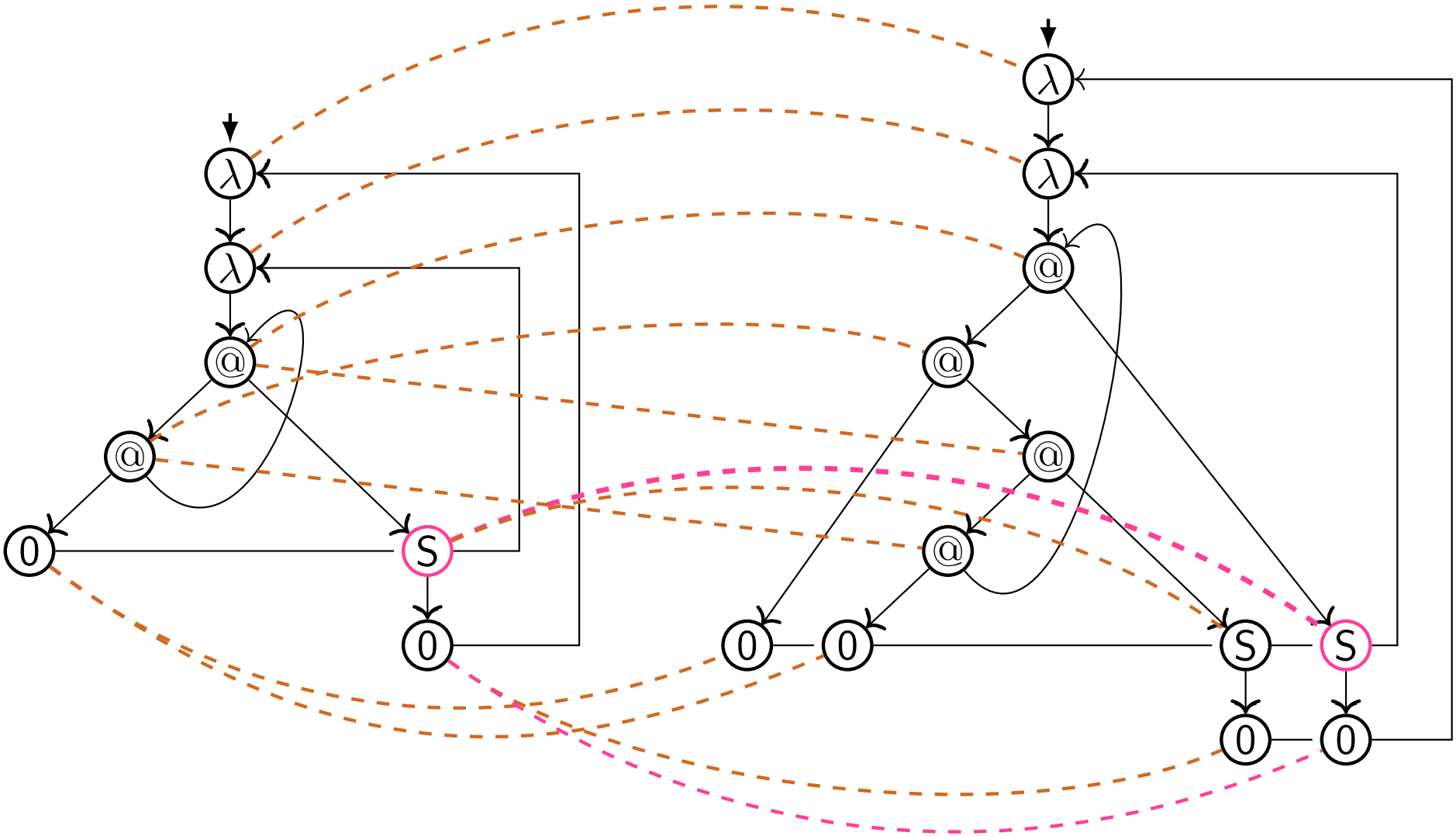
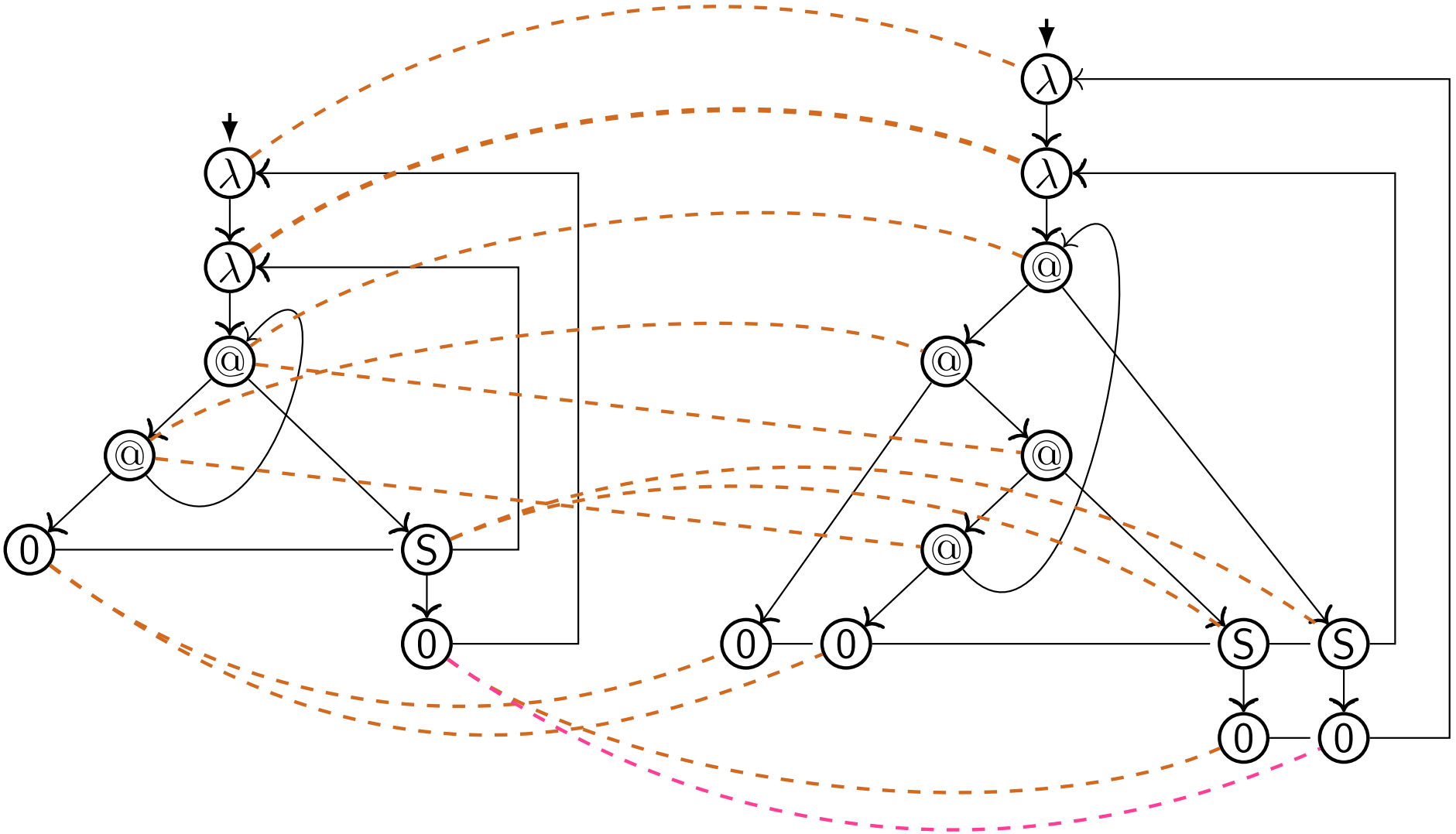
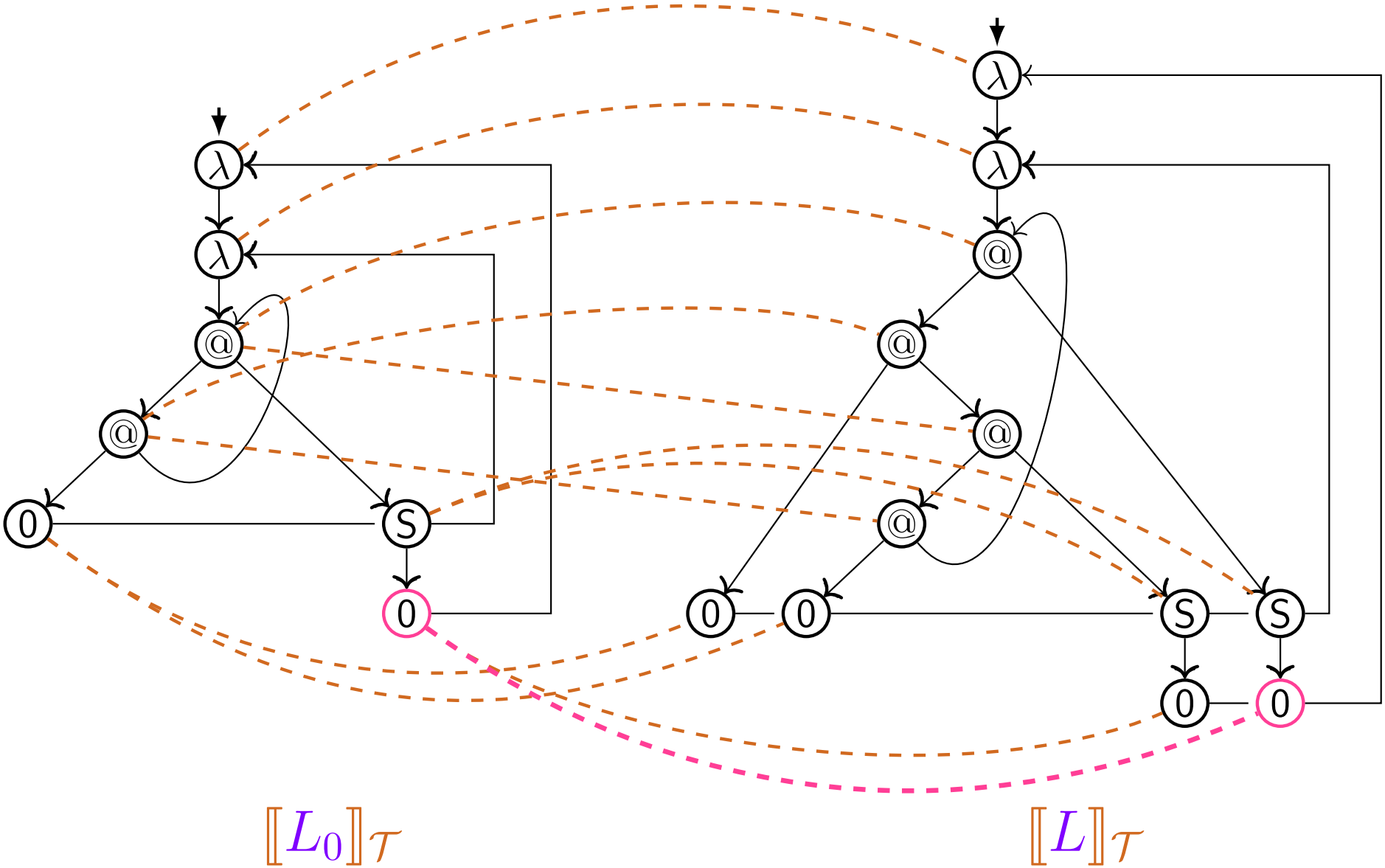


bisimulation check between λ-term-graphs

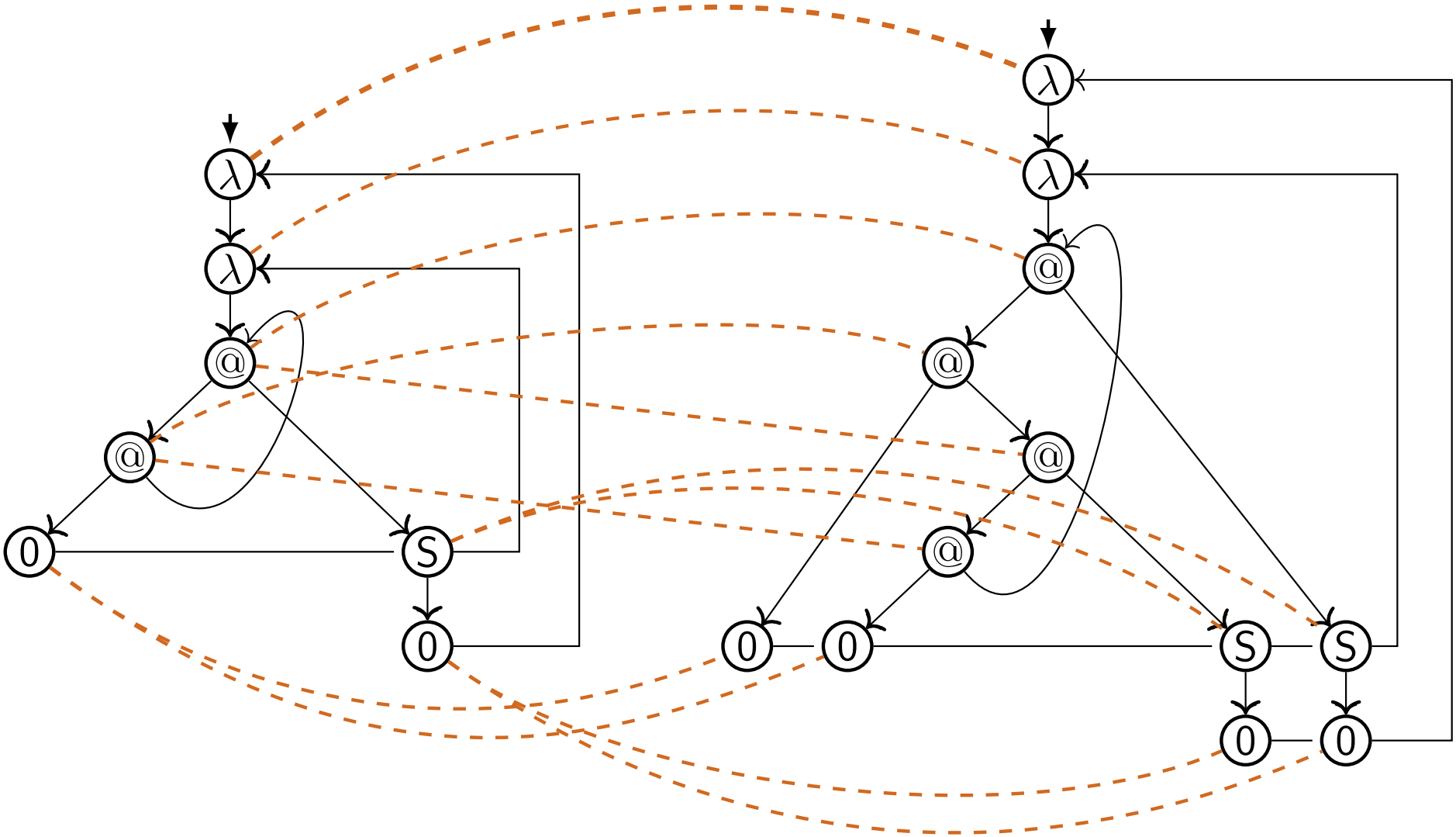


$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation check between λ-term-graphs

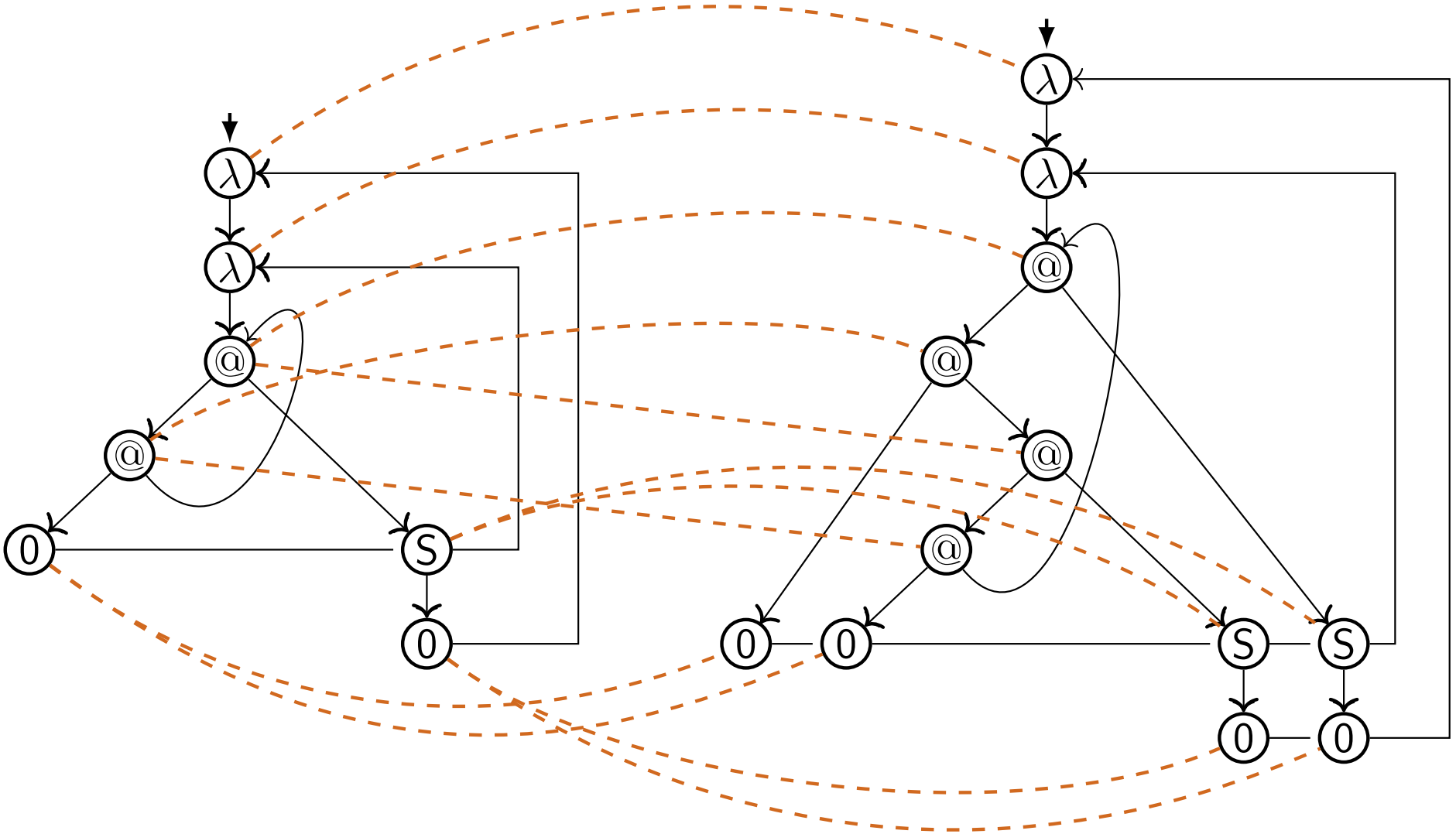


bisimulation check between λ-term-graphs



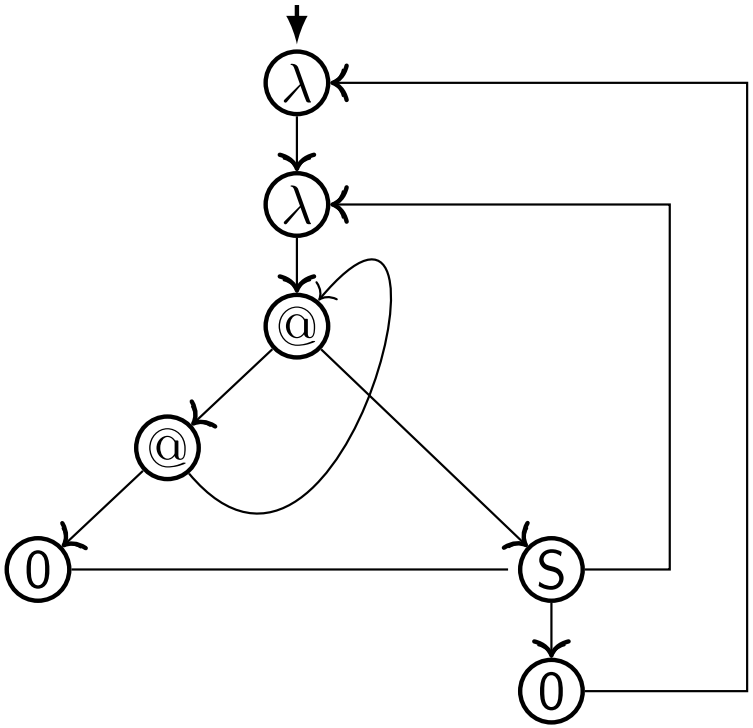
$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimulation between λ-term-graphs

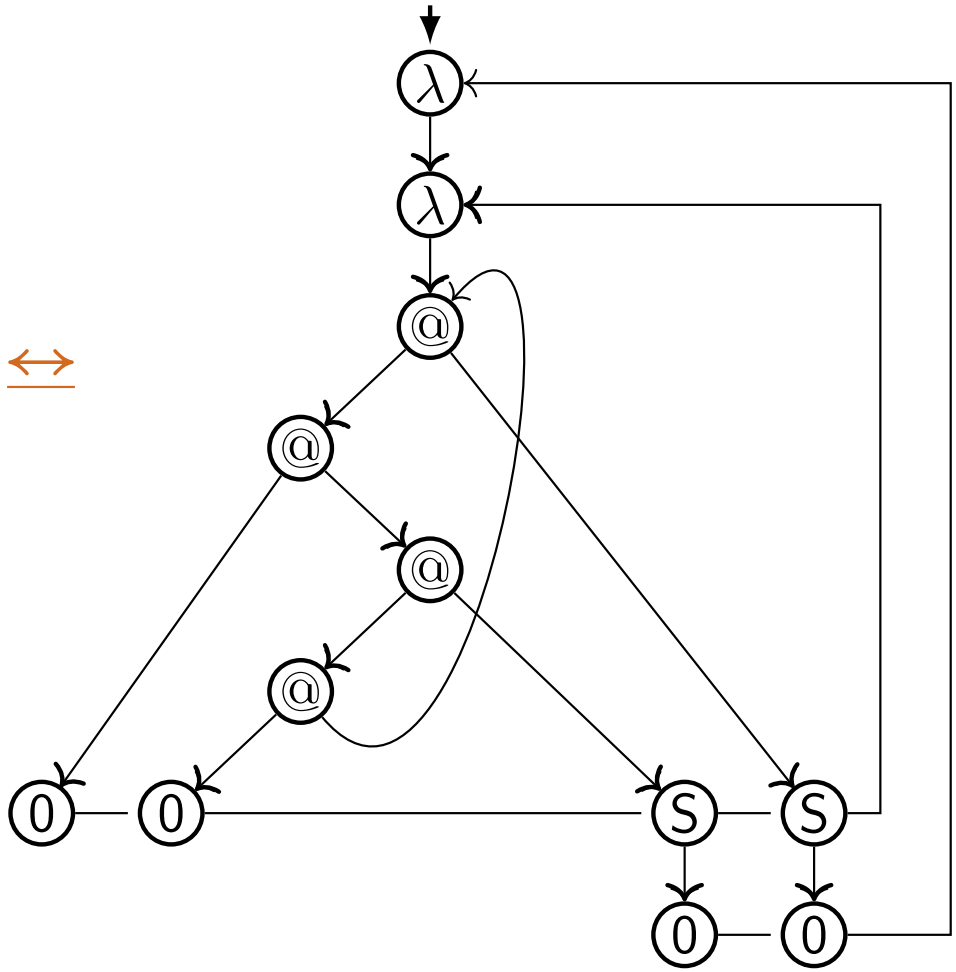


$[\![L_0]\!]\tau$ $[\![L]\!]\tau$

bisimilarity between λ-term-graphs

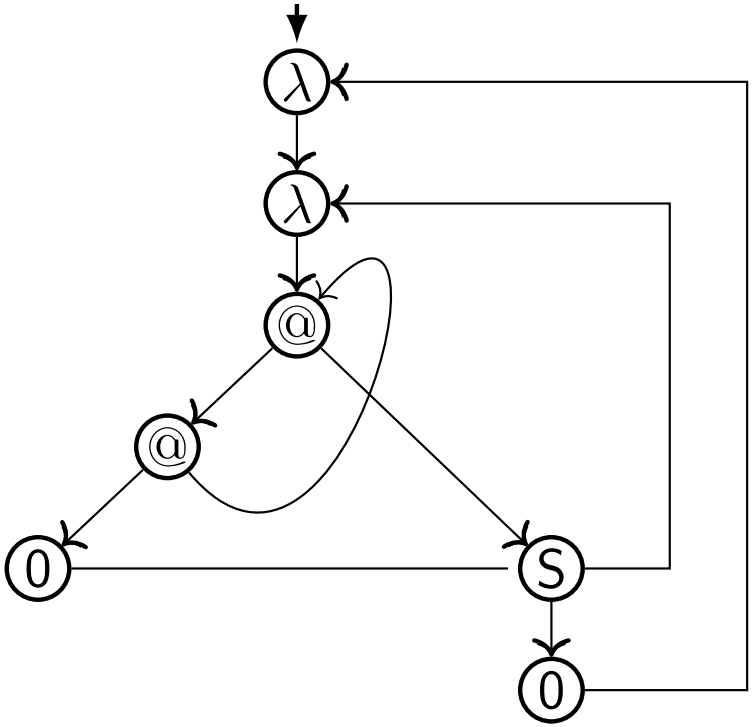


$\leftrightarrow$

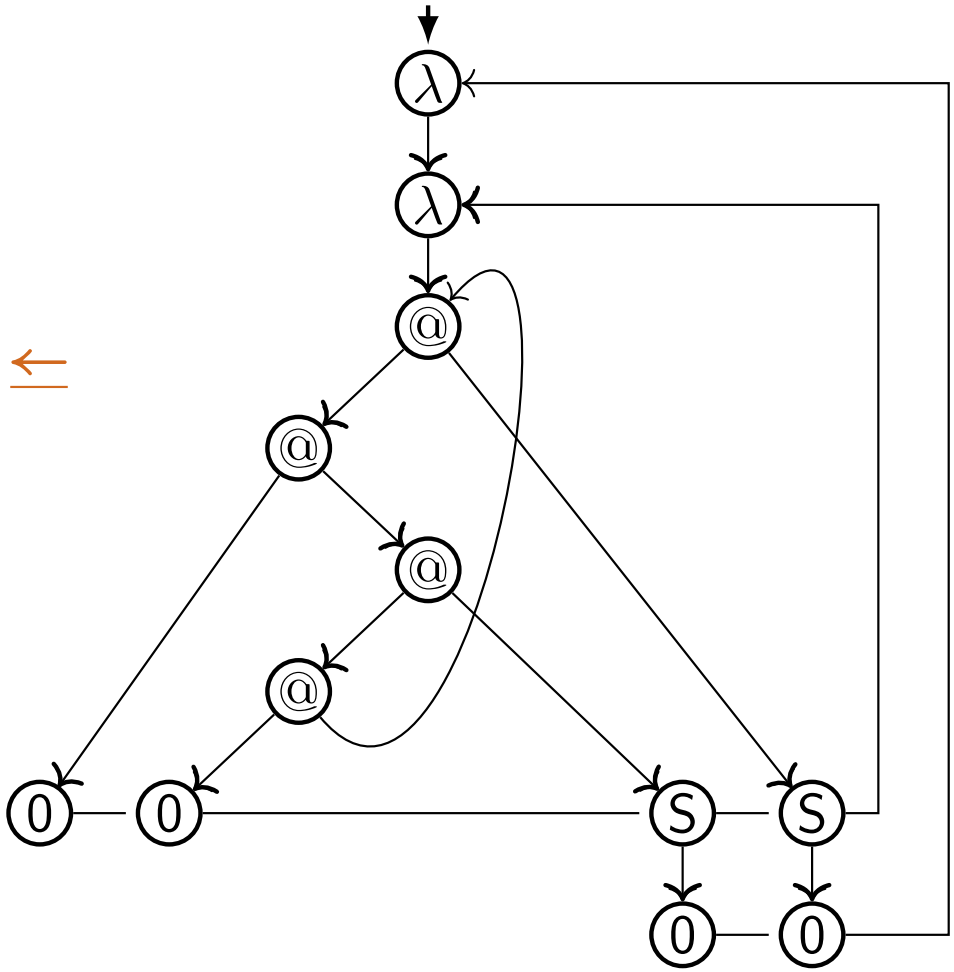


$[\![L_0]\!]\tau$ $\leftrightarrow$ $[\![L]\!]\tau$

functional bisimilarity and bisimulation collapse



$\leftarrow$



$[\![L_0]\!]\tau$ $\leftarrow$ $[\![L]\!]\tau$

bisimulation collapse: property

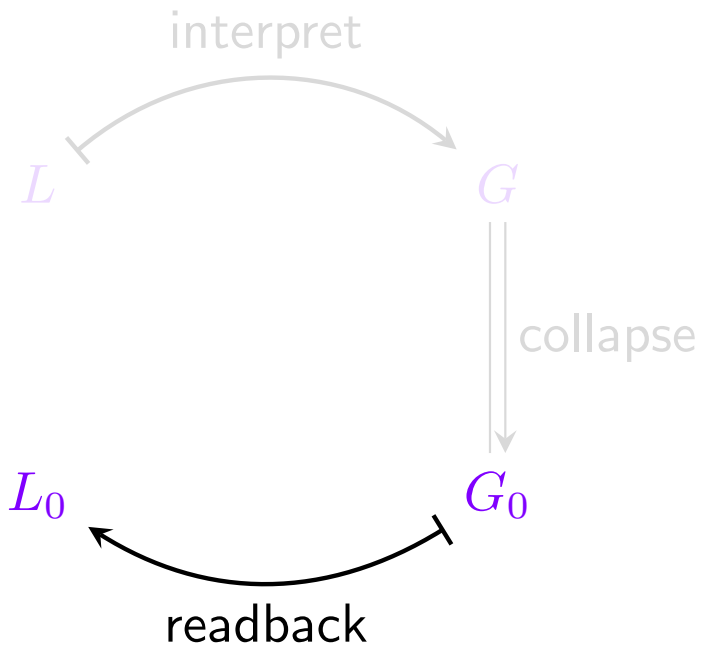
Theorem

The class of eager-scope λ-term-graphs
is closed under functional bisimilarity $\rightrightarrows$.

$\Longrightarrow$ For a $\boldsymbol{\lambda}_{\mathsf{letrec}}$-term L

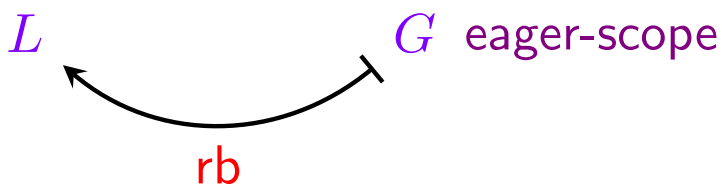
the bisimulation collapse of $[\![L]\!]_{\mathcal{T}}$ is again an eager-scope λ-term-graph.

readback



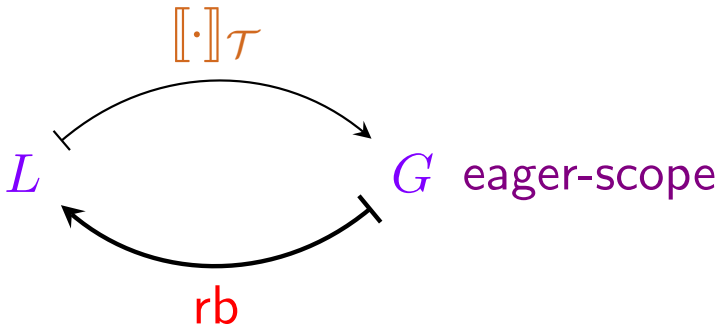
readback

defined with property:



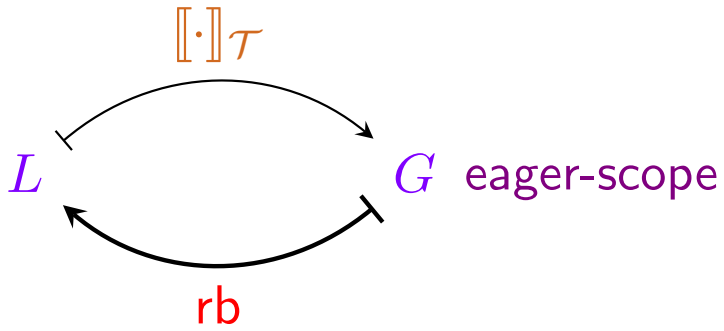
readback

defined with property:



readback

defined with property:



Theorem

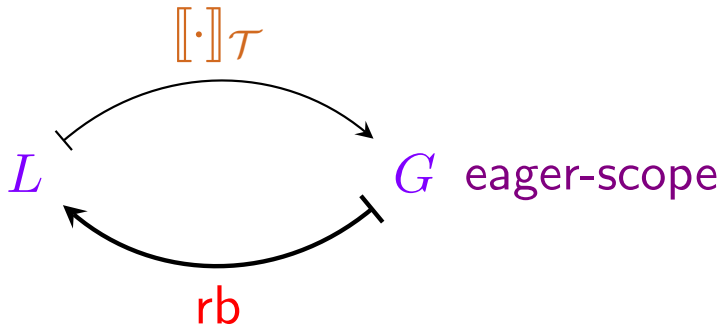
For all eager-scope λ-term-graphs G:

$$([\![\cdot]\!]_\tau \circ \mathsf{rb})(G) \simeq G$$

The readback rb is a right-inverse of $[\![\cdot]\!]_\tau$ modulo isomorphism $\simeq$.

readback

defined with property:



Theorem

For all eager-scope λ-term-graphs G:

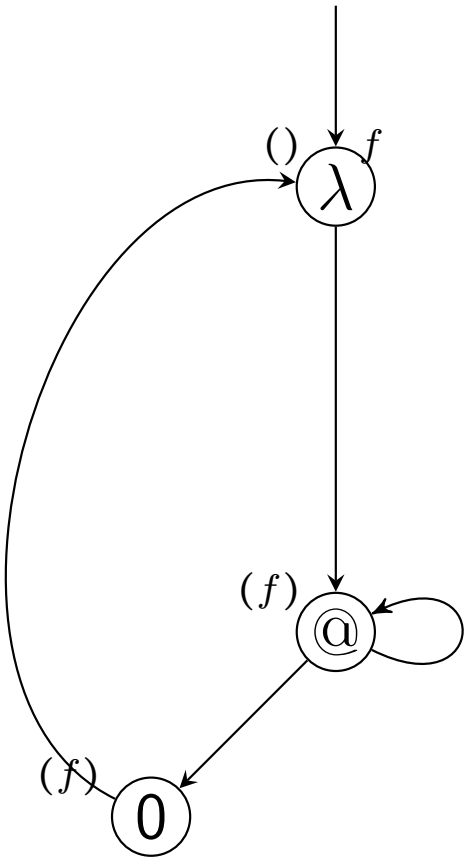
$$([\![\cdot]\!]_\mathcal{T} \circ \mathsf{rb})(G) \simeq G$$

The readback rb is a right-inverse of $[\![\cdot]\!]_\mathcal{T}$ modulo isomorphism $\simeq$.

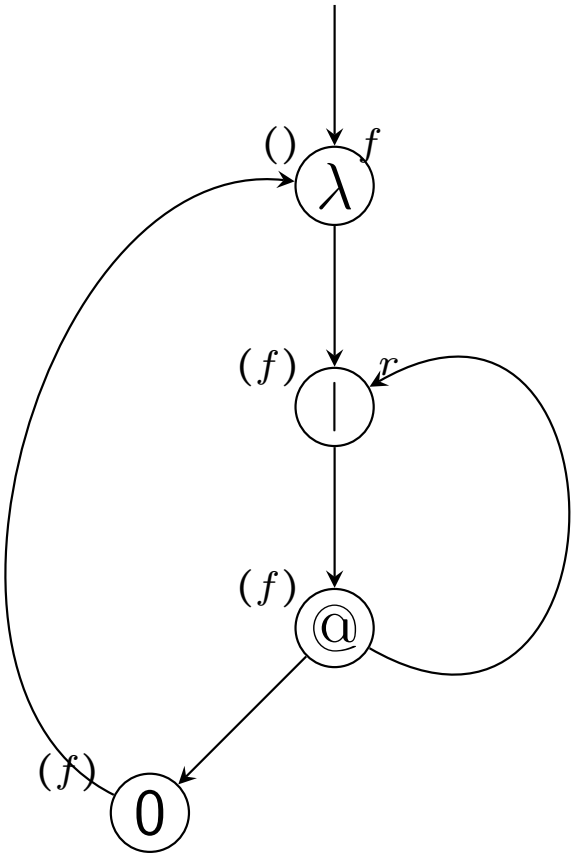
idea:

1. construct a spanning tree T of G
2. using local rules, in a bottom-up traversal of T synthesize $L = \mathsf{rb}(G)$

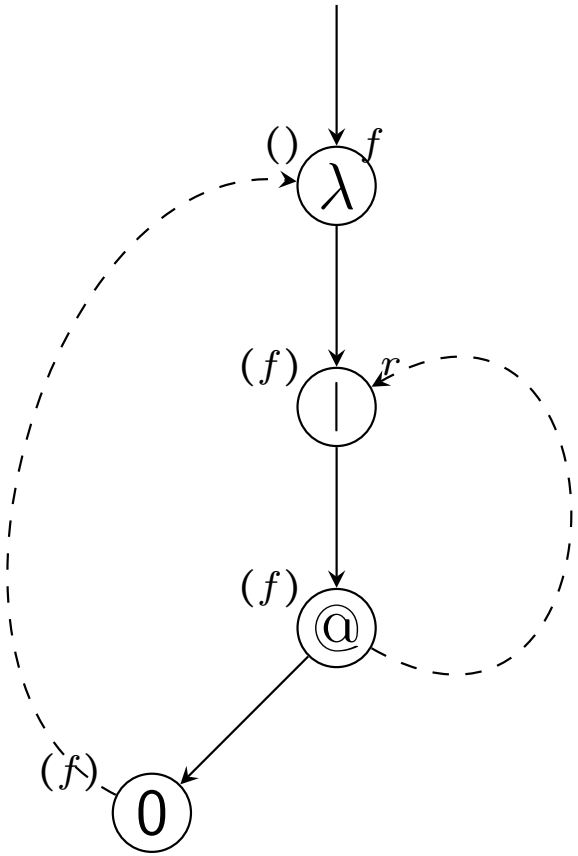
readback: example (fix)



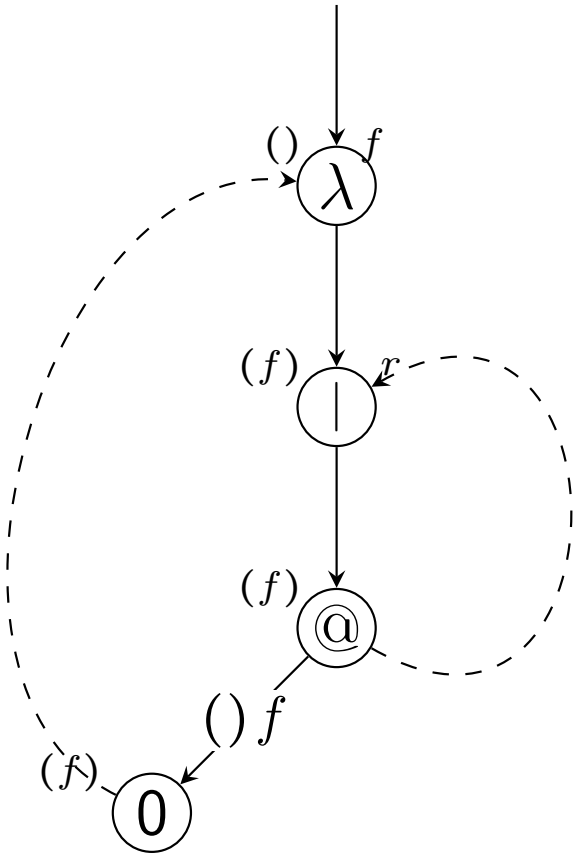
readback: example (fix)



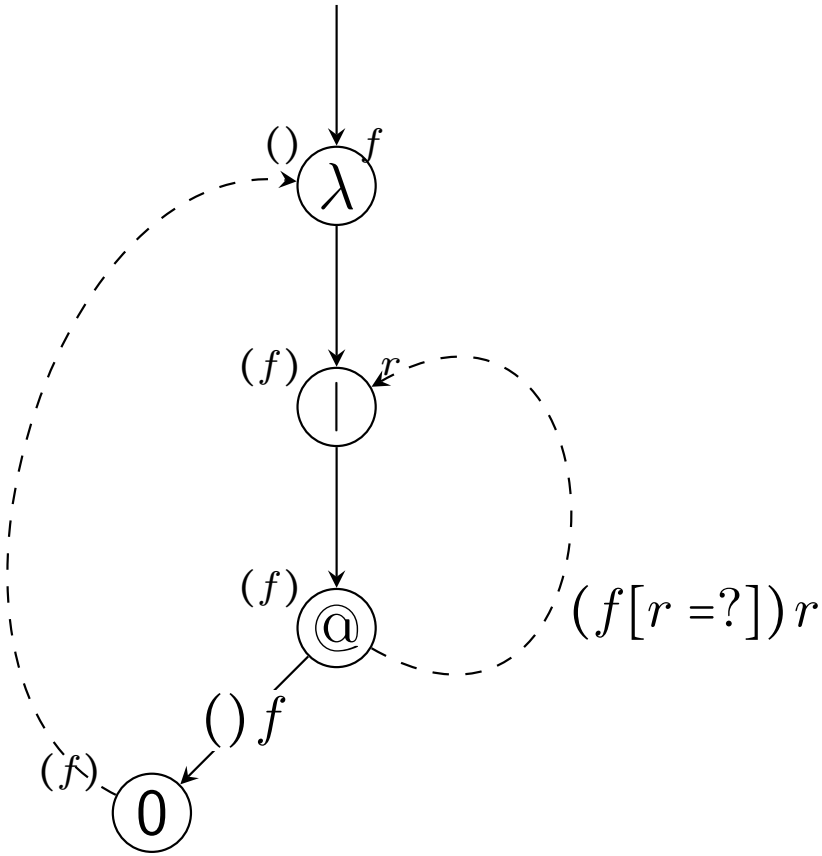
readback: example (fix)



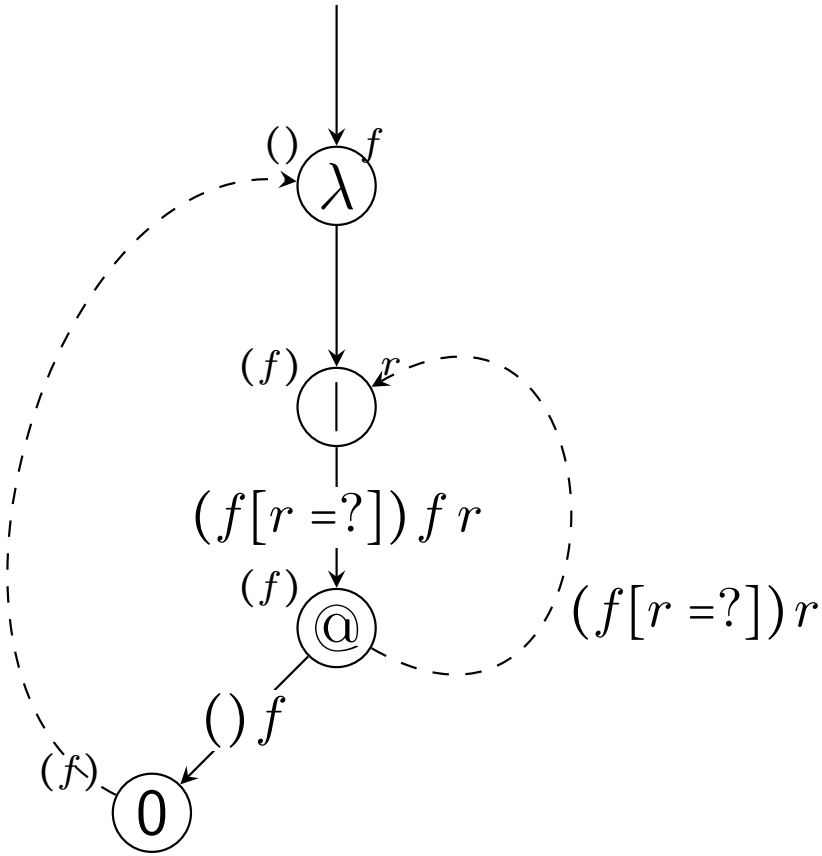
readback: example (fix)



readback: example (fix)



readback: example (fix)



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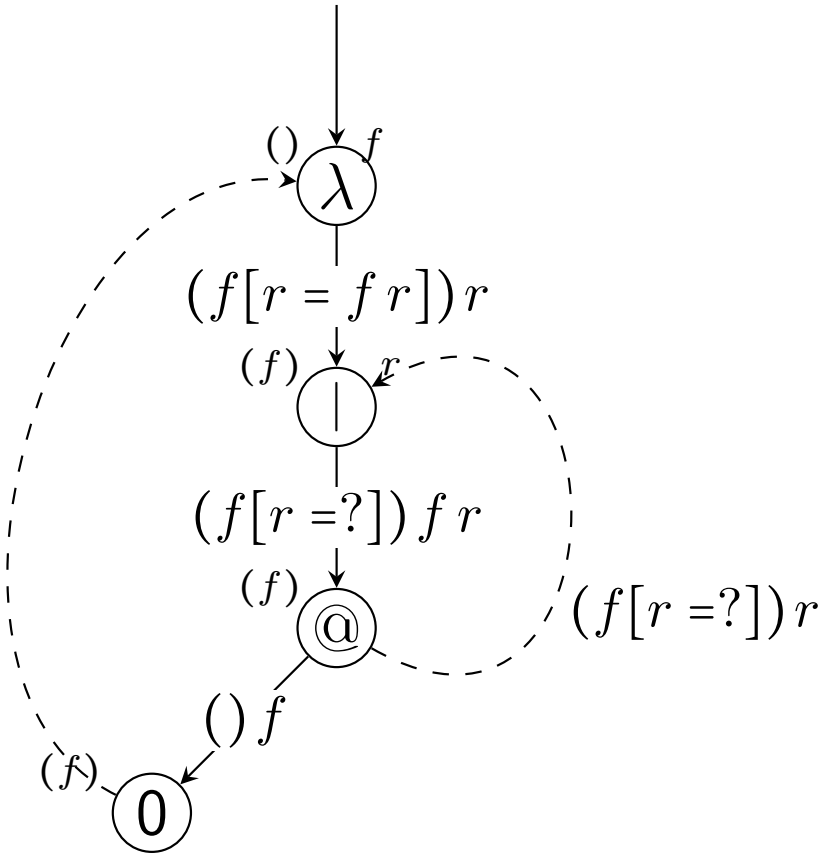
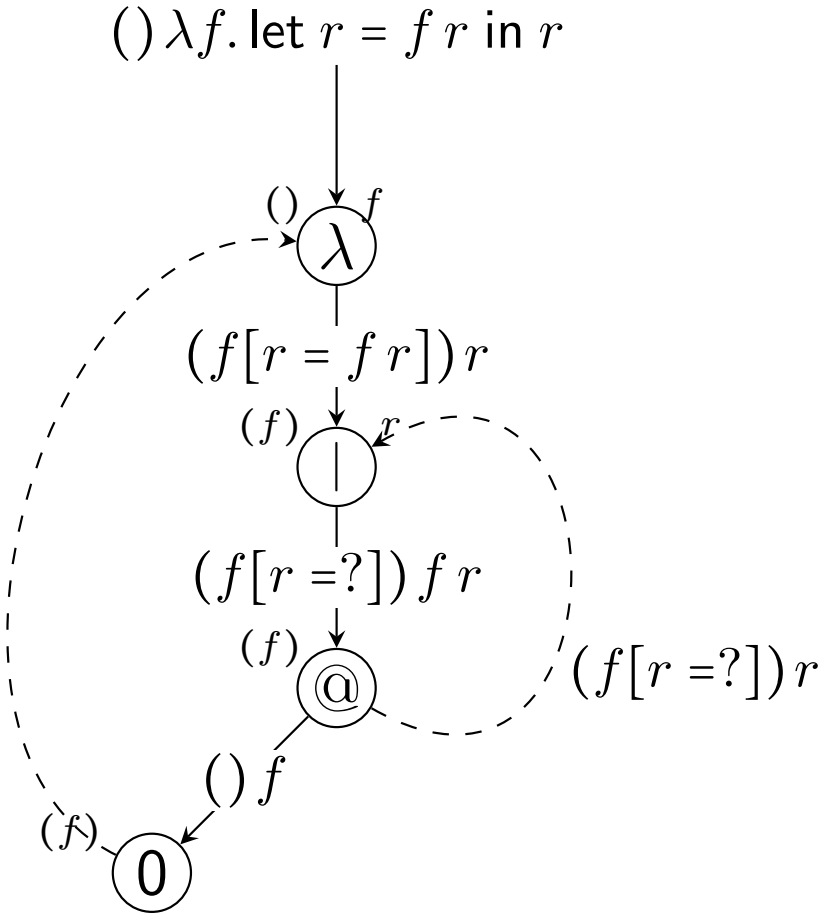


readback: example (fix)



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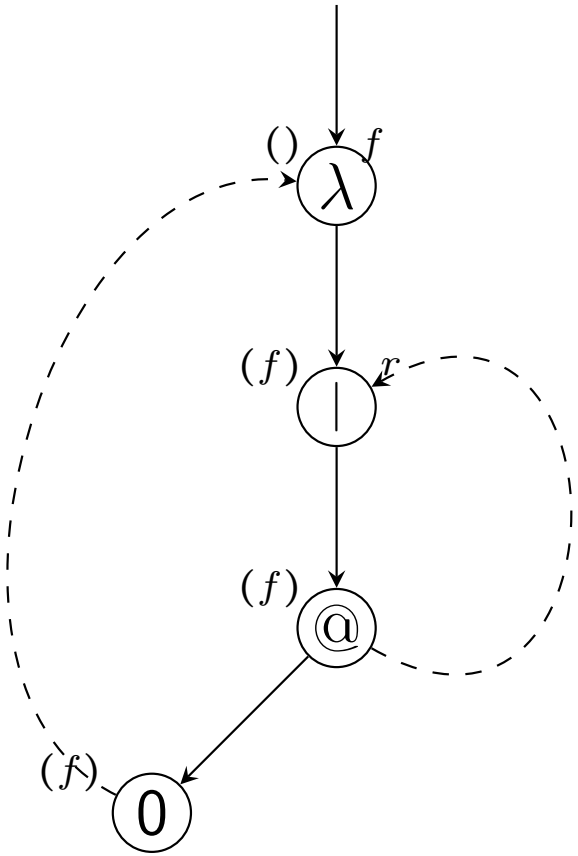

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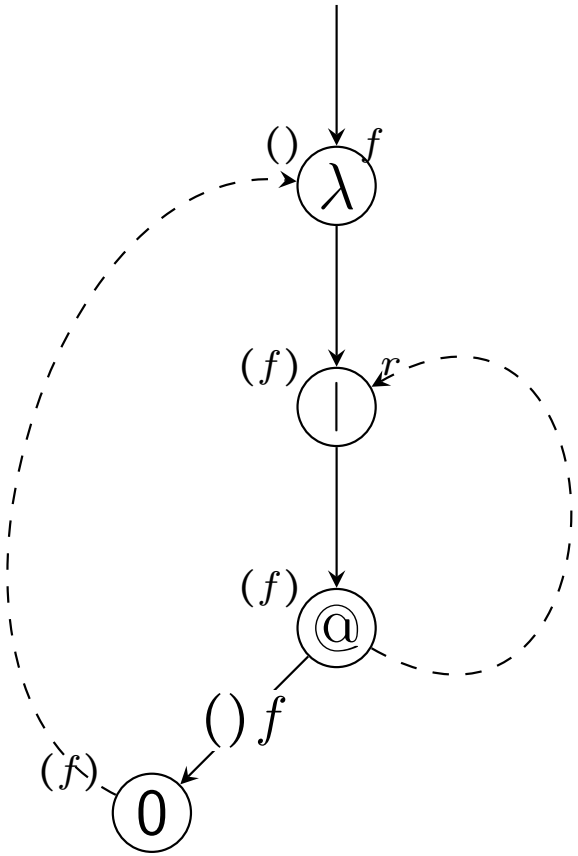

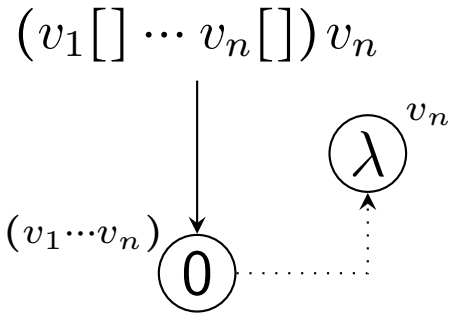

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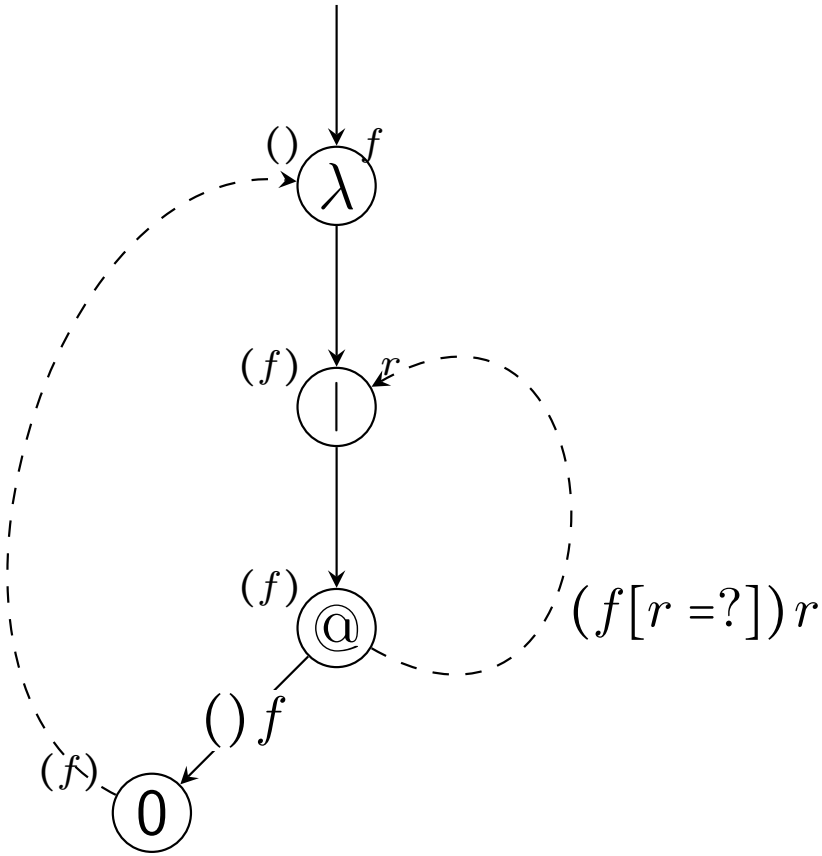

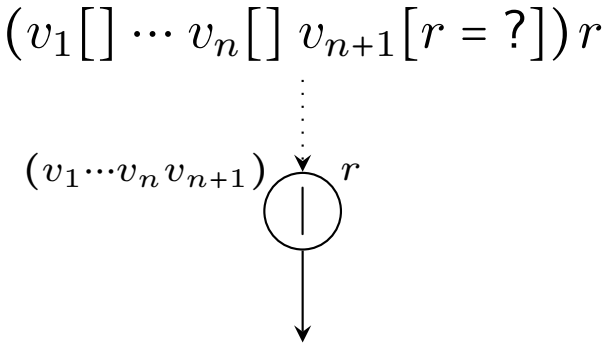

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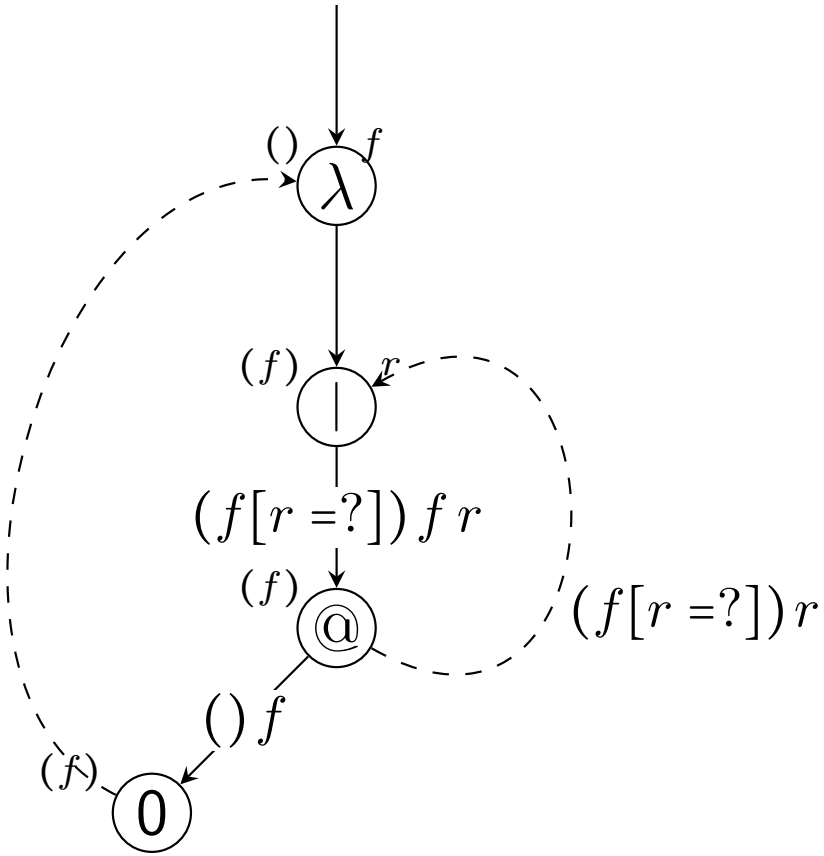

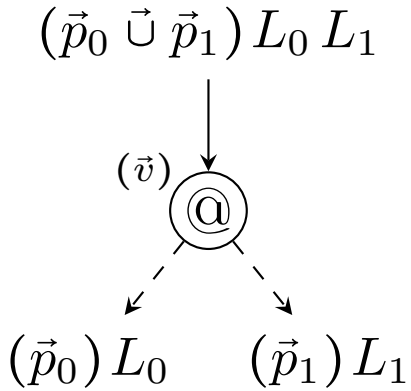

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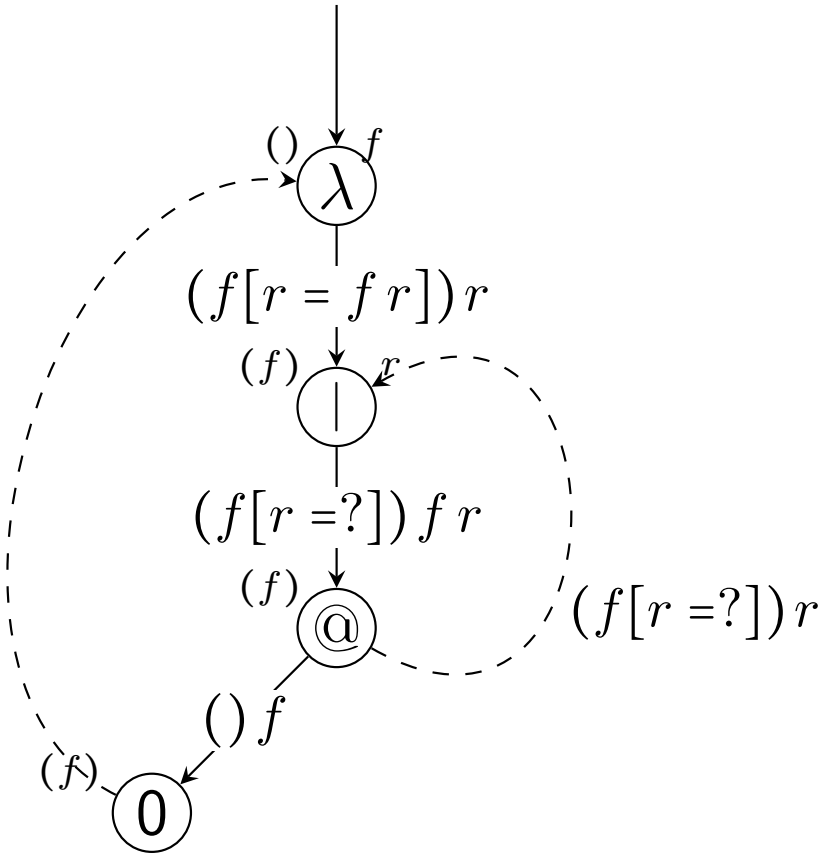

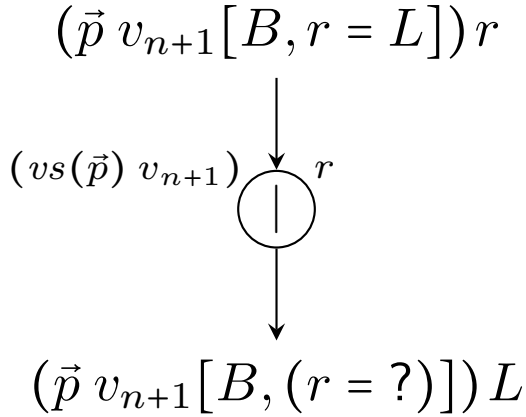

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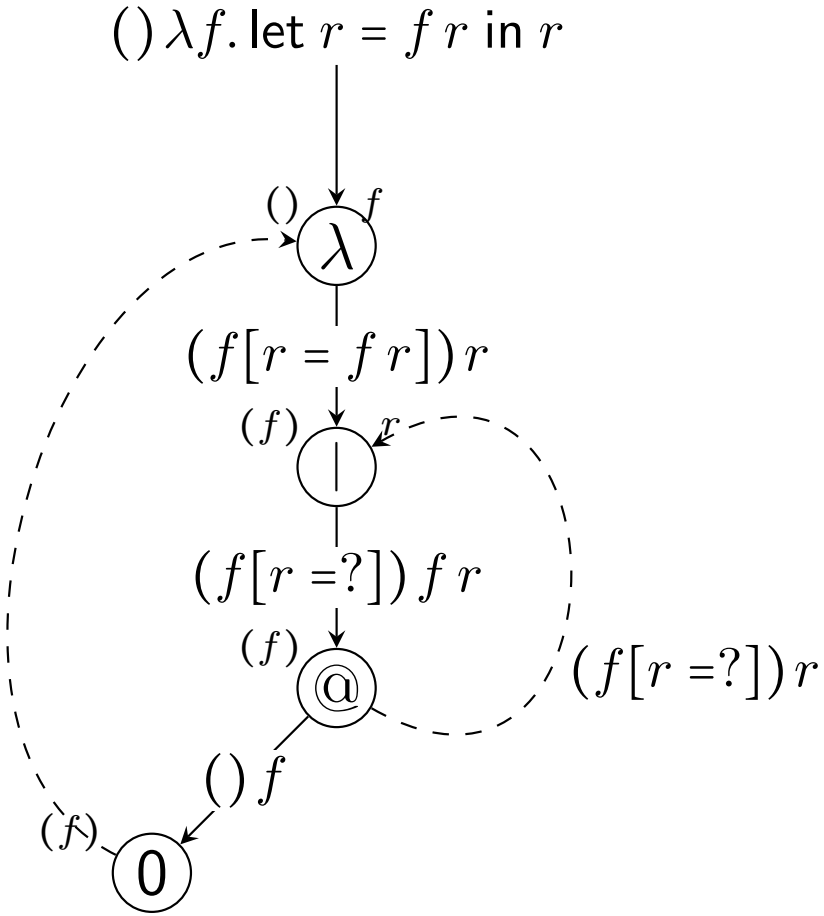
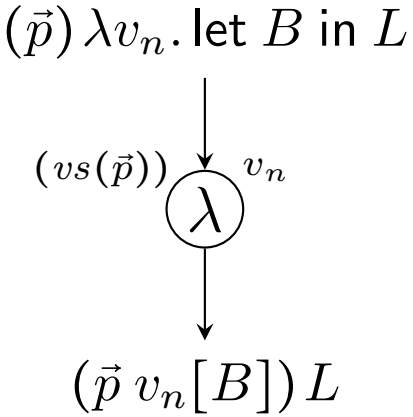
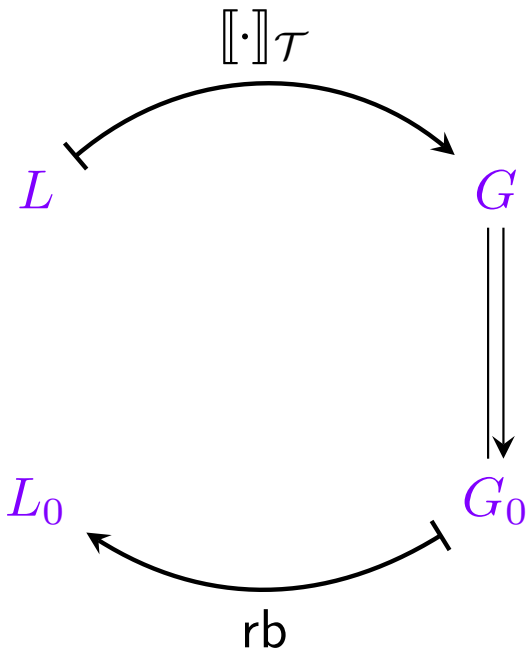




maximal sharing: complexity



1. interpretation

   of λ_{letrec}-term L with $|L| = n$

   as λ-term-graph $G = [\![L]\!]_{\mathcal{T}}$

   - in time $O(n^2)$, size $|G| \in O(n^2)$.

2. bisimulation collapse $|\downarrow$

   of f-o term graph G into G_0

   - in time $O(|G| \log |G|) = O(n^2 \log n)$

3. readback rb

   of f-o term graph G_0

   yielding λ_{letrec}-term $L_0 = \text{rb}(G_0)$.

   - in time $O(|G| \log |G|) = O(n^2 \log n)$

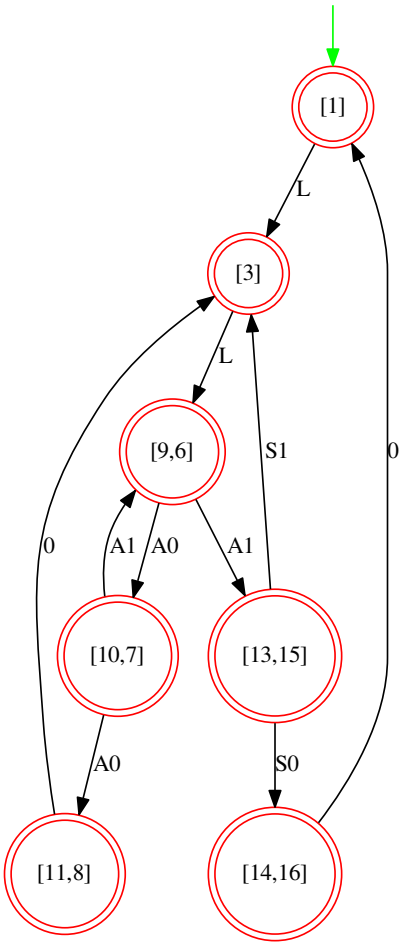
Theorem

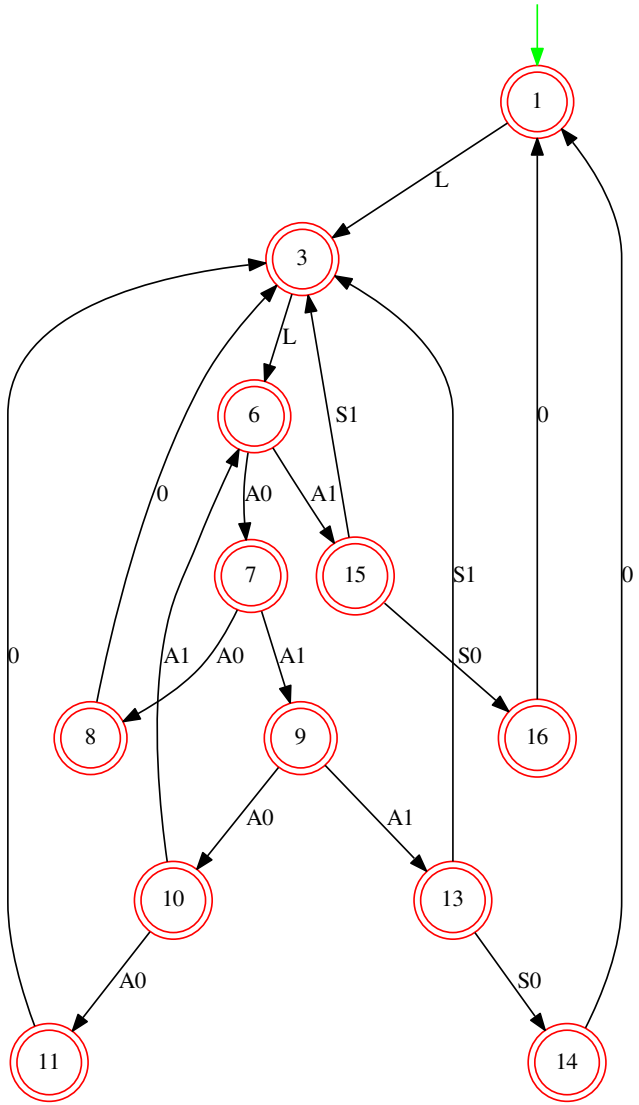
Computing a maximally compact form $L_0 = (\text{rb} \circ |\downarrow \circ [\![\cdot]\!]_{\mathcal{T}})(L)$ *of* L *for a* λ_{letrec}*-term* L *requires time* $O(n^2 \log n)$*, where* $|L| = n$.

Demo: console output

```
jan:~/papers/maxsharing-ICFP/talks/ICFP-2014> maxsharing running.l
λ-letrec-term:
λx. λf. let r = f (f r x) x in r

derivation:
              ---------- 0                 ------- 0
              (x f[r]) f    (x f[r]) r    (x) x
              ----------------------- @   ---------- S
              (x f[r]) f r                (x f[r]) x
------------ 0 ---------------------------------------- @    ------- 0
(x f[r]) f      (x f[r]) f r x                              (x) x
------------------------------------------------------- @   ---------- S
(x f[r]) f (f r x)                                         (x f[r]) x
-------------------------------------------------------------------- @
(x f[r]) f (f r x) x                                                    (x f[r]) r
------------------------------------------------------------------------------------ let
(x f) let r = f (f r x) x in r
------------------------------------------------------------------------------------- λ
(x) λf. let r = f (f r x) x in r
-------------------------------------------------------------------------------------- λ
() λx. λf. let r = f (f r x) x in r

writing DFA to file: running-dfa.pdf

readback of DFA:
λx. λy. let F = y (y F x) x in F

writing minimised DFA to file: running-mindfa.pdf

readback of minimised DFA:
λx. λy. let F = y F x in F
jan:~/papers/maxsharing-ICFP/talks/ICFP-2014>
```

Demo: generated λ-NFAs



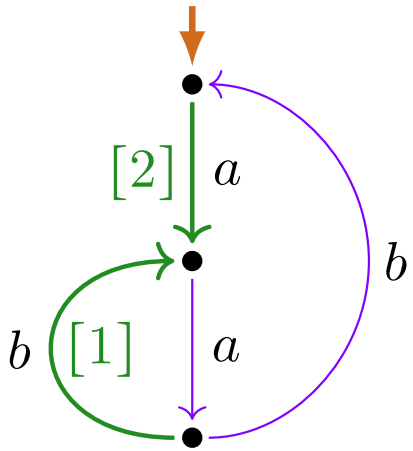


Resources (maximal sharing)

- tool maxsharing on `hackage.haskell.org`
- papers and reports
  - Maximal Sharing in the Lambda Calculus with Letrec
    - ICFP 2014 paper
    - accompanying report arXiv:1401.1460
  - Term Graph Representations for Cyclic Lambda Terms
    - TERMGRAPH 2013 proceedings
    - extended report arXiv:1308.1034
  - Vincent van Oostrom, CG: Nested Term Graphs
    - TERMGRAPH 2014 post-proceedings in EPTCS 183
- thesis Jan Rochel
  - Unfolding Semantics of the Untyped λ-Calculus with letrec
    - Ph.D. Thesis, Utrecht University, 2016

Process interpretation of regular expressions

(current work with Wan Fokkink)



Regular Expressions

Definition

The set $\mathrm{Reg}(A)$ of regular expressions over alphabet A is defined by the grammar:

$$e, f \ ::= \ 0 \mid 1 \mid a \mid (e + f) \mid (e \cdot f) \mid (e^\star) \qquad (\text{for } a \in A).$$

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Definition

The set $\mathrm{Reg}(A)$ of regular expressions over alphabet A is defined by the grammar:

$$e, f \ ::= \ 0 \mid 1 \mid a \mid (e + f) \mid (e \cdot f) \mid (e^\star) \qquad (\text{for } a \in A).$$

Note, here:

- symbol 0 instead of $\varnothing$
- symbol 1 used (often dropped, definable as $0^\star$)
- no complementation operation $\overline{e}$
  - is not expressible under language interpretation

Language interpretation $[\![\cdot]\!]_L$ *(S.C. Kleene, 1951)*

$\mathbf{0} \xmapsto{[\![\cdot]\!]_L}$ empty language $\varnothing$

$\mathbf{1} \xmapsto{[\![\cdot]\!]_L} \{\epsilon\}$ (ϵ the empty word)

$a \xmapsto{[\![\cdot]\!]_L} \{a\}$

Language interpretation $[\![\cdot]\!]_L$ *(S.C. Kleene, 1951)*

$\mathbf{0} \overset{[\![\cdot]\!]_L}{\longmapsto}$ empty language $\varnothing$

$\mathbf{1} \overset{[\![\cdot]\!]_L}{\longmapsto} \{\epsilon\}$ (ϵ the empty word)

$a \overset{[\![\cdot]\!]_L}{\longmapsto} \{a\}$

$e + f \overset{[\![\cdot]\!]_L}{\longmapsto}$ union of $[\![e]\!]_L$ and $[\![f]\!]_L$

$e \cdot f \overset{[\![\cdot]\!]_L}{\longmapsto}$ element-wise concatenation of $[\![e]\!]_L$ and $[\![f]\!]_L$

$e^* \overset{[\![\cdot]\!]_L}{\longmapsto}$ set of words formed by concatenating words in $[\![e]\!]_L$, and adding the empty word ϵ

Process interpretation $[\![\cdot]\!]_P$ *(R. Milner, 1984)*

$\mathbf{0} \overset{[\![\cdot]\!]_P}{\longmapsto}$ deadlock δ, no termination

$\mathbf{1} \overset{[\![\cdot]\!]_P}{\longmapsto}$ empty process ϵ, then terminate

$a \overset{[\![\cdot]\!]_P}{\longmapsto}$ atomic action a, then terminate

Process interpretation $[\![\cdot]\!]_P$ *(R. Milner, 1984)*

$\mathbf{0} \overset{[\![\cdot]\!]_P}{\longmapsto}$ deadlock δ, no termination

$\mathbf{1} \overset{[\![\cdot]\!]_P}{\longmapsto}$ empty process ϵ, then terminate

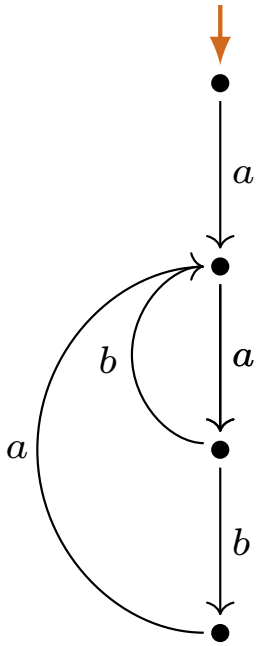
$a \overset{[\![\cdot]\!]_P}{\longmapsto}$ atomic action a, then terminate

$e + f \overset{[\![\cdot]\!]_P}{\longmapsto}$ alternative composition of $[\![e]\!]_P$ and $[\![f]\!]_P$

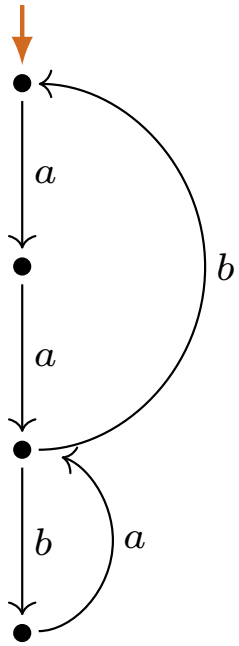
$e \cdot f \overset{[\![\cdot]\!]_P}{\longmapsto}$ sequential composition of $[\![e]\!]_P$ and $[\![f]\!]_P$

$e^* \overset{[\![\cdot]\!]_P}{\longmapsto}$ unbounded iteration of $[\![e]\!]_P$, option to terminate

Process interpretation of regular expressions

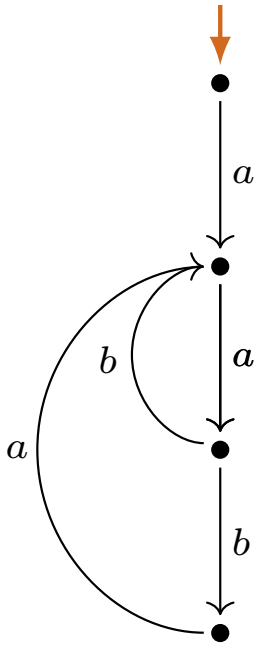


$$a(a(b+ba))^*0$$

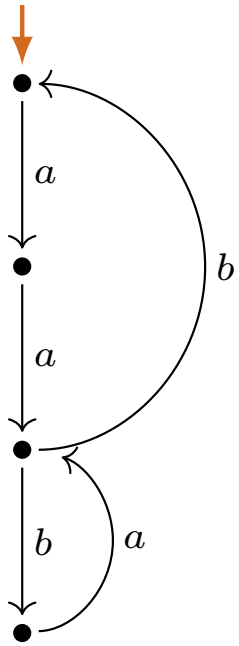


$$(aa(ba)^*b)^*0$$

Process interpretation of regular expressions

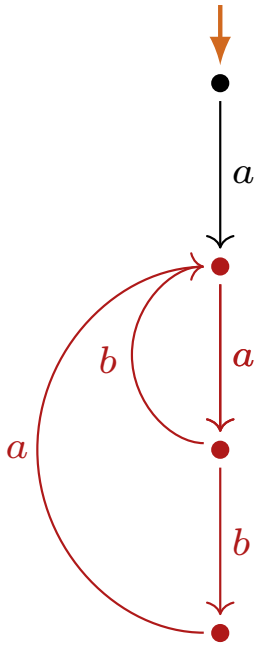


$$a \cdot (a \cdot (b + b \cdot a))^* \cdot 0$$



$$(a \cdot a \cdot (b \cdot a)^* \cdot b)^* \cdot 0$$

Process interpretation of regular expressions



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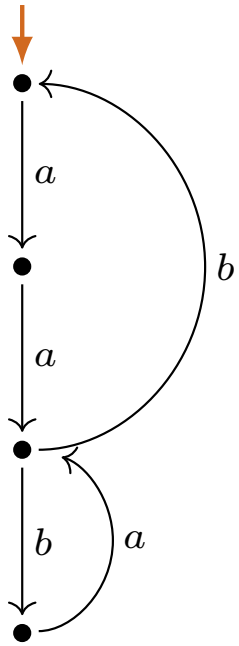

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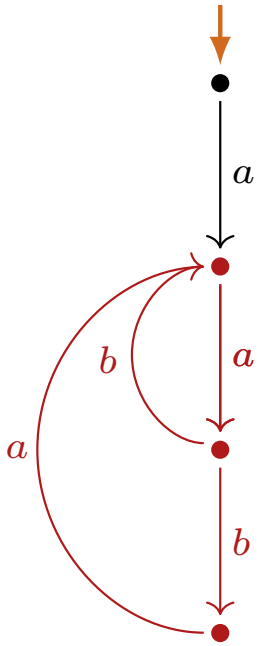

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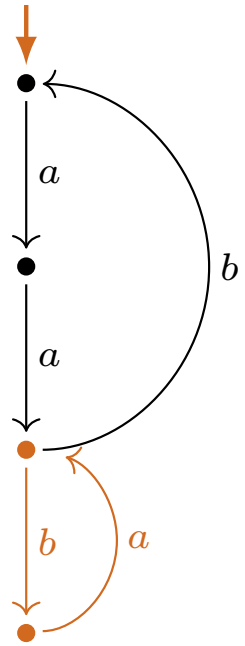

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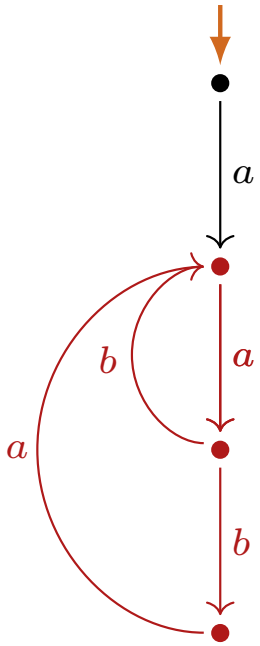
Process interpretation of regular expressions



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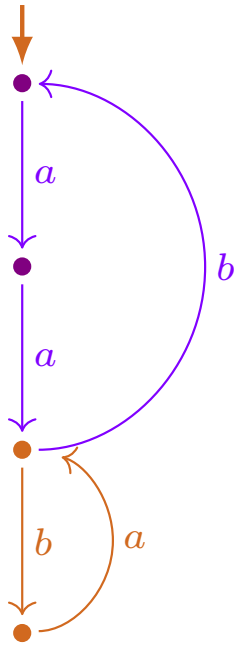

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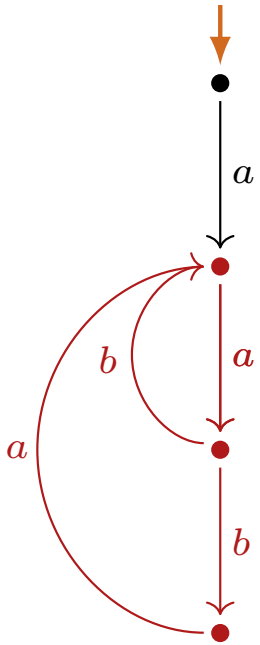
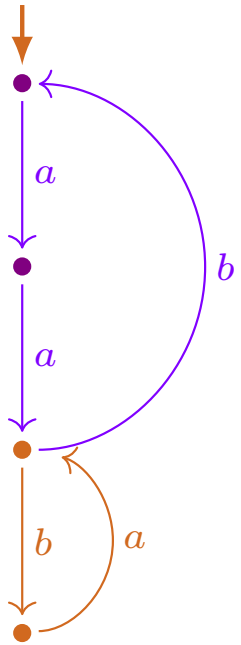
Process interpretation of regular expressions



$$[\![a \cdot (a \cdot (b + b \cdot a))^* \cdot 0]\!]_{\boldsymbol{P}}$$



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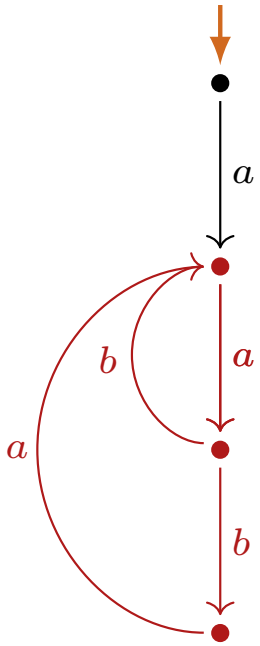

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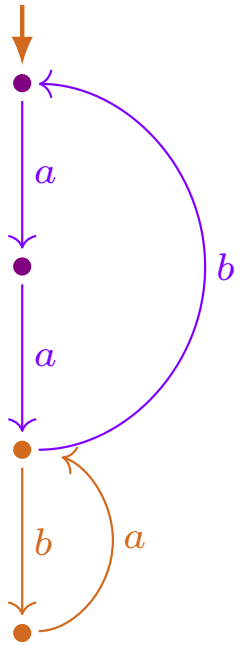

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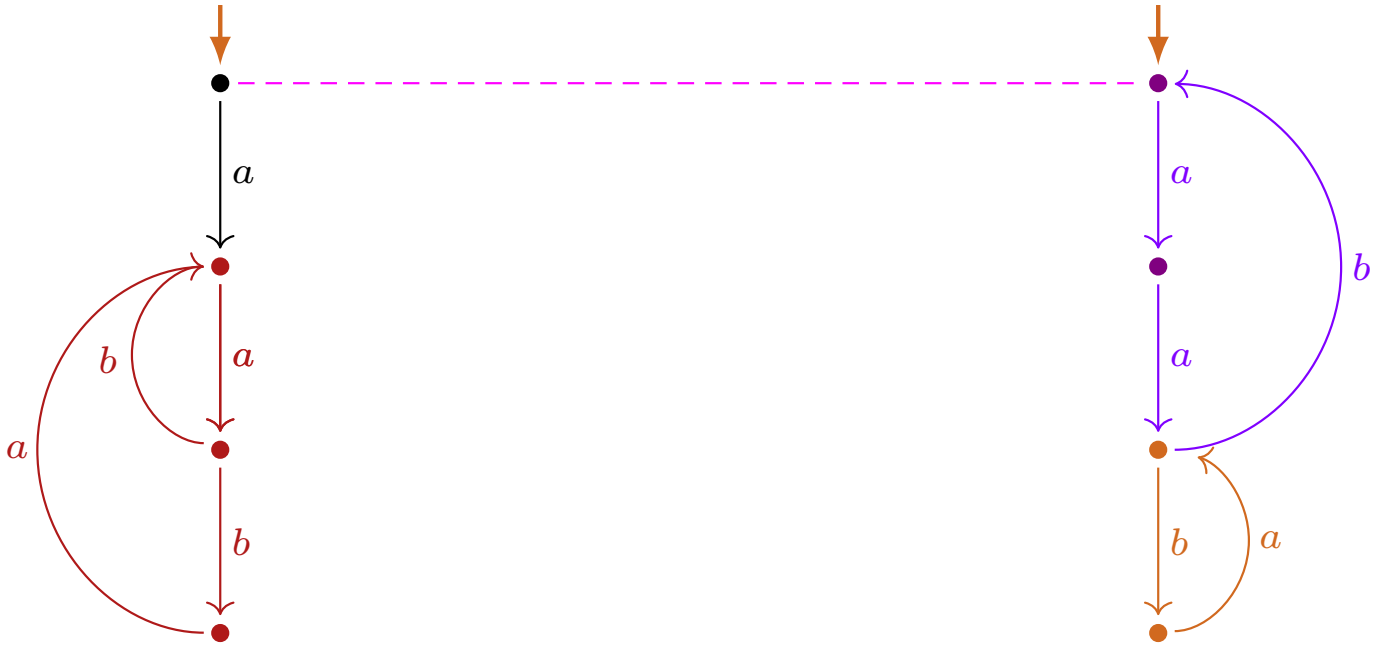

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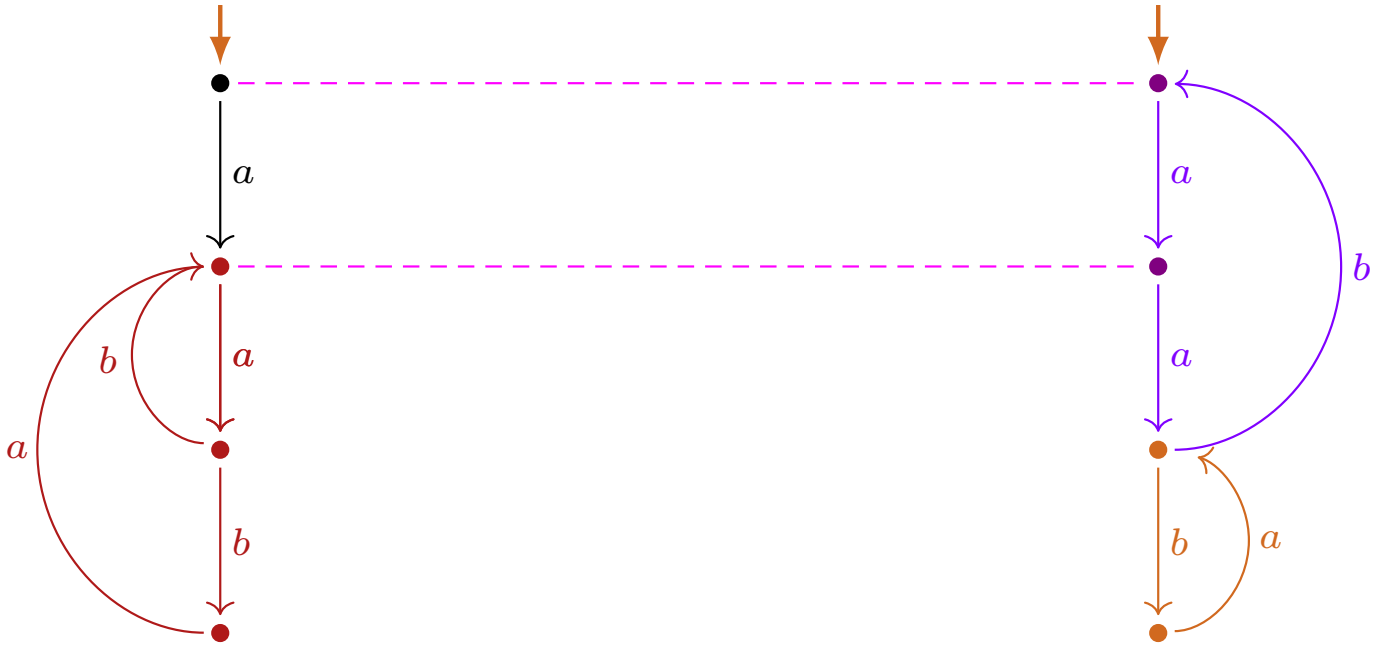

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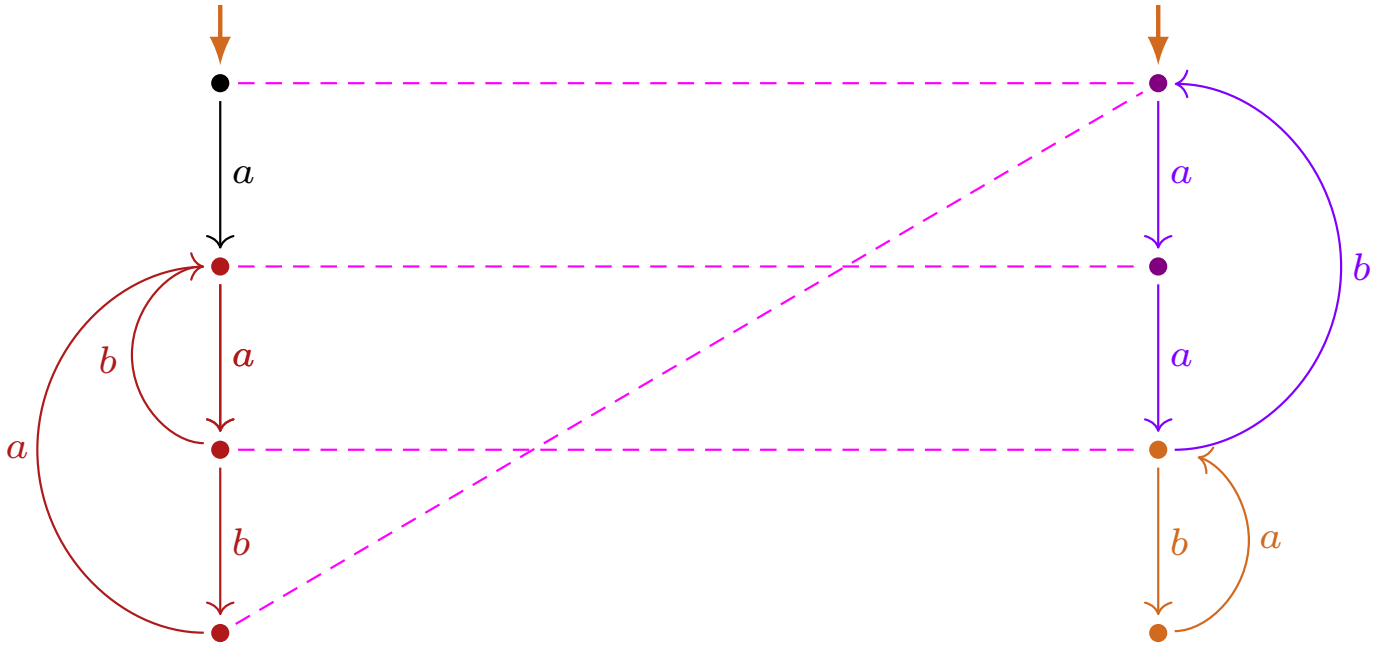

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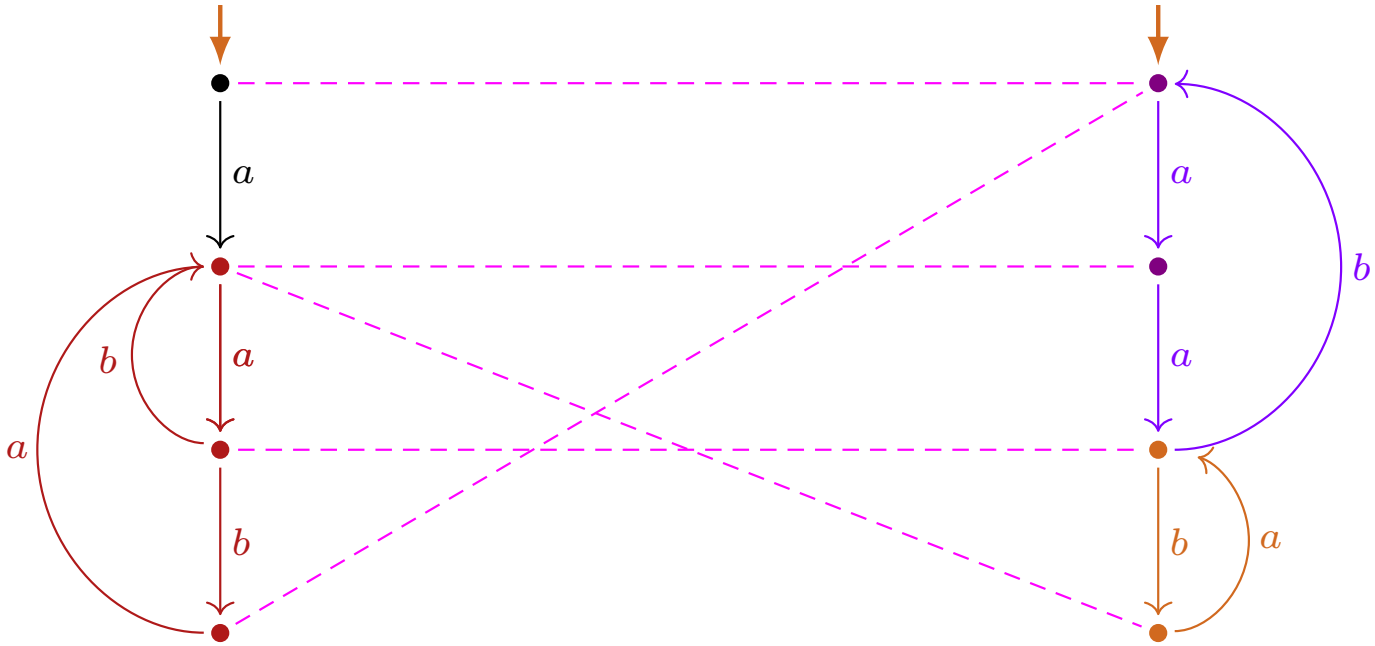

$[\![a(a(b+ba))^*0]\!]_{\boldsymbol{P}}$

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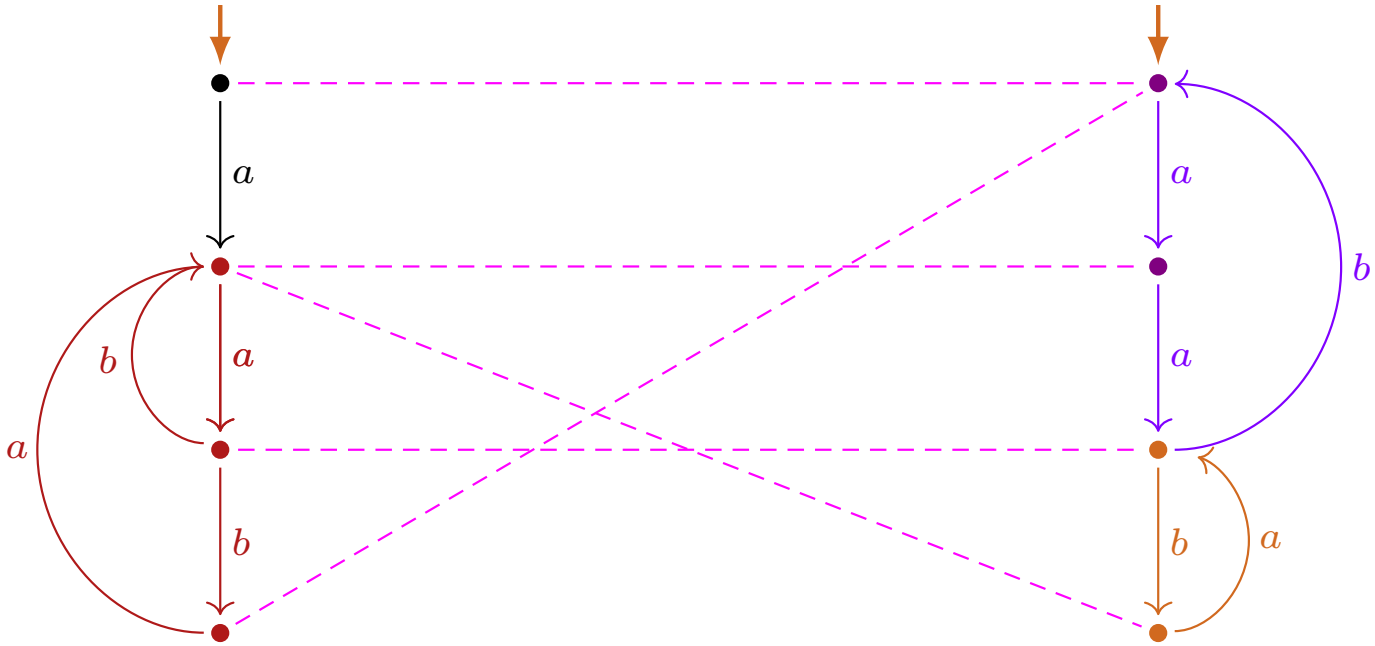
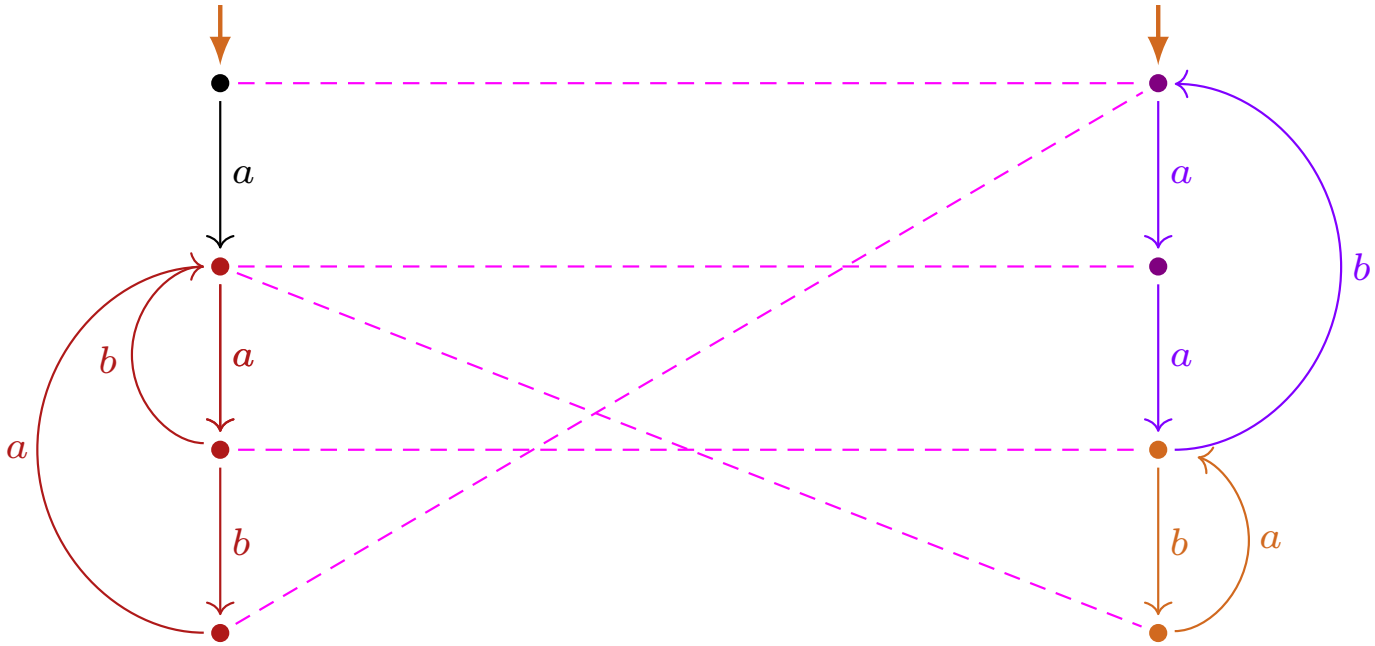


$$[\![a(a(b+ba))^*0]\!]_{\boldsymbol{P}} \quad \leftrightarrow \quad [\![(aa(ba)^*b)^*0]\!]_{\boldsymbol{P}}$$

Process interpretation of regular expressions



$$a(a(b+ba))^*0 \quad \underline{\leftrightarrow}_{P} \quad (aa(ba)^*b)^*0$$

Process graphs and NFAs

Definition

A process graph over actions in A is a tuple $G = \langle V, v_s, T, E \rangle$ where:

- V is a set of *vertices*,
- $v_s \in V$ is the *start vertex*,
- $T \subseteq V \times A \times V$ the set of *transitions*,
- $E \subseteq V \times \{\downarrow\}$ the set of *termination extensions*.

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that are studied under bisimulation, not under language equivalence.

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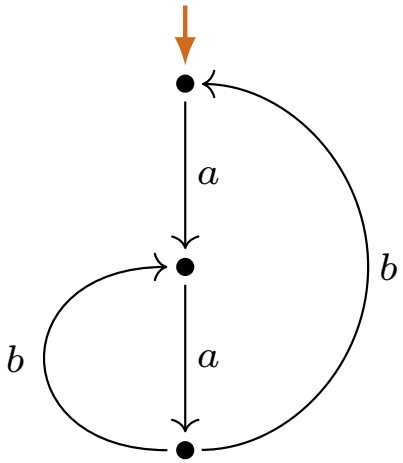
Antimirov (1996): NFA-definition of $[\![\cdot]\!]_P$ via partial derivatives.

Expressible process graphs (under bisimulation $\leftrightarrows$)



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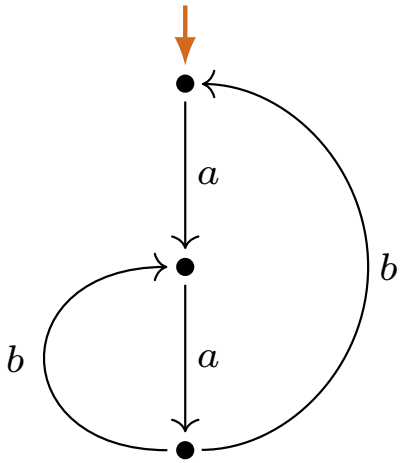

$$\notin im([\![\cdot]\!]_{\boldsymbol{P}})$$

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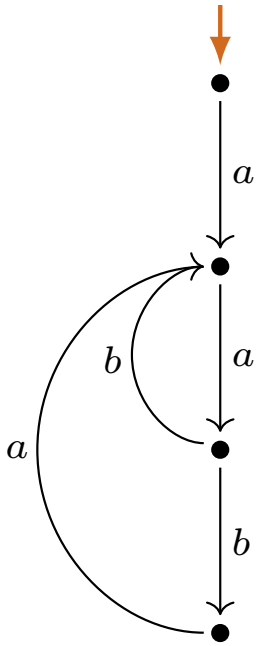
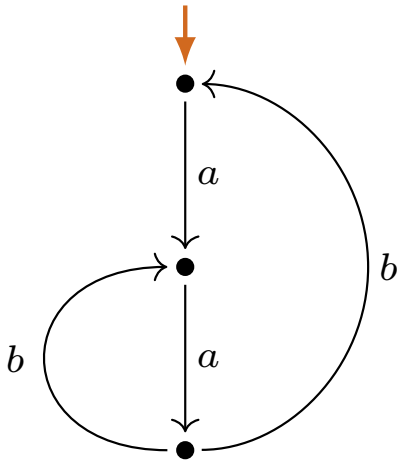


$\in im([\![\cdot]\!]_{\boldsymbol{P}})$

$[\![\cdot]\!]_{\boldsymbol{P}}$-expressible



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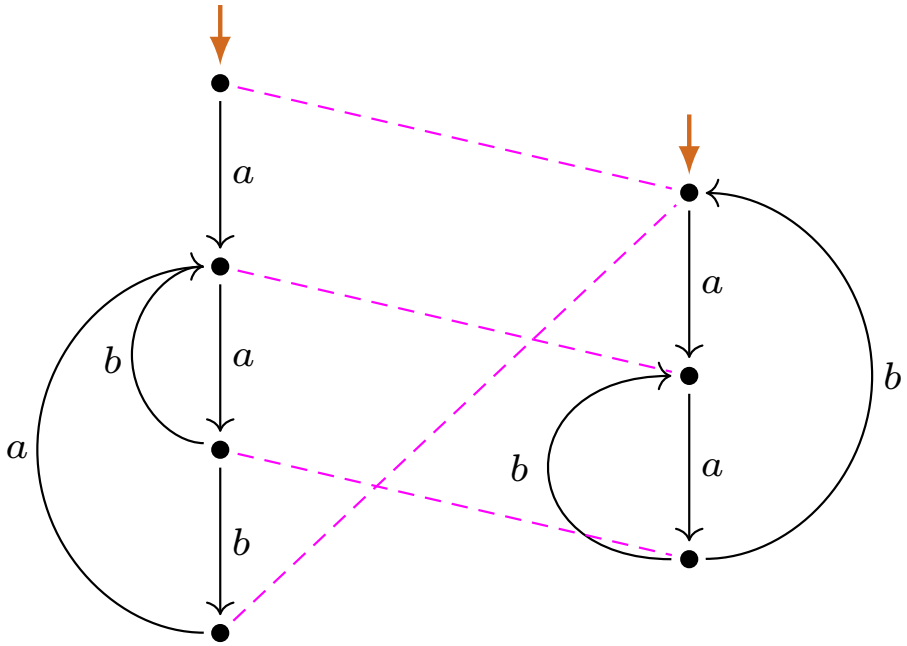
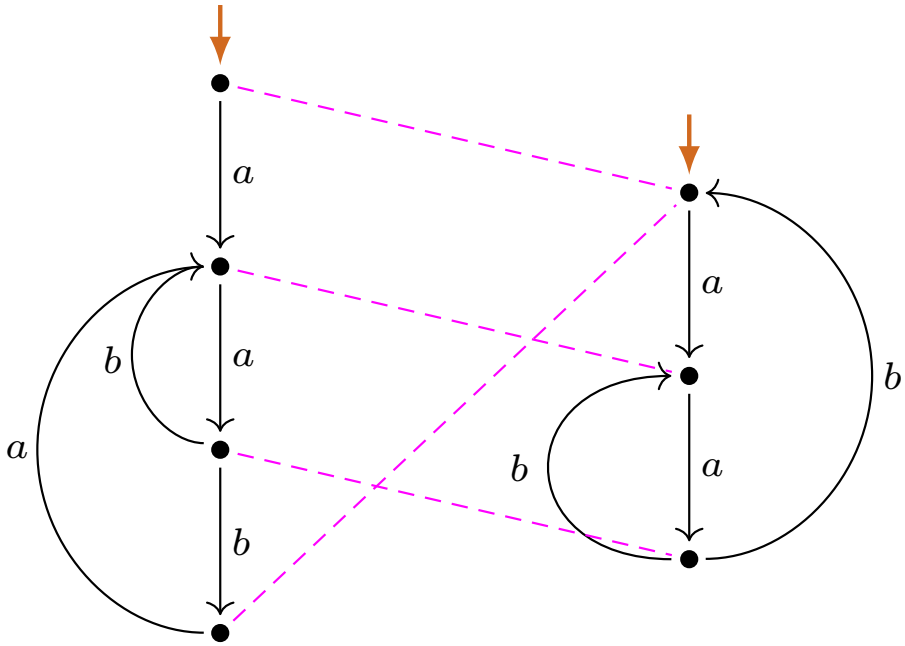


$\in im([\![\cdot]\!]_P)$ $\quad\quad$ $\notin im([\![\cdot]\!]_P)$

$[\![\cdot]\!]_P$-expressible

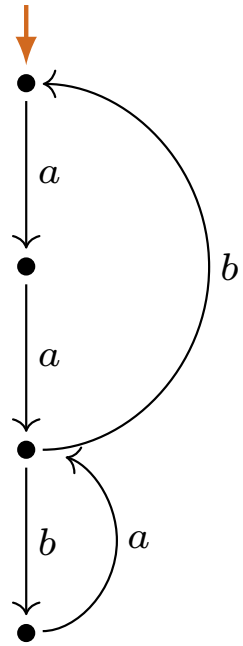
Expressible process graphs (under bisimulation $\leftrightarrows$)



$\in im(\llbracket\cdot\rrbracket_{\boldsymbol{P}})$

$\llbracket\cdot\rrbracket_{\boldsymbol{P}}$-expressible

$\notin im(\llbracket\cdot\rrbracket_{\boldsymbol{P}})$



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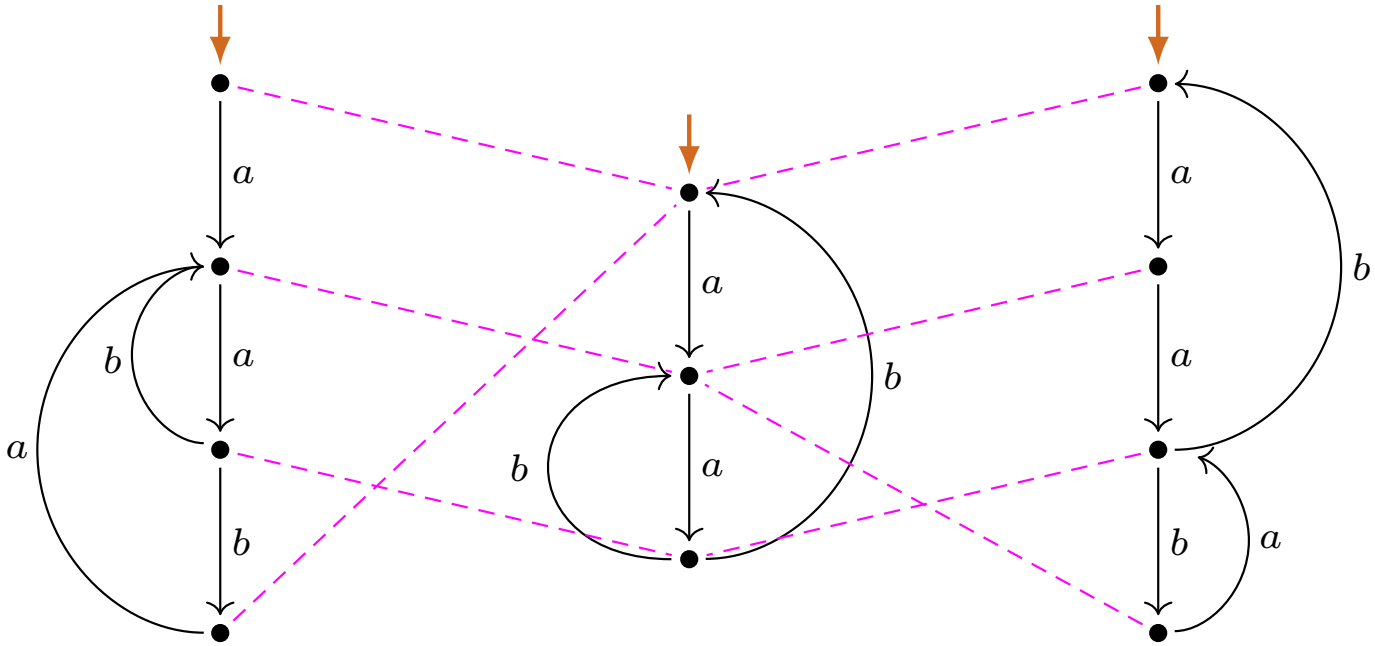

$\in im([\![\cdot]\!]_P)$ $\quad \notin im([\![\cdot]\!]_P)$ $\quad \in im([\![\cdot]\!]_P)$

$[\![\cdot]\!]_P$-expressible $\qquad$ $[\![\cdot]\!]_P$-expressible

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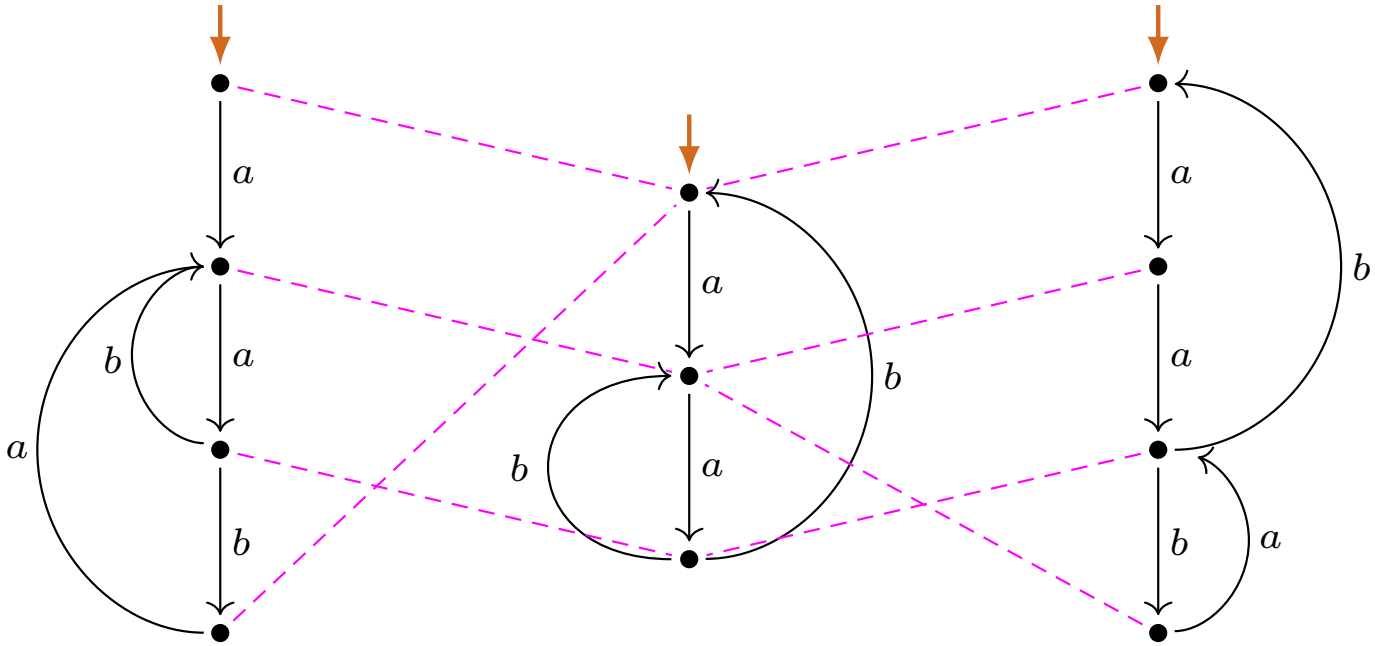
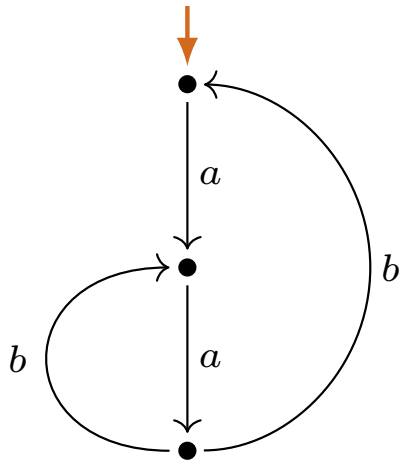


$\in im([\![\cdot]\!]_P)$ $\quad$ $\notin im([\![\cdot]\!]_P)$ $\quad$ $\in im([\![\cdot]\!]_P)$

$[\![\cdot]\!]_P$-expressible $\quad$ $[\![\cdot]\!]_P$-expressible modulo $\leftrightarrows$ $\quad$ $[\![\cdot]\!]_P$-expressible

Properties of P

- Not every finite-state process is $[\![\cdot]\!]_P$-expressible.



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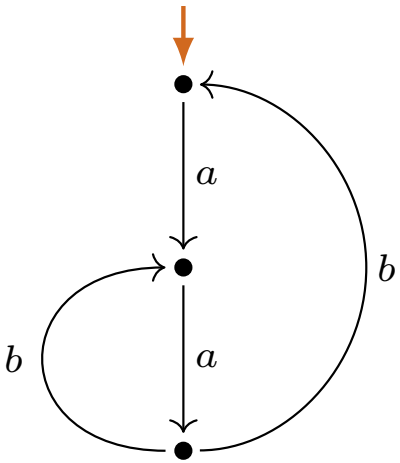

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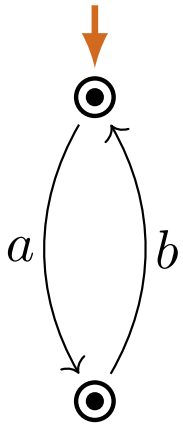

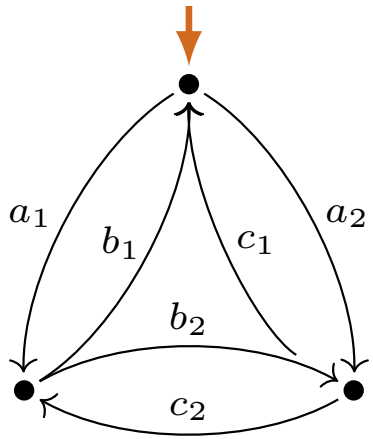

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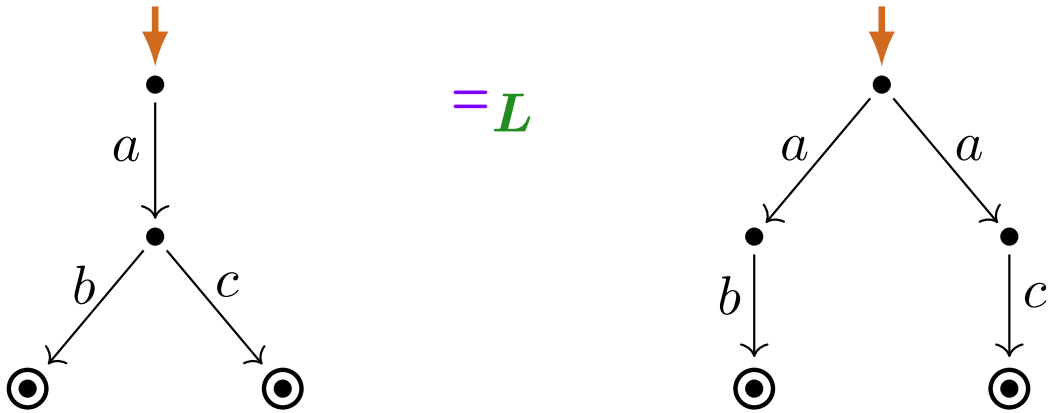

$a \cdot (b + c)$ $=_L$ $a \cdot b + a \cdot c$

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$a \cdot (b + c) \quad \not\underline{\leftrightarrow}_P \quad a \cdot b + a \cdot c$

Salomaa's axiomatization of $=_L$ (products commuted)

Axioms :

(B1) $e + (f + g) = (e + f) + g$

(B2) $(e \cdot f) \cdot g = e \cdot (f \cdot g)$

(B3) $e + f = f + e$

(B4) $(e + f) \cdot g = e \cdot g + f \cdot g$

(B5) $e \cdot (f + g) = e \cdot f + e \cdot g$

(B6) $e + e = e$

(B7) $e \cdot 1 = e$

(B8) $e \cdot 0 = 0$

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Inference rules : equational logic *plus*

$$\frac{e = f \cdot e + g}{e = f^* \cdot g}\ \text{FIX} \quad (\text{if } \underbrace{\{\epsilon\} \notin [\![f]\!]_L}_{\textit{non-empty-word property}})$$

Sound and unsound axioms with respect to $\underline{\leftrightarrow}_P$

Axioms :

(B1)	$e + (f + g) = (e + f) + g$	(B7)	$e \cdot 1 = e$
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(B5)	$e \cdot (f + g) = e \cdot f + e \cdot g$	(B11)	$e^* = (1 + e)^*$
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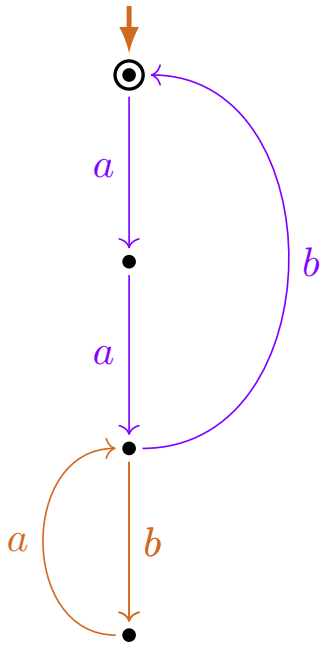
- Mil$^-$ + one of two stronger rules (than RSP*) is complete *(G, 2006)*
  - with a coinductive rule (based on *Antimirov's* partial derivatives)
  - with a unique solvability principle USP

Well-behaved form, looping palm trees



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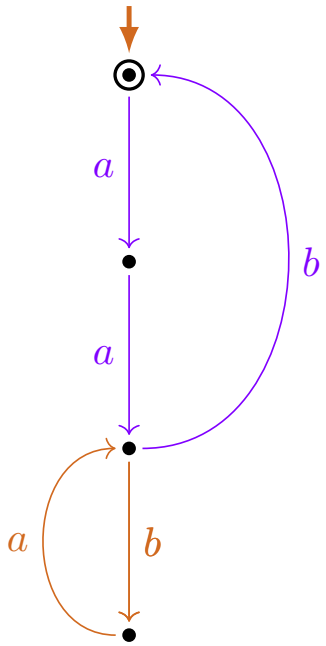

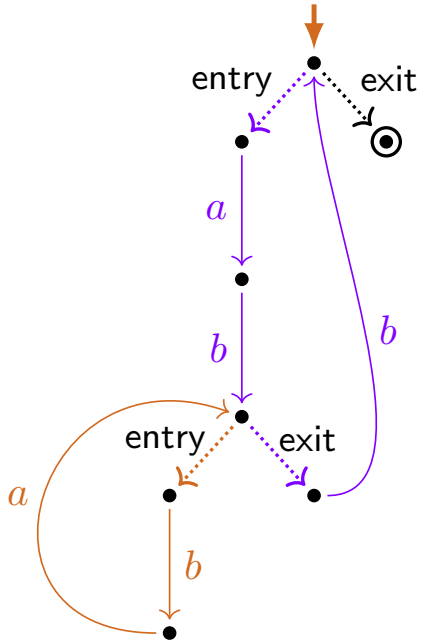

well-behaved form

(Corradini, Baeten)

$[\![(aa(ba)^*b)^*]\!]_{\boldsymbol{P}}$ $[\![(1 \cdot aa(1 \cdot ba)^* \cdot 1 \cdot b)^*(1 \cdot 1)]\!]_{\boldsymbol{P}}$

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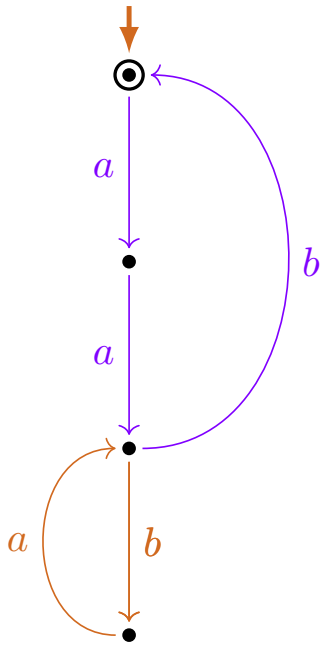

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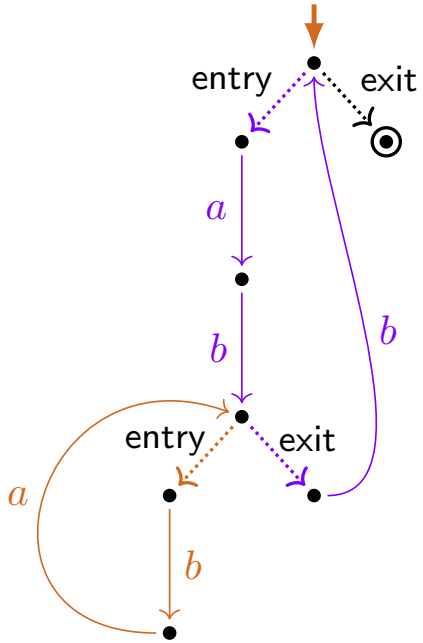

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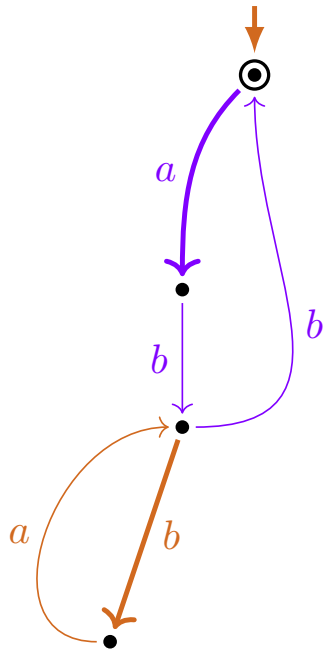

looping palm tree

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Loop chart

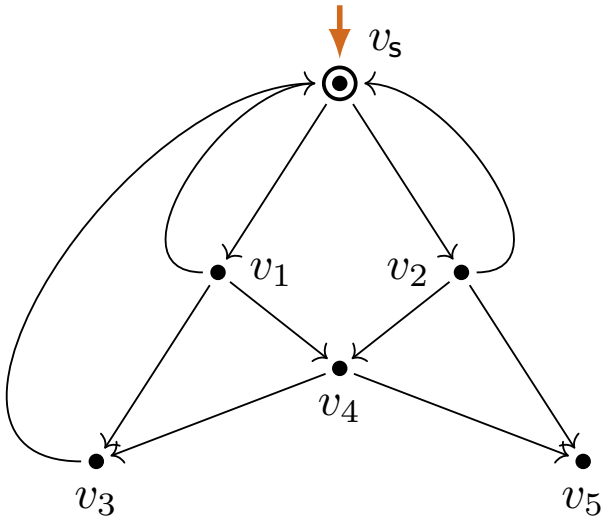
Definition

A process graph is a loop chart if:

L-1.

L-2.

L-3.



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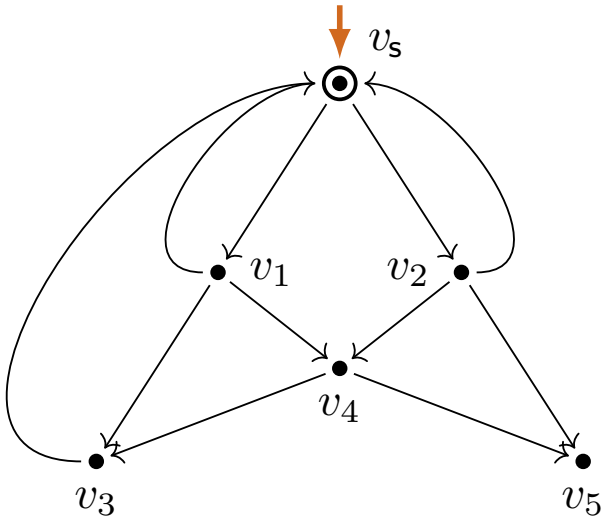

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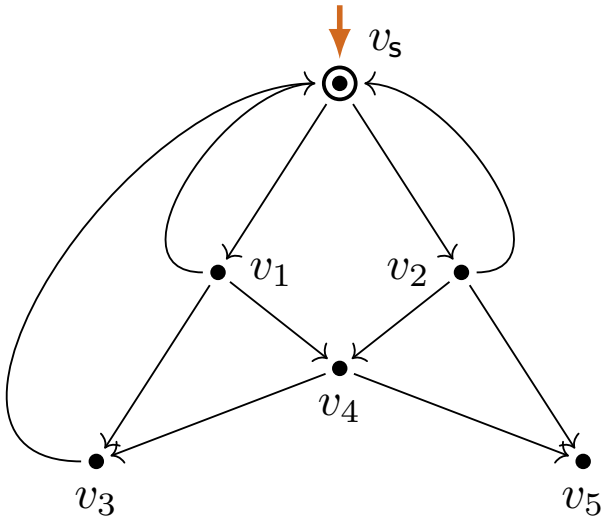

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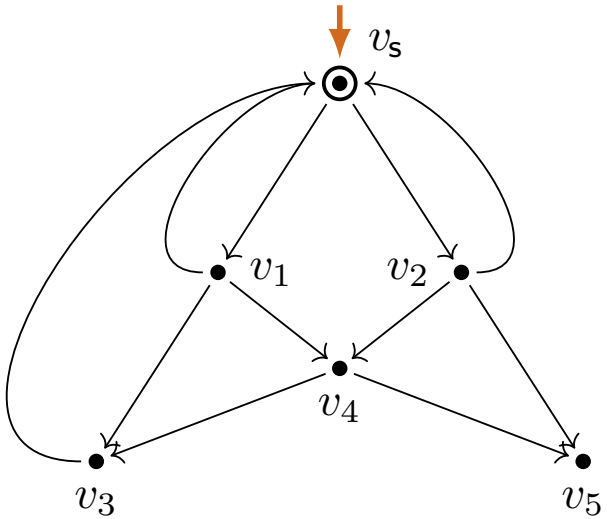

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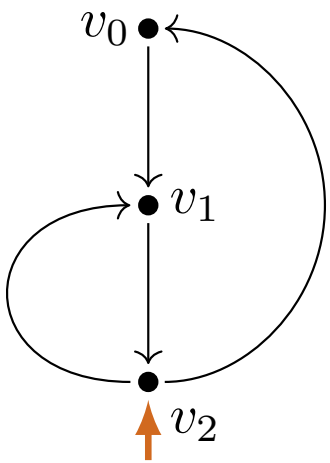

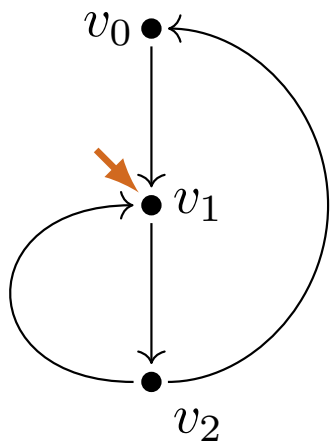

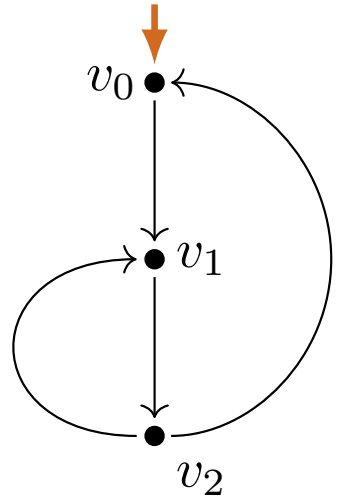

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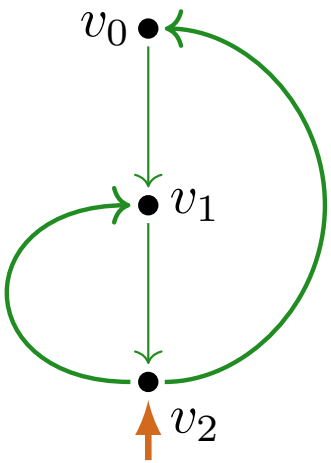

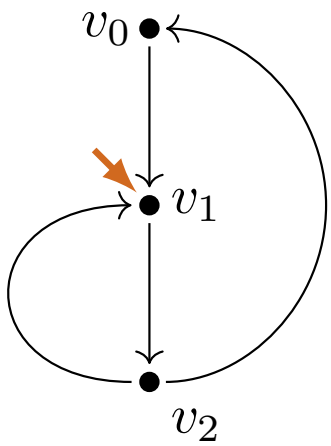

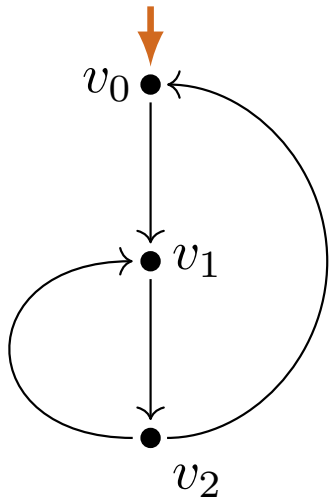

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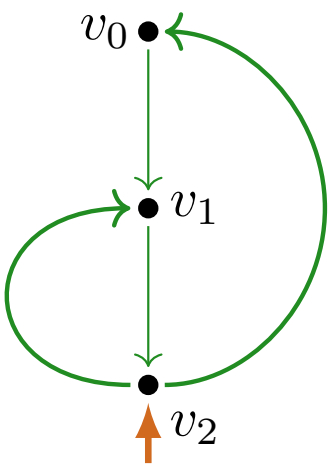

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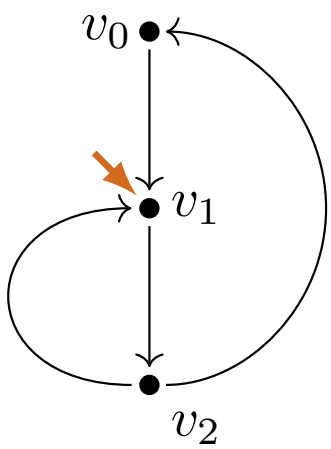

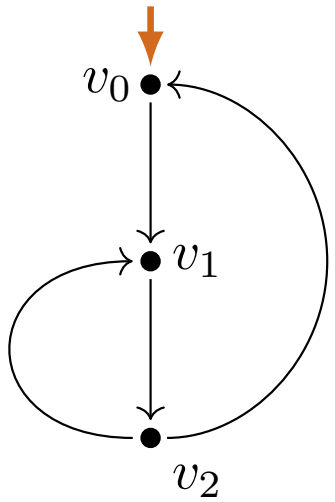

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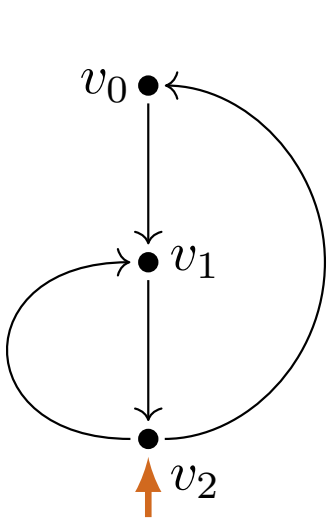

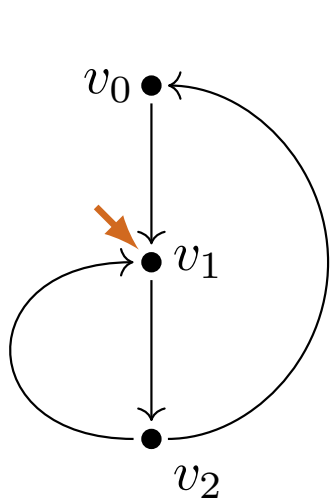

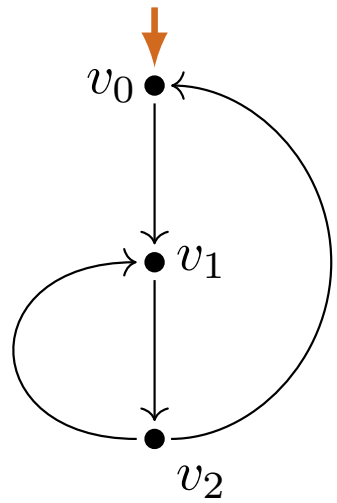

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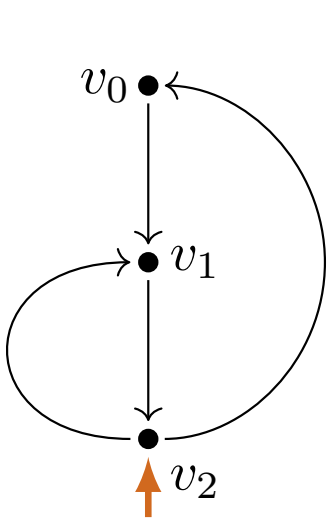

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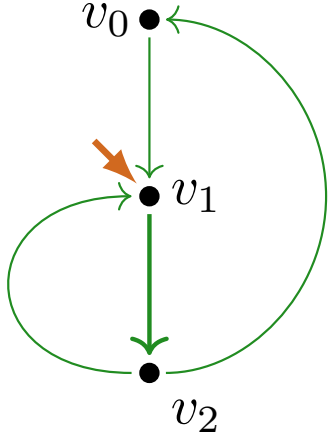

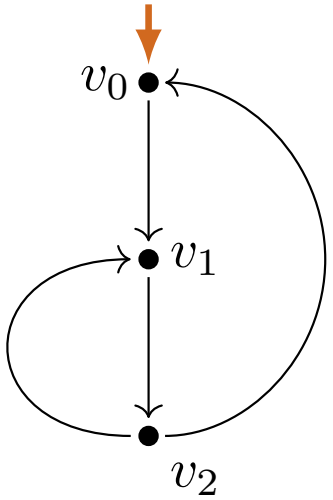

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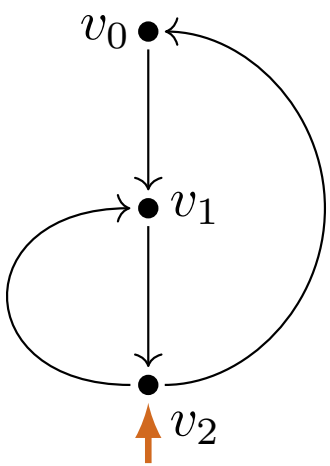

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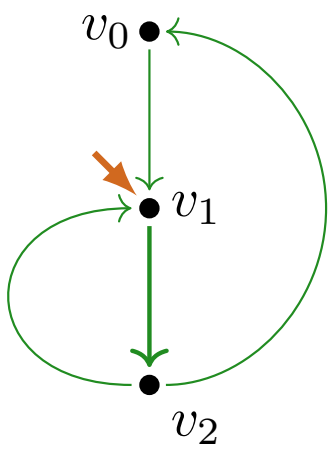

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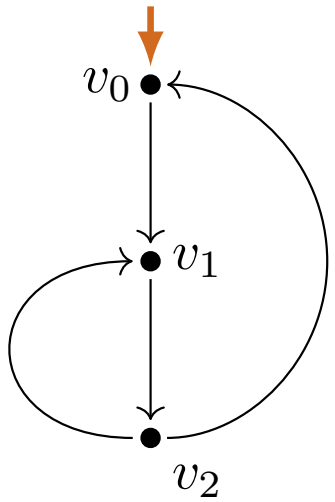

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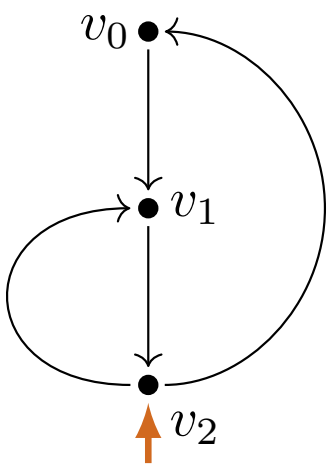
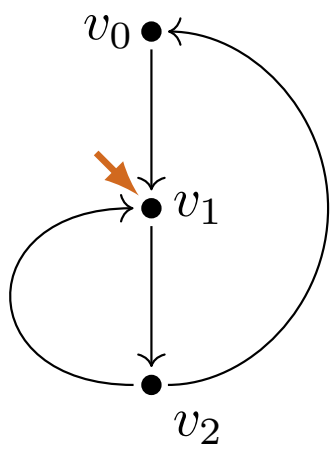


loop chart



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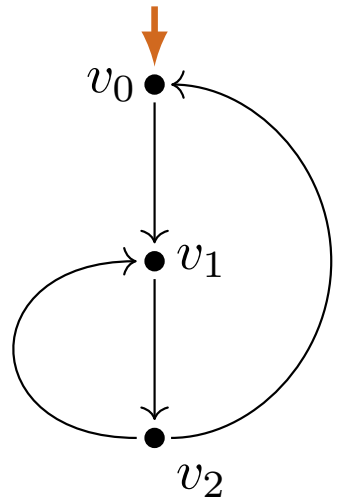

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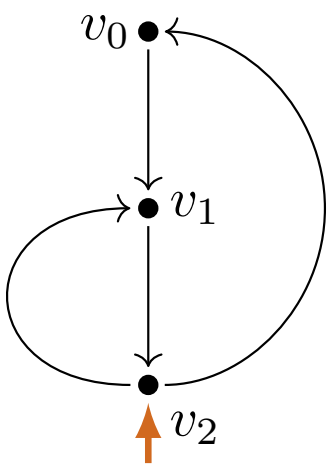

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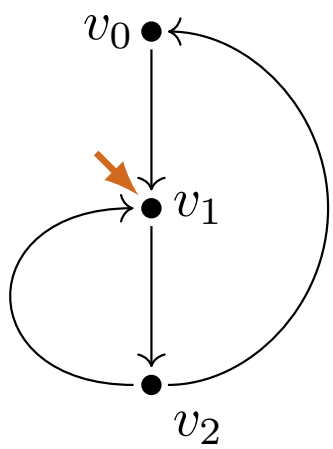

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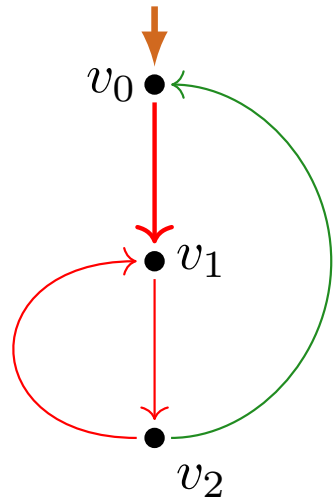

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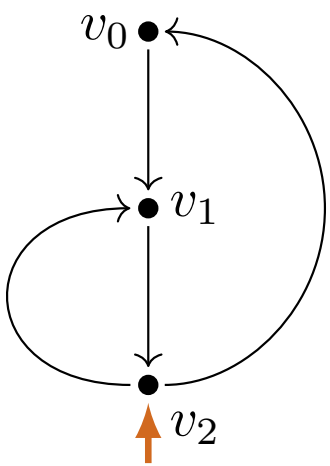

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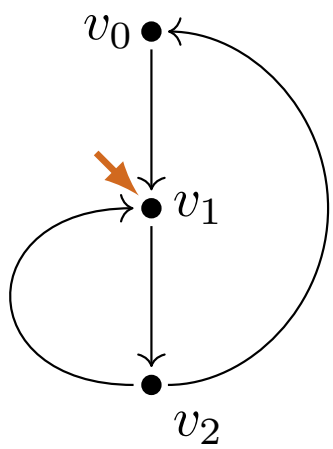

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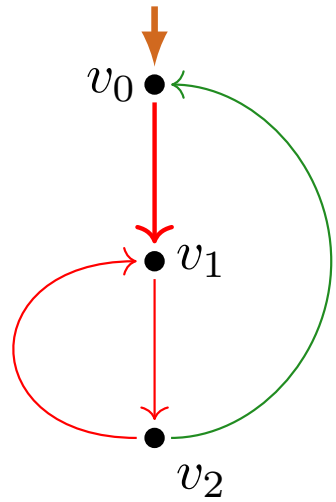

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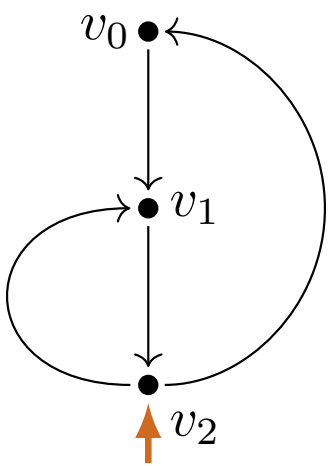

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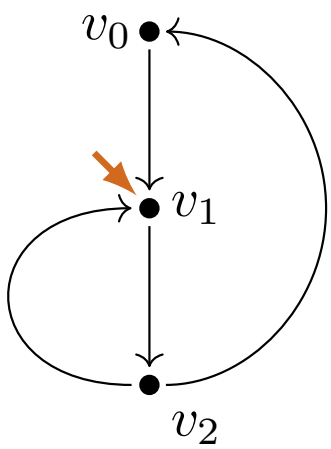

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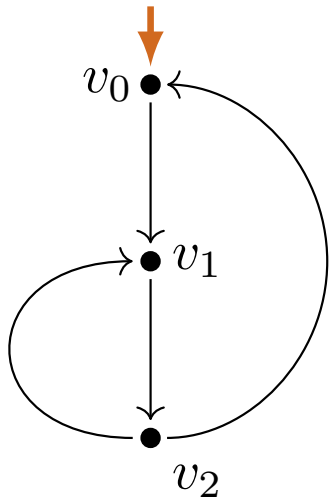

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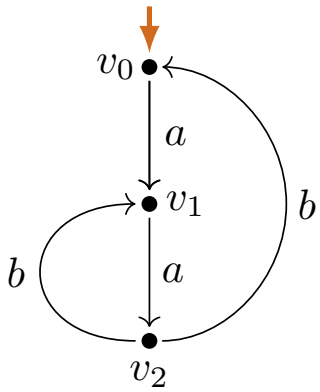
Loop elimination



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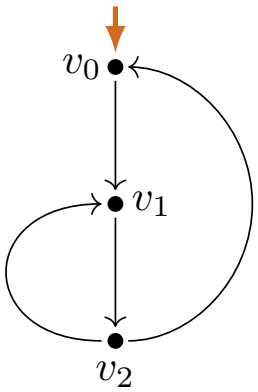

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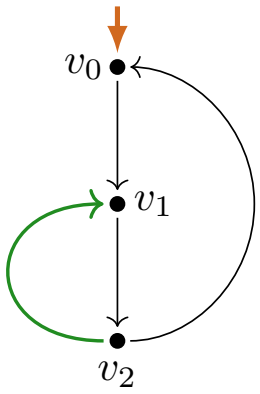

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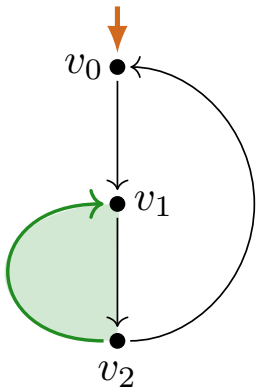

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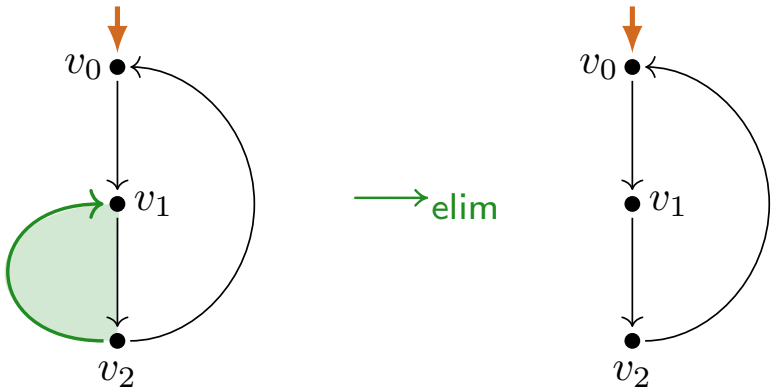

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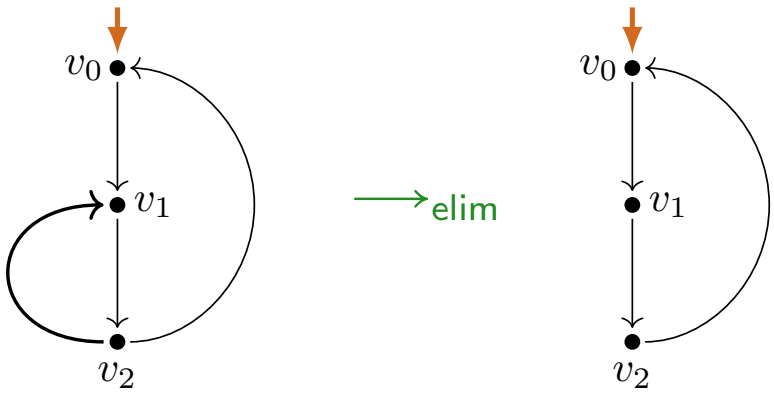

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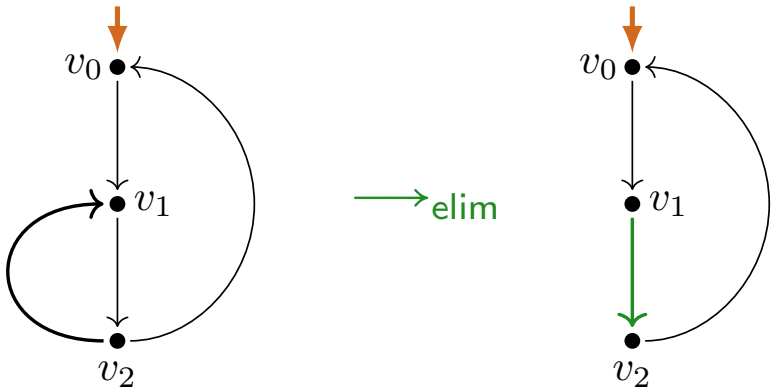

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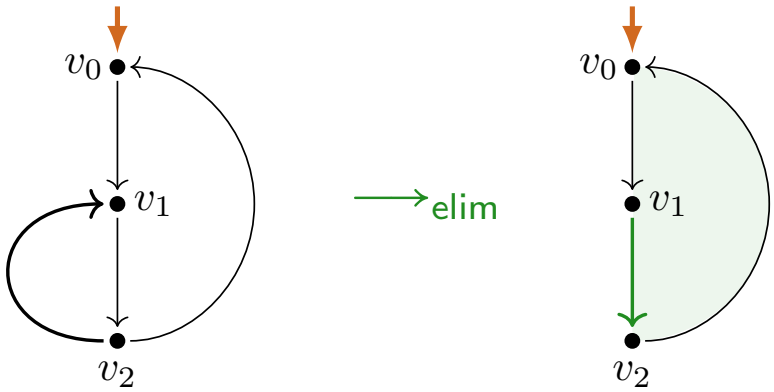

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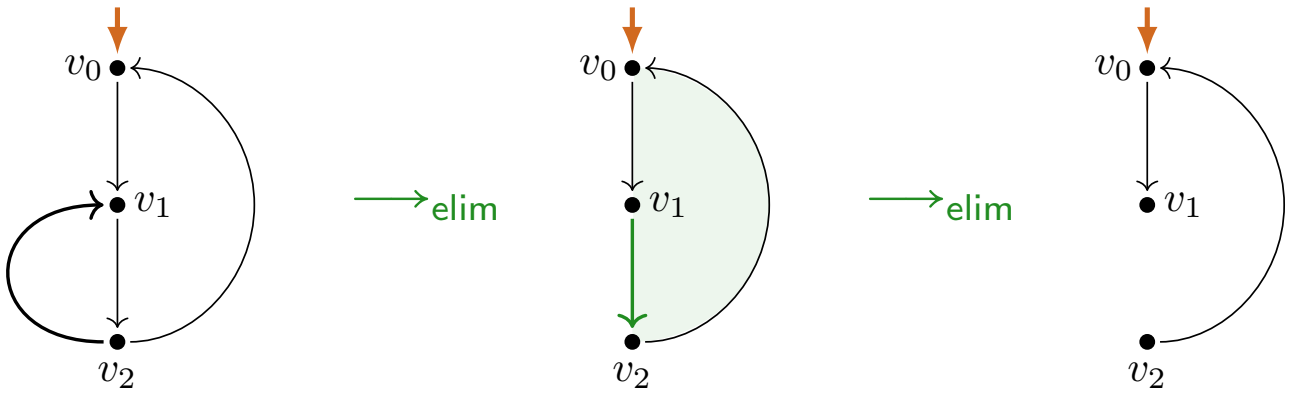

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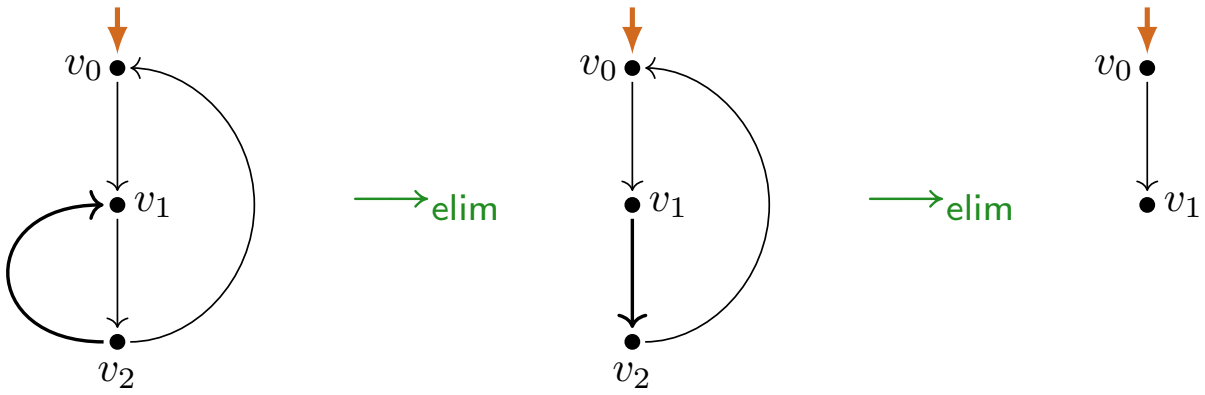

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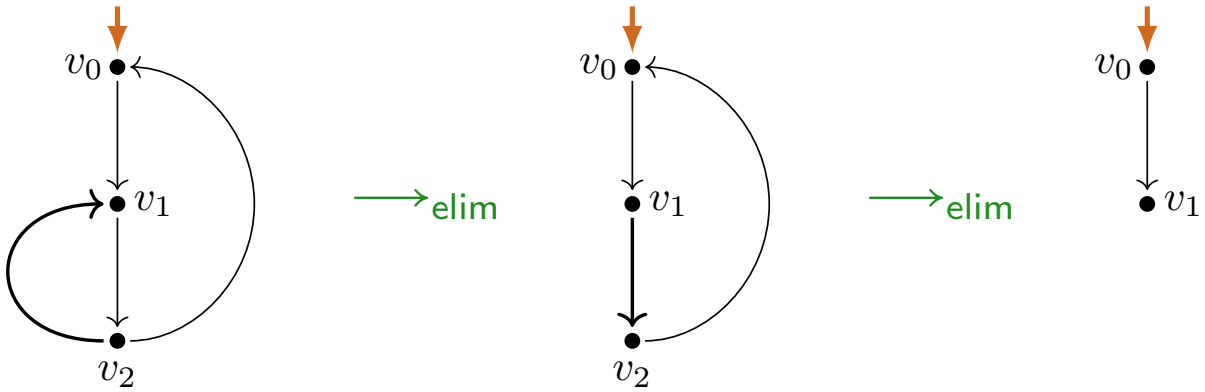

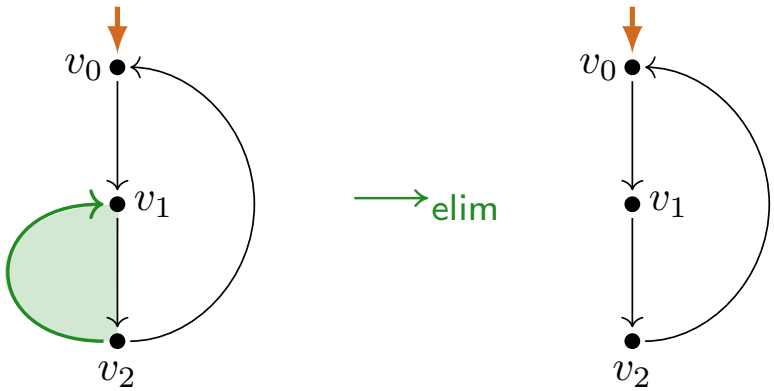

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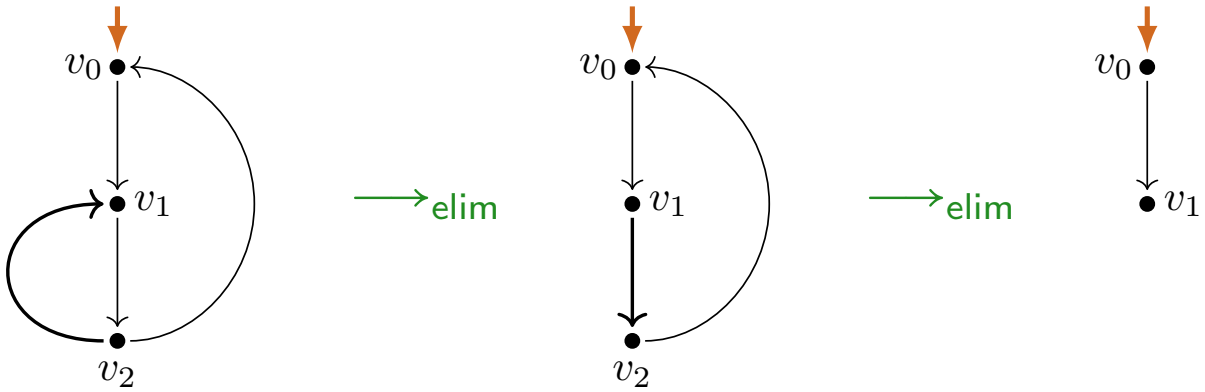

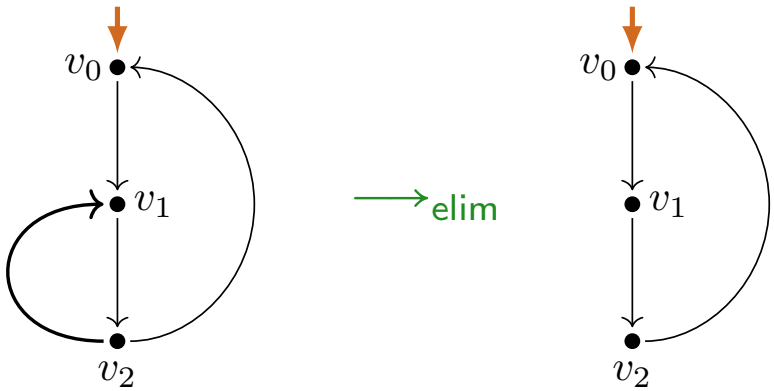

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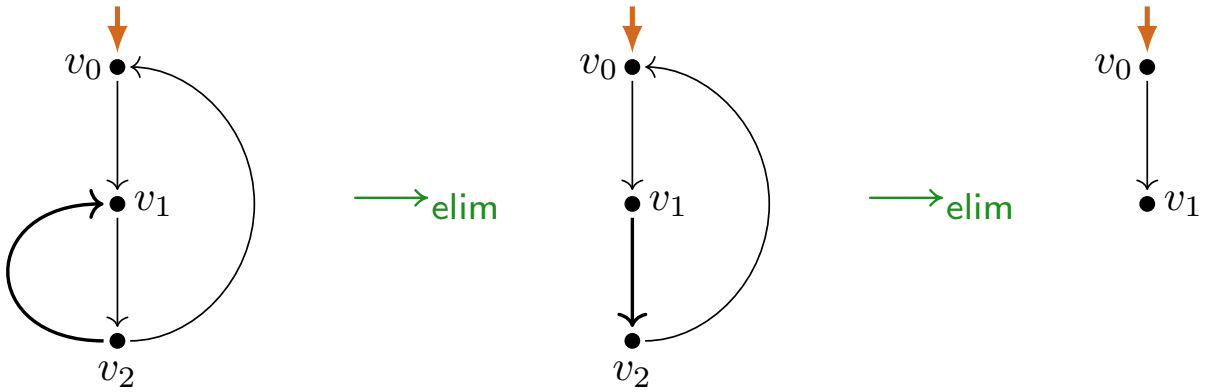

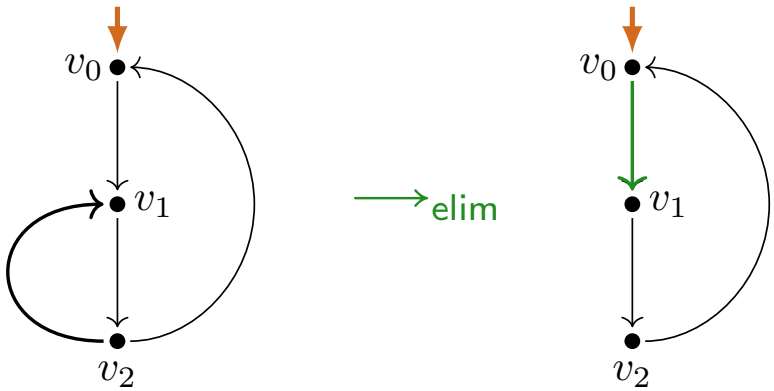

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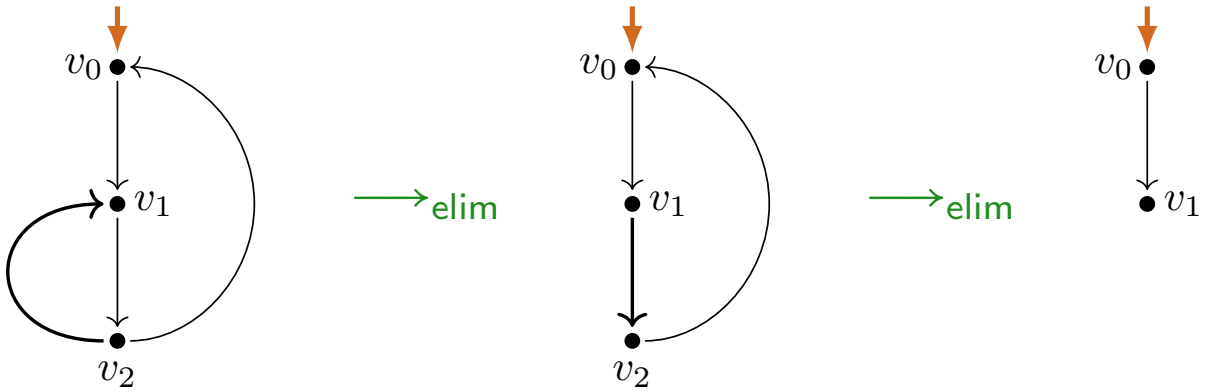

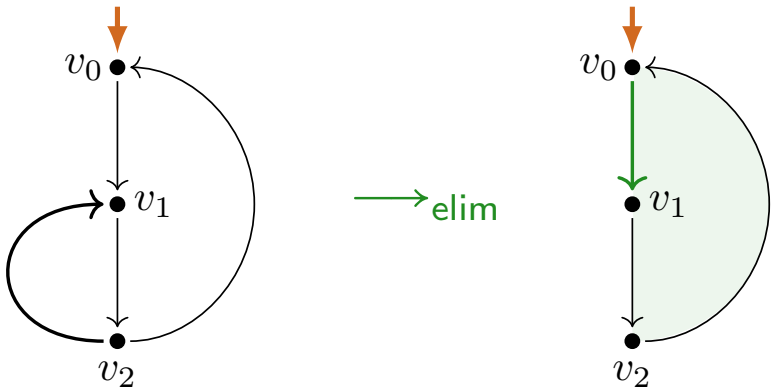

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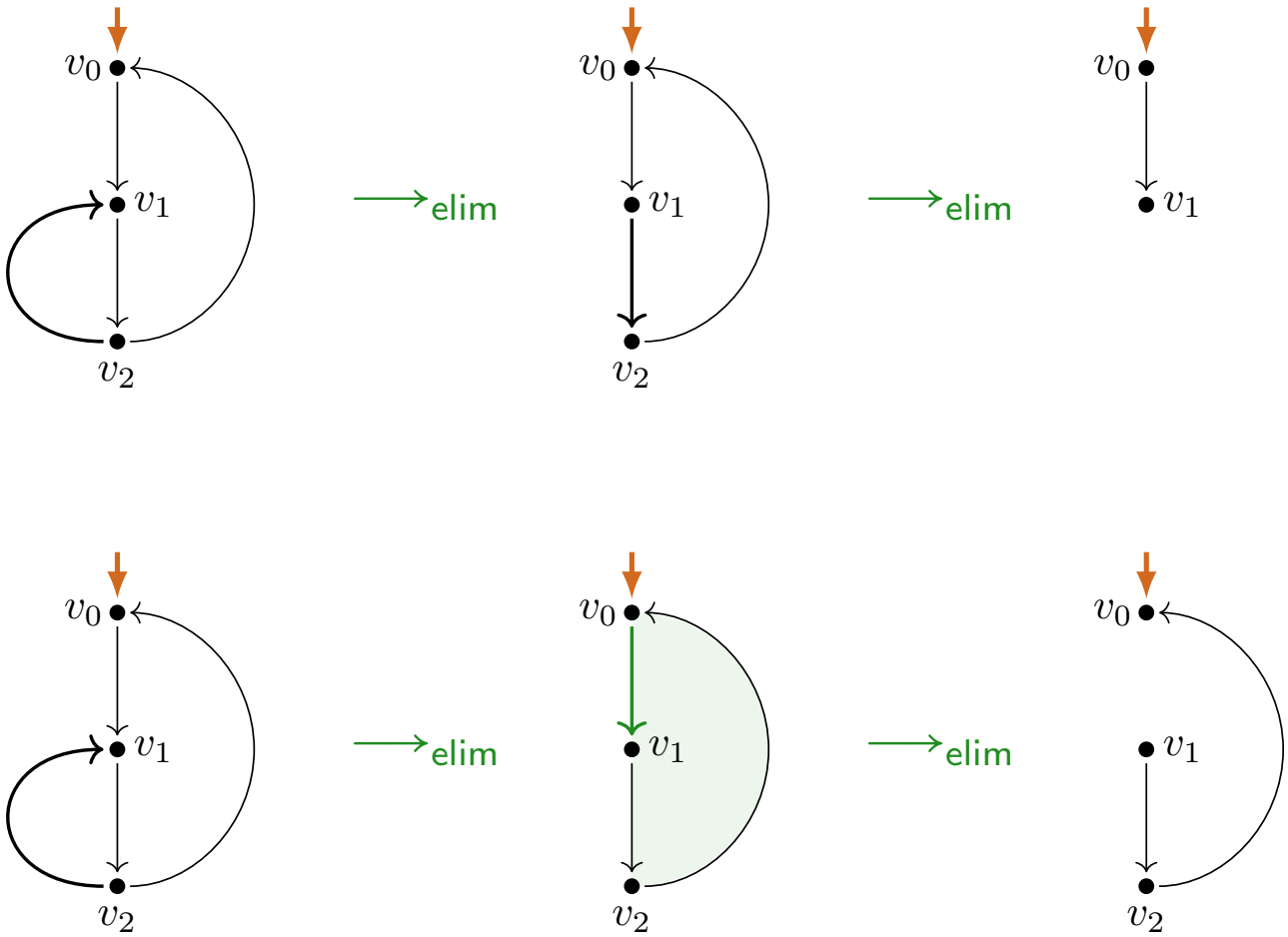

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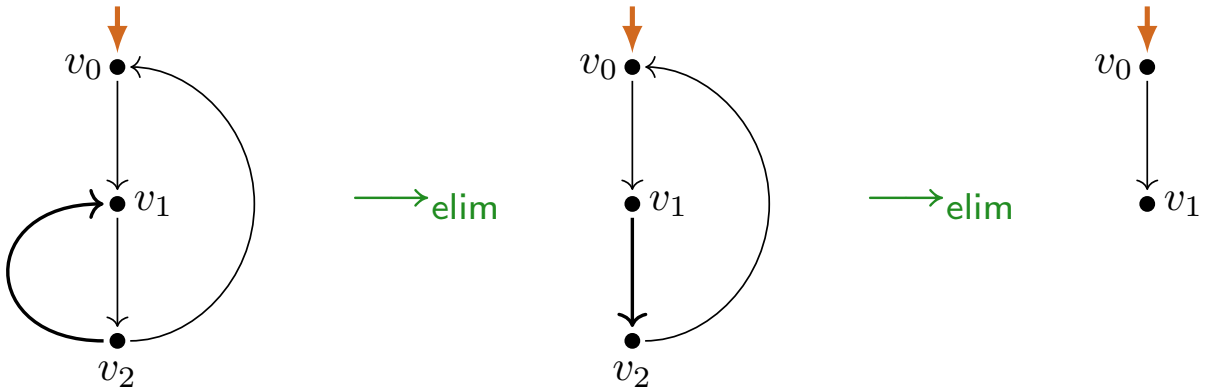

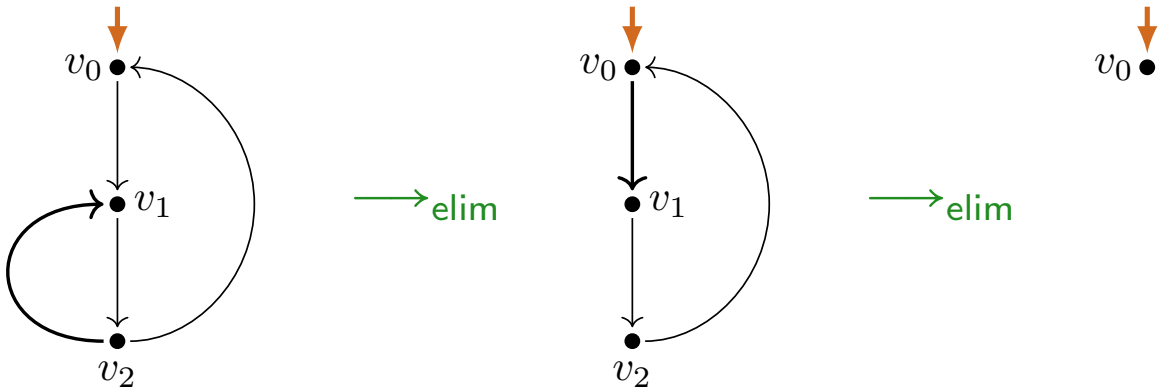

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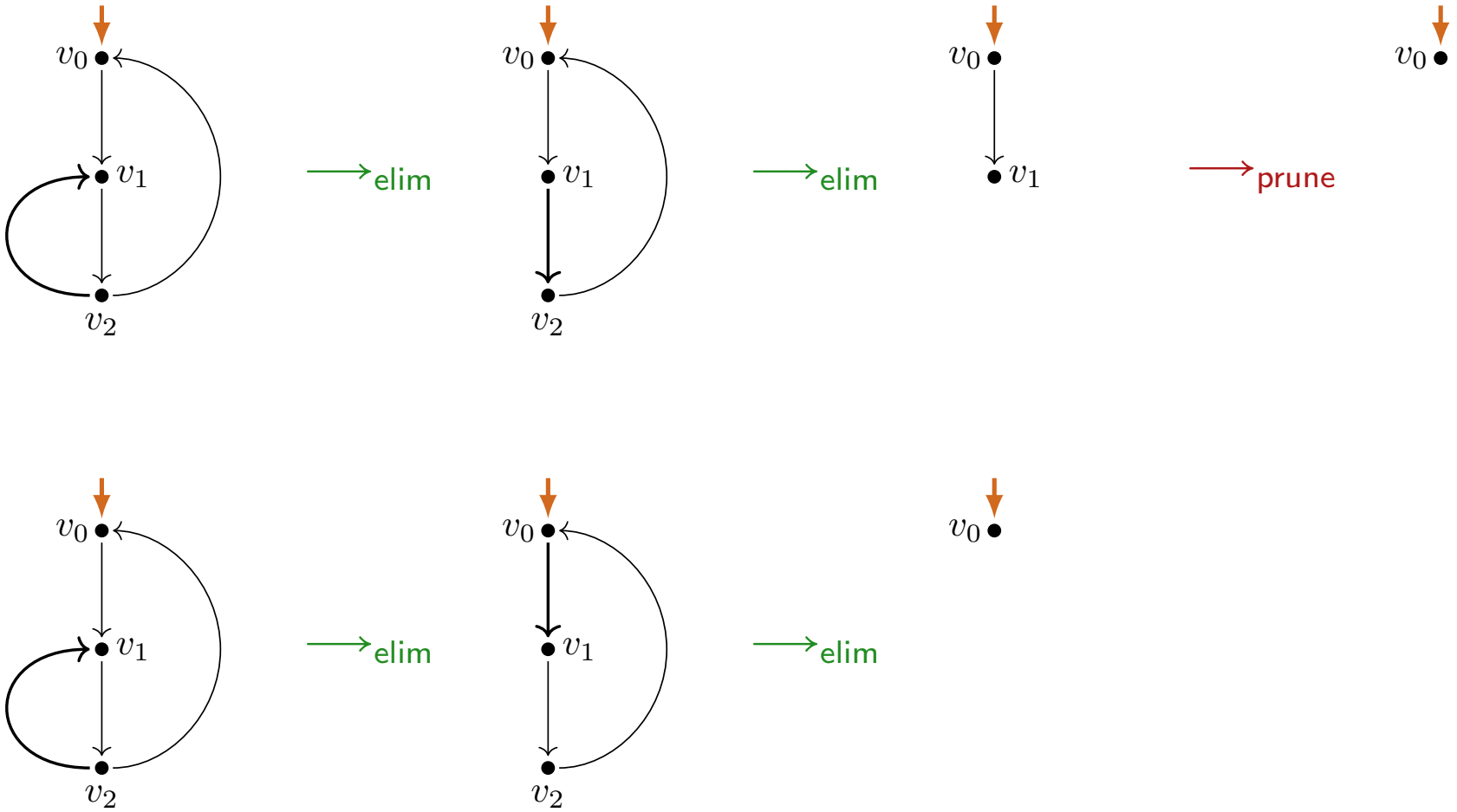

Loop elimination, and properties

$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:

- removing the loop-entry transition(s)
- garbage collection

$\longrightarrow_{\text{prune}}$: remove a transition to a deadlocking state

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Lemma

(i) $\longrightarrow_{\text{elim}}$ *is terminating.*

(ii) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating and confluent.*

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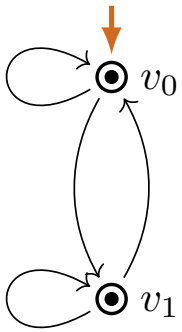

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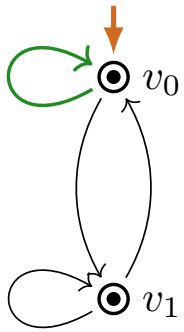

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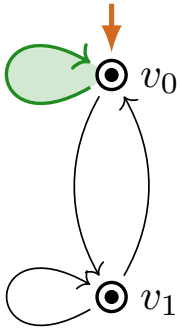

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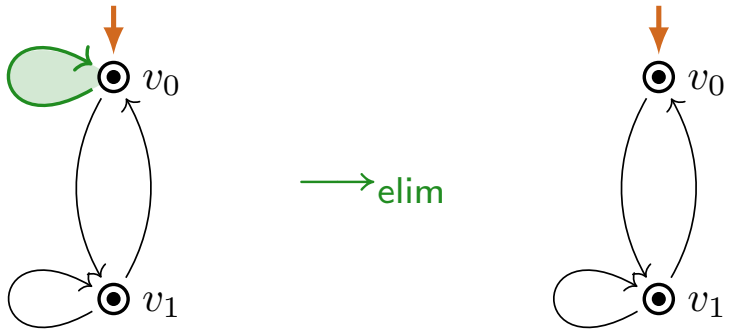

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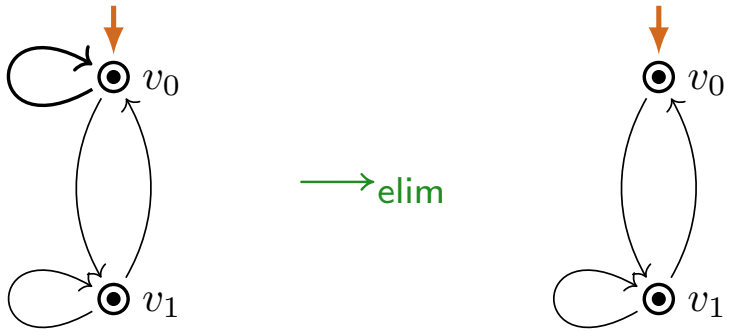

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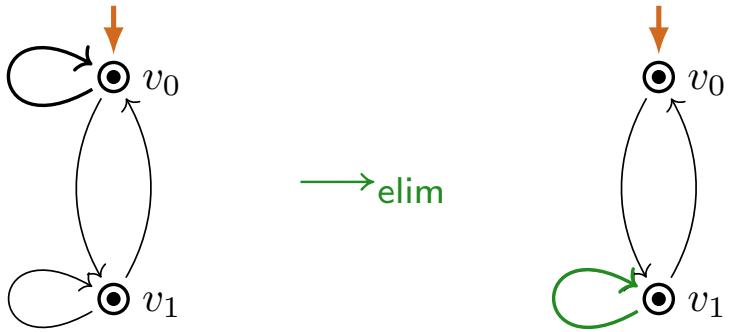

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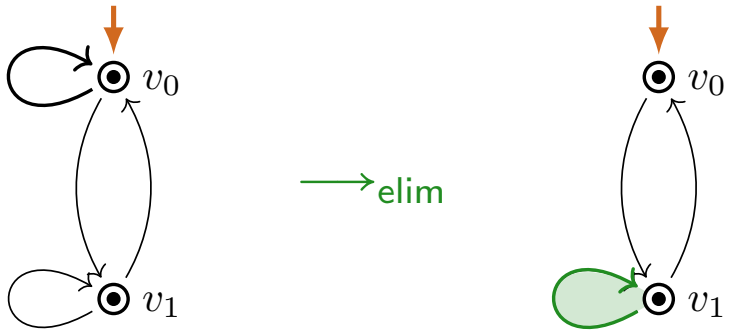

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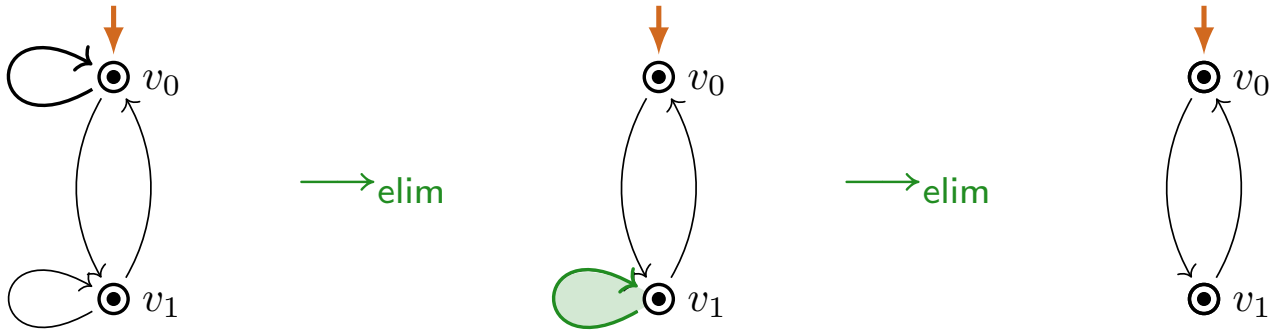

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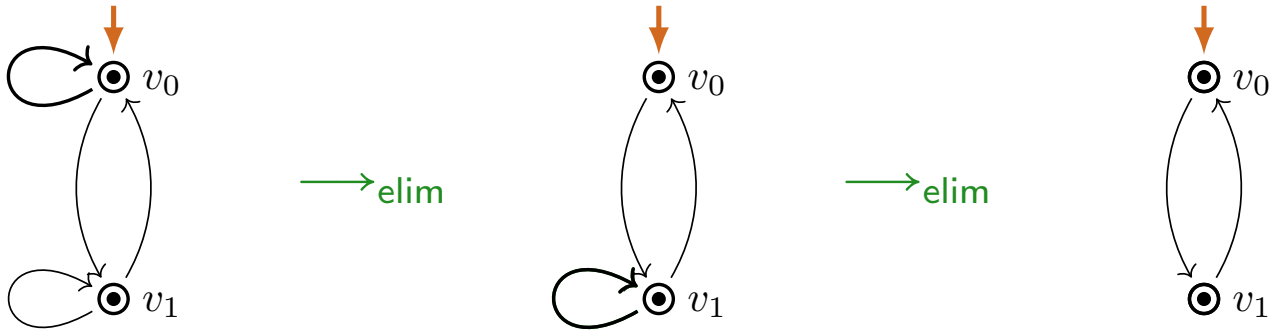

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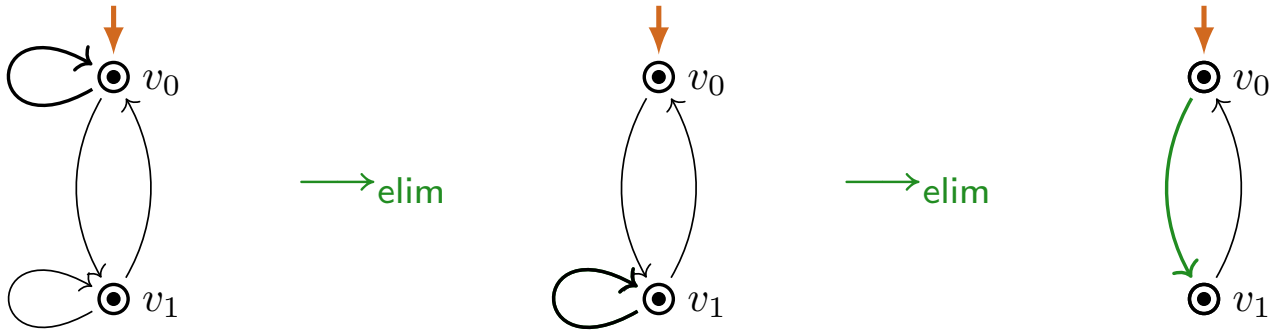
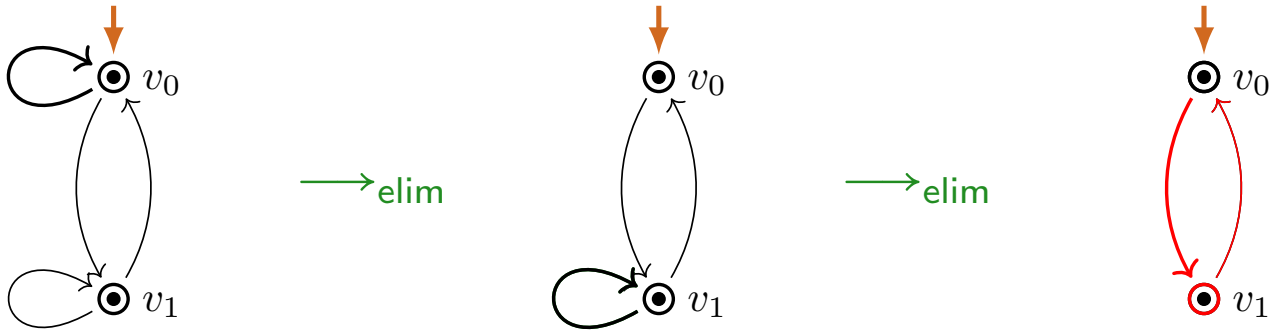


Loop elimination



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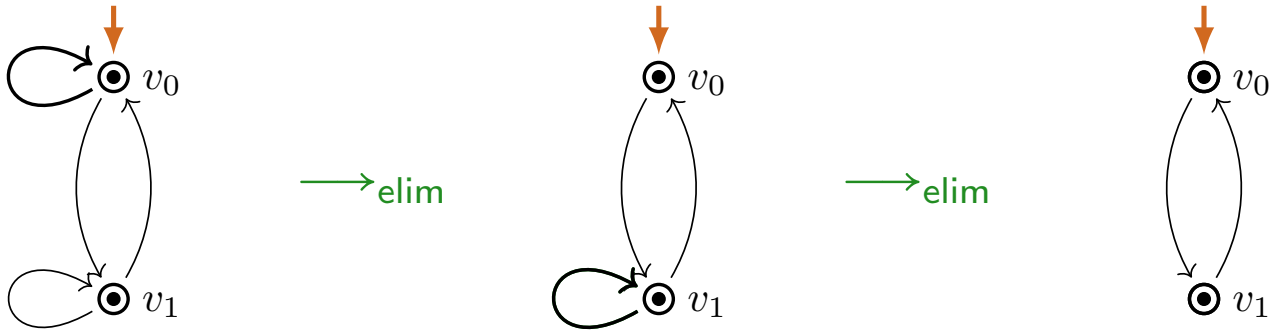

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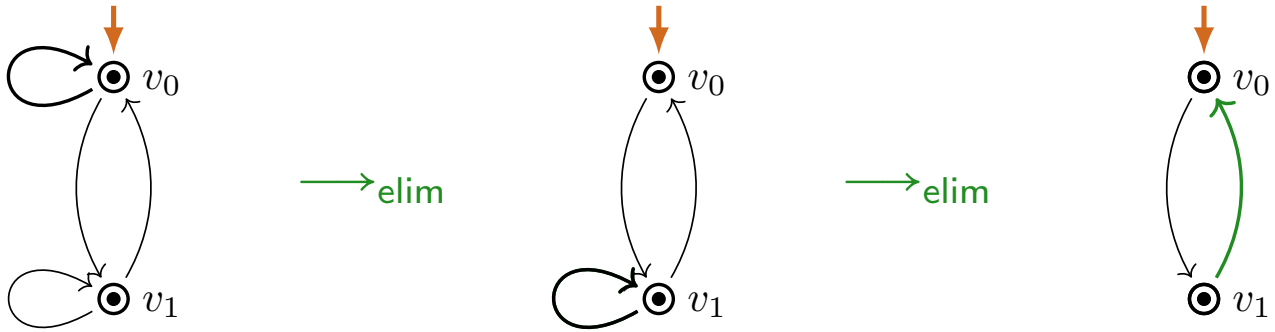

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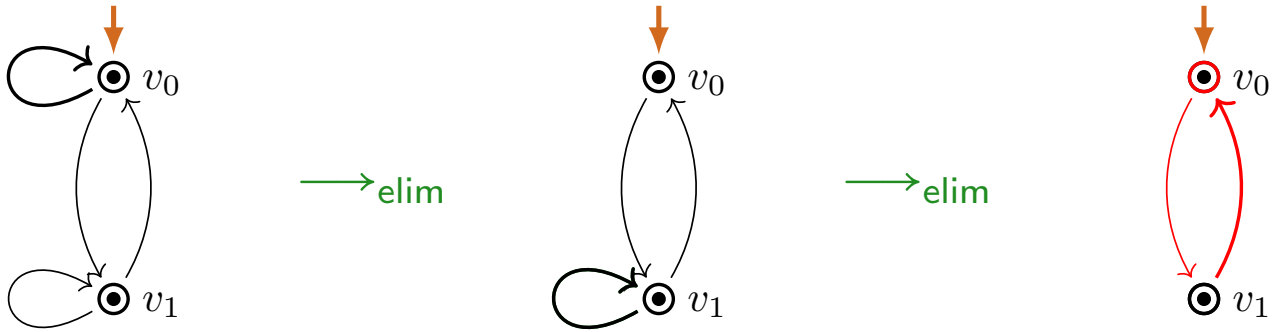
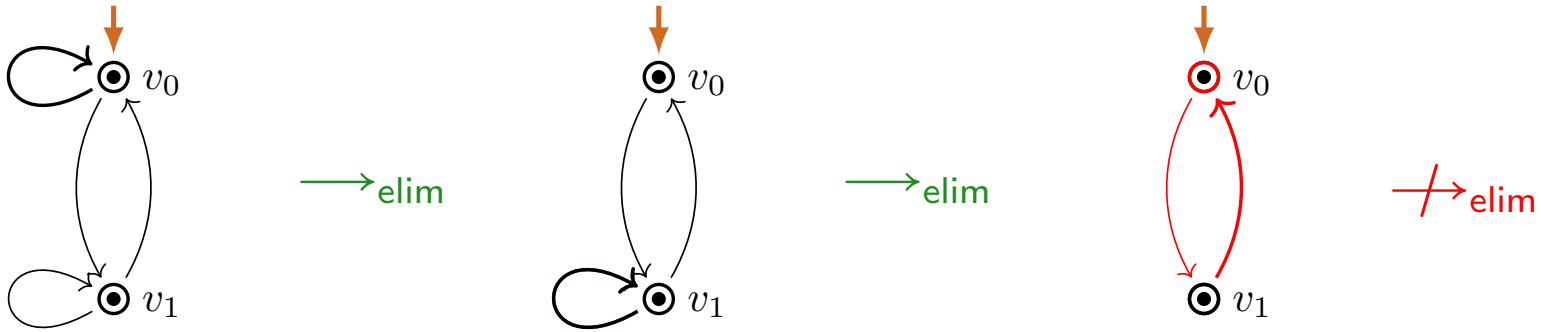


Loop elimination



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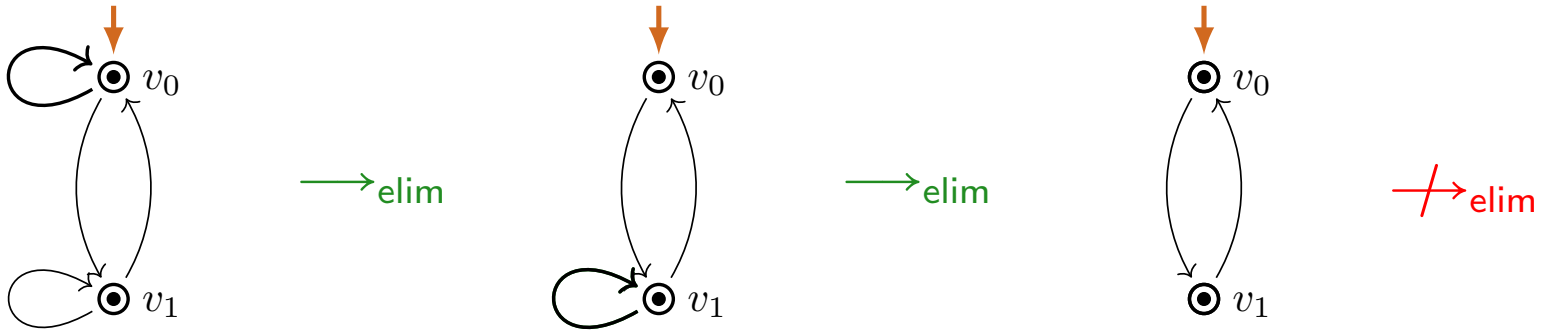

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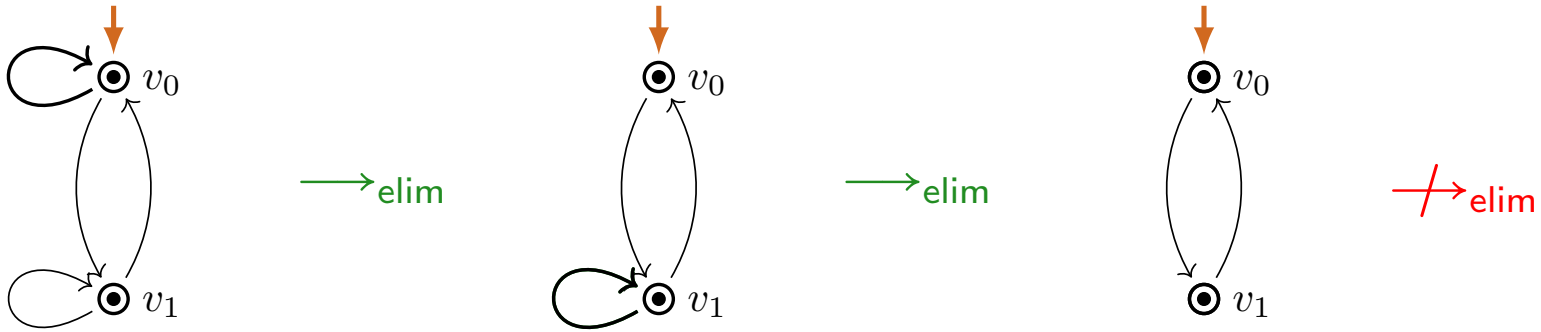

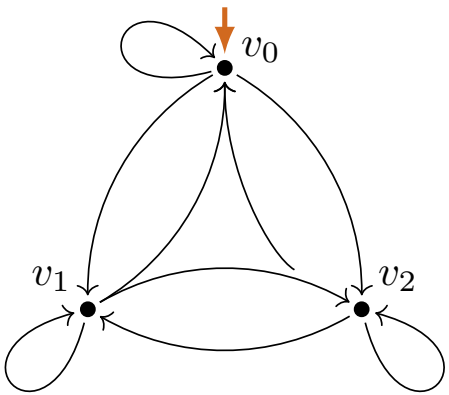

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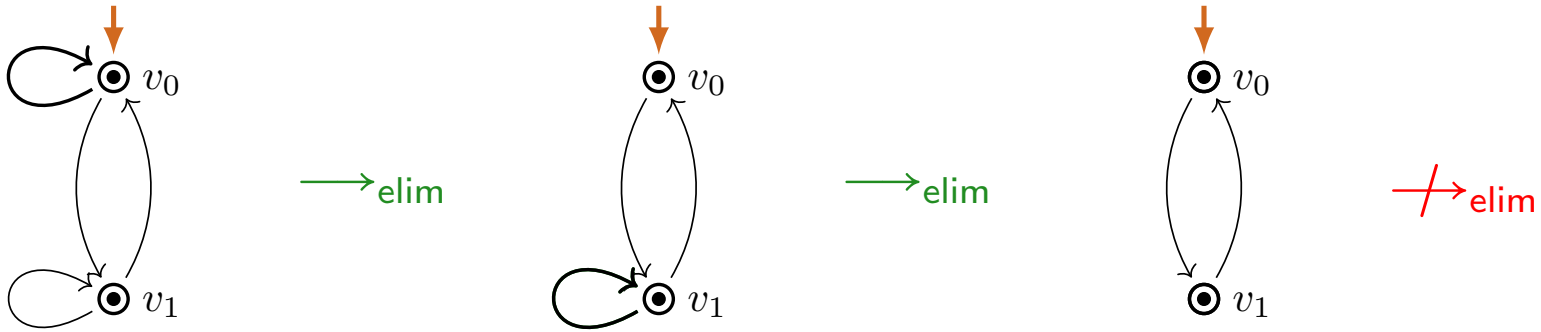

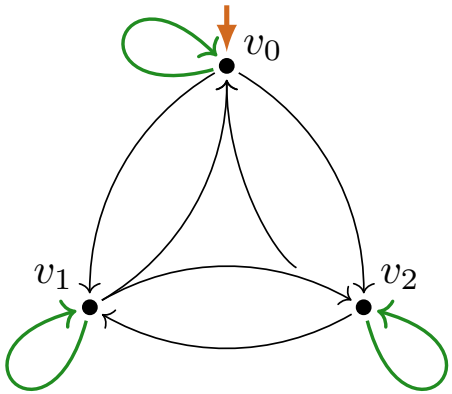

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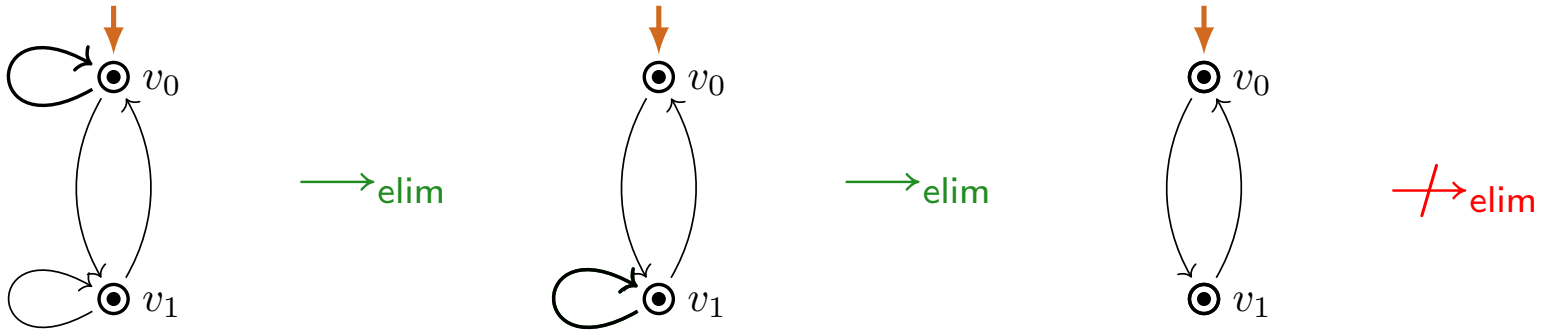

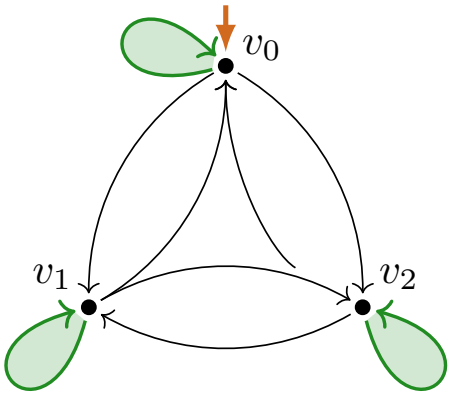

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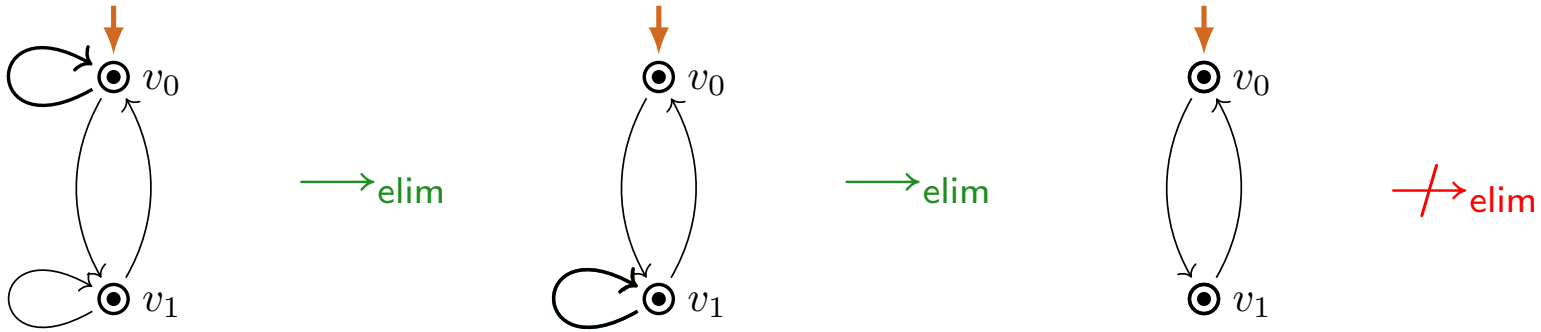

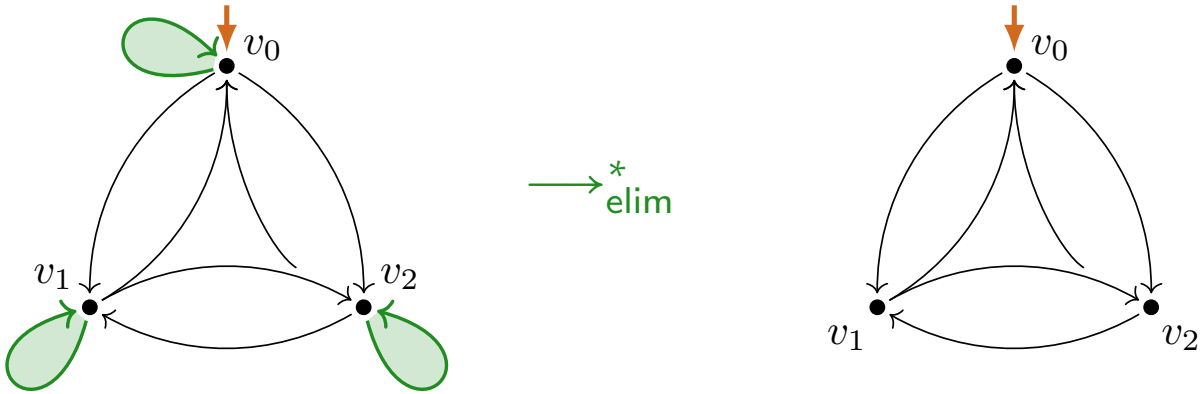

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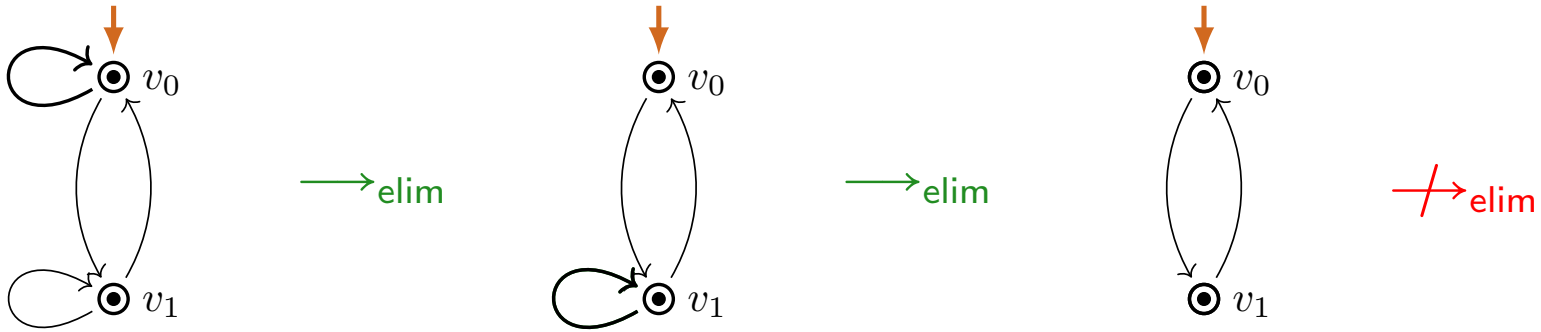

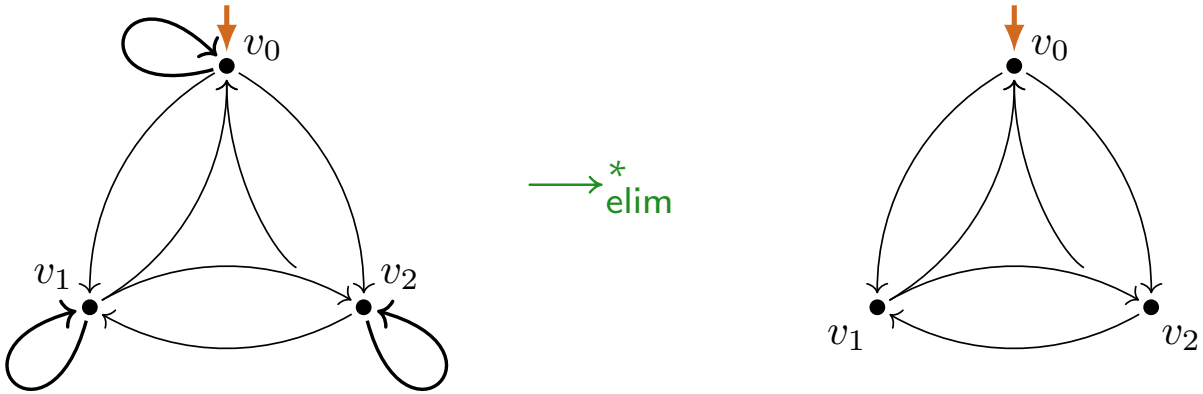

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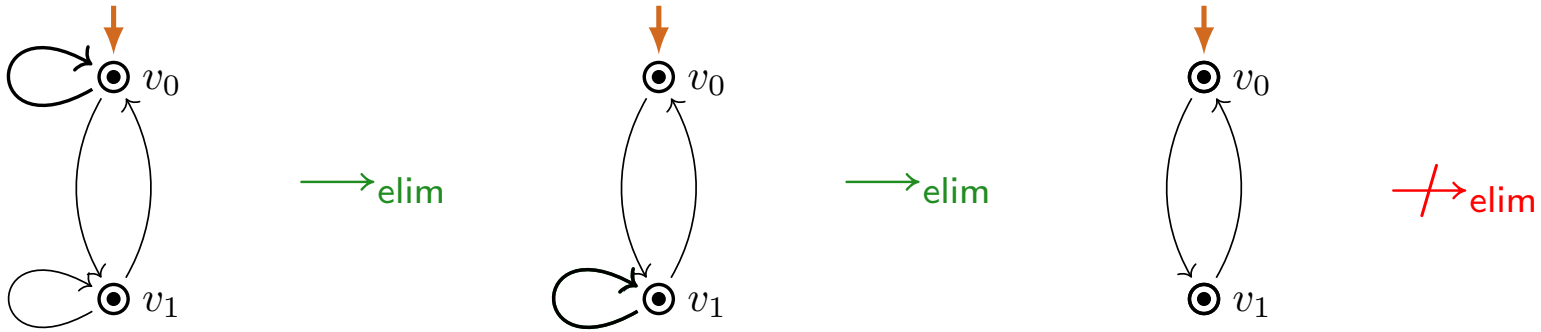
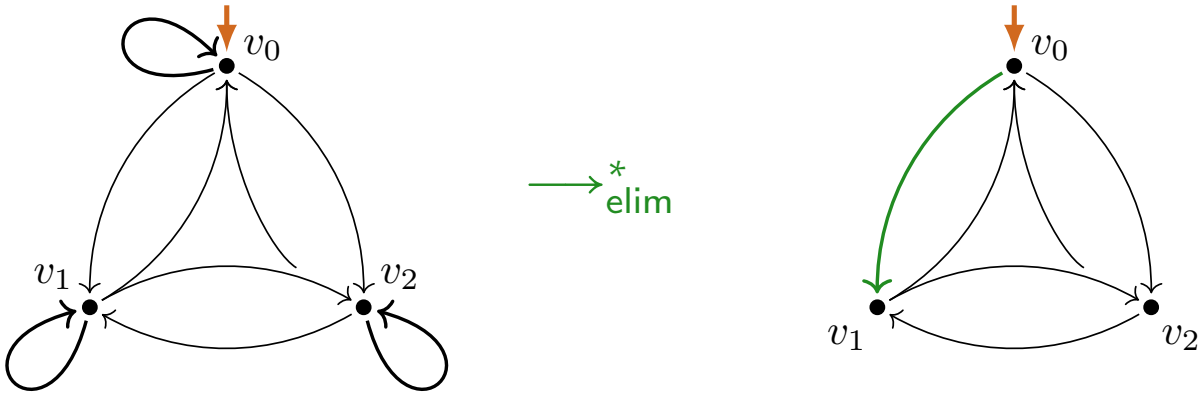
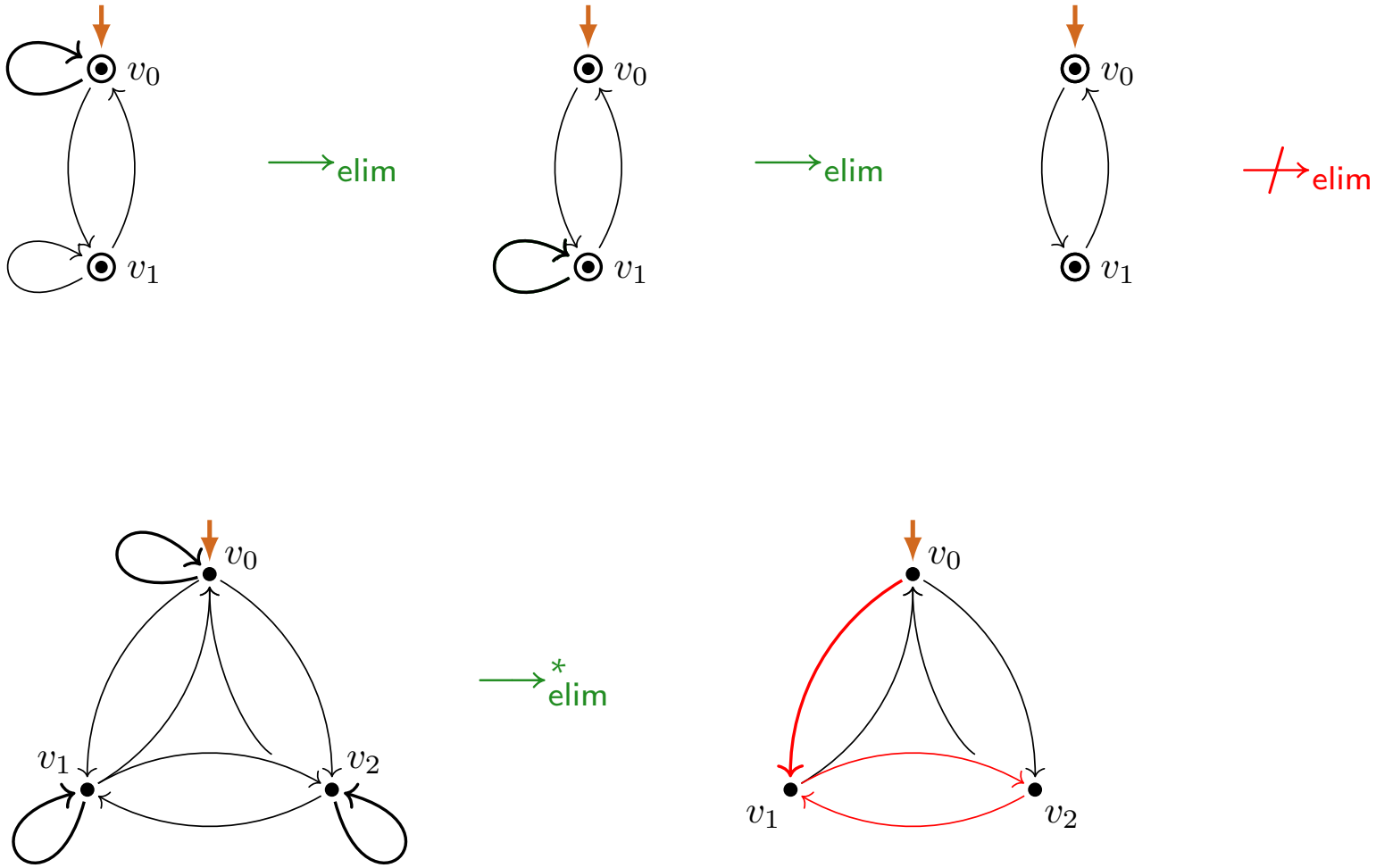




Loop elimination



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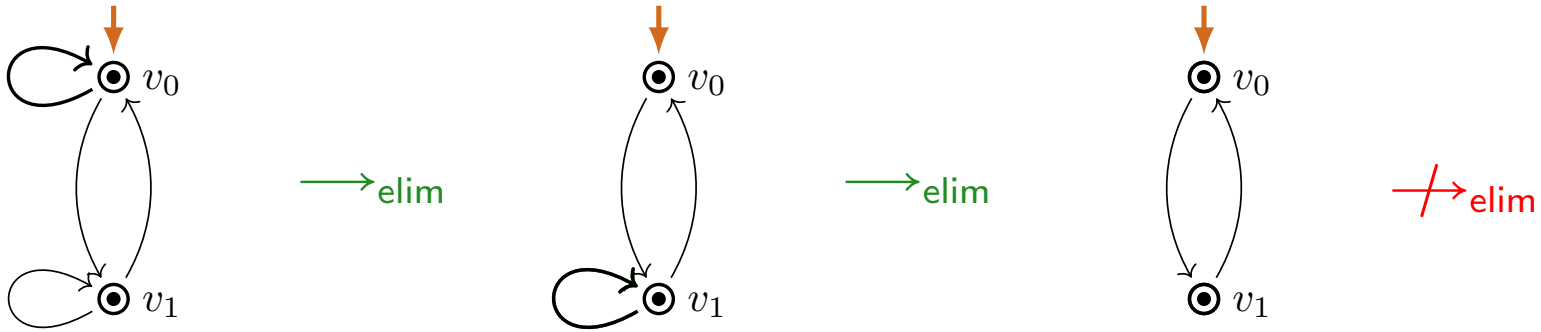

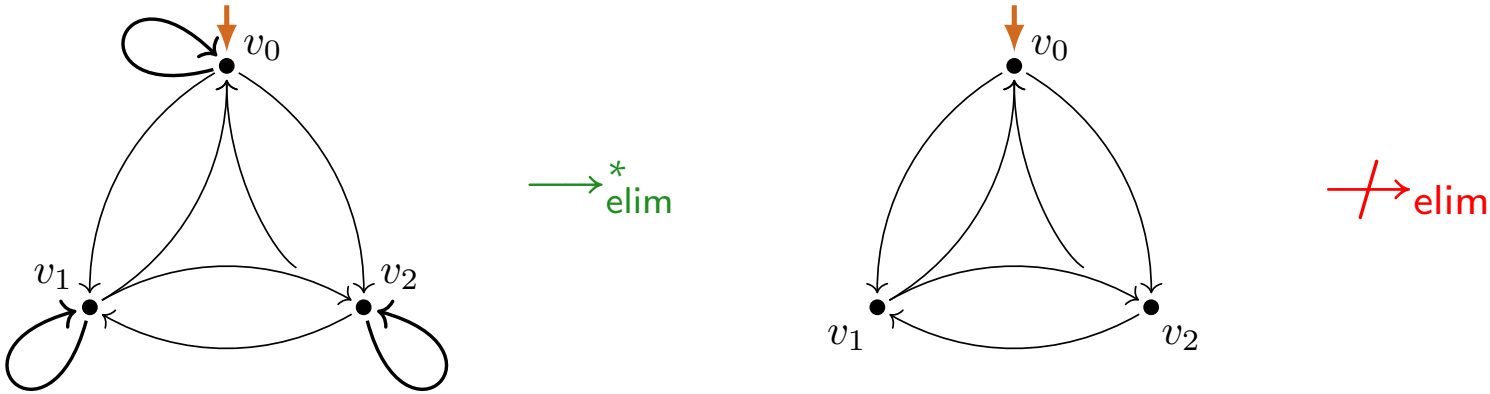

Structure property LEE

Definition

A process graph G satisfies LEE (*loop existence and elimination*) if:

$$\exists G_0 \left(G \longrightarrow^*_{\text{elim}} G_0 \not\longrightarrow_{\text{elim}} \right.$$
$$\left. \wedge\ G_0 \text{ has no infinite trace} \right) .$$

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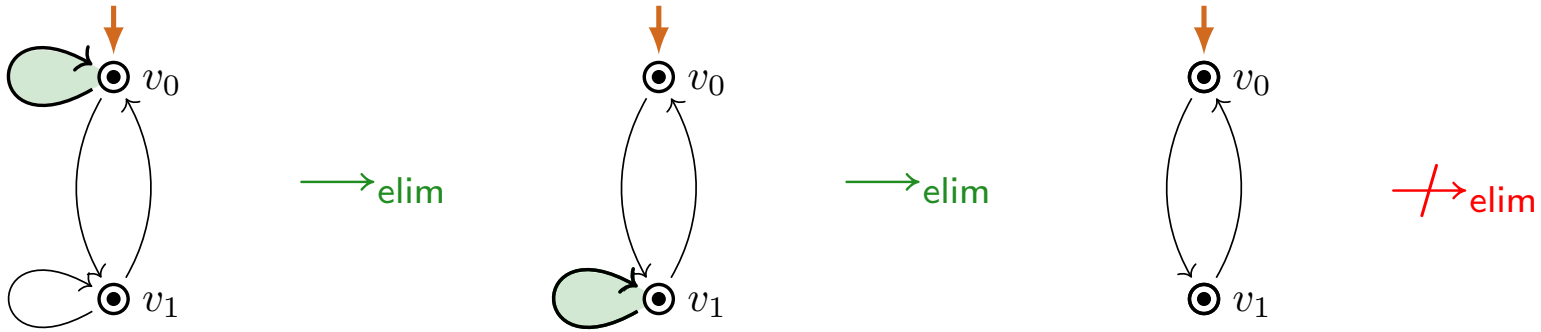
LEE fails



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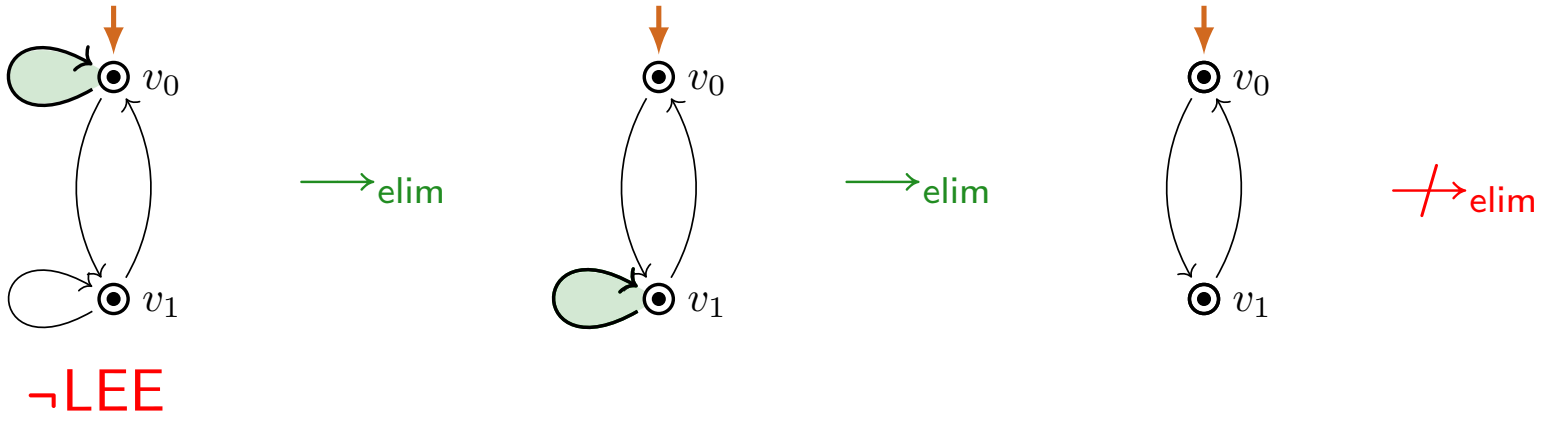
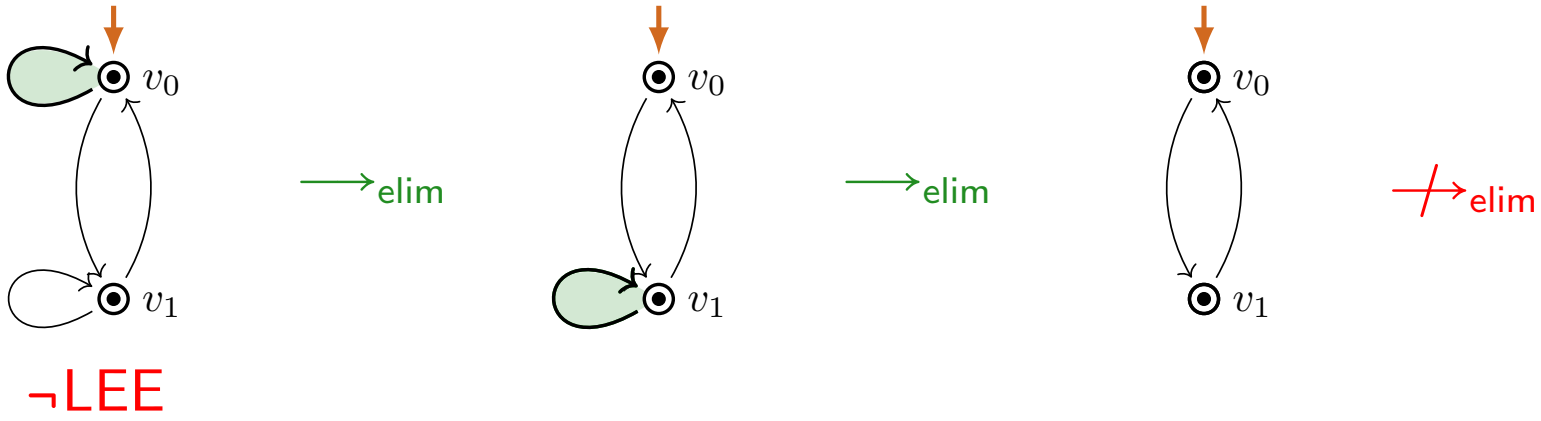
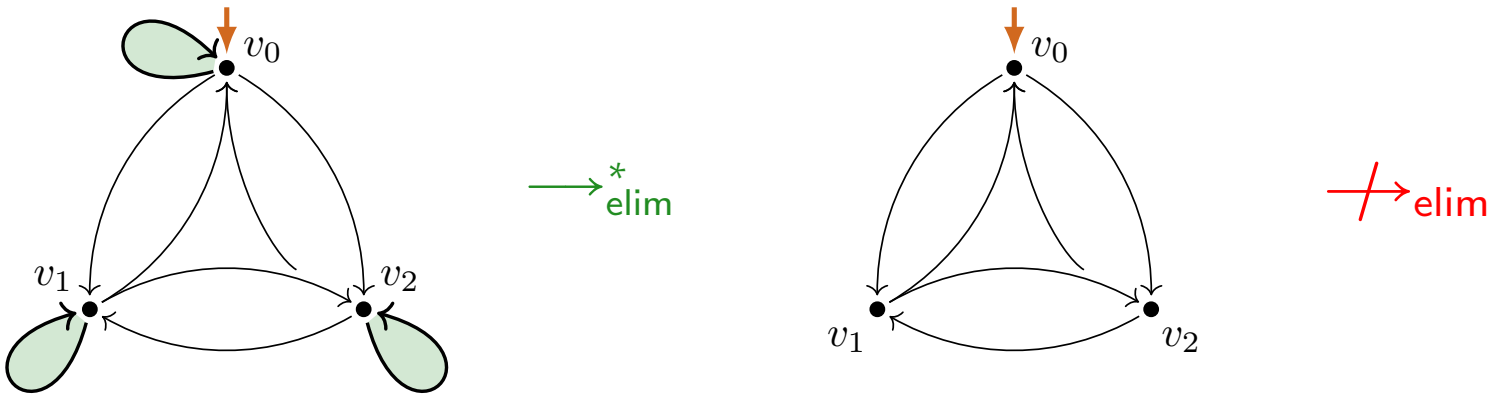


¬LEE

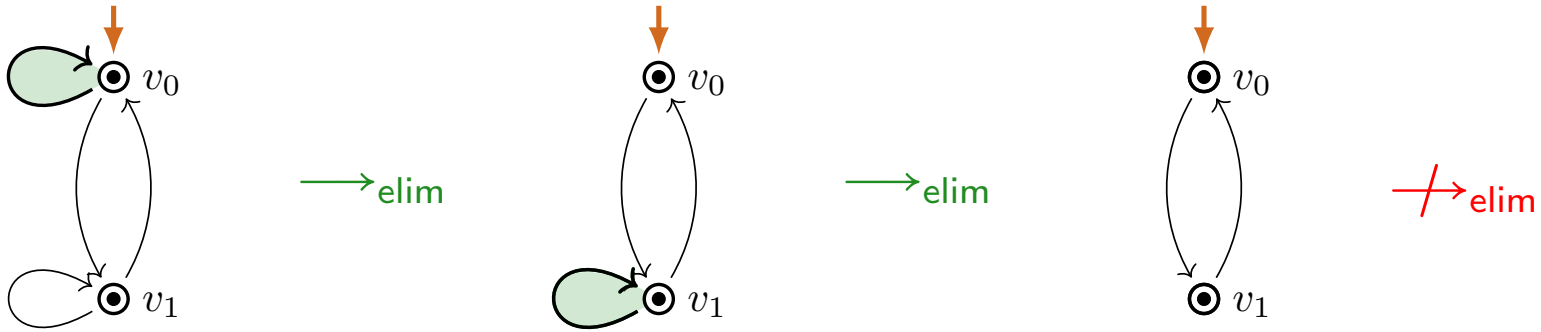
LEE fails



¬LEE



LEE fails



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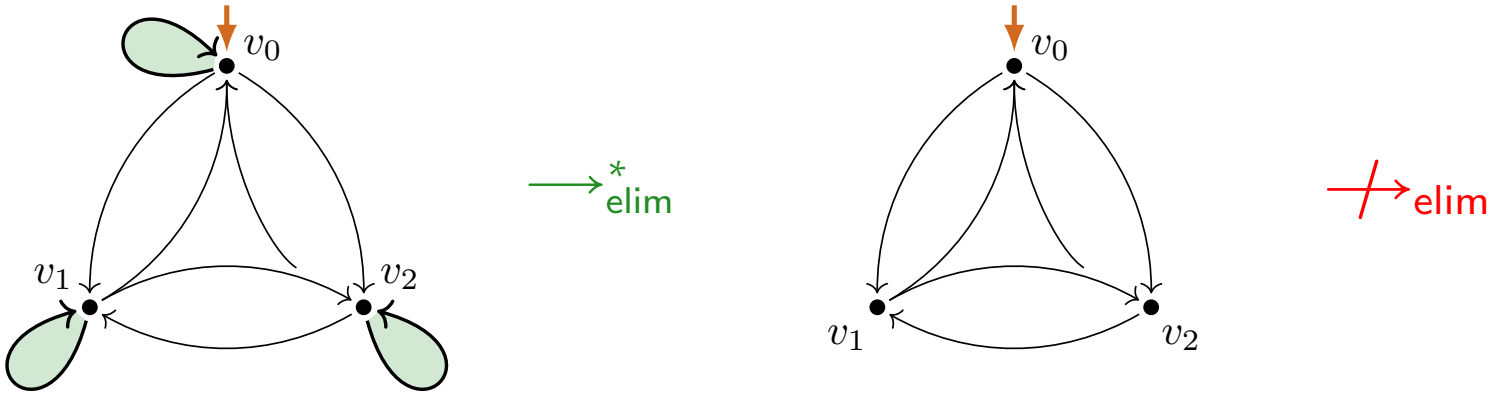

¬LEE

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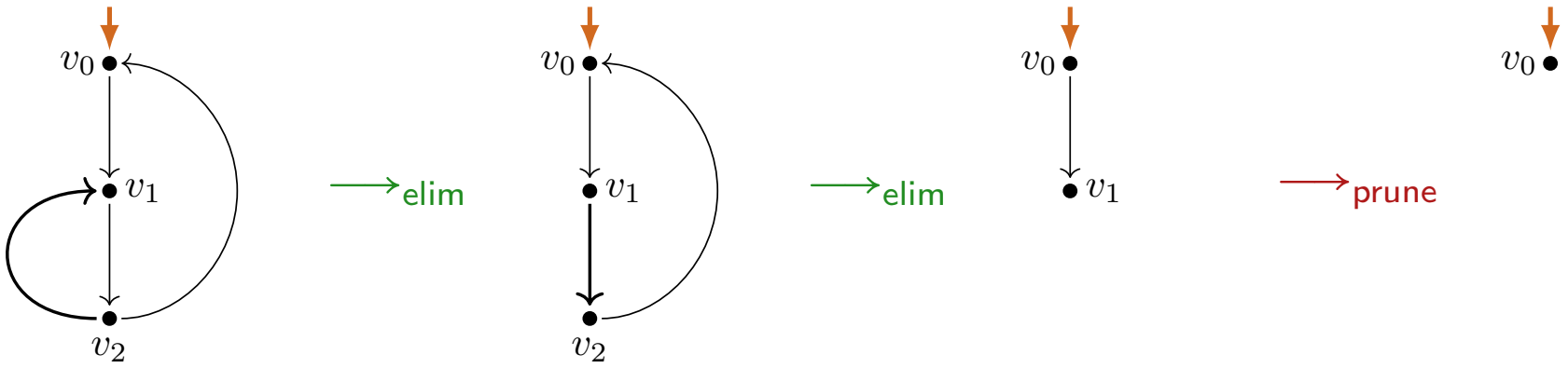

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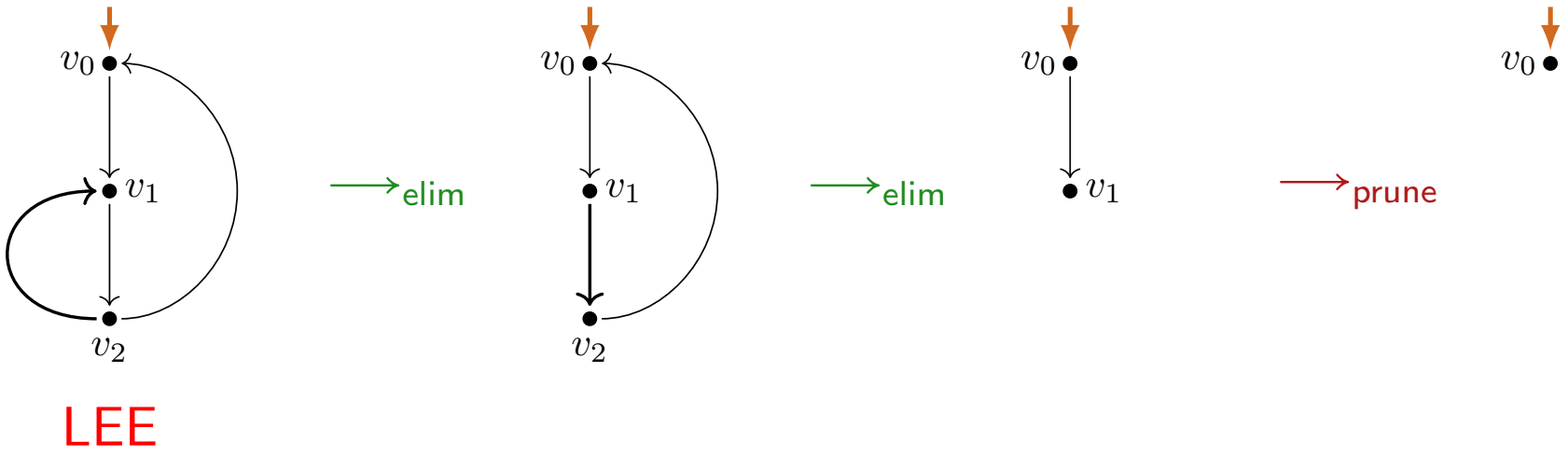
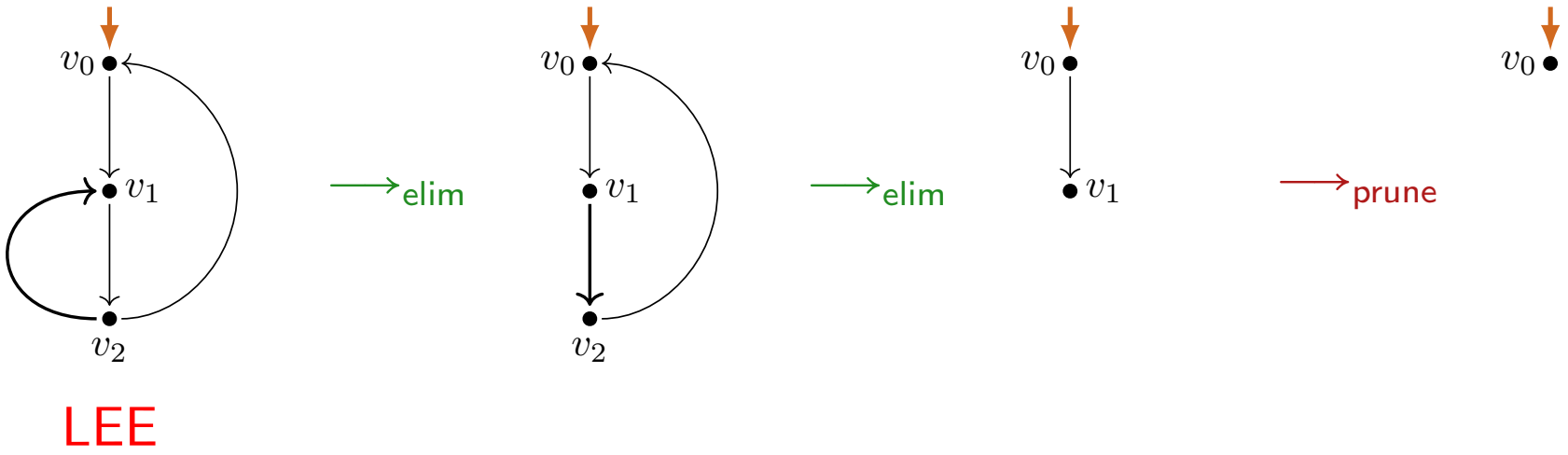


LEE

LEE holds / Recording loop elimination



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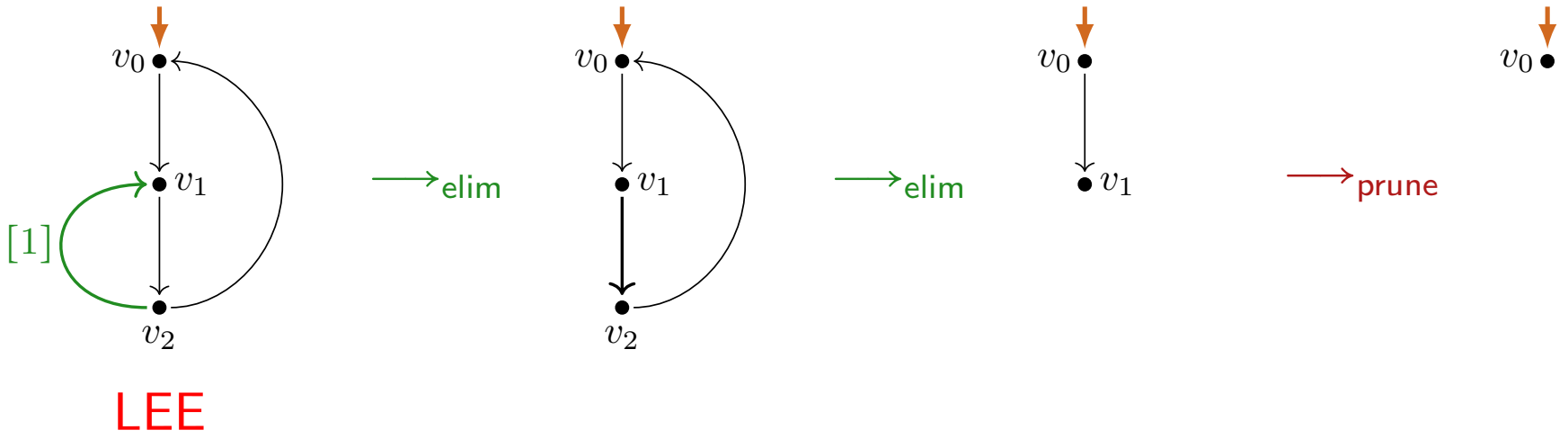

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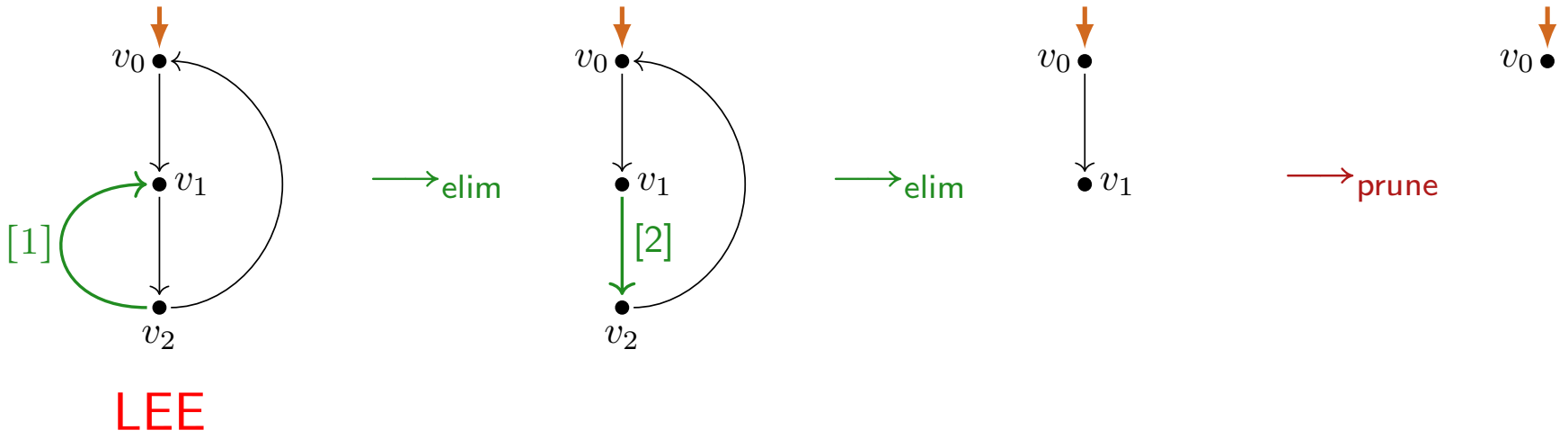

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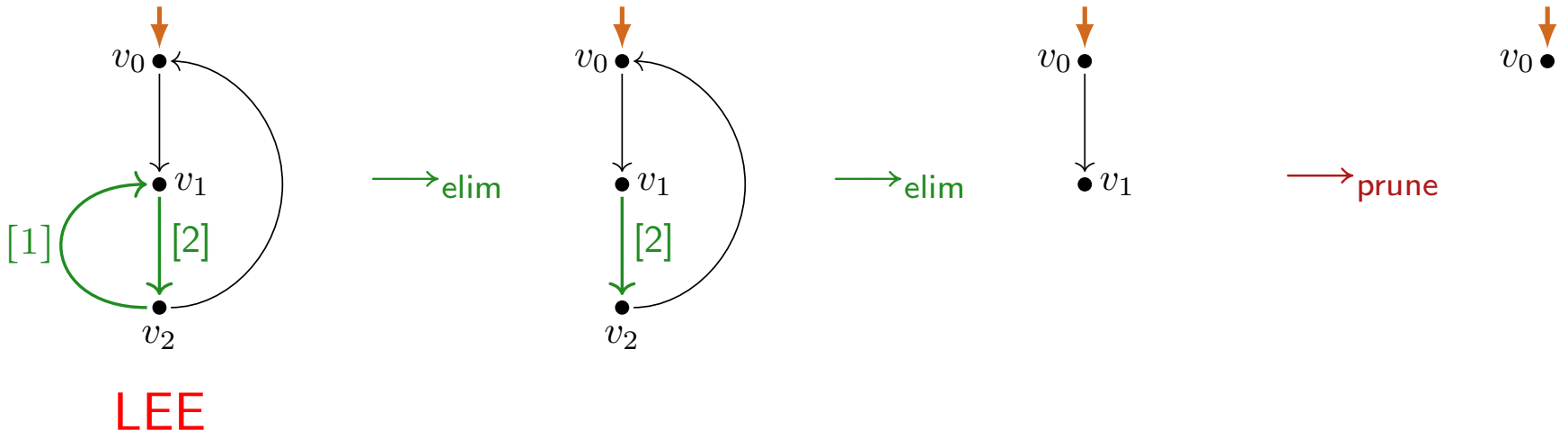

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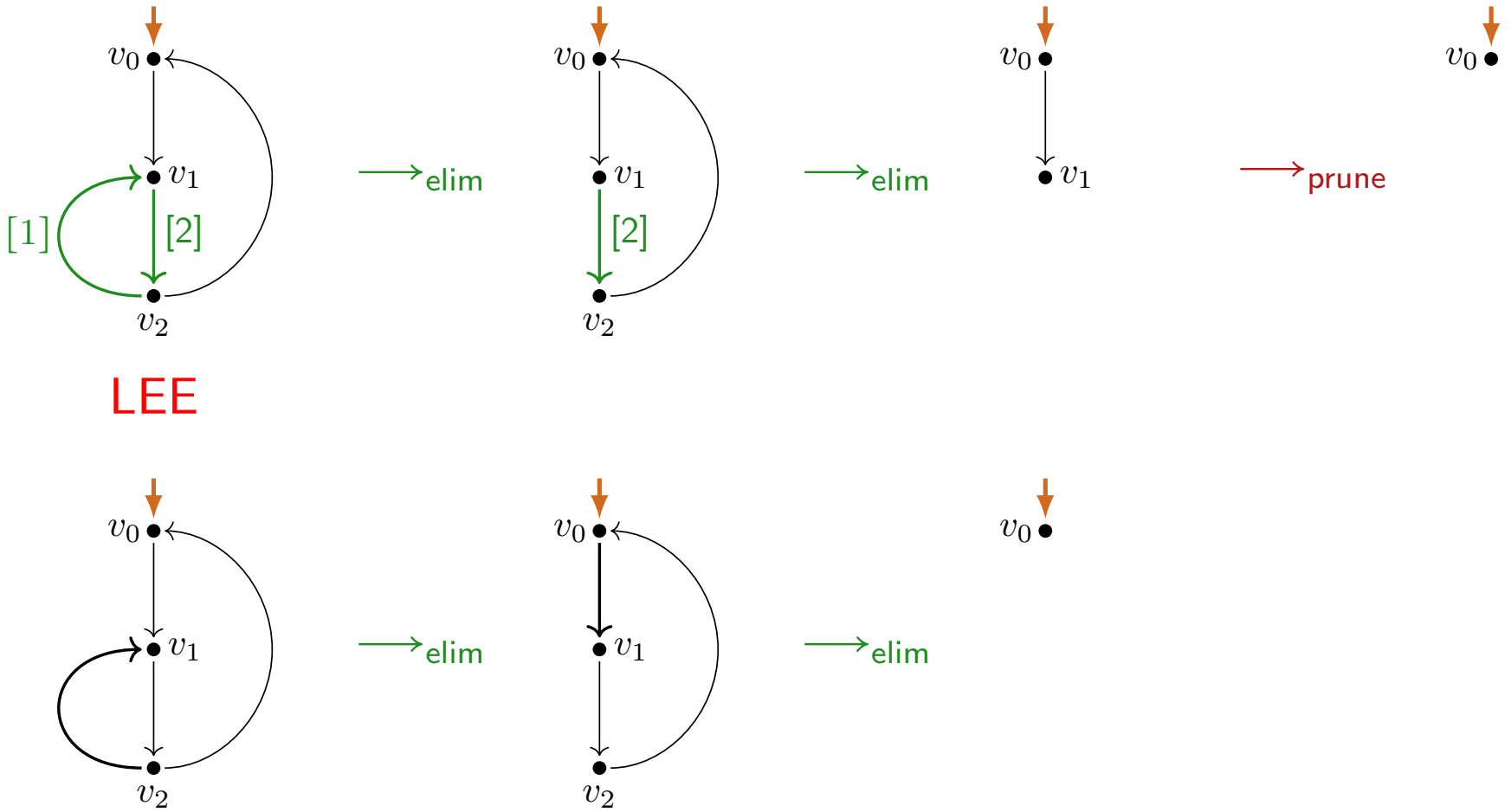

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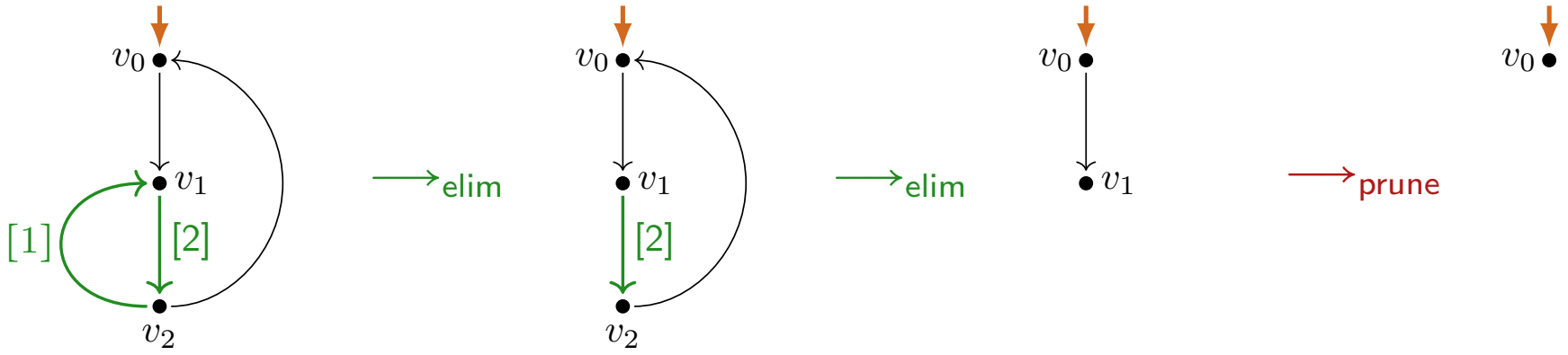

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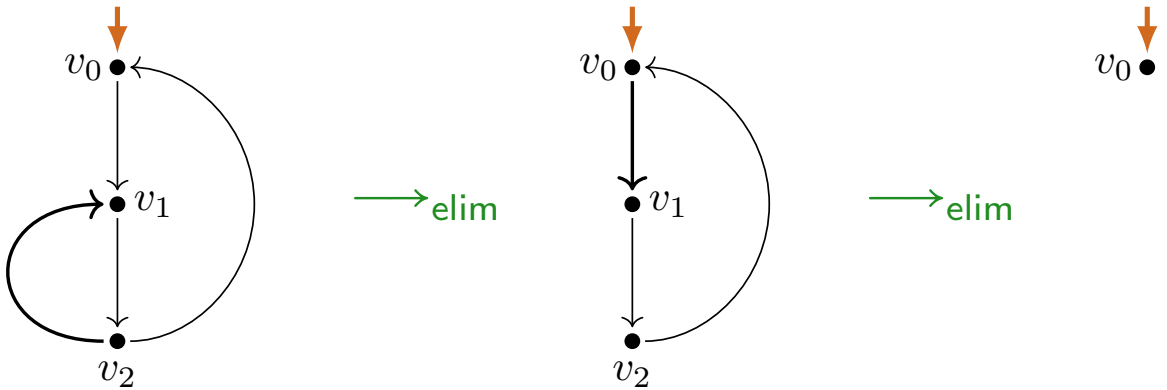

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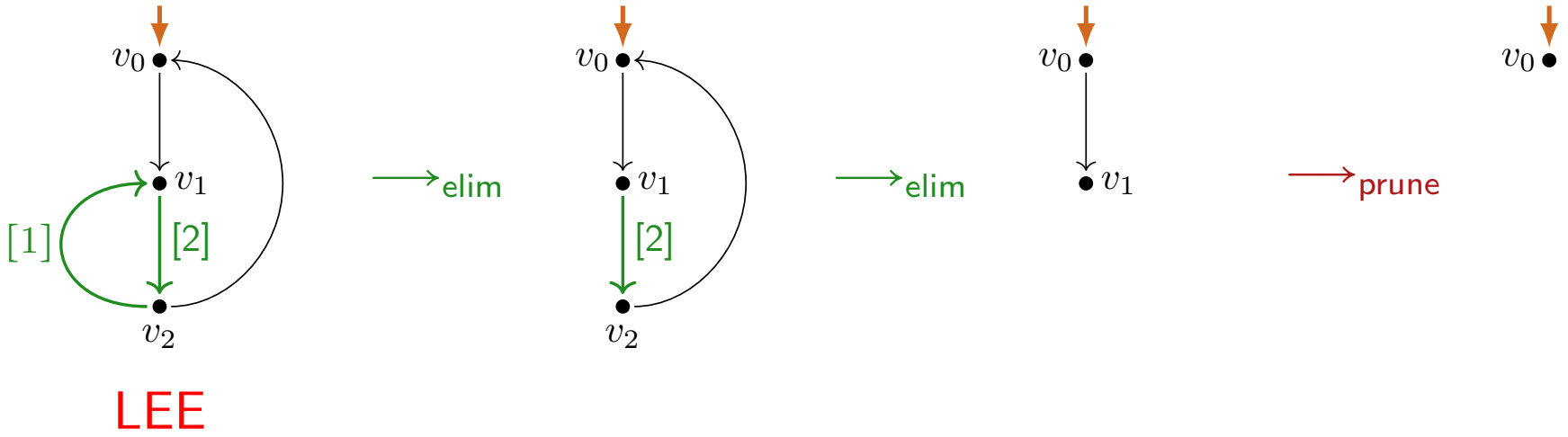

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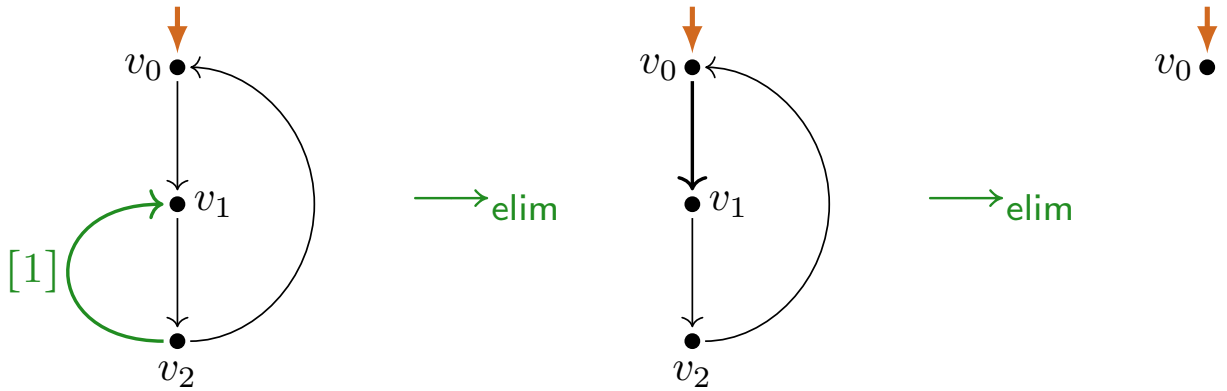

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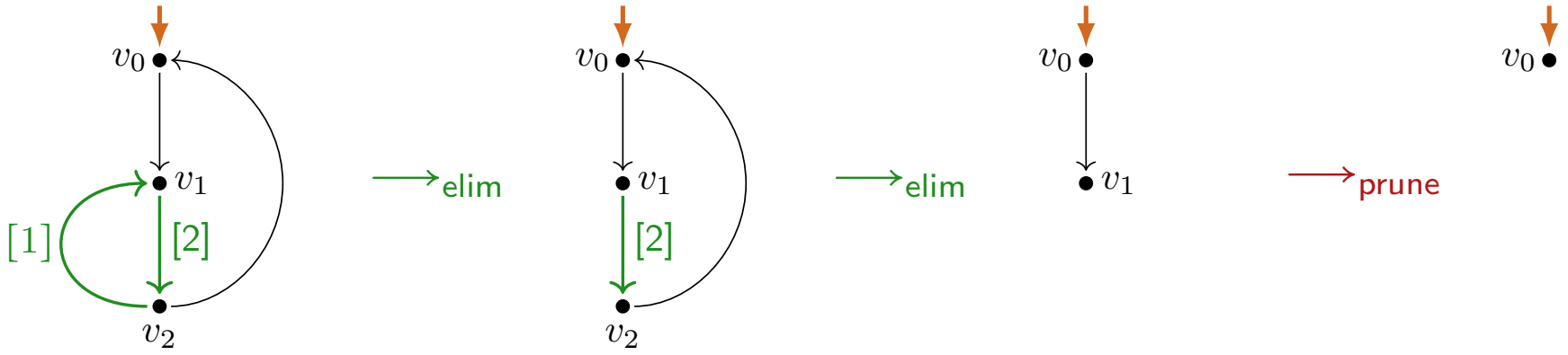

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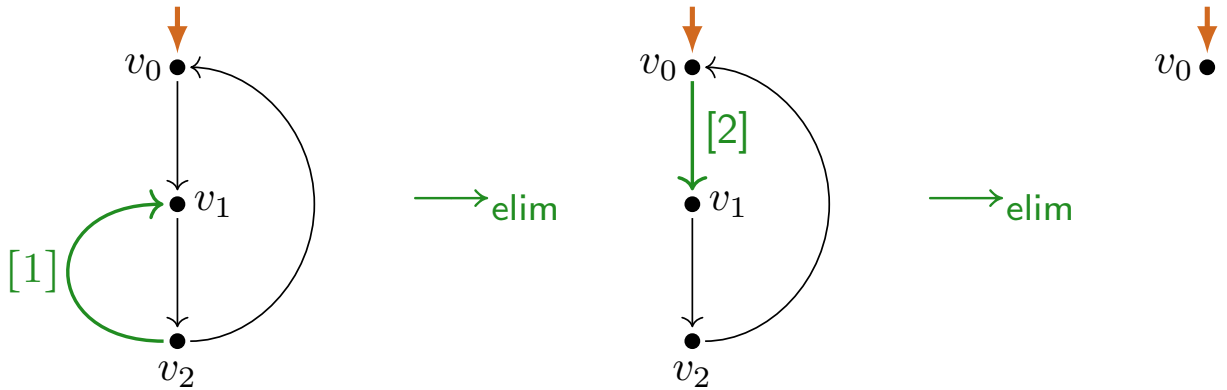

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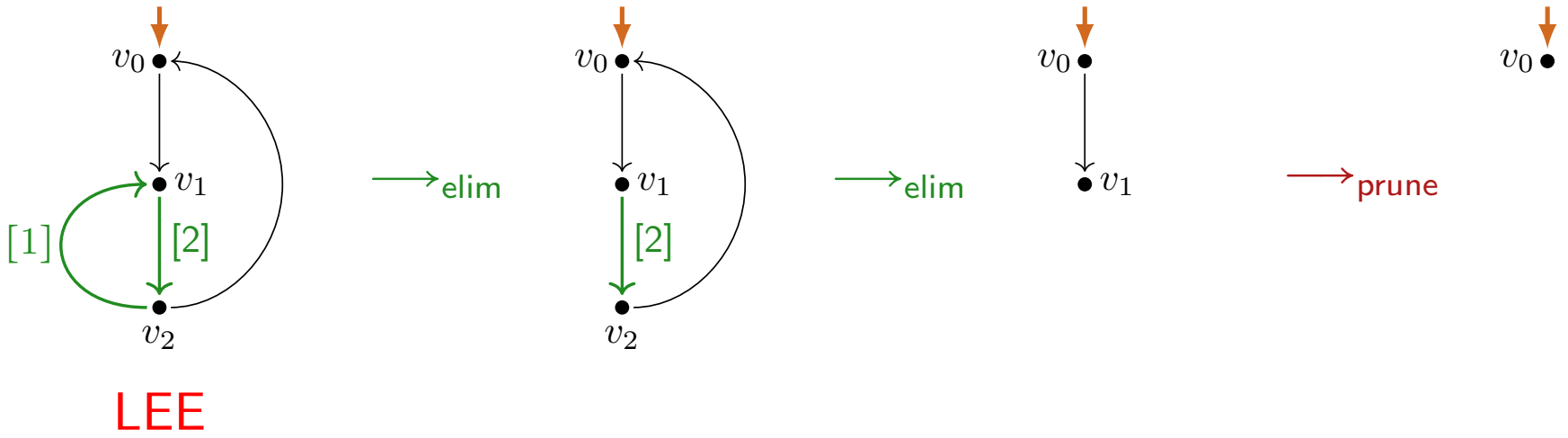

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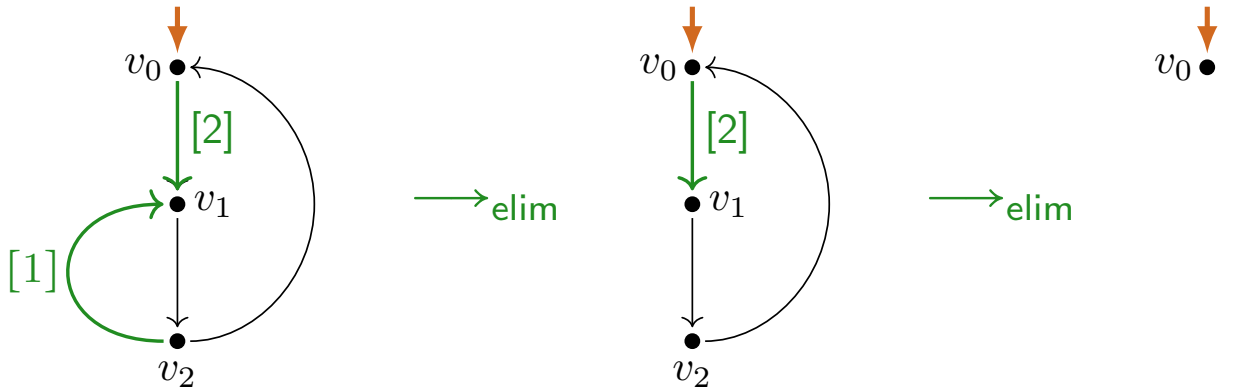
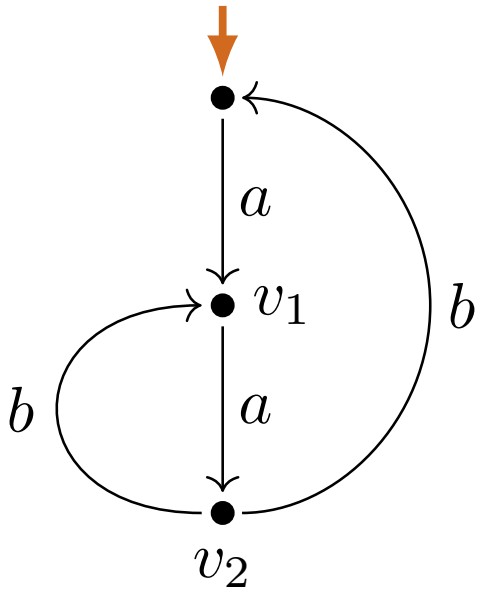


LEE

LEE-witness



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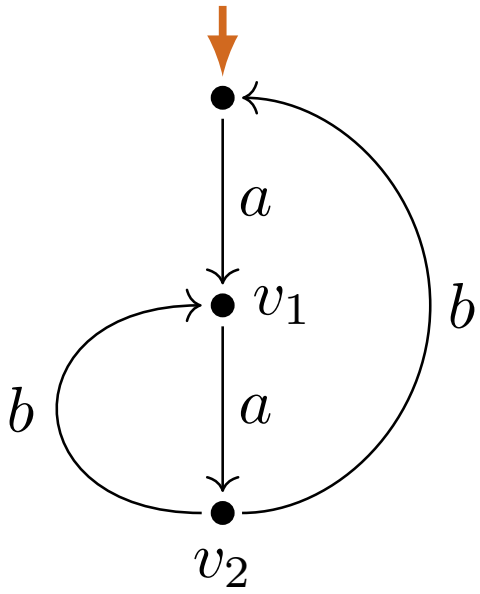
loop–branch labeling: marking transitions $\overset{a}{\to}$ as:



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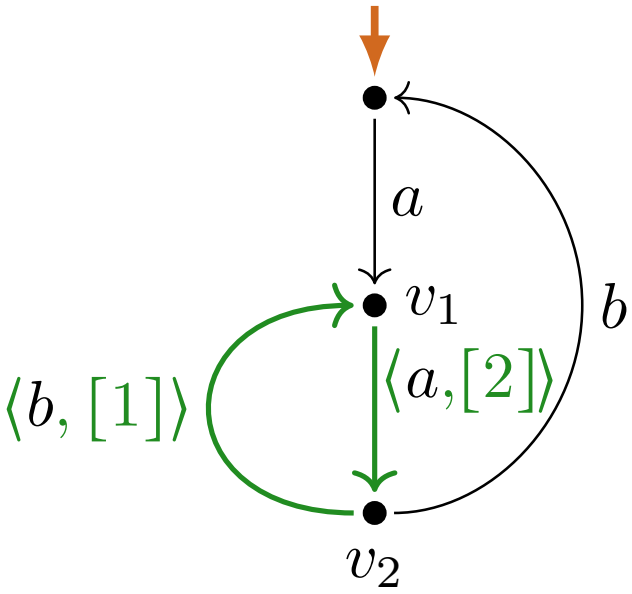

LEE-witness

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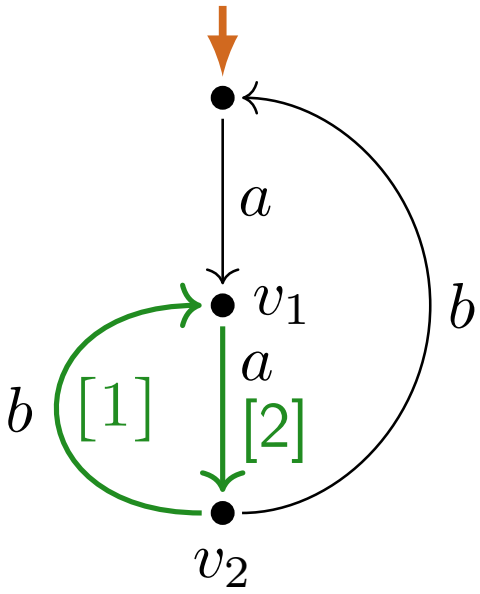

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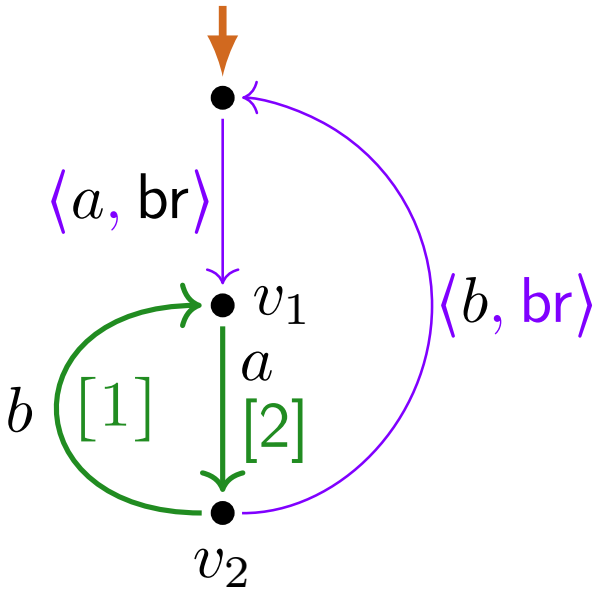

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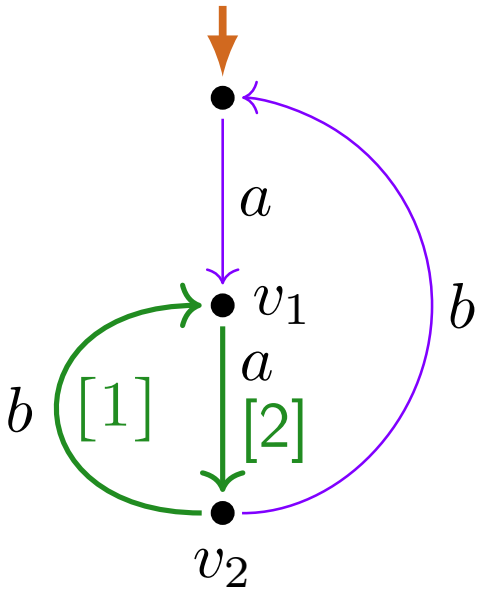
LEE-witness



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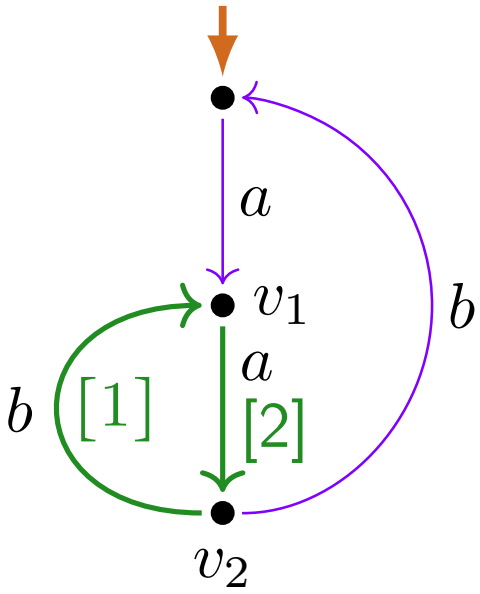

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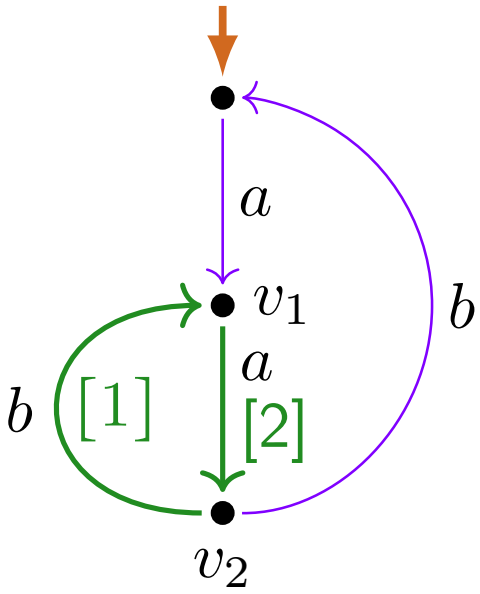

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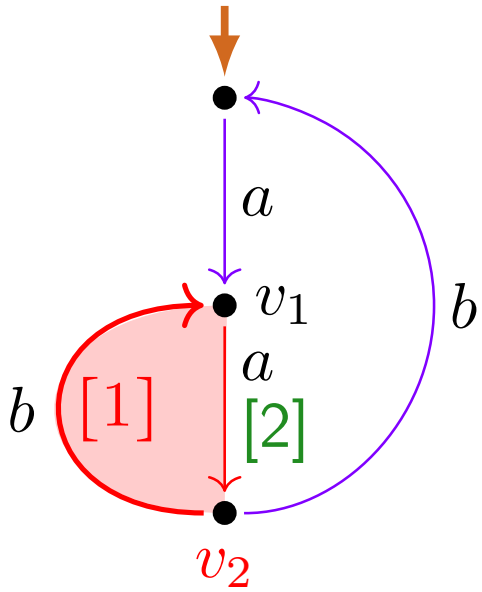

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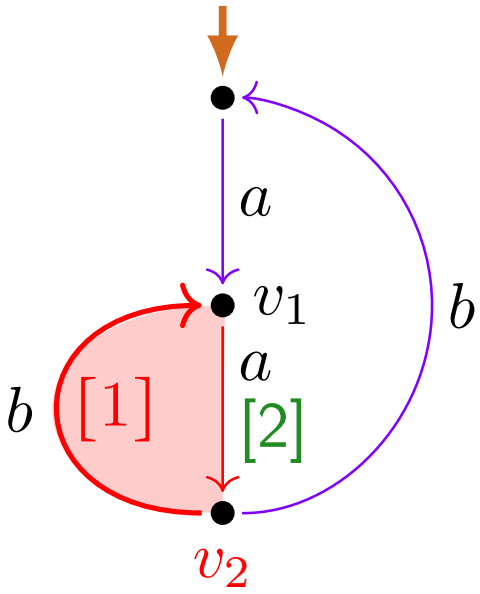

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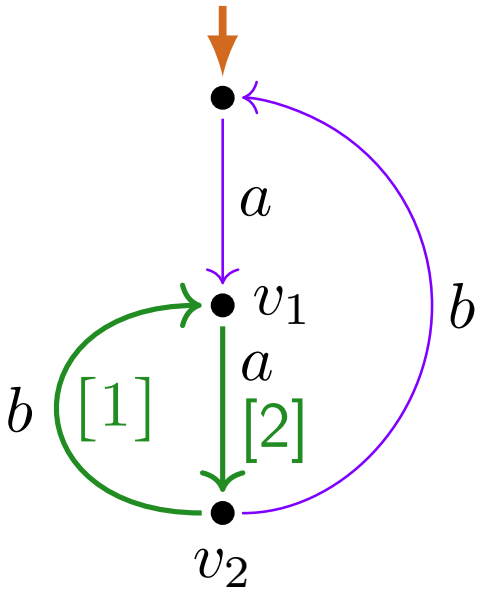

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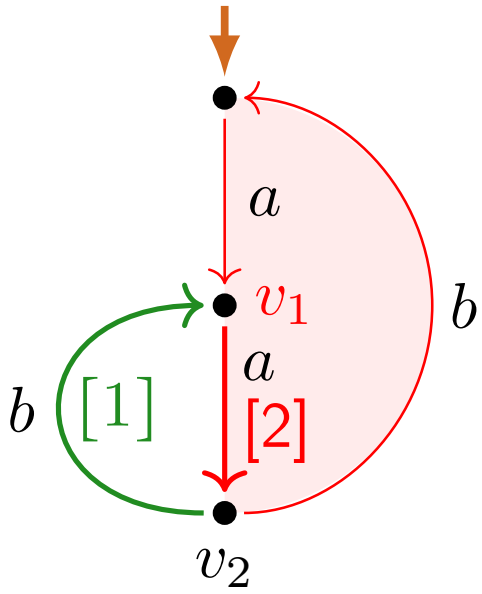

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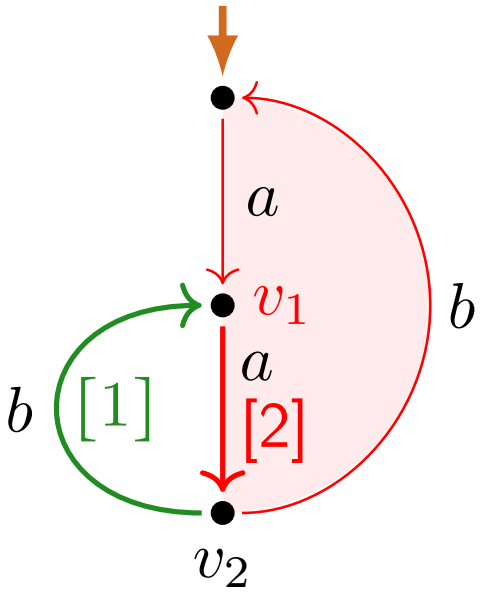

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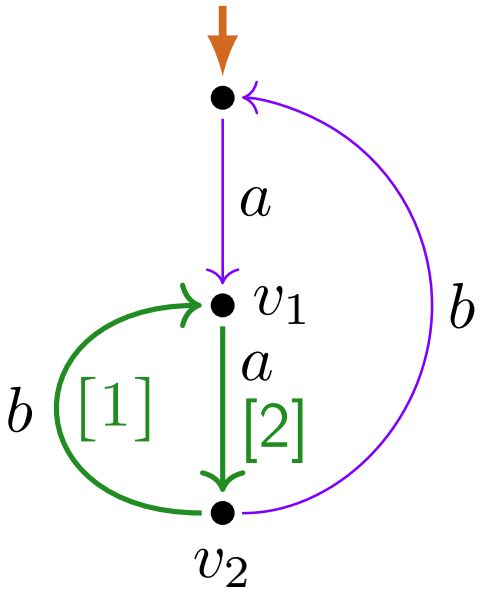

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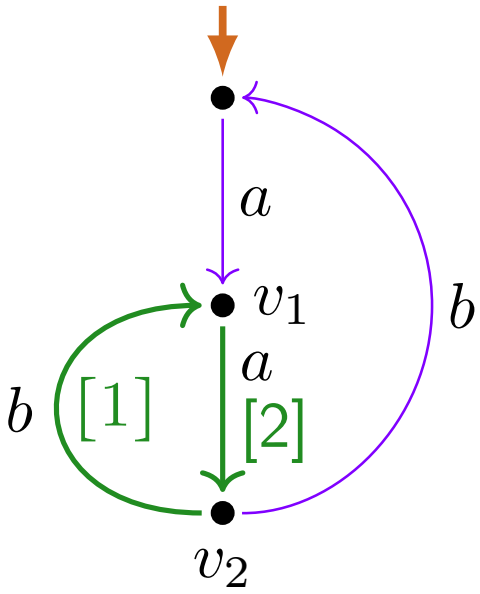

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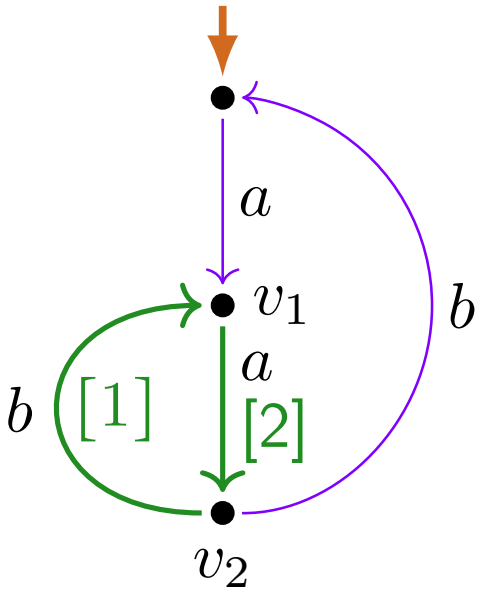

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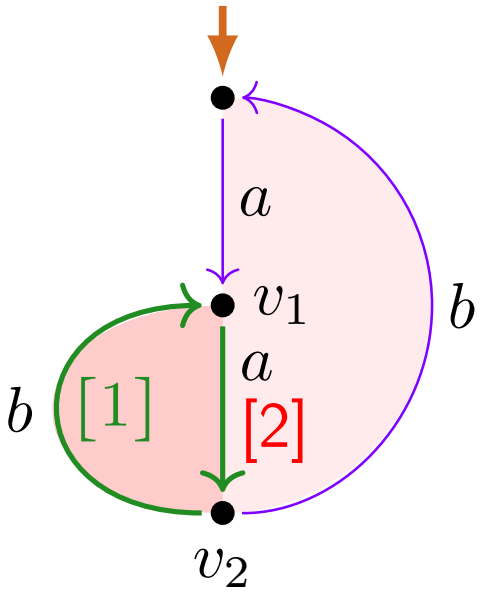

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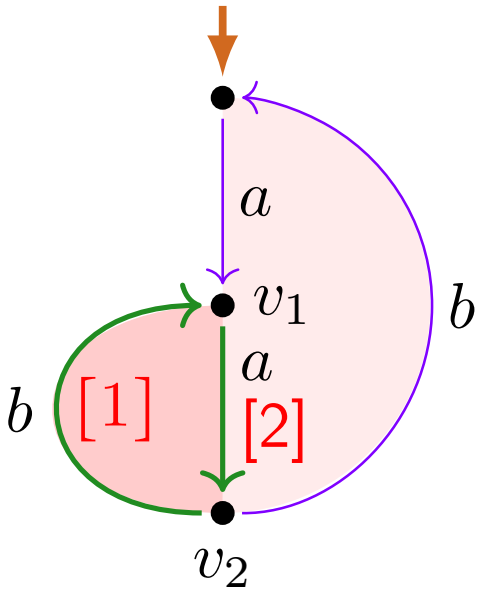

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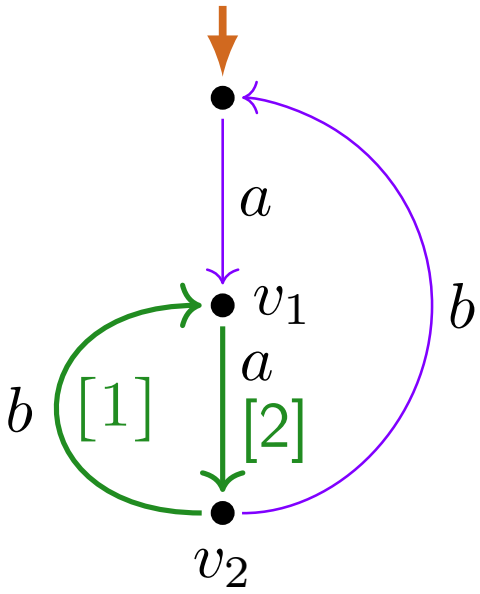

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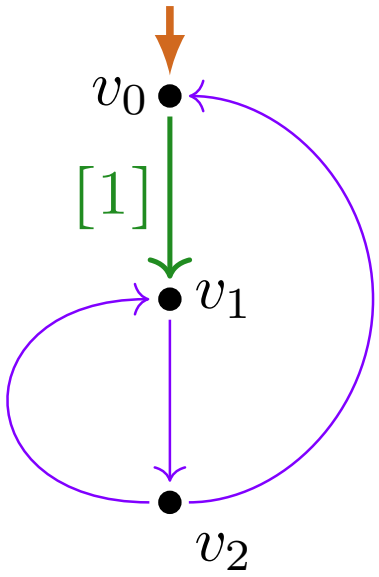

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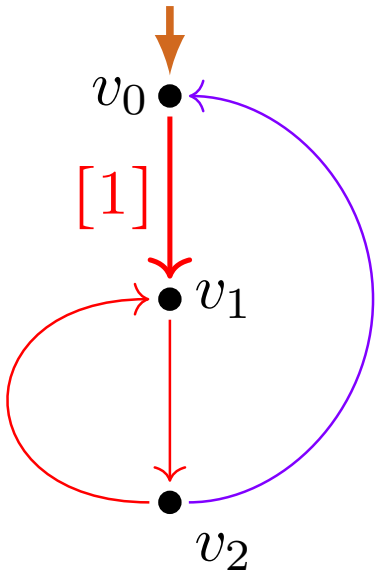

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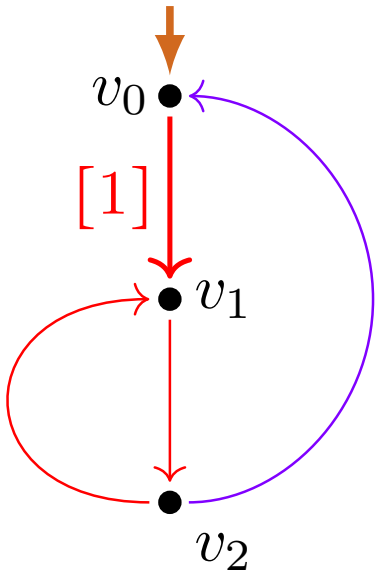

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not a loop chart

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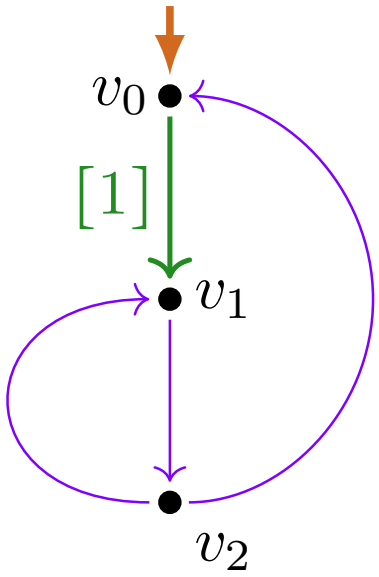

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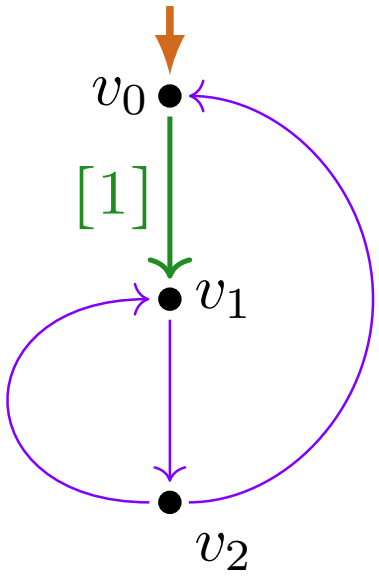

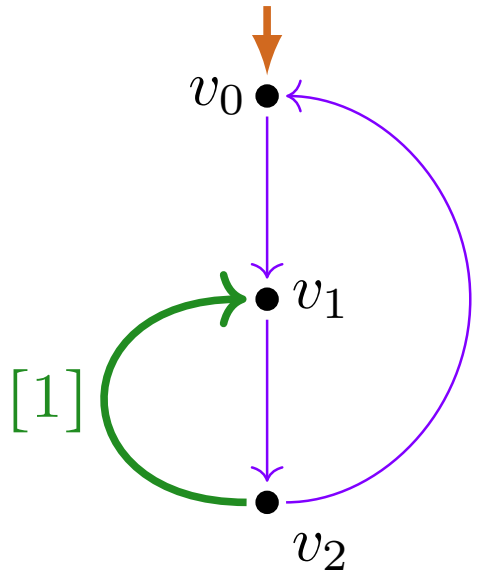

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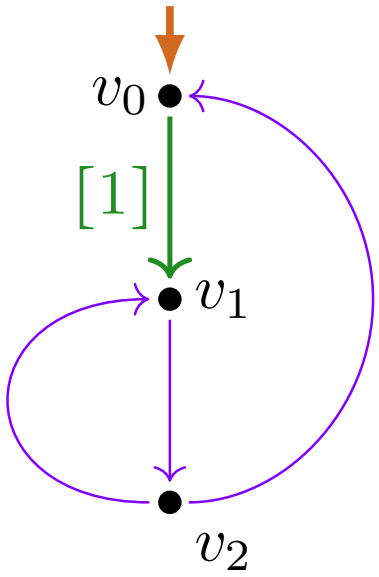

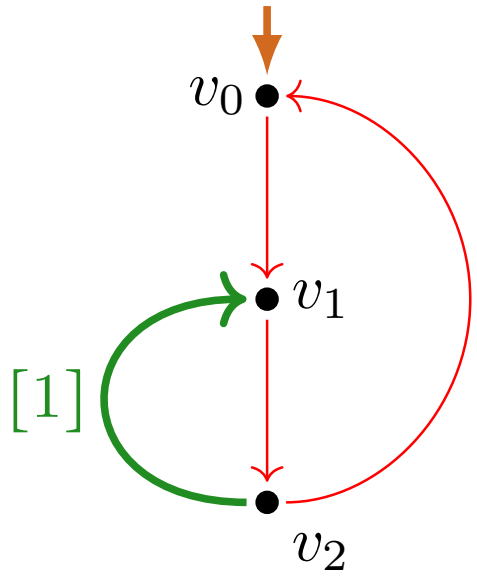

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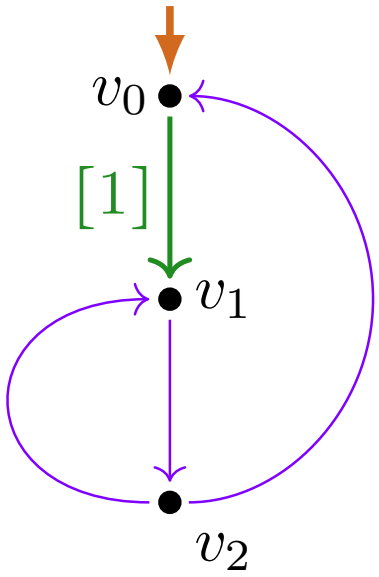

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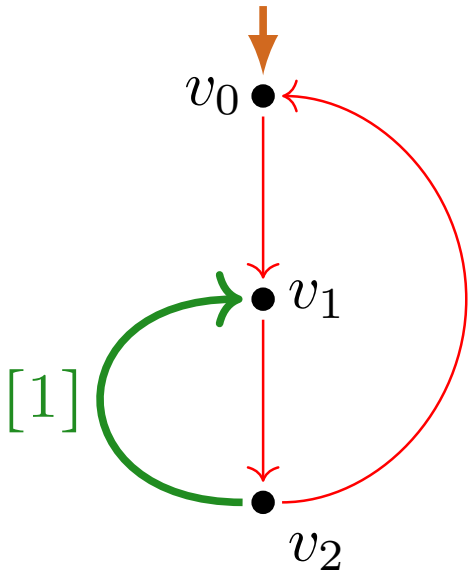

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(L2.) violated:

infinite $\rightarrow_{\mathsf{br}}$ path

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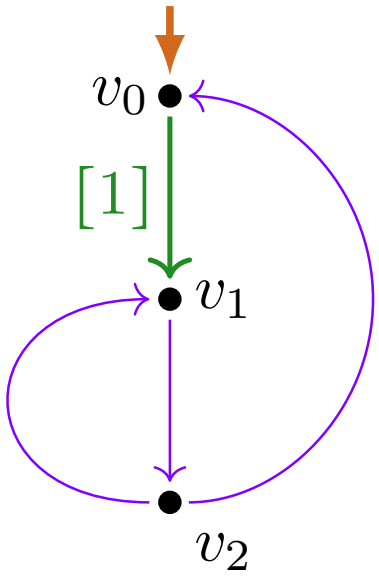
LEE-witness ?



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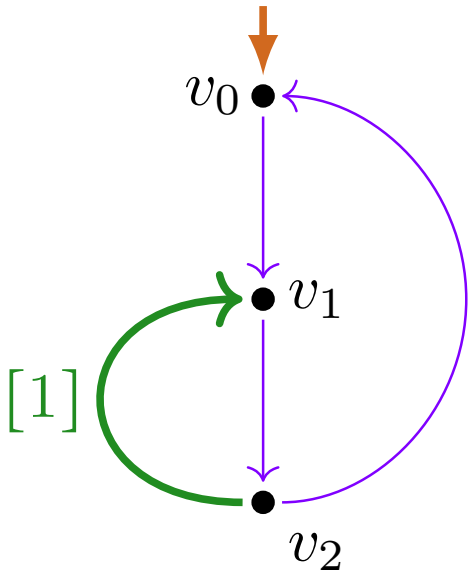

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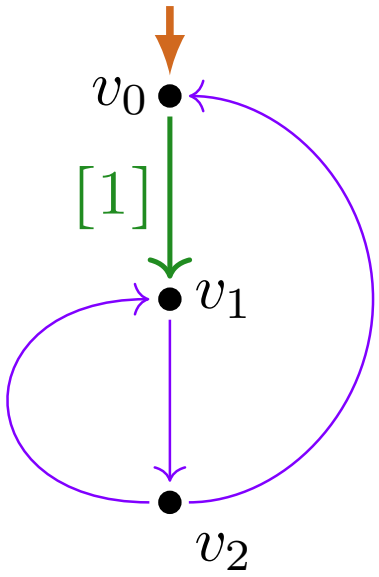
from start vertex

LEE-witness ?



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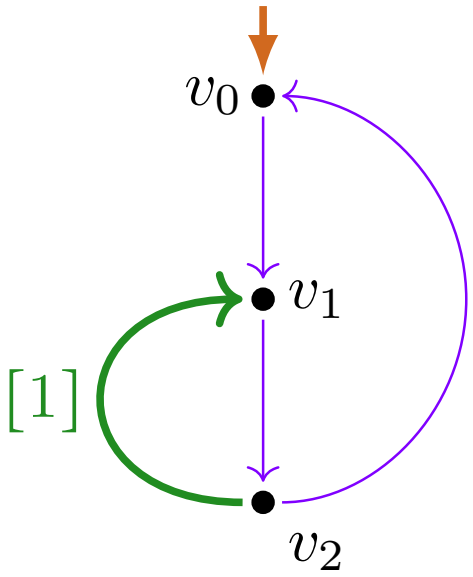
$\mathcal{L}(v_0, \rightarrow_{[1]}, \rightarrow_{\mathsf{br},[>1]})$

not a loop chart

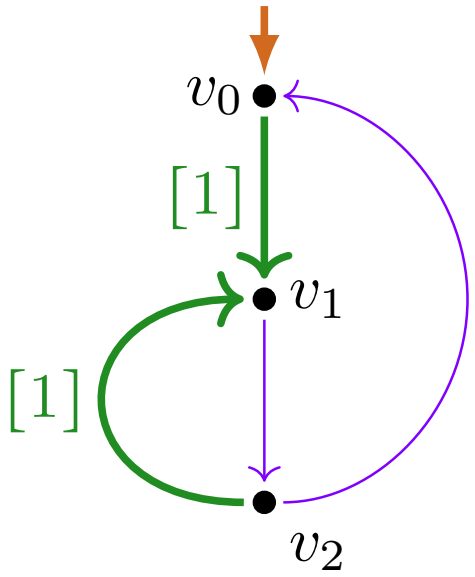


no!

(L2.) violated:

infinite $\rightarrow_{\mathsf{br}}$ path

from start vertex



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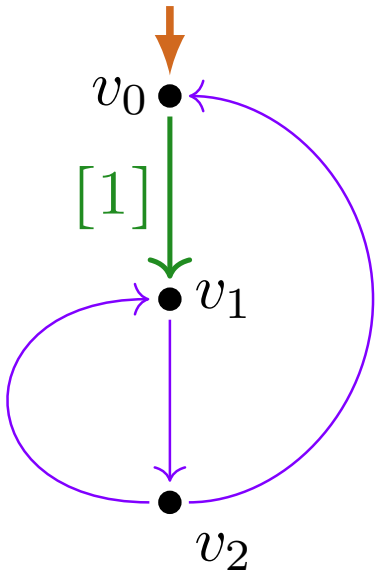

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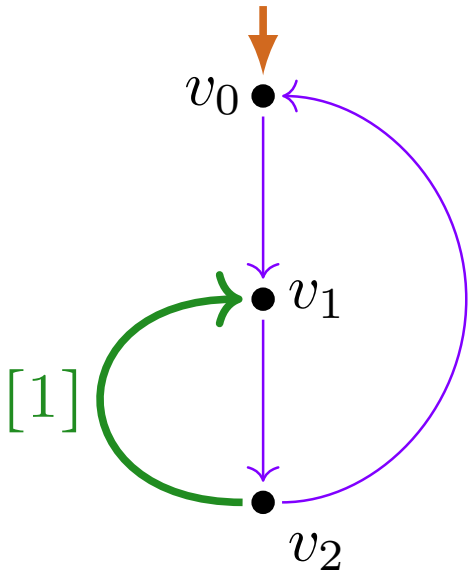

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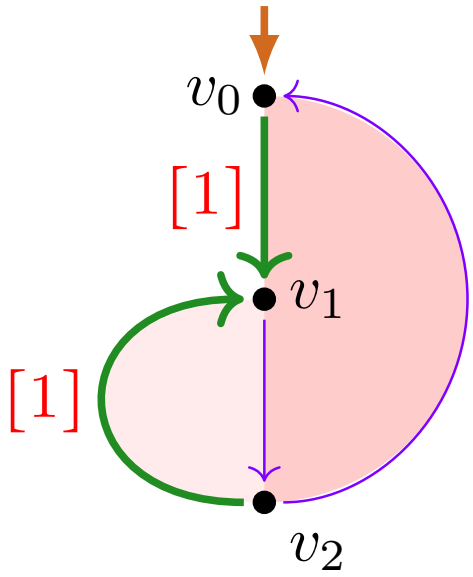

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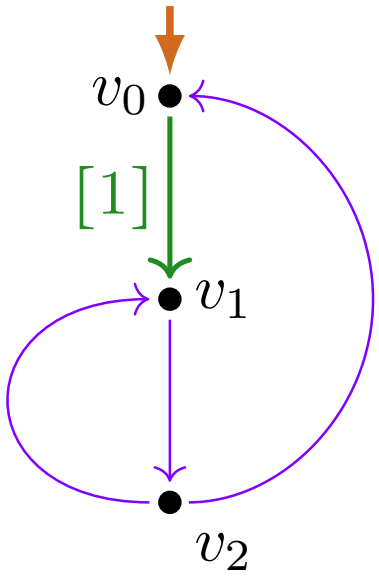

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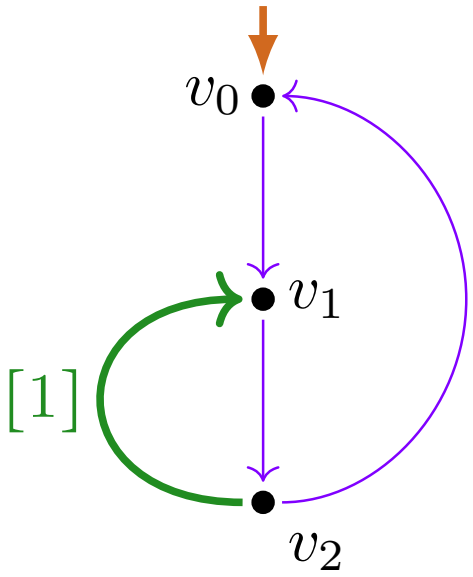

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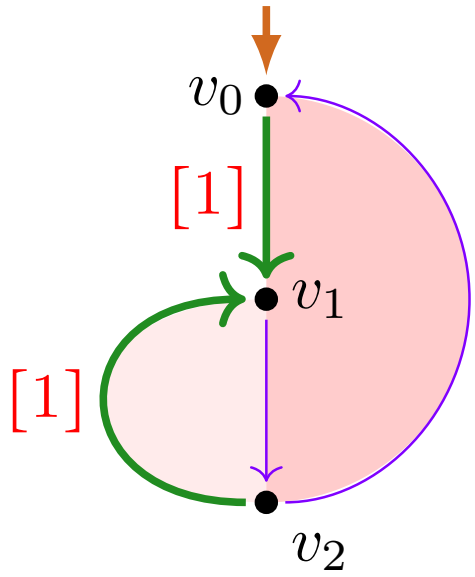


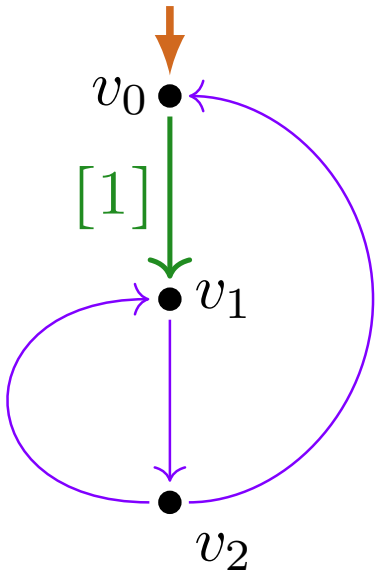
no!

(L3.) violated:

overlapping loop charts

have same level

LEE-witness ?



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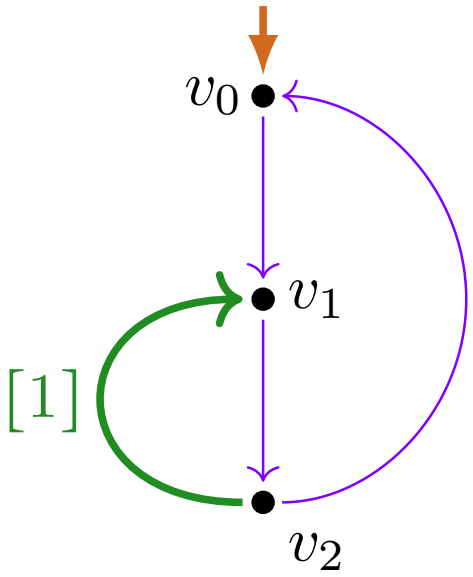
$\mathcal{L}(v_0, \rightarrow_{[1]}, \rightarrow_{\mathsf{br},[>1]})$

not a loop chart



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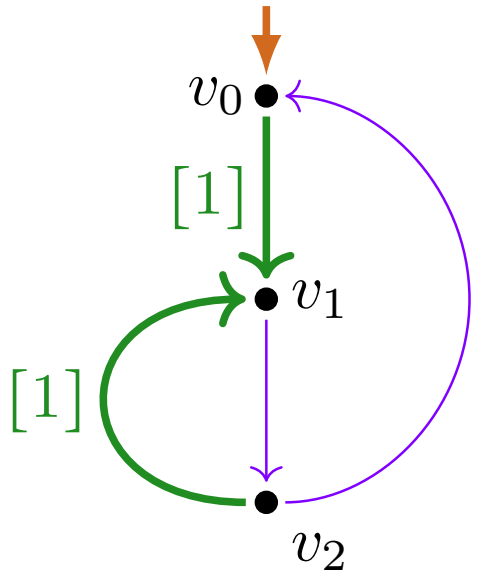

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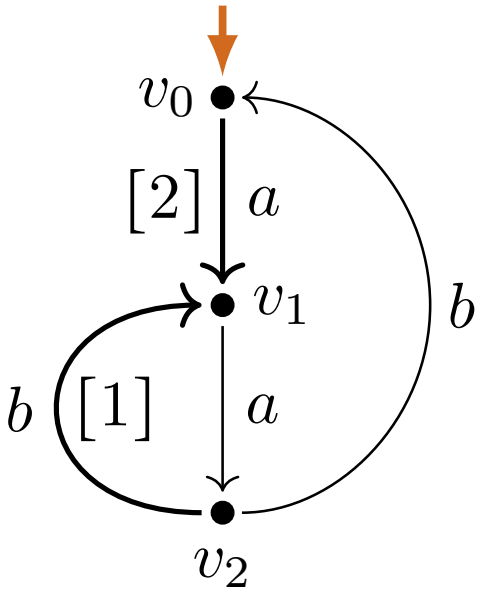

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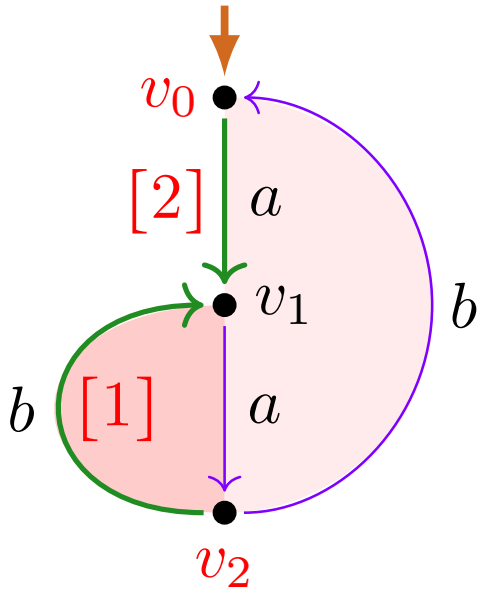

$\mathcal{L}(v_2, \to_{[1]}, \to_{\mathsf{br},[>1]})$

$\mathcal{L}(v_0, \to_{[2]}, \to_{\mathsf{br},[>2]})$

LEE-witness

loop–branch labeling: marking transitions $\xrightarrow{a}$ as:

- entry steps $\xrightarrow{\langle a,[n]\rangle}$ for $n \in \mathbb{N}$, written $\xrightarrow{a}_{[n]}$,
- branch steps $\xrightarrow{\langle a,\mathsf{br}\rangle}$, written $\xrightarrow{a}_{\mathsf{br}}$ or $\xrightarrow{a}$.

Definition

A loop–branch labeling is a LEE-witness, if:

L1. $\forall n \in \mathbb{N} \forall v \in V\Big(\begin{array}{c}\mathcal{L}(v, \to_{[n]}, \to_{\mathsf{br},[>n]}) \\ \text{is a loop subchart, or trivial}\end{array}\Big)$.

L2. No infinite $\to_{\mathsf{br}}$ path from start vertex.

L3. $\mathcal{L}(w_i, \to_{[n_i]}, \to_{\mathsf{br},[>n_i]})$ for $i \in \{1,2\}$ loop charts
$\wedge\ w_1 \neq w_2 \ \wedge\ w_1 \in \mathcal{L}(w_2, \ldots, \ldots) \implies n_1 \neq n_2$.

$\mathcal{L}(v, \to_{[n]}, \to_{\mathsf{br},[>n]})$:= subchart induced
by entry steps $\to_{[n]}$ from v
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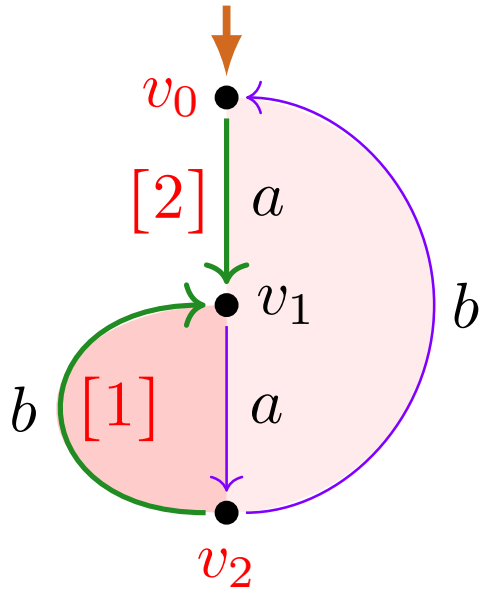
or entry steps $\to_{[m]}$ with $m > n$,
until v is reached again

Layered LEE-witness



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Definition

A loop–branch labeling is a layered LEE-witness, if:

I-L1. $\forall n \in \mathbb{N} \forall v \in V\Big(v \to_{[n]} \Rightarrow \mathcal{L}(v, \to_{[n]}, \to_{\mathsf{br},[>n]}) \text{ is a loop subchart} \Big)$.

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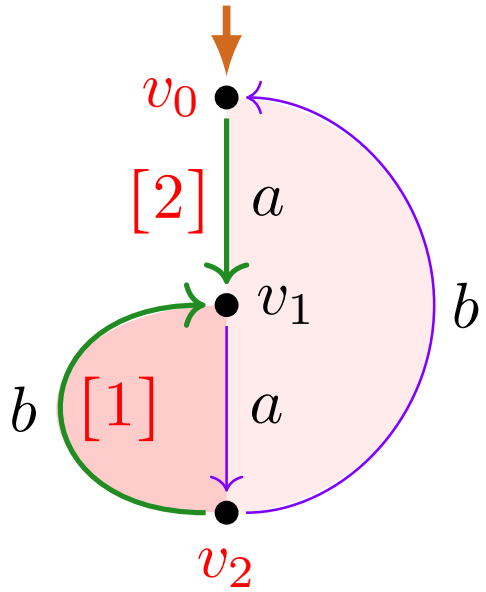

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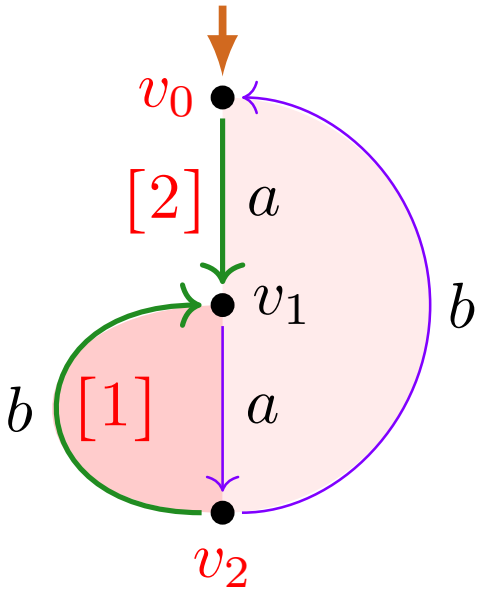

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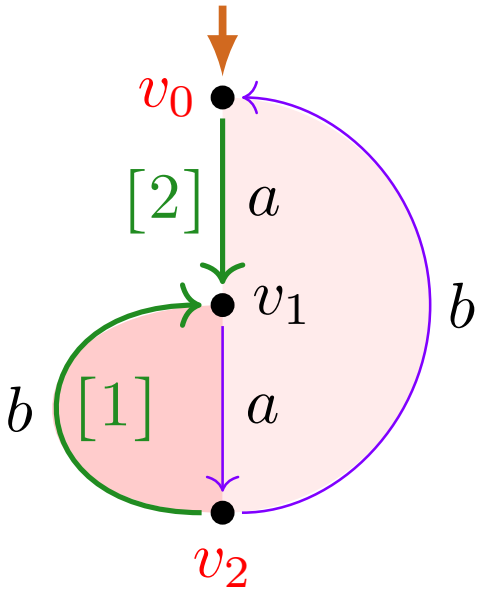

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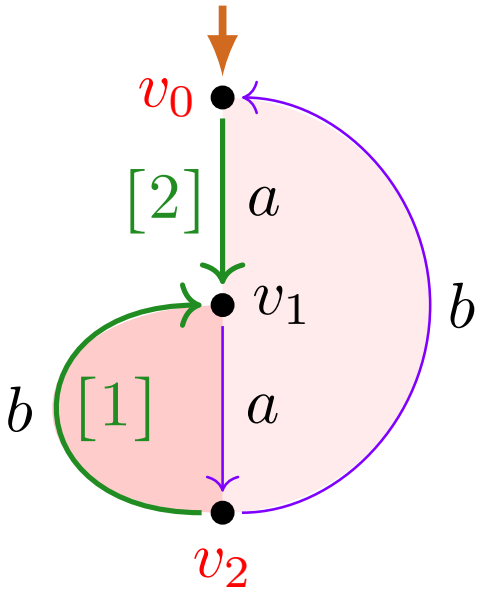

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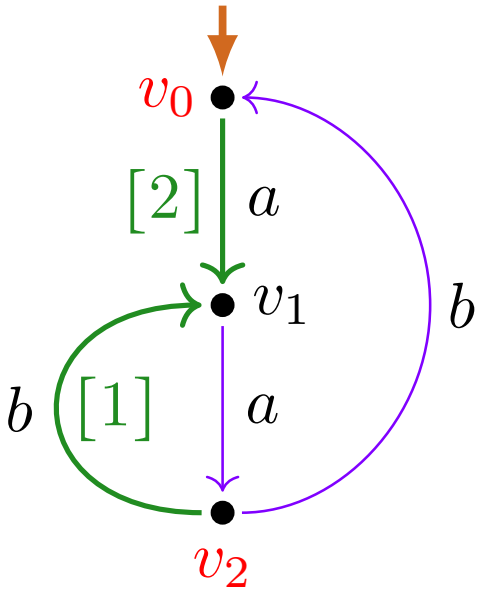

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LEE versus LEE-witness

Theorem

For every process graph G :

$$\mathrm{LEE}(G) \iff G \textit{ has a LEE-witness.}$$

LEE versus LEE-witness

Theorem

For every process graph G *:*

$$\mathrm{LEE}(G) \iff G \text{ has a LEE-witness.}$$

Proof.

$\Rightarrow$: record loop elimination

LEE versus LEE-witness

Theorem

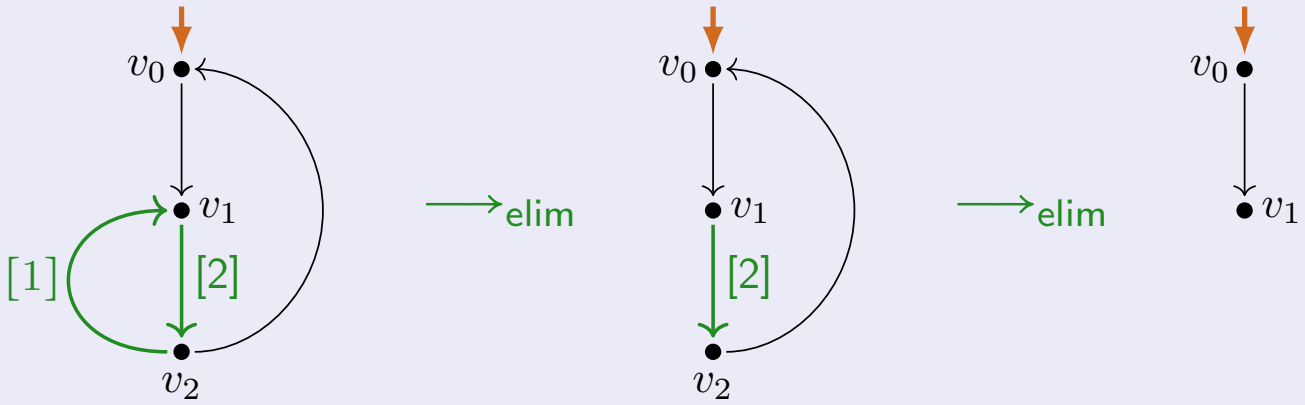
For every process graph G :

$$\mathrm{LEE}(G) \iff G \textit{ has a LEE-witness.}$$

Proof.

$\Rightarrow$: record loop elimination

$\Leftarrow$: carry out loop-elimination as indicated in the LEE-witness, in *inside–out* direction, e.g.:



LEE and (layered) LEE-witness

Lemma

Every layered LEE-witness is a LEE-witness.

Lemma

Every LEE-witness $\widehat{G}$ of a process graph G
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Every LEE-witness $\widehat{G}$ of a process graph G can be transformed by an effective procedure (cut-elimination-like) into a layered LEE-witness $\widehat{G}'$ of G.

Lemma

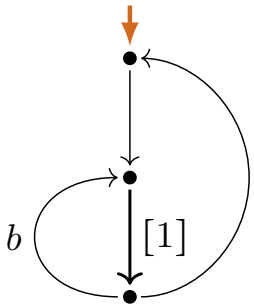
For every process graph G the following are equivalent:

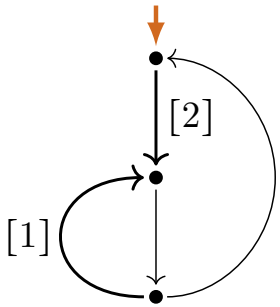
(i) LEE(G).

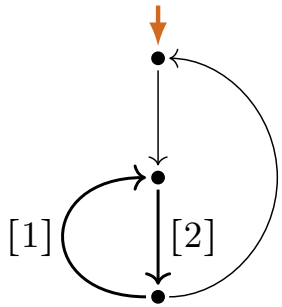
(ii) *G has a LEE-witness.*

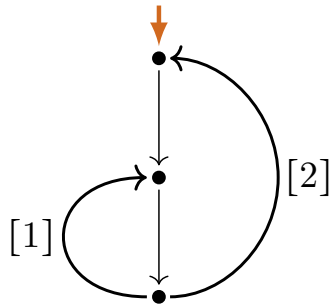
(iii) *G has a layered LEE-witness.*

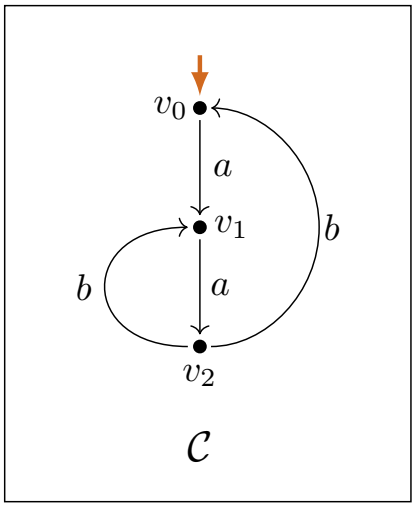
7 LEE-witnesses

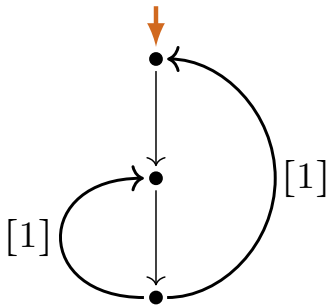


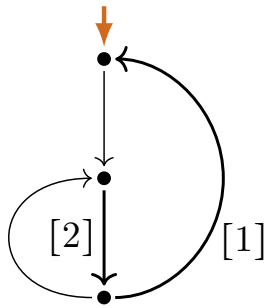


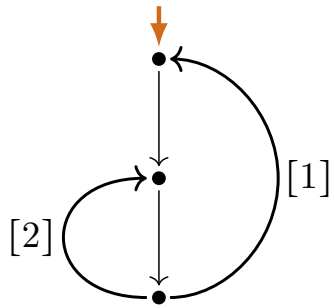




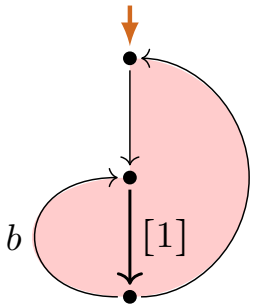


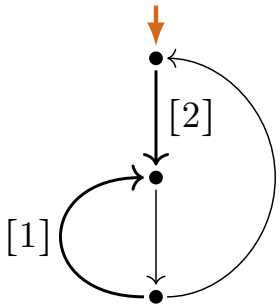


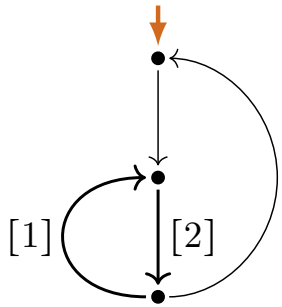


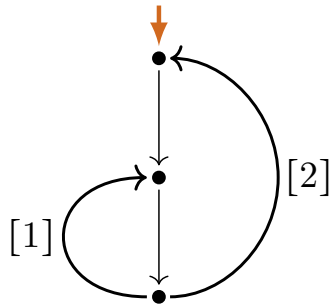


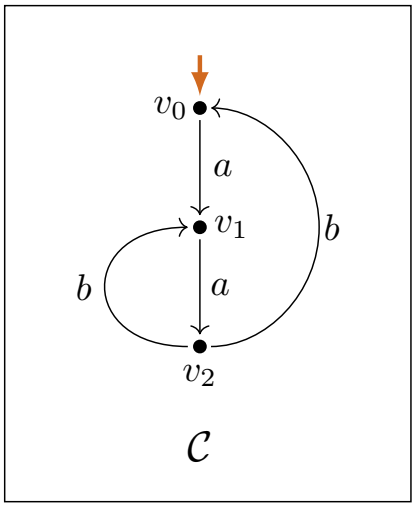
7 LEE-witnesses

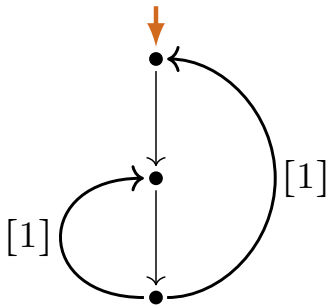


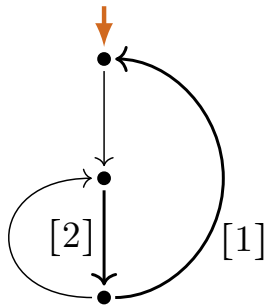


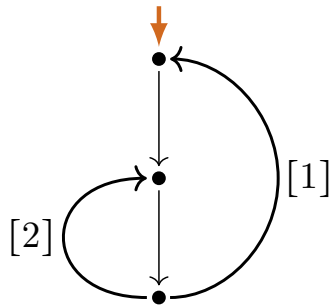




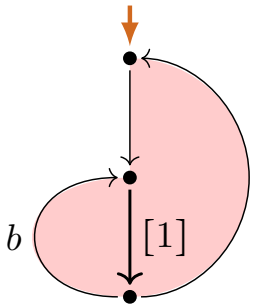


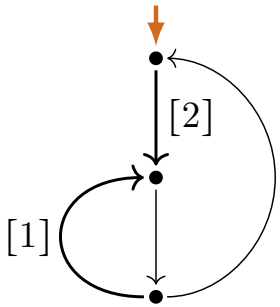


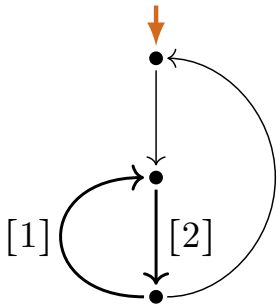


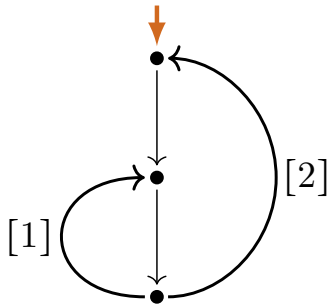


7 LEE-witnesses









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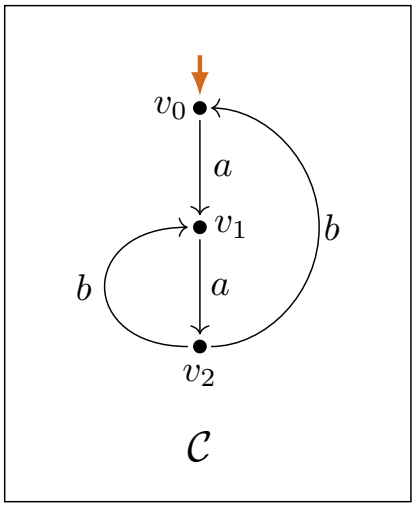
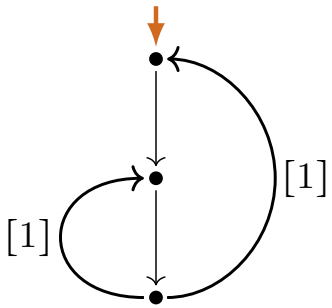
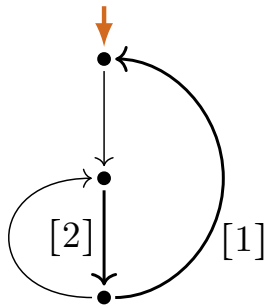
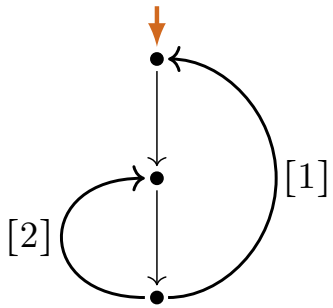
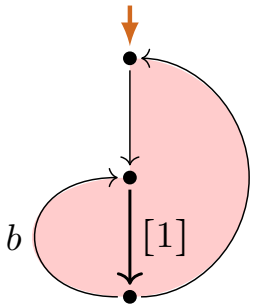


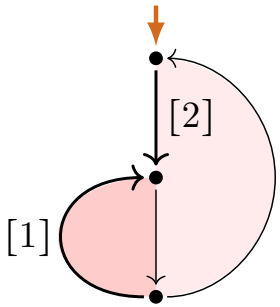


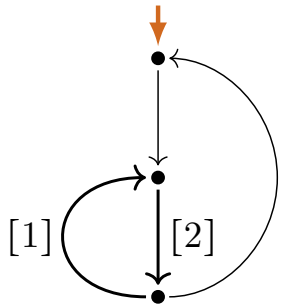


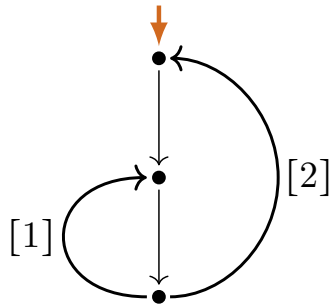


7 LEE-witnesses









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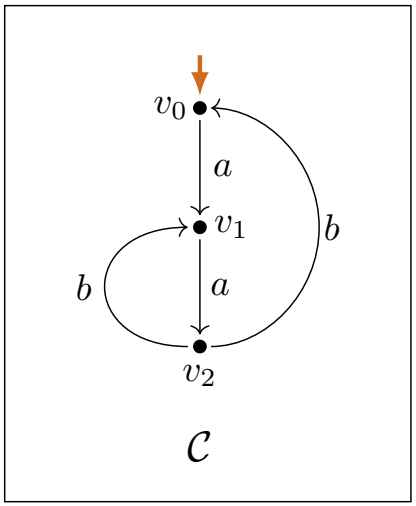
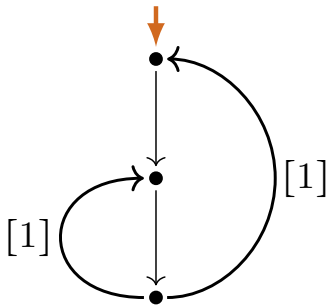
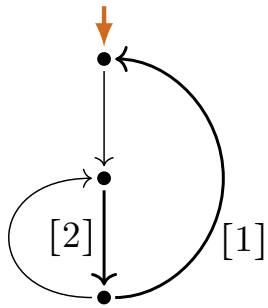
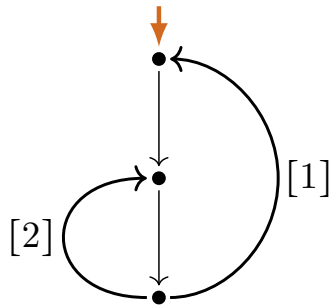
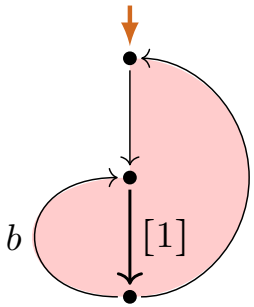








7 LEE-witnesses



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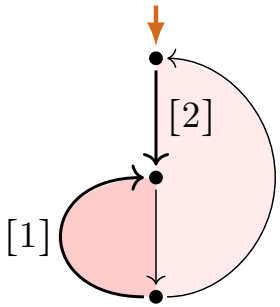

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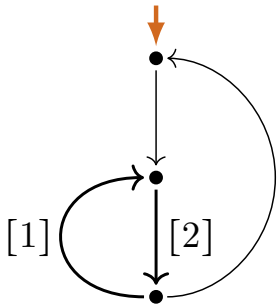
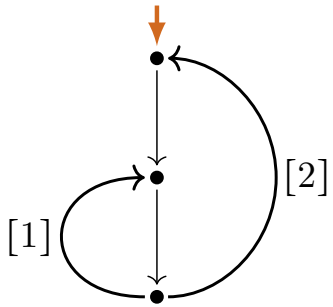
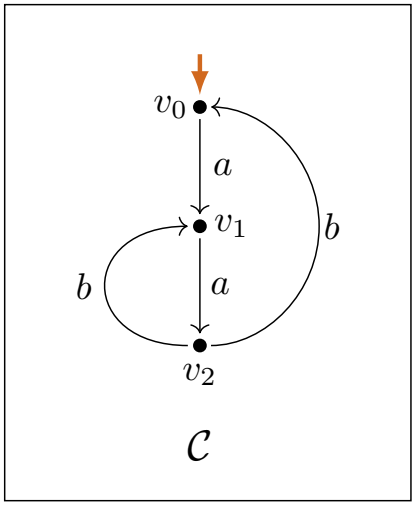
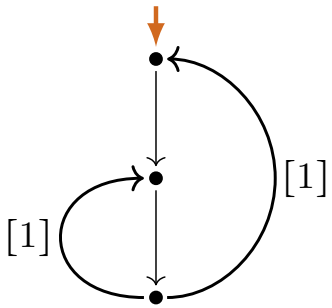
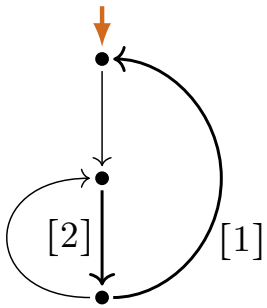
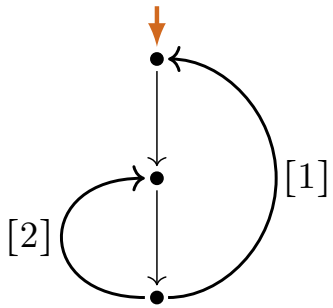




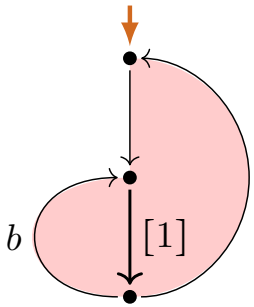








7 LEE-witnesses



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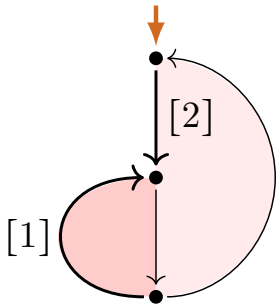

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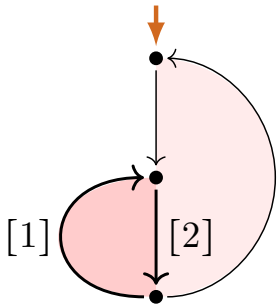

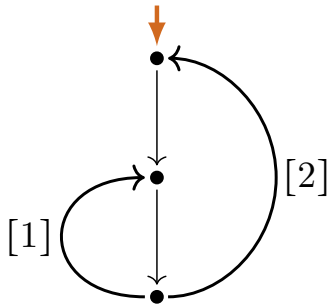

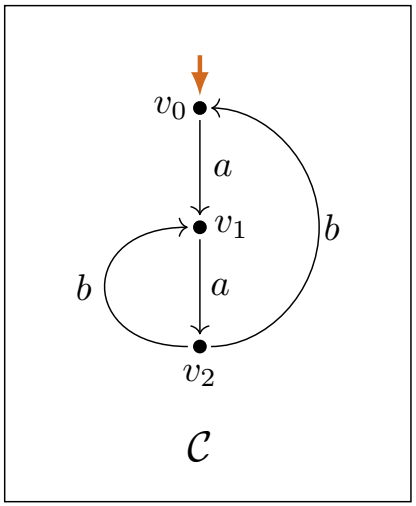

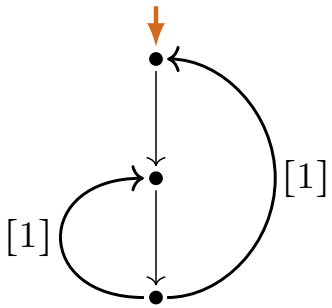

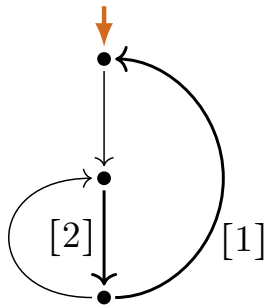

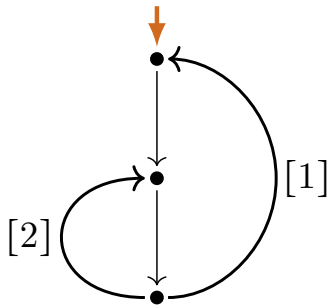

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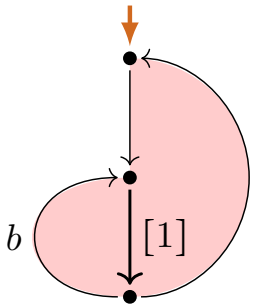

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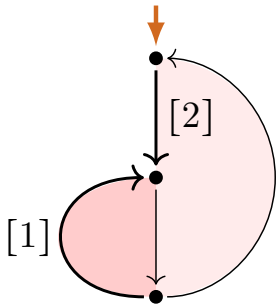

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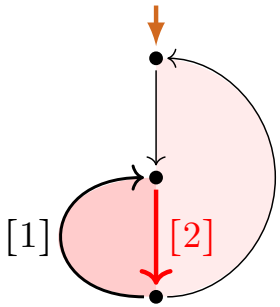

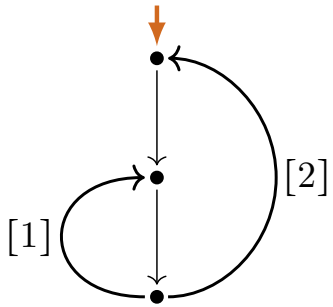

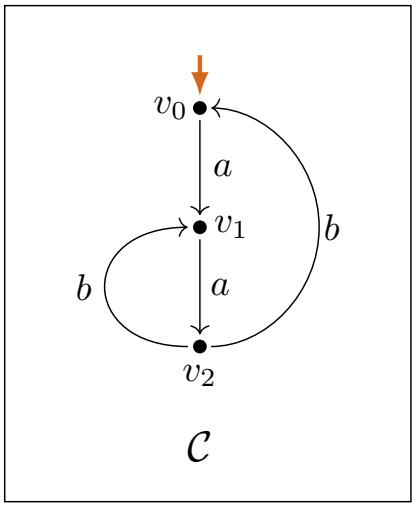

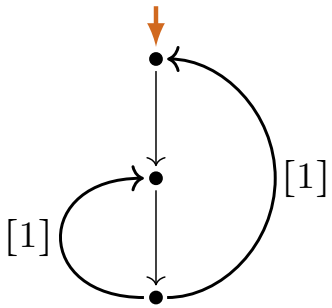

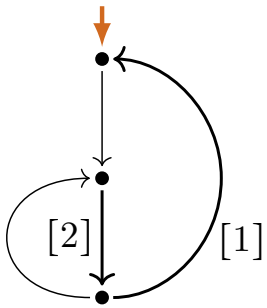

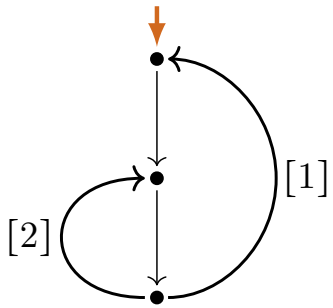

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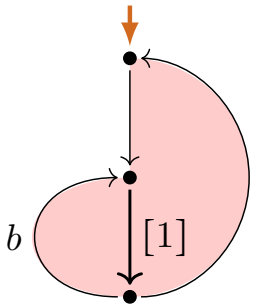

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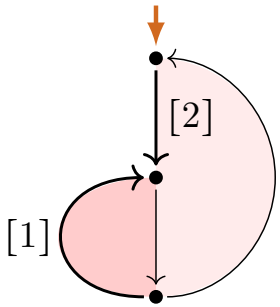

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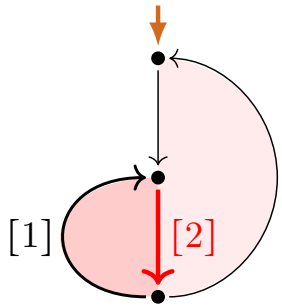

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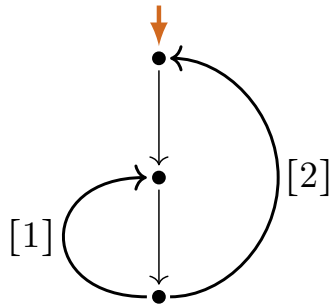
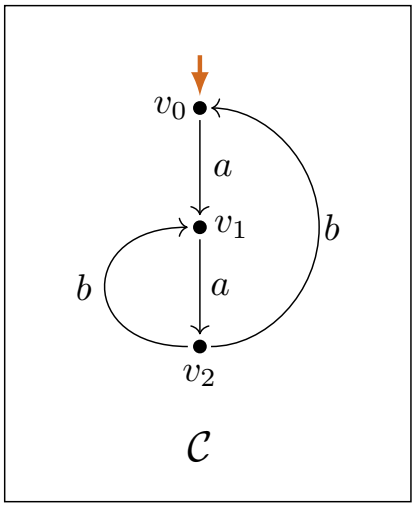
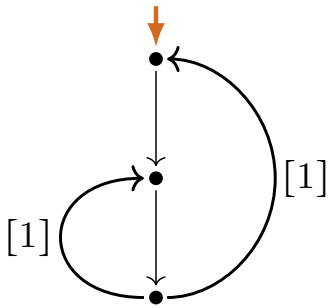
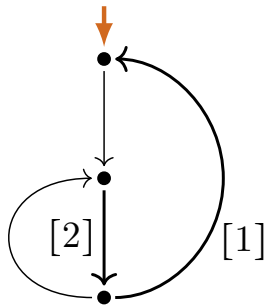
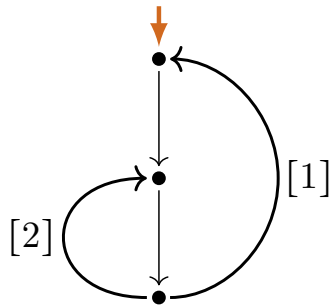


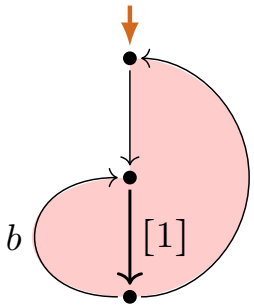








7 LEE-witnesses



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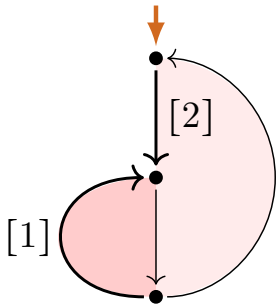

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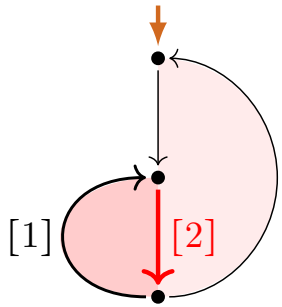

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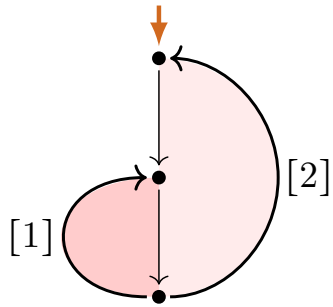
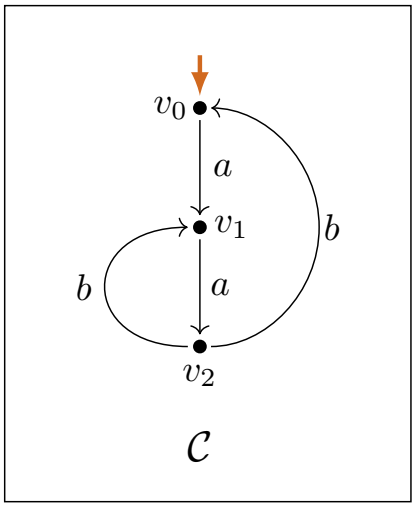
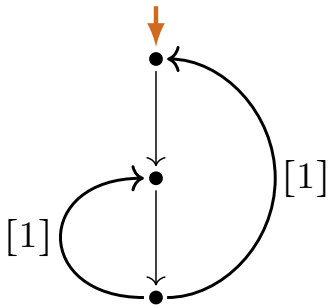
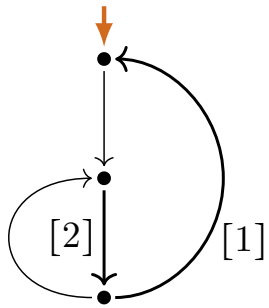
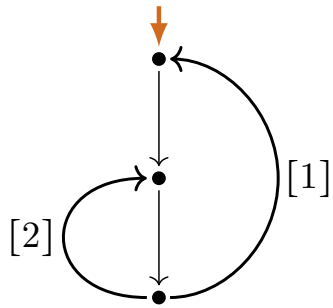


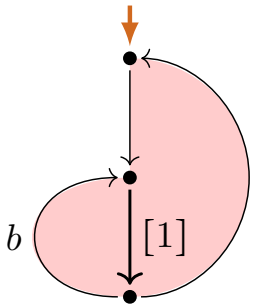








7 LEE-witnesses



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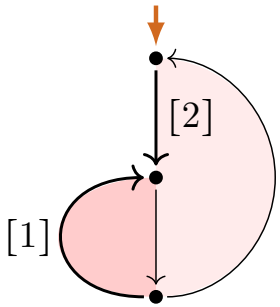

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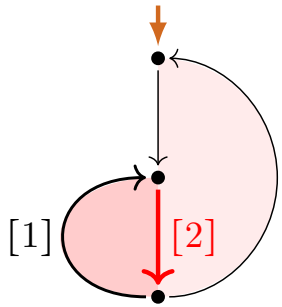

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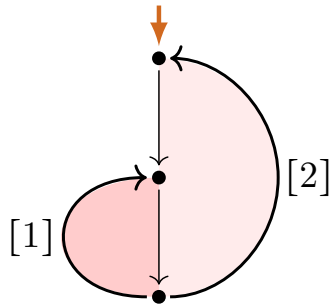

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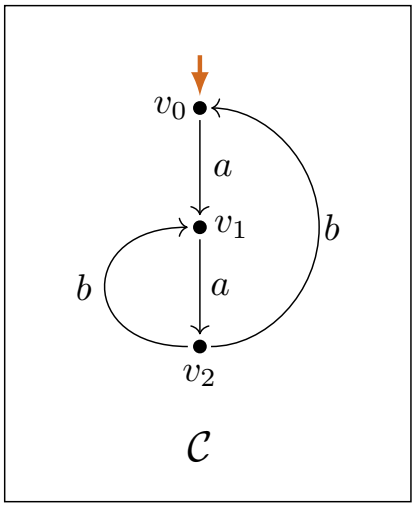
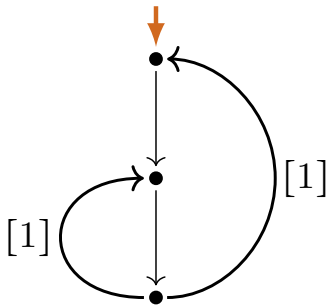
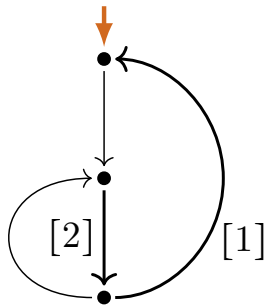
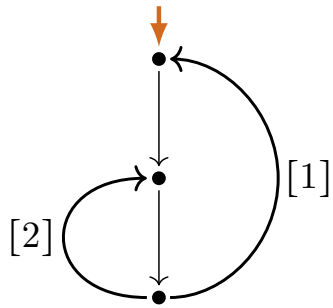
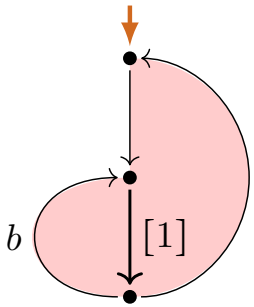








7 LEE-witnesses



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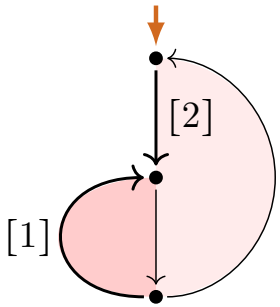

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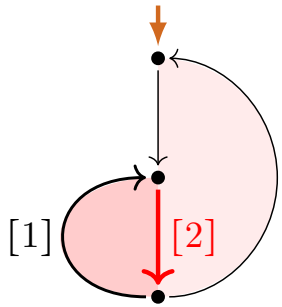

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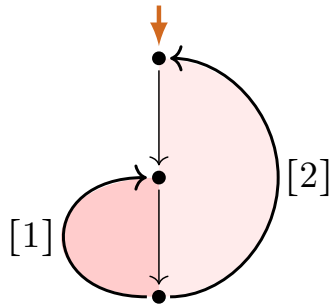

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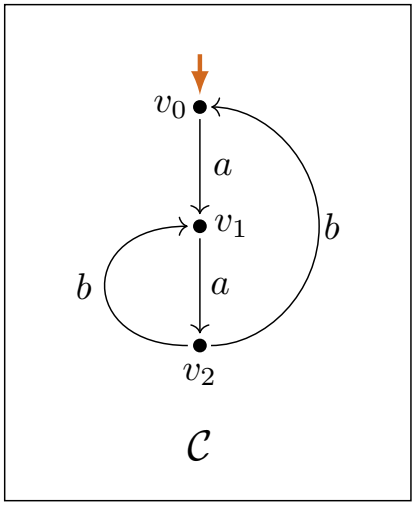
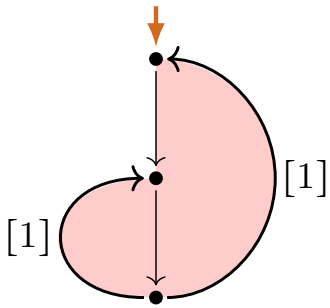
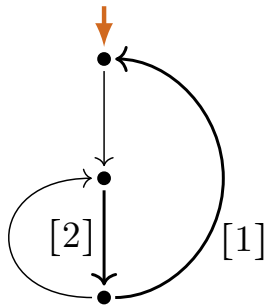
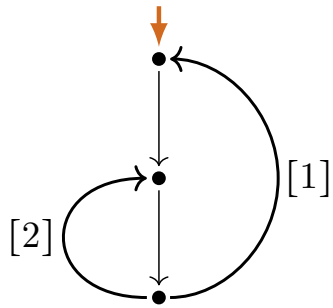
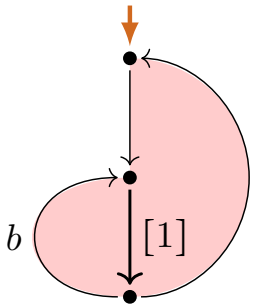








7 LEE-witnesses



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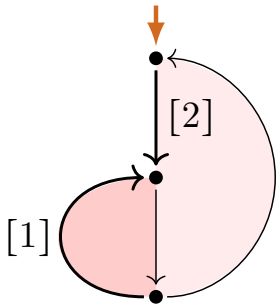

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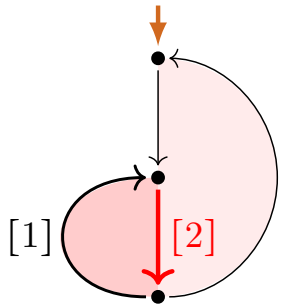

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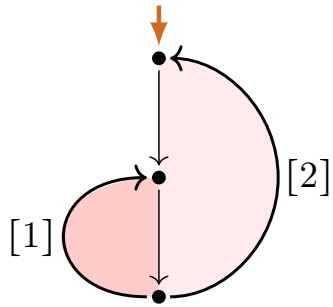

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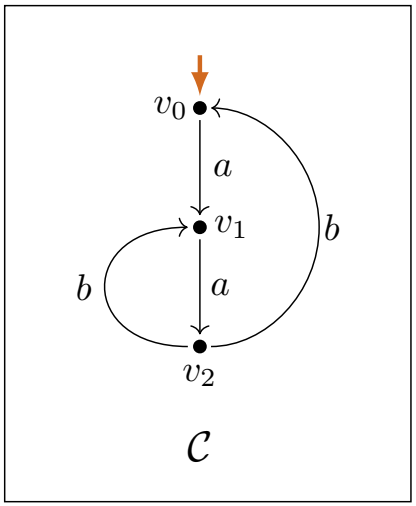

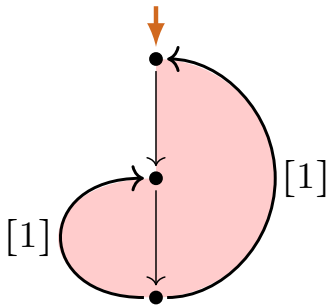

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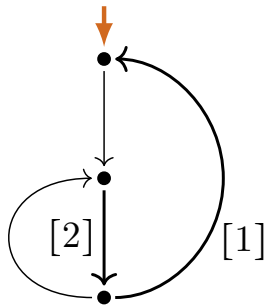

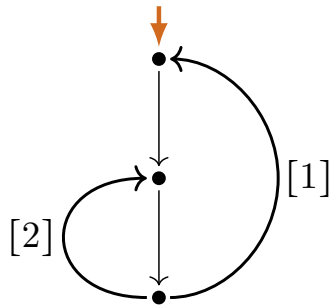

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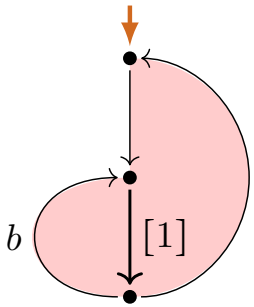

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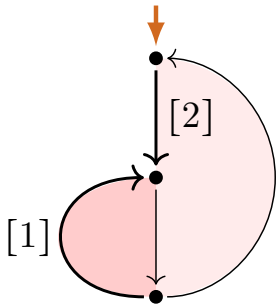

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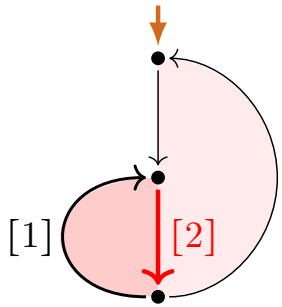

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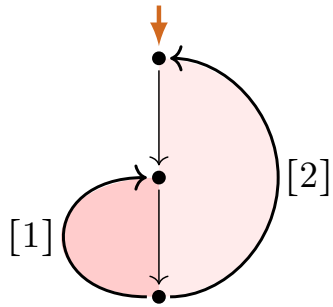

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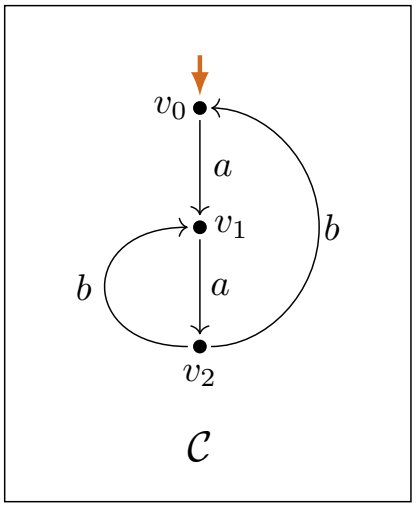

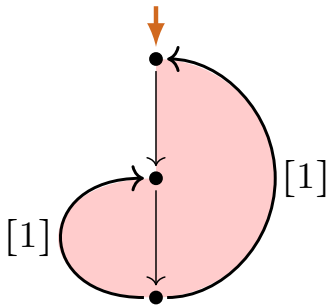

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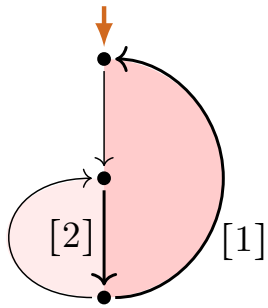

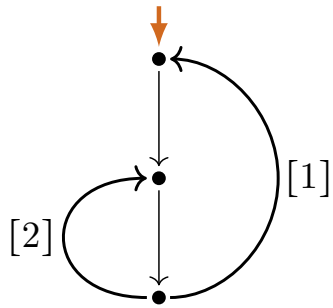

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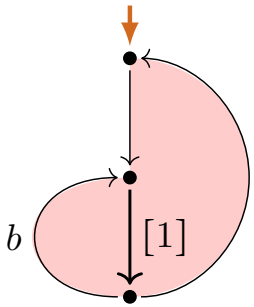

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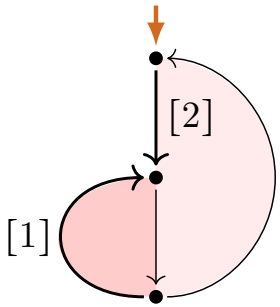

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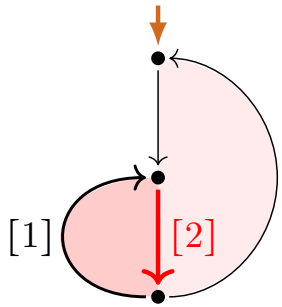

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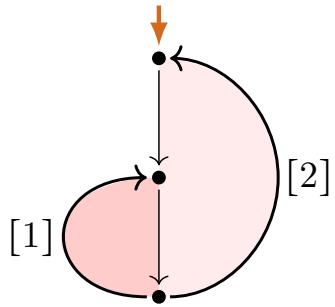

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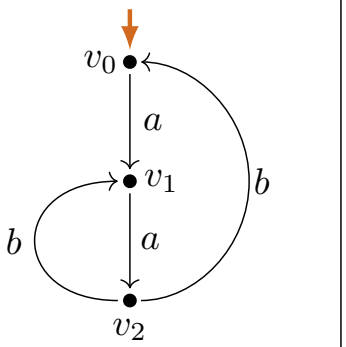

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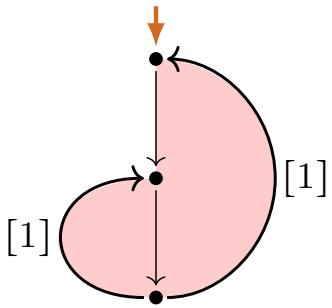

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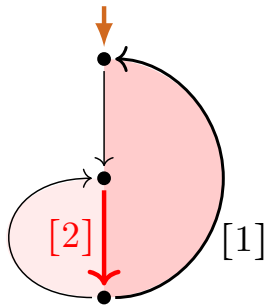
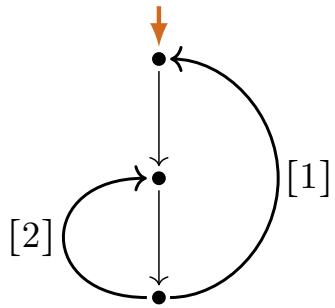
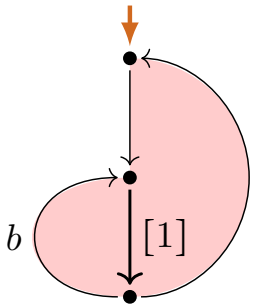




7 LEE-witnesses



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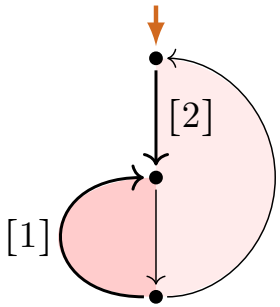

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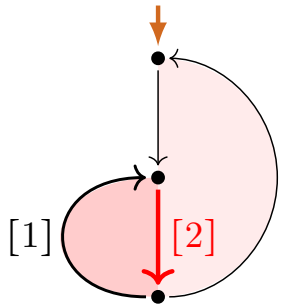

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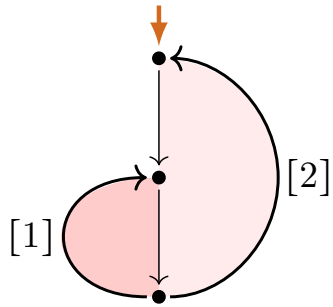

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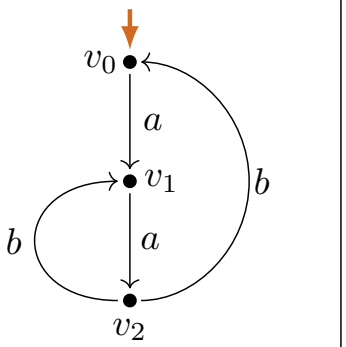

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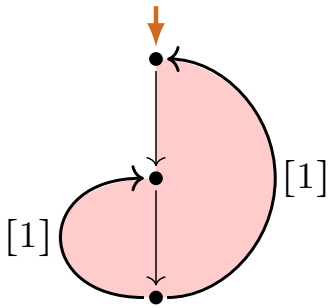

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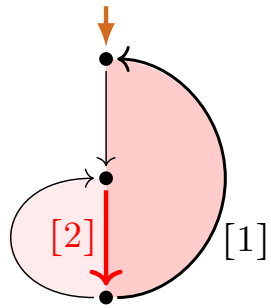

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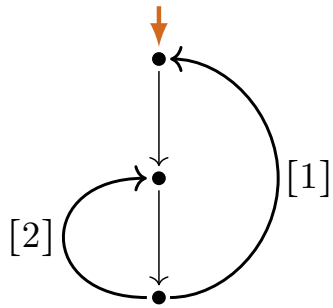
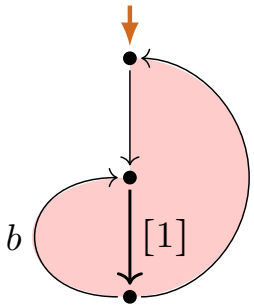


7 LEE-witnesses



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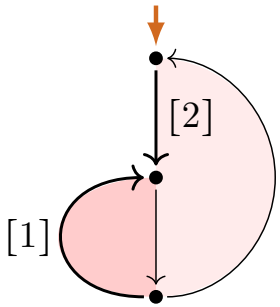

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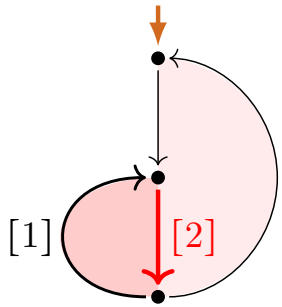

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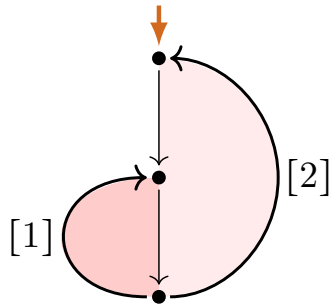

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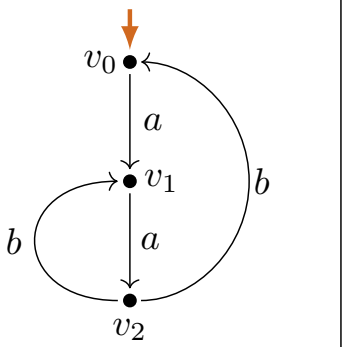

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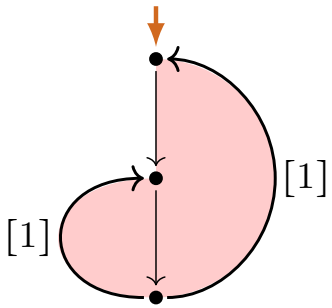

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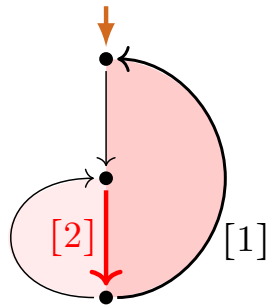

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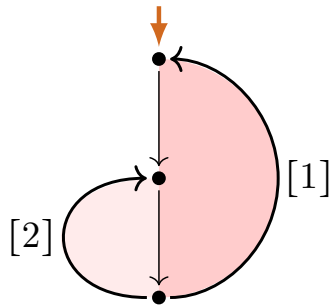
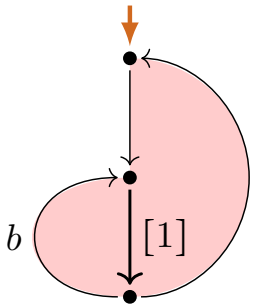


7 LEE-witnesses



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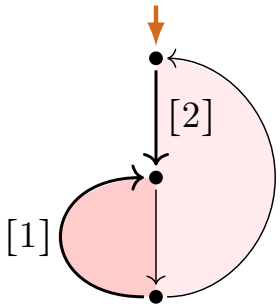

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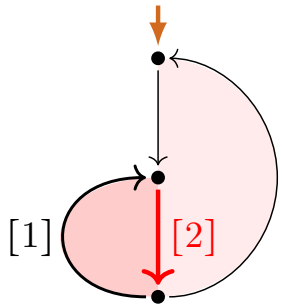

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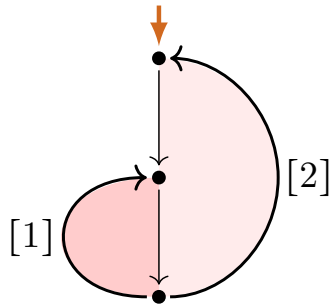

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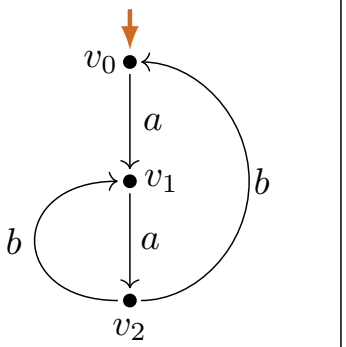

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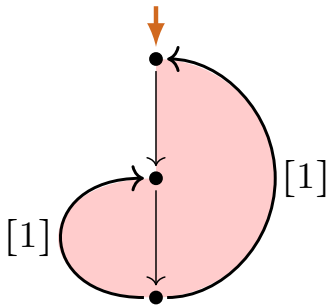

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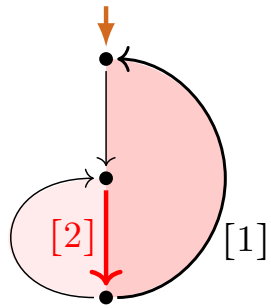

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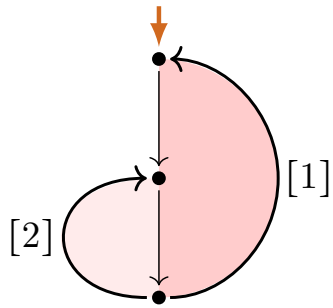

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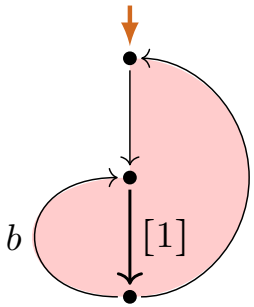
7 LEE-witnesses



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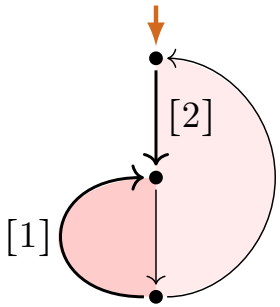

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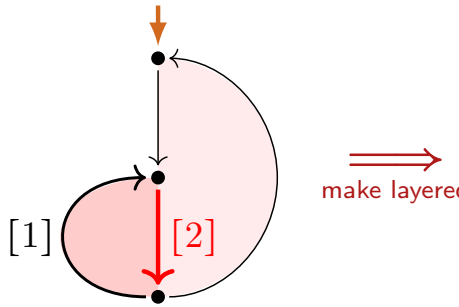

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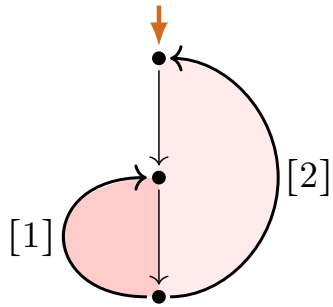

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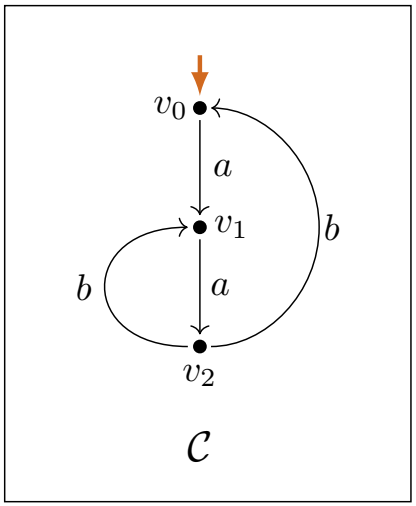

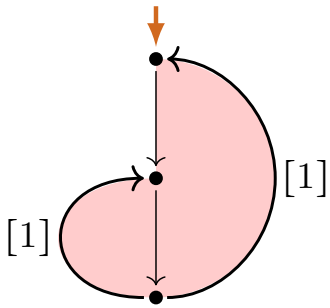

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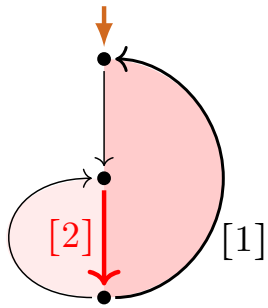

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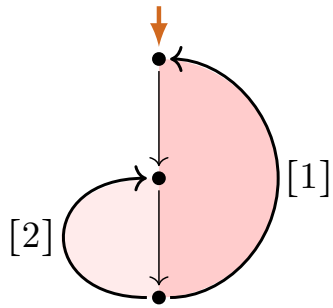

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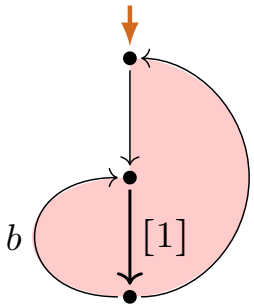
7 LEE-witnesses



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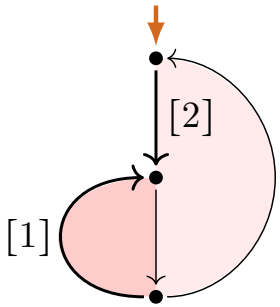

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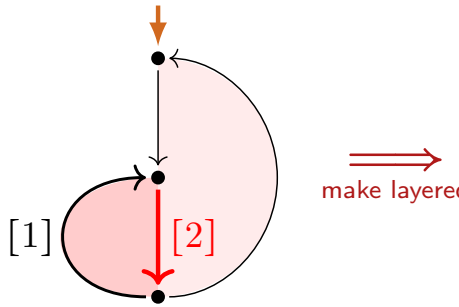

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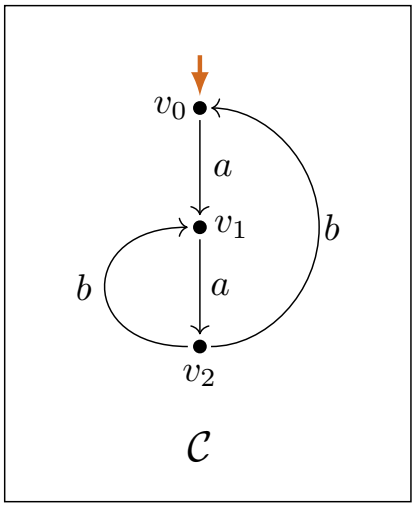

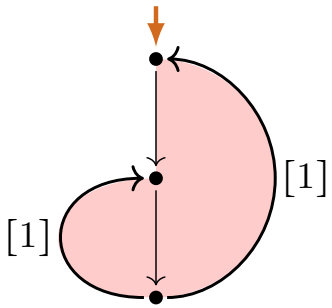

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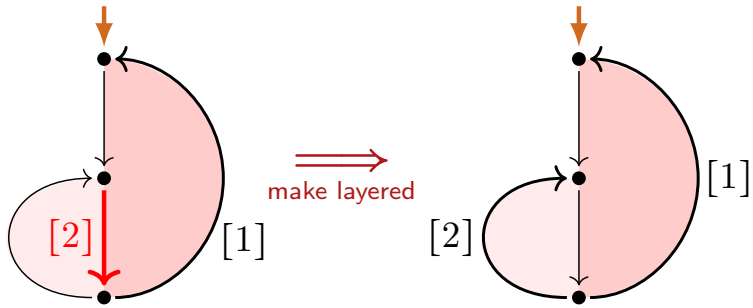

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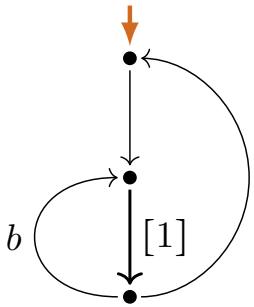
7 LEE-witnesses



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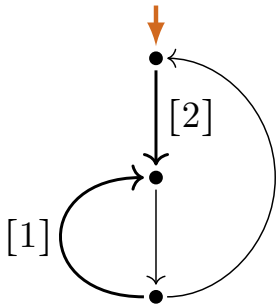

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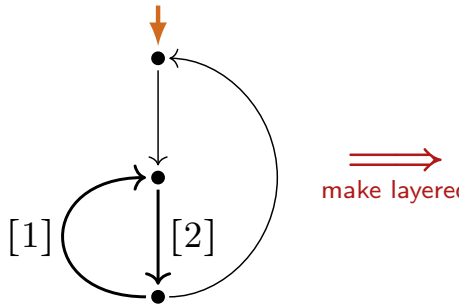

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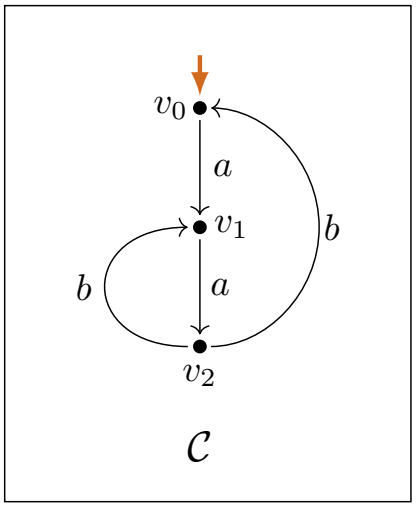

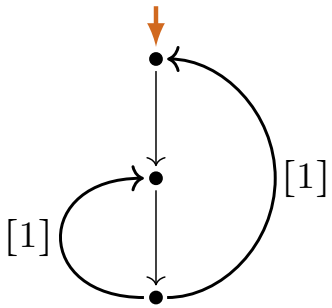

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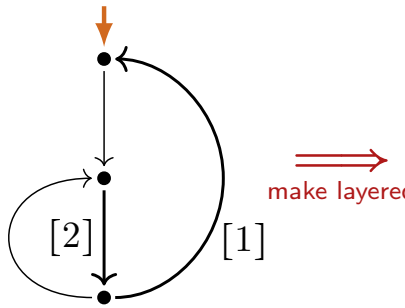

not layered layered

LEE under bisimulation?

LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.

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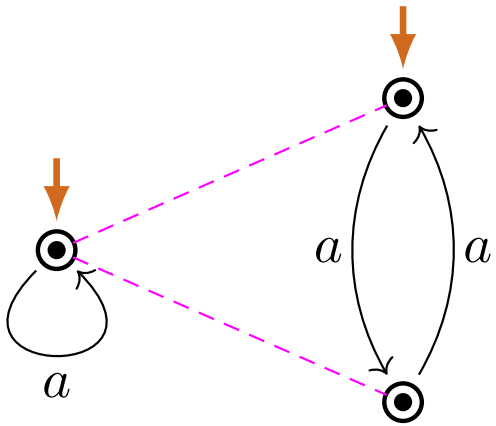
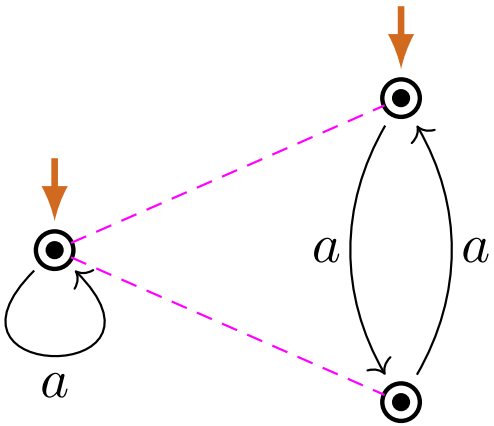


LEE ¬LEE

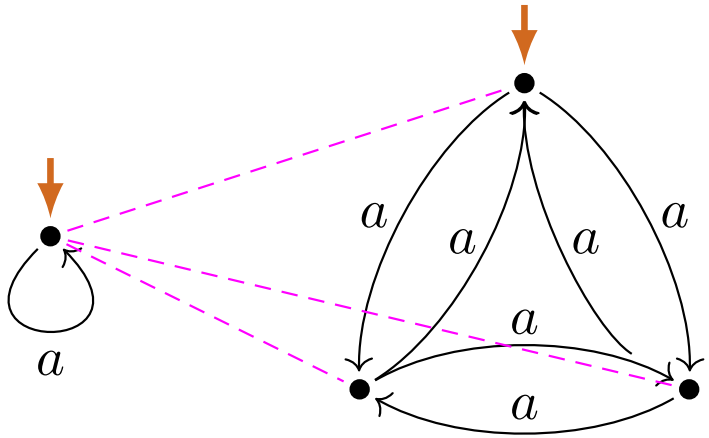
LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.



LEE ¬LEE

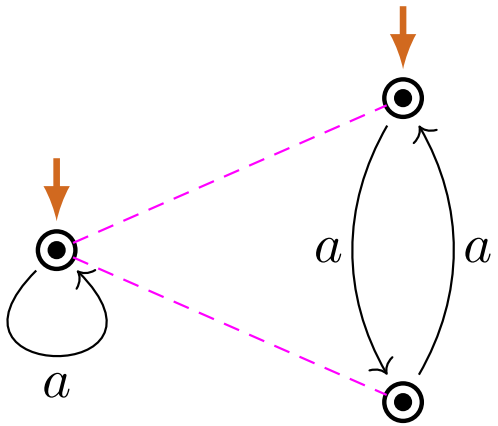


LEE ¬LEE

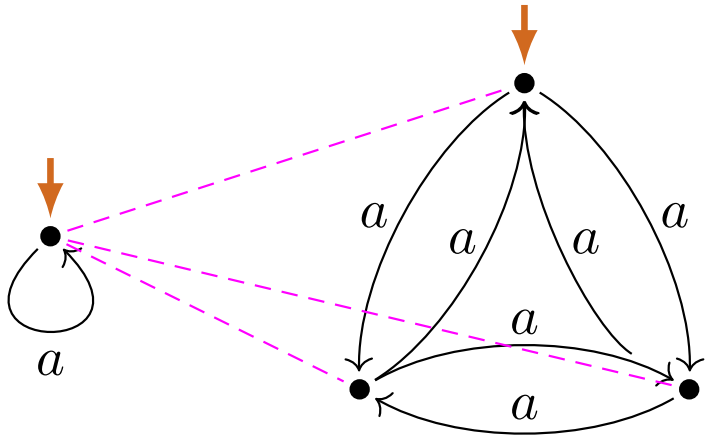
LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.
- LEE is not preserved by converse functional bisimulation.



LEE ¬LEE



LEE ¬LEE

LEE under functional bisimulation

Lemma

(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \twoheadrightarrow G_2 \implies \mathsf{LEE}(G_2) \,.$$

LEE under functional bisimulation

Lemma

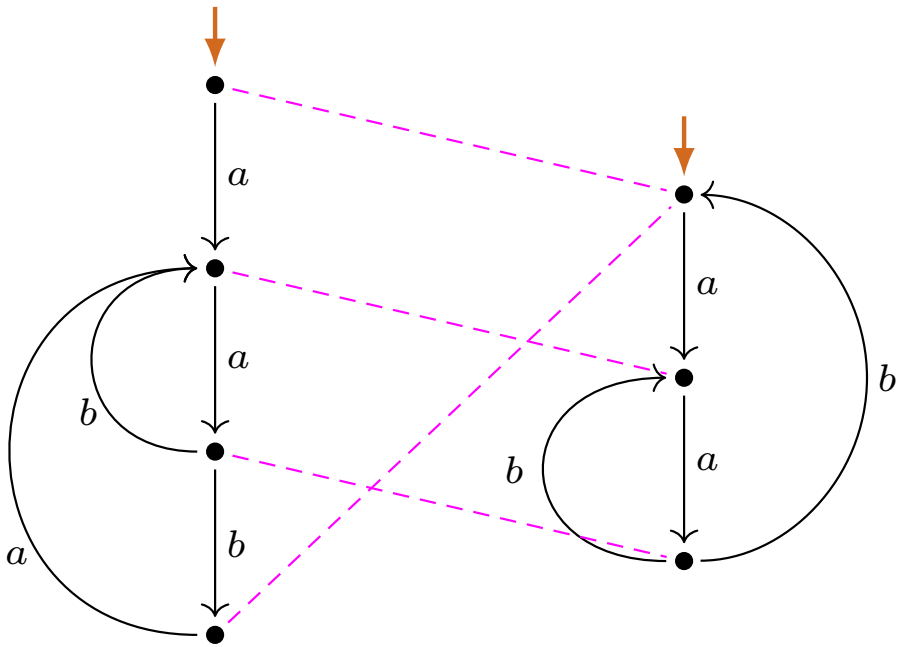
(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \twoheadrightarrow G_2 \implies \mathsf{LEE}(G_2)\ .$$

Proof (Idea).

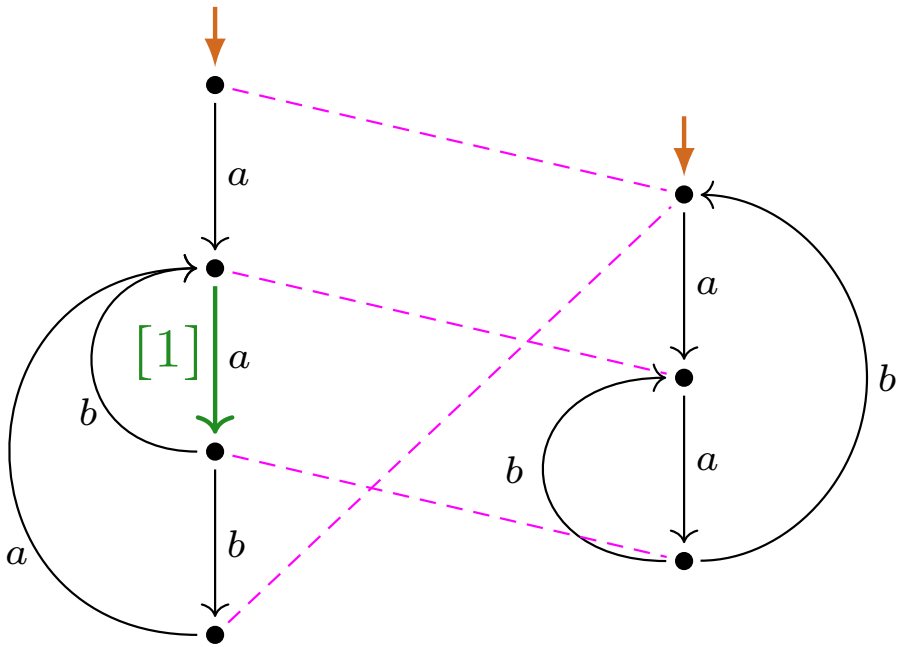
Use loop elimination in G_1 to carry out loop elimination in G_2.

Collapsing LEE-witnesses



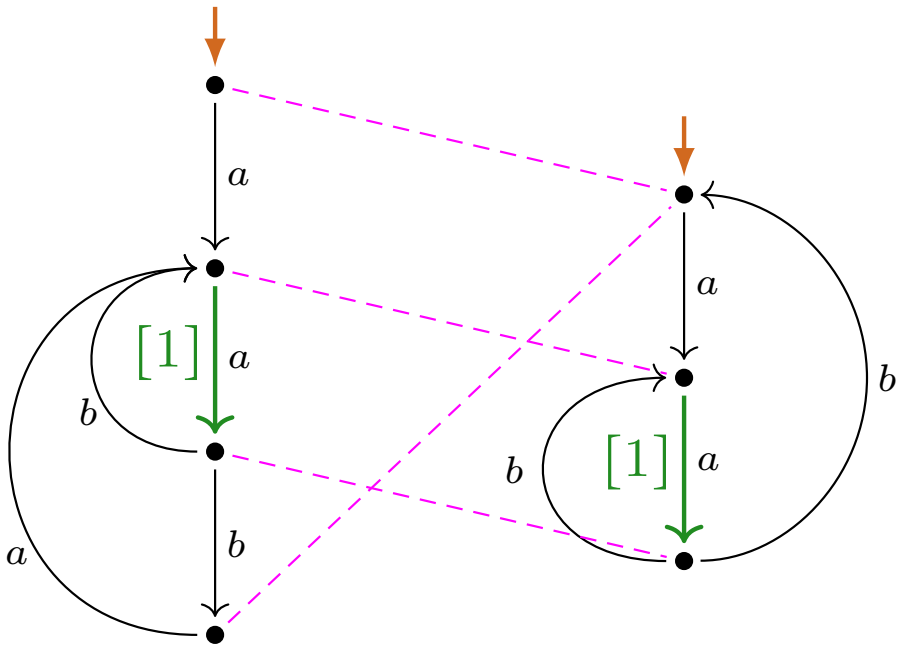
$[\![a(a(b+ba))^*0]\!]_{\boldsymbol{P}}$

Collapsing LEE-witnesses



$$[\![a(a(b+ba))^*0]\!]_{\boldsymbol{P}}$$

Collapsing LEE-witnesses



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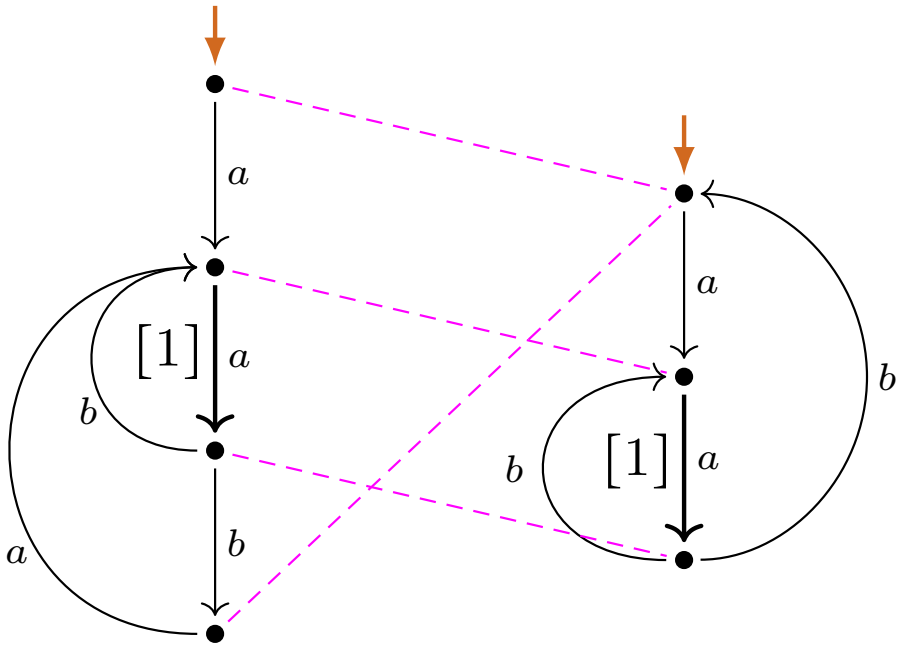

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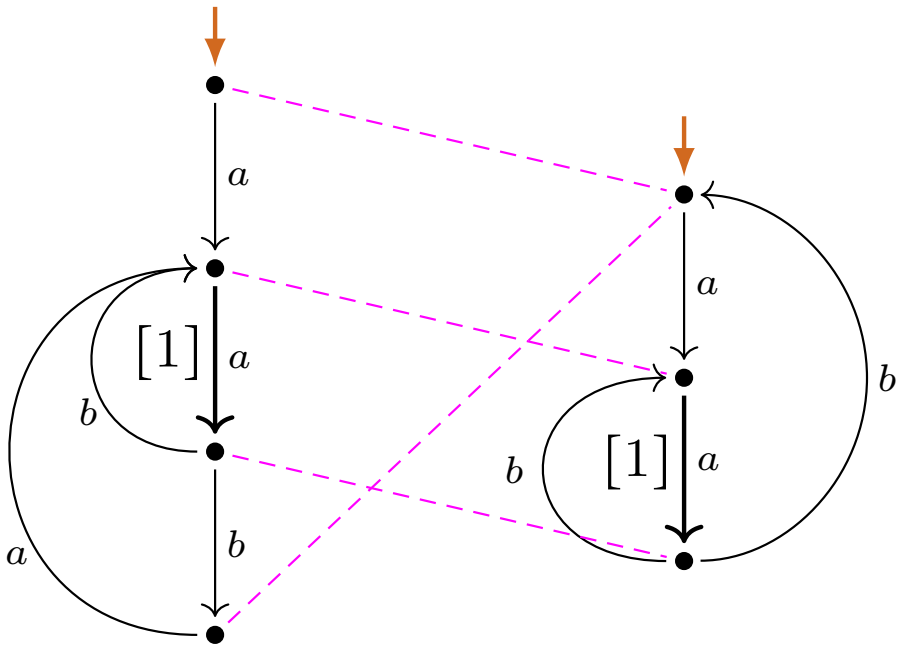
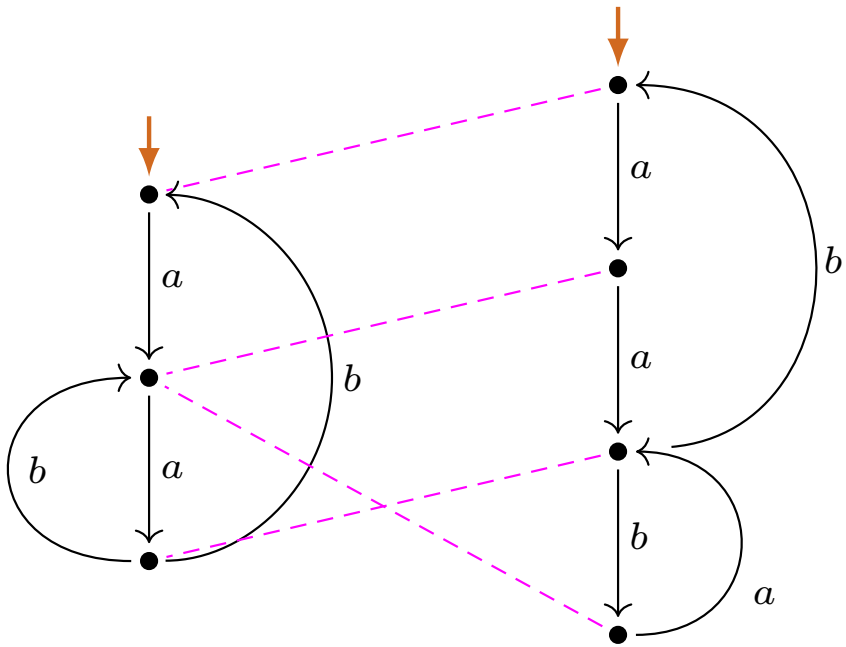
Collapsing LEE-witnesses

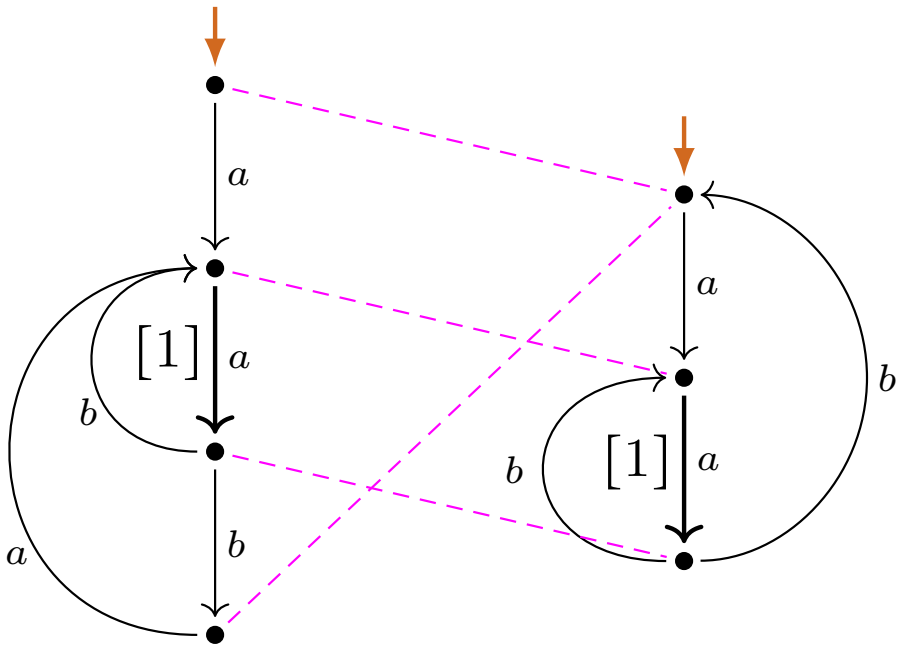


$[\![a(a(b+ba))^*0]\!]_{\boldsymbol{P}}$



$[\![(aa(ba)^*b)^*0]\!]_{\boldsymbol{P}}$

Collapsing LEE-witnesses



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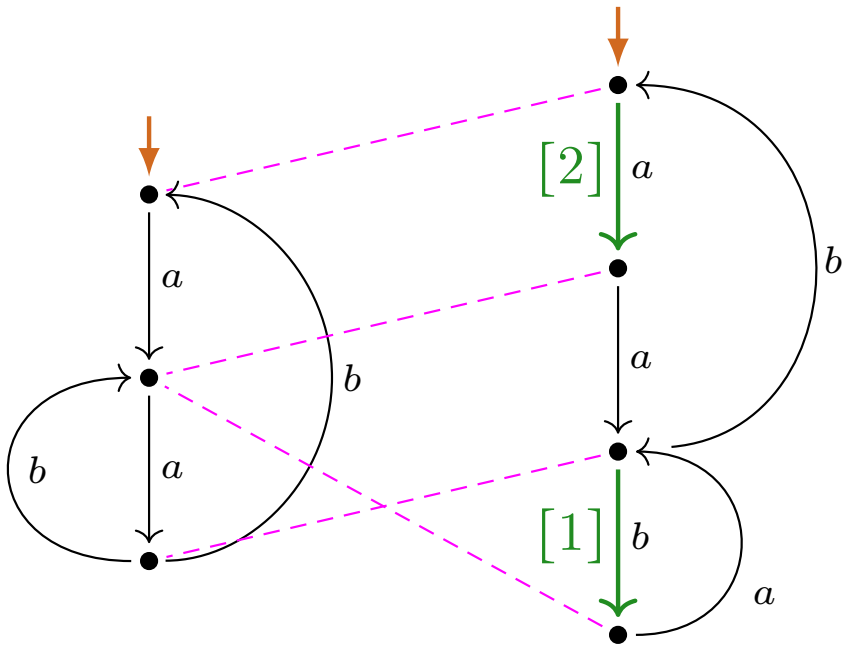

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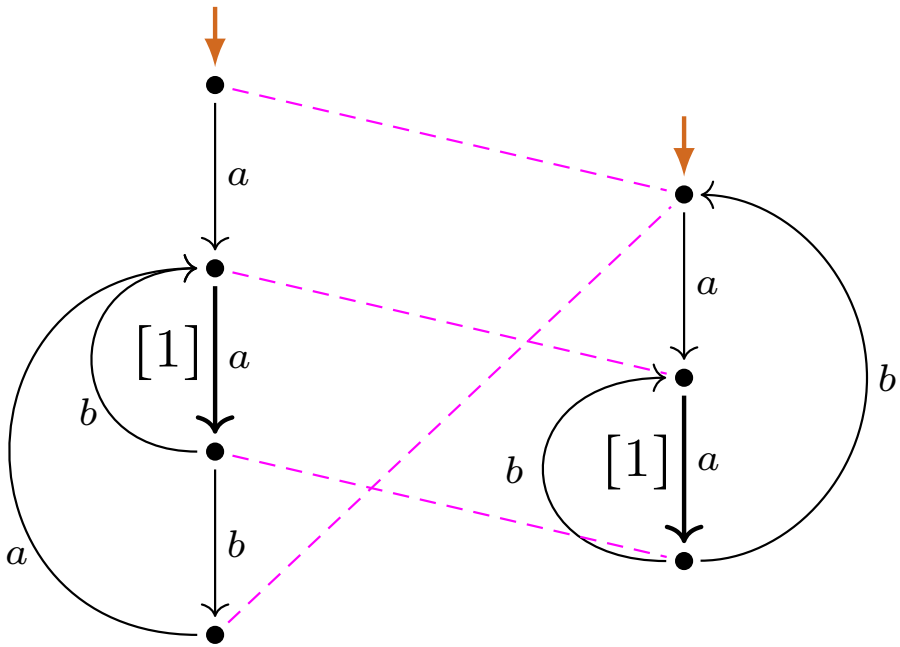
Collapsing LEE-witnesses



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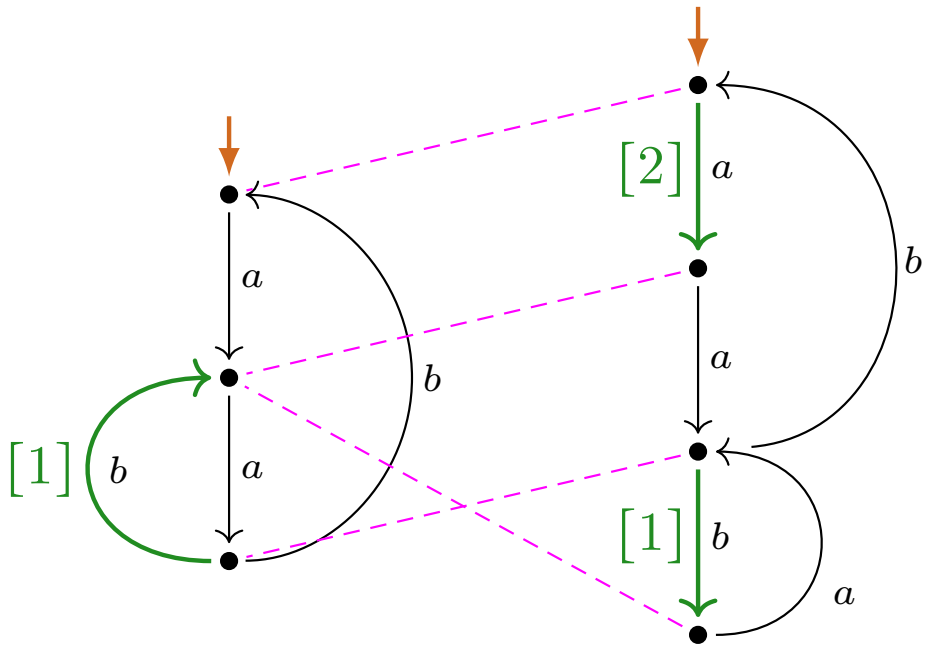

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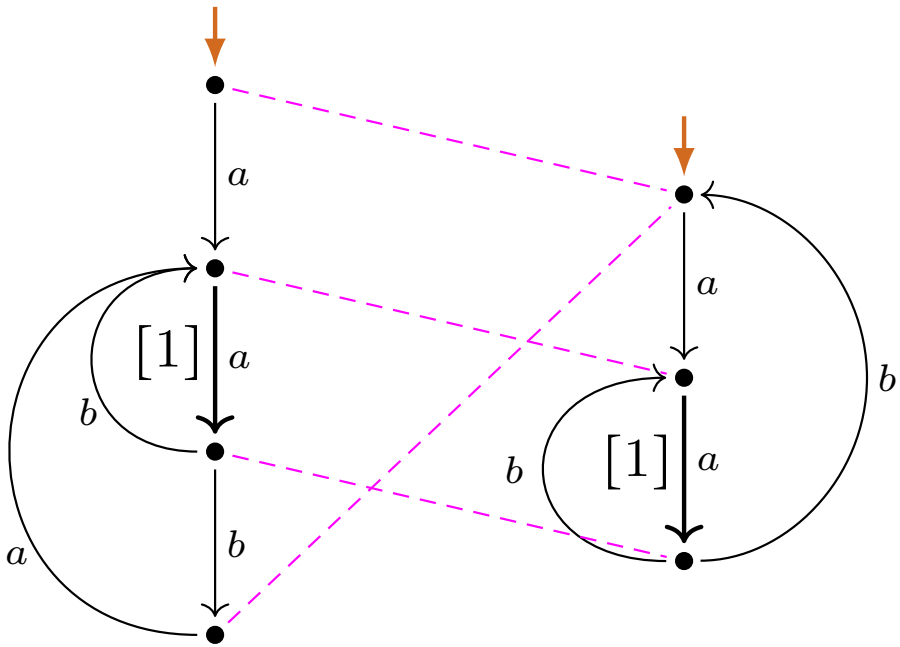

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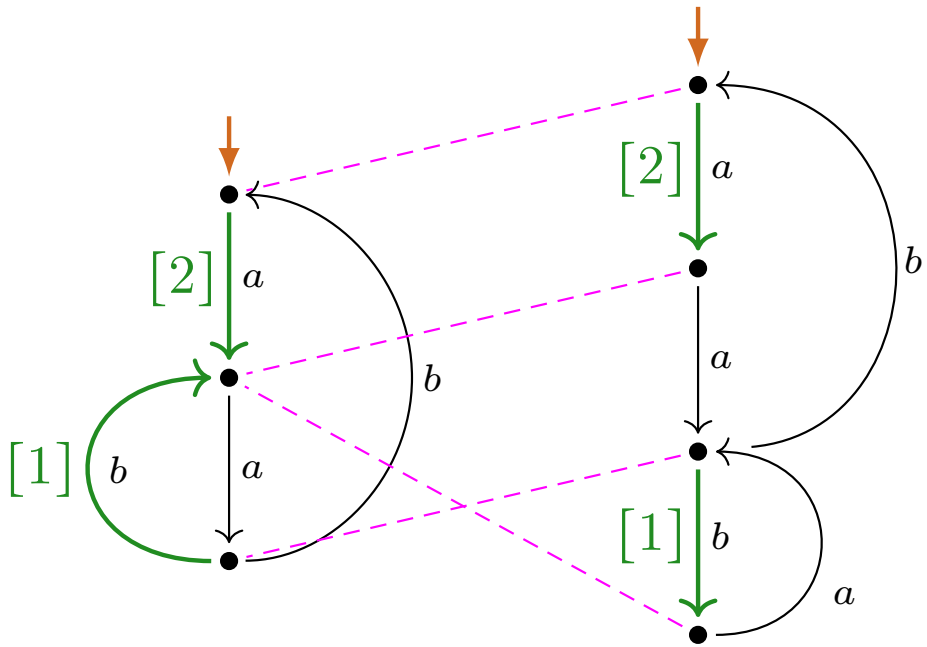

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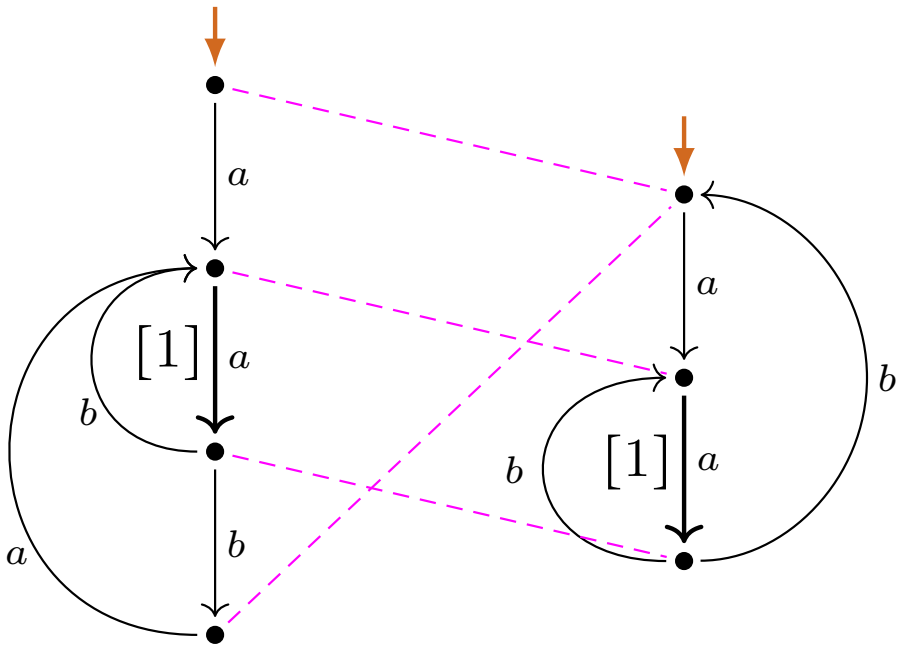
Collapsing LEE-witnesses



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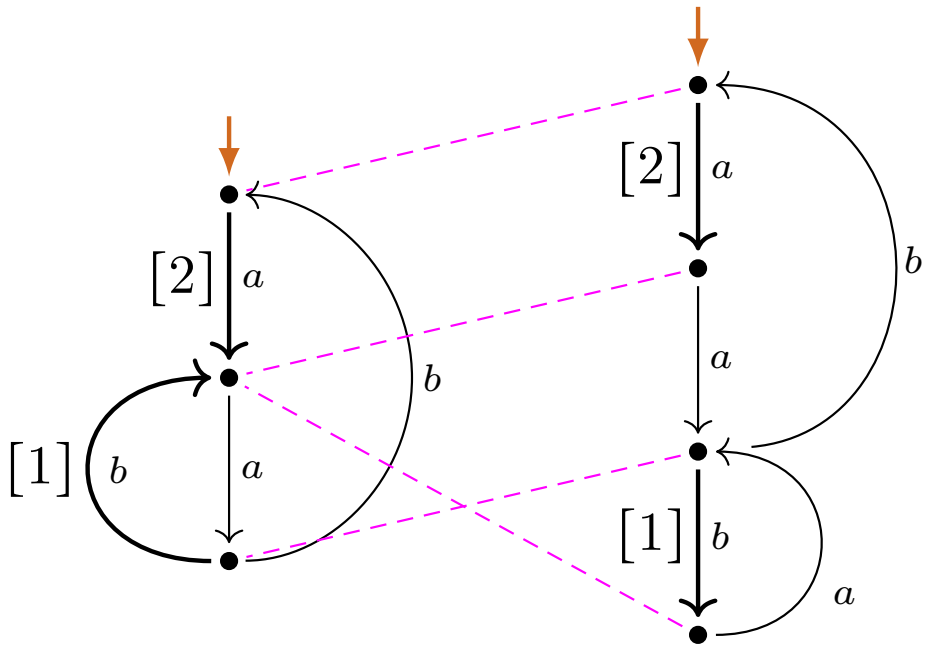

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LEE under functional bisimulation

Lemma

(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \twoheadrightarrow G_2 \implies \mathsf{LEE}(G_2)\ .$$

Idea of Proof for (i)

Use loop elimination in G_1 to carry out loop elimination in G_2.

LEE under functional bisimulation / bisimulation collapse

Lemma

(i) LEE *is preserved by functional bisimulations:*

$$\mathrm{LEE}(G_1) \wedge G_1 \twoheadrightarrow G_2 \implies \mathrm{LEE}(G_2) \ .$$

(ii) LEE *is preserved from a process graph to its bisimulation collapse:*

$$\mathrm{LEE}(G) \wedge C \text{ is bisimulation collapse of } G \implies \mathrm{LEE}(C) \ .$$

Idea of Proof for (i)

Use loop elimination in G_1 to carry out loop elimination in G_2.

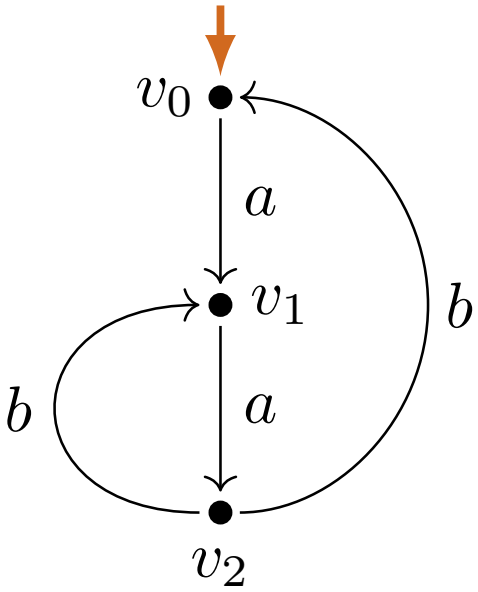
Readback

Lemma

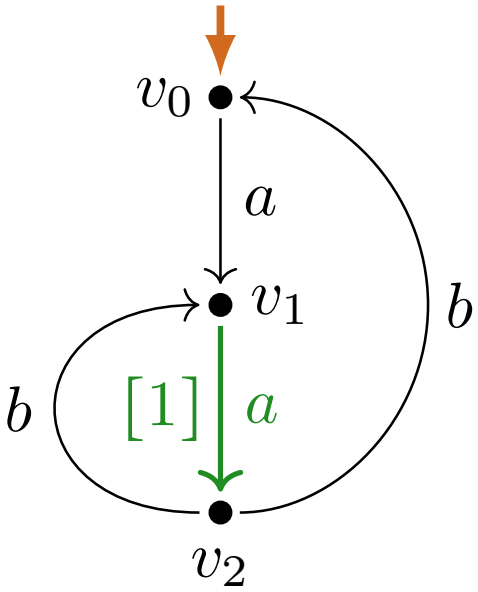
Process graphs with LEE are $[\![\cdot]\!]_{\boldsymbol{P}}$-expressible:

$$\mathsf{LEE}(G) \implies \exists\, e \in \mathsf{Reg}(A) \left(G \leftrightarrows [\![e]\!]_{\boldsymbol{P}} \right).$$

Readback from layered LEE-witness (example)

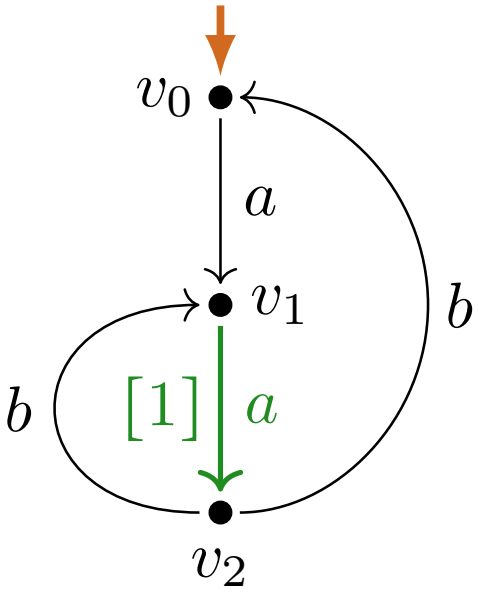


Readback from layered LEE-witness (example)



layered
LEE-witness

Readback from layered LEE-witness (example)

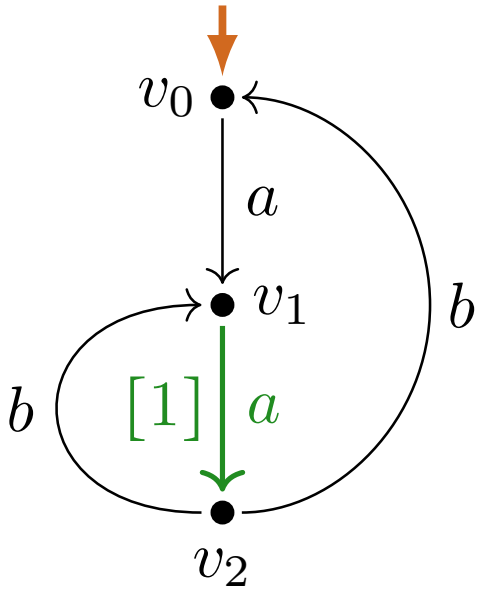


layered
LEE-witness

$$
\begin{aligned}
s(v_0) &= 0^* \cdot a \cdot s(v_1) \\
&=_{\text{Mil}^-} a \cdot s(v_1) \\
&=_{\text{Mil}^-} a \cdot \big(a \cdot (b + b \cdot a)\big)^* \cdot 0 \\
s(v_1) &= \big(a \cdot s(v_2, v_1)\big)^* \cdot 0 \\
&=_{\text{Mil}^-} \big(a \cdot (b + b \cdot a)\big)^* \cdot 0 \\
s(v_2, v_1) &= 0^* \cdot \big(b \cdot s(v_1, v_1) + b \cdot s(v_0, v_1)\big) \\
&=_{\text{Mil}^-} 0^* \cdot \big(b \cdot 1 + b \cdot a\big) \\
&=_{\text{Mil}^-} b + b \cdot a \\
s(v_1, v_1) &= 1 \\
s(v_0, v_1) &= 0^* \cdot a \cdot s(v_1, v_1) \\
&= 0^* \cdot a \cdot 1 \\
&=_{\text{Mil}^-} a
\end{aligned}
$$

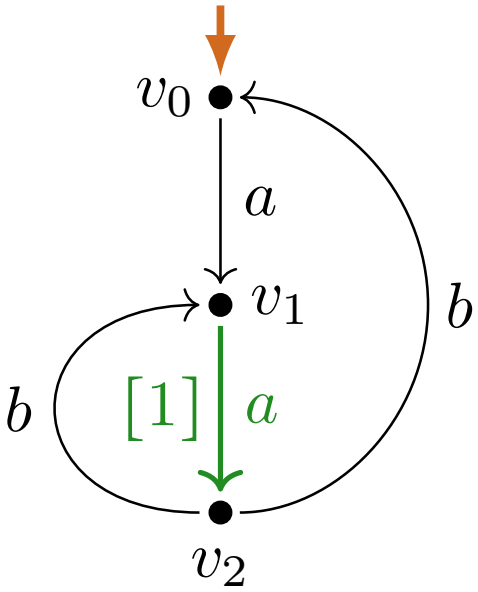
Readback from layered LEE-witness (example)

$$s(v_0) = \quad 0^* \cdot a \cdot s(v_1)$$



layered
LEE-witness

Readback from layered LEE-witness (example)

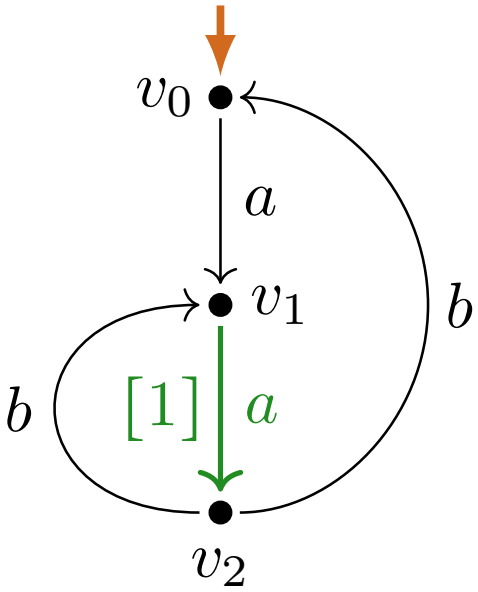


layered
LEE-witness

$$s(v_0) = \quad 0^* \cdot a \cdot s(v_1)$$

$$s(v_1) = \quad \big(a \cdot s(v_2, v_1)\big)^* \cdot 0$$

Readback from layered LEE-witness (example)



layered
LEE-witness

$$s(v_0) = \quad 0^* \cdot a \cdot s(v_1)$$

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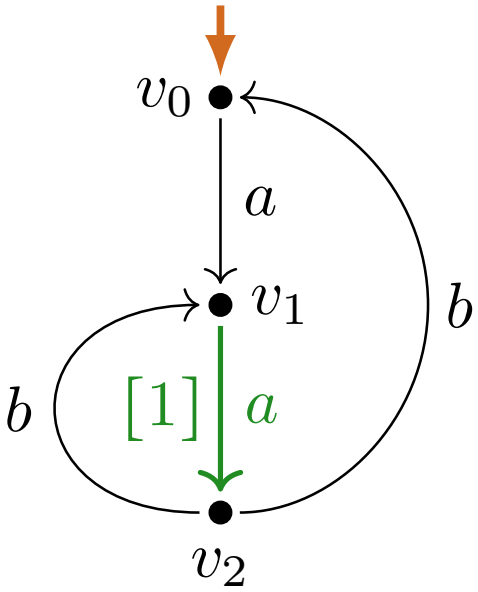
$$s(v_2, v_1) = \quad 0^* \cdot \big(b \cdot s(v_1, v_1) + b \cdot s(v_0, v_1)\big)$$

Readback from layered LEE-witness (example)



layered LEE-witness

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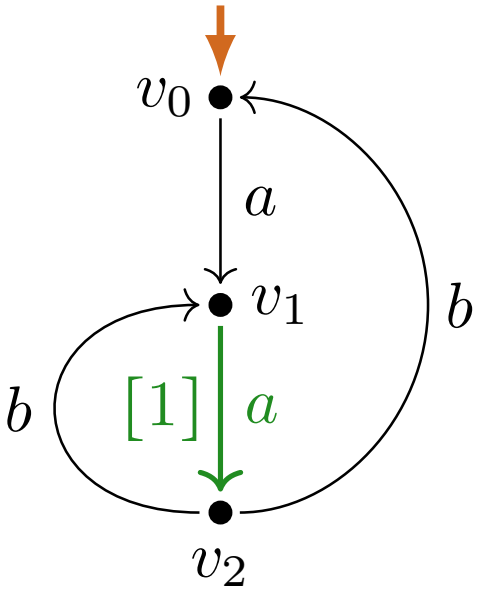
$$s(v_1, v_1) = \quad 1$$

Readback from layered LEE-witness (example)



layered LEE-witness

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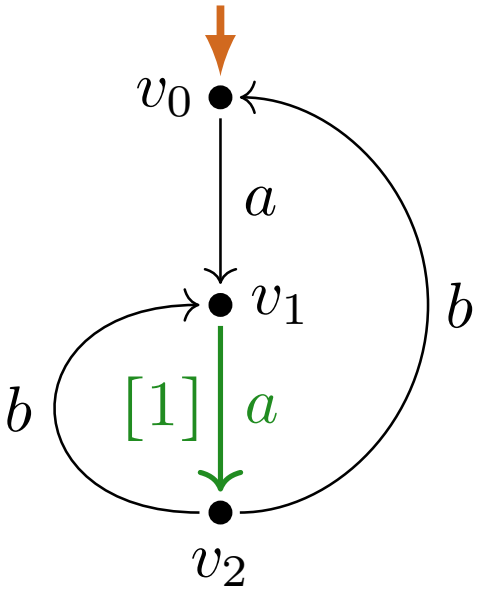
Readback from layered LEE-witness (example)



layered LEE-witness

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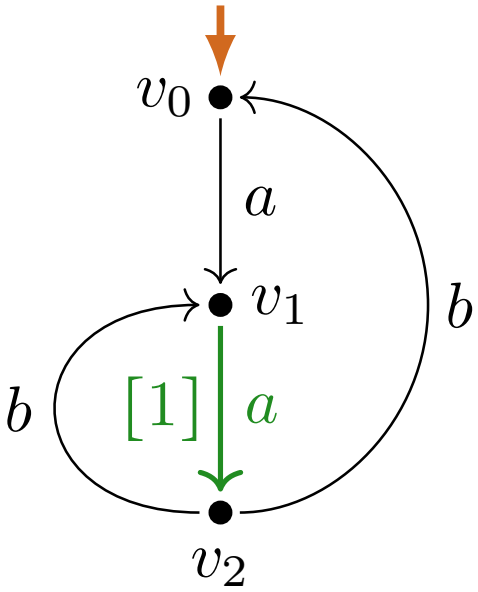
$$\begin{aligned} s(v_1, v_1) &= \quad 1 \\ s(v_0, v_1) &= \quad 0^* \cdot a \cdot s(v_1, v_1) \\ &= \quad 0^* \cdot a \cdot 1 \end{aligned}$$

Readback from layered LEE-witness (example)



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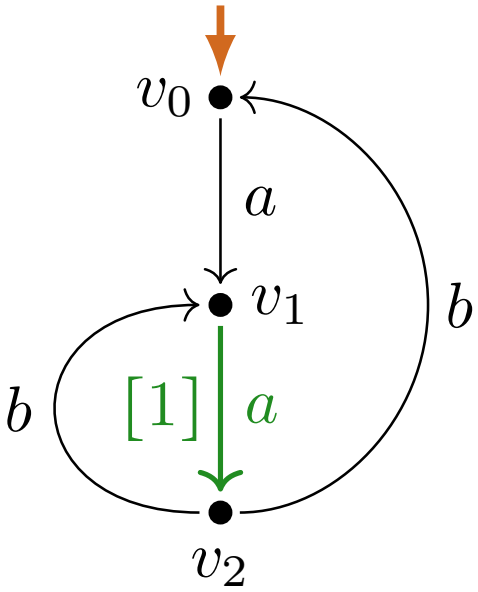
Readback from layered LEE-witness (example)



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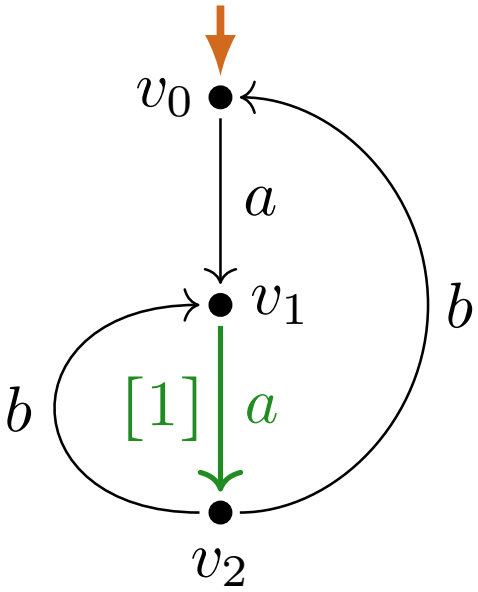
Readback from layered LEE-witness (example)



layered LEE-witness

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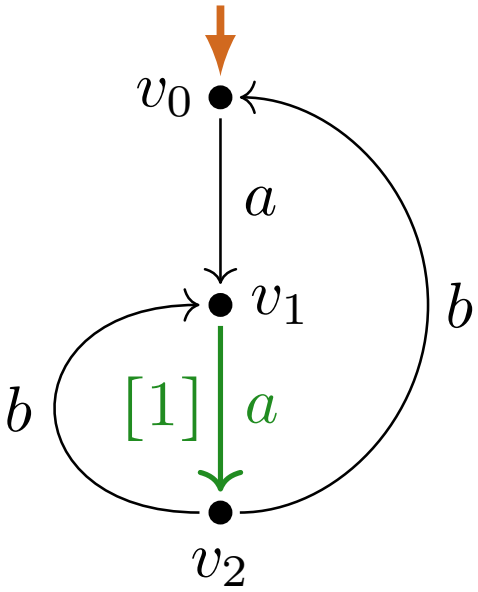
$$

Readback from layered LEE-witness (example)



layered LEE-witness

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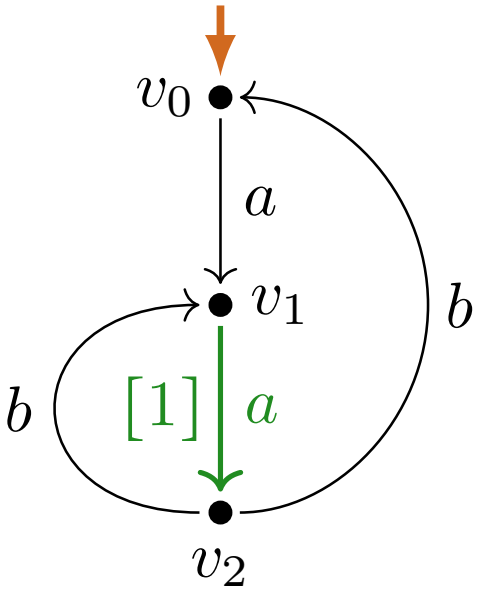
&=_{\text{Mil}^-} \big(a \cdot (b + b \cdot a)\big)^* \cdot 0 \\
s(v_2, v_1) &= 0^* \cdot \big(b \cdot s(v_1, v_1) + b \cdot s(v_0, v_1)\big) \\
&=_{\text{Mil}^-} 0^* \cdot \big(b \cdot 1 + b \cdot a\big) \\
&=_{\text{Mil}^-} b + b \cdot a \\
s(v_1, v_1) &= 1 \\
s(v_0, v_1) &= 0^* \cdot a \cdot s(v_1, v_1) \\
&= 0^* \cdot a \cdot 1 \\
&=_{\text{Mil}^-} a
\end{aligned}
$$

Readback from layered LEE-witness (example)



layered LEE-witness

$$
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$$
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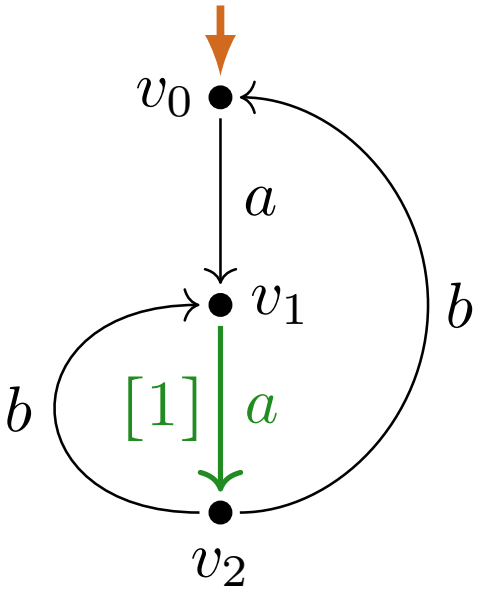

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1-return-less regular expressions

Lemma

Process graphs with LEE are $[\![\cdot]\!]_{\boldsymbol{P}}$-expressible:

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Characterization of expressibility$^{\cancel{1r}\backslash\star}$ modulo $\underline{\leftrightarrow}$

Theorem

For every process graph G with bisimulation collapse C the following are equivalent:

(i) G *is* $[\![\cdot]\!]_{\boldsymbol{P}}^{\cancel{1r}\backslash\star}$*-expressible modulo* $\underline{\leftrightarrow}$*.*

(ii) LEE(C).

(iii) C *has a* LEE*-witness.*

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Milners characterization question:

Q1. Which structural property of finite process graphs characterizes $[\![\cdot]\!]_P$-expressibility modulo $\underline{\leftrightarrow}$?

Characterization of expressibility $^{\not{1r}\backslash\star}$ modulo $\underline{\leftrightarrow}$

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Milners characterization question restricted:

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Milners characterization question restricted, and adapted:

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Answering Milners characterization question restricted, and adapted:

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- The loop-existence and elimination property LEE.

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Also yields: efficient decision method of $[\![\cdot]\!]_P^{\not{1r}\backslash\star}$-expressibility modulo $\underline{\leftrightarrow}$.

Structure constrained finite process graphs

graphs with LEE / a (layered) LEE-witness

Benefits of the class of process graphs with LEE:

- is closed under $\twoheadrightarrow$
- forth-/back-correspondence with 1-return-less regular expressions

Structure constrained finite process graphs

graphs with LEE / a (layered) LEE-witness

$\subsetneq$ graphs whose collapse satisfies LEE

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Structure constrained finite process graphs

loop–exit palm trees $\subsetneq$ $[\![\cdot]\!]_P^{\not{1r}\backslash\star}$-expressible graphs

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Application to Milner's questions yields partial results:

Q1: characterization/efficient decision of $[\![\cdot]\!]_P^{\not{1r}\backslash\star}$-expressibility modulo $\leftrightarrow$

Q2: alternative compl. proof of Mil on 1-return-less expressions (C/DN/L)

Comparison results: structure-constrained graphs

λ-calculus with letrec under $=_{\boldsymbol{\lambda^\infty}}$

Not available: graph interpretation that is studied under $\underline{\leftrightarrow}$

Regular expressions under $\underline{\leftrightarrow}_{\boldsymbol{P}}$

Given: graph interpretation $[\![\cdot]\!]_{\boldsymbol{P}}$, studied under bisimulation $\underline{\leftrightarrow}$

- not closed under $\underline{\rightarrow}$, and $\underline{\leftrightarrow}$, incomplete under $\underline{\leftrightarrow}$

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Defined: int's $[\![\cdot]\!]_{\mathcal{H}}/[\![\cdot]\!]_{\mathcal{T}}$ as higher-order/first-order λ-term graphs

- closed under $\underline{\rightarrow}$ (hence under collapse)
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  - efficient translation and readback
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- not closed under $\underline{\twoheadrightarrow}$, and $\underline{\leftrightarrow}$, incomplete under $\underline{\leftrightarrow}$

Defined: class of process graphs with LEE / (layered) LEE-witness

- closed under $\underline{\twoheadrightarrow}$ (hence under collapse)
- back-/forth correspondence with 1-return-less expr's
- contains the collapse of a process graph G
  $\Longleftrightarrow$ G is $[\![\cdot]\!]_{\boldsymbol{P}}^{\cancel{\text{1r}}\backslash^\star}$-expressible modulo $\underline{\leftrightarrow}$