The process semantics of regular expressions

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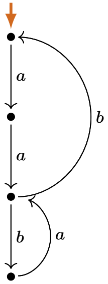
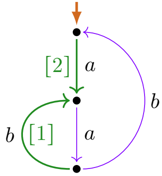


Postdoc Seminar

November 19, 2018



Regular Expressions *(Copi–Elgot–Wright, 1958; based on Kleene, 1951)*

Definition

The set $\mathsf{Reg}(A)$ of regular expressions over alphabet A is defined by the grammar:

$$e, f \; ::= \; 0 \mid 1 \mid a \mid (e + f) \mid (e \cdot f) \mid (e^{\star}) \qquad \text{(for } a \in A\text{)}.$$

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Note, here:

- symbol 0 instead of $\varnothing$
- symbol 1 used (often dropped, definable as $0^{\star}$)
- no complementation operation $\overline{e}$

Language semantics $[\![\cdot]\!]_L$ *(Copi–Elgot–Wright, 1958)*

$\mathbf{0} \xmapsto{[\![\cdot]\!]_L}$ empty language $\varnothing$

$\mathbf{1} \xmapsto{[\![\cdot]\!]_L} \{\epsilon\}$ (ϵ the empty word)

$a \xmapsto{[\![\cdot]\!]_L} \{a\}$

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$a \xmapsto{[\![\cdot]\!]_L} \{a\}$

$e + f \xmapsto{[\![\cdot]\!]_L}$ union of $[\![e]\!]_L$ and $[\![f]\!]_L$

$e \cdot f \xmapsto{[\![\cdot]\!]_L}$ element-wise concatenation of $[\![e]\!]_L$ and $[\![f]\!]_L$

$e^* \xmapsto{[\![\cdot]\!]_L}$ set of words formed by concatenating words in $[\![e]\!]_L$ plus the empty word ϵ

Process semantics $[\![\cdot]\!]_P$ *(Milner, 1984)*

$\mathbf{0} \xmapsto{[\![\cdot]\!]_P}$ deadlock $\boldsymbol{\delta}$, no termination

$\mathbf{1} \xmapsto{[\![\cdot]\!]_P}$ empty process $\boldsymbol{\epsilon}$, then terminate

$a \xmapsto{[\![\cdot]\!]_P}$ atomic action a, then terminate

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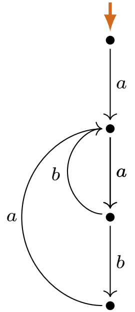
$a \xmapsto{[\![\cdot]\!]_{\boldsymbol{P}}}$ atomic action a, then terminate

$e + f \xmapsto{[\![\cdot]\!]_{\boldsymbol{P}}}$ alternative composition of $[\![e]\!]_{\boldsymbol{P}}$ and $[\![f]\!]_{\boldsymbol{P}}$

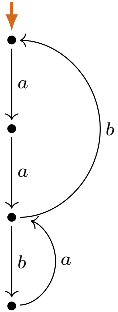
$e \cdot f \xmapsto{[\![\cdot]\!]_{\boldsymbol{P}}}$ sequential composition of $[\![e]\!]_{\boldsymbol{P}}$ and $[\![f]\!]_{\boldsymbol{P}}$

$e^* \xmapsto{[\![\cdot]\!]_{\boldsymbol{P}}}$ unbounded iteration of $[\![e]\!]_{\boldsymbol{P}}$, option to terminate

Process semantics $[\![\cdot]\!]_P$ (examples)



$$a(a(b+ba))^*0$$



$$(aa(ba)^*b)^*0$$

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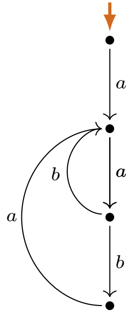
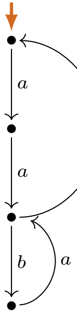


$$a \cdot (a \cdot (b + b \cdot a))^* \cdot 0$$



$$(a \cdot a \cdot (b \cdot a)^* \cdot b)^* \cdot 0$$

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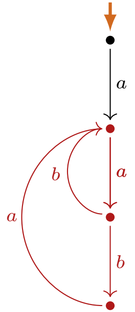

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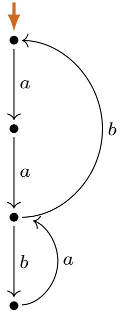

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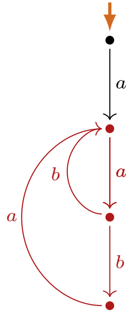

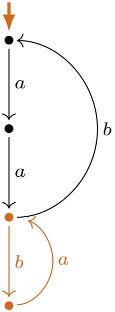

$$a \cdot (a \cdot (b + b \cdot a))^* \cdot 0 \qquad\qquad (a \cdot a \cdot (b \cdot a)^* \cdot b)^* \cdot 0$$

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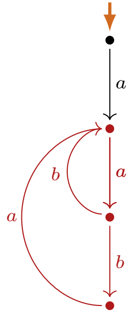

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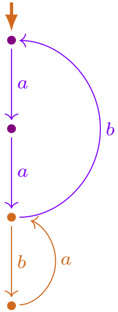

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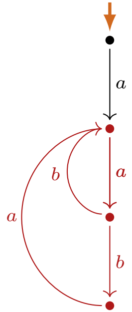

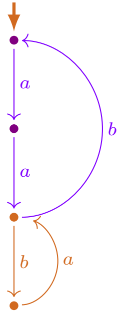

$[\![a \cdot (a \cdot (b + b \cdot a))^* \cdot 0]\!]_P$ $[\![(a \cdot a \cdot (b \cdot a)^* \cdot b)^* \cdot 0]\!]_P$

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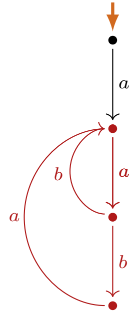
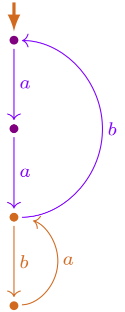
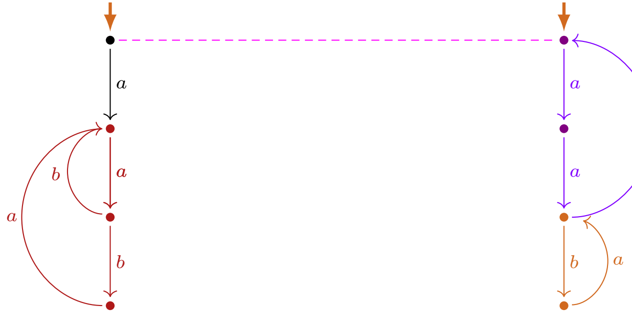




$$[\![a(a(b+ba))^*0]\!]_P$$

$$[\![(aa(ba)^*b)^*0]\!]_P$$

Process semantics $[\![\cdot]\!]_P$ (examples)



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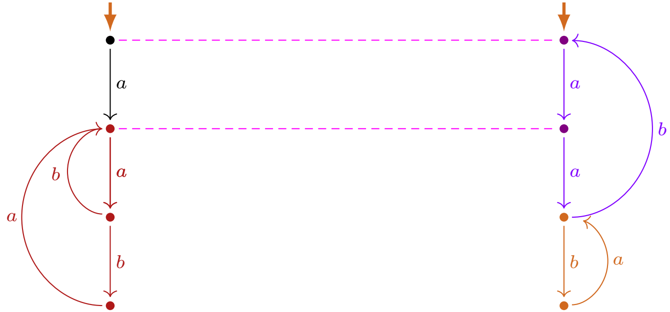

$[\![a(a(b+ba))^{*}0]\!]_{P}$

$[\![(aa(ba)^{*}b)^{*}0]\!]_{P}$

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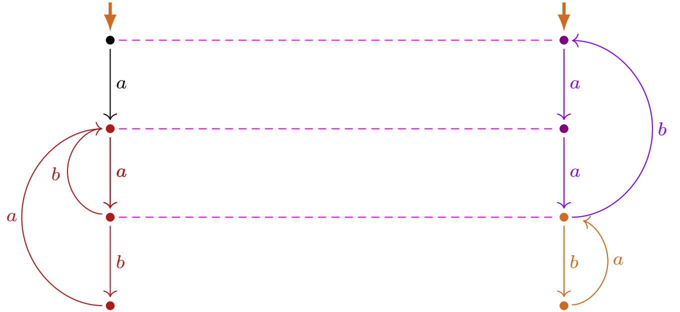

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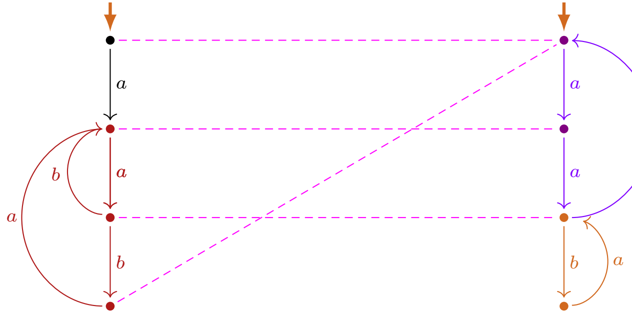

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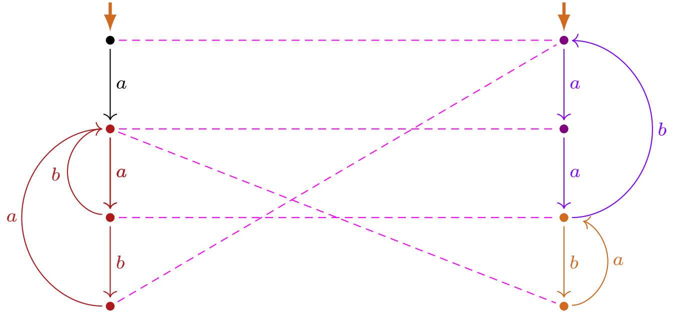

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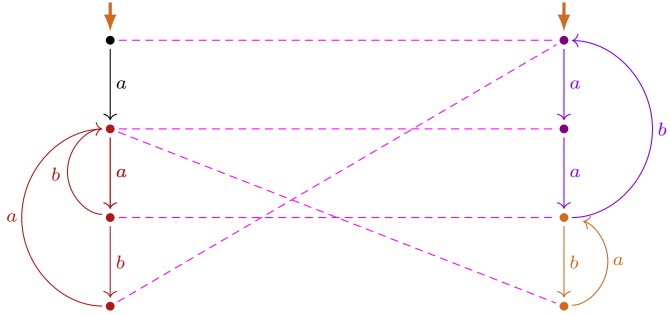
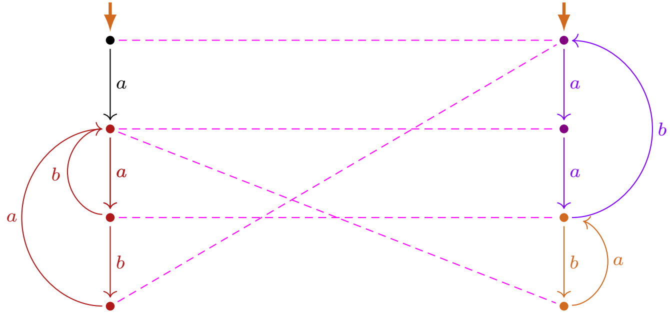


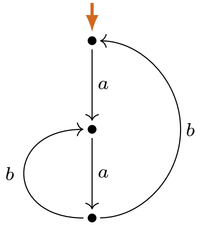
$$[\![a(a(b+ba))^*0]\!]_P \quad \underline{\leftrightarrow} \quad [\![(aa(ba)^*b)^*0]\!]_P$$

Process semantics $[\![\cdot]\!]_P$ (examples)



$$a(a(b+ba))^*0 \quad \underline{\leftrightarrow}_P \quad (aa(ba)^*b)^*0$$

Expressible process graphs (under bisimulation $\underline{\leftrightarrow}$)



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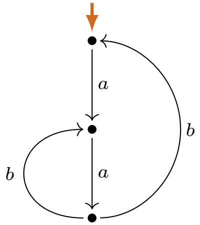
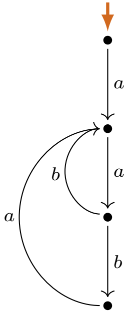
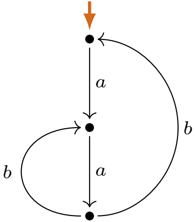


$$\notin im(\llbracket \cdot \rrbracket_{\boldsymbol{P}})$$

Expressible process graphs (under bisimulation $\leftrightarrows$)

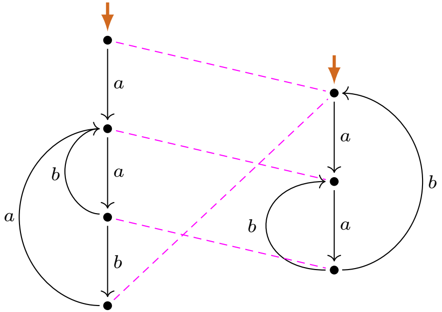




$\in im(\llbracket\cdot\rrbracket_{\boldsymbol{P}})$ $\notin im(\llbracket\cdot\rrbracket_{\boldsymbol{P}})$

$\llbracket\cdot\rrbracket_{\boldsymbol{P}}$-expressible

Expressible process graphs (under bisimulation $\underline{\leftrightarrow}$)



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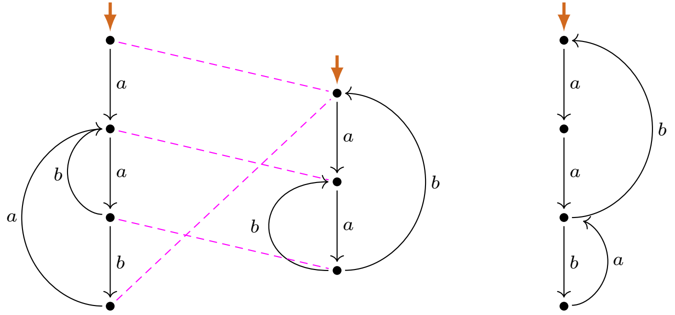
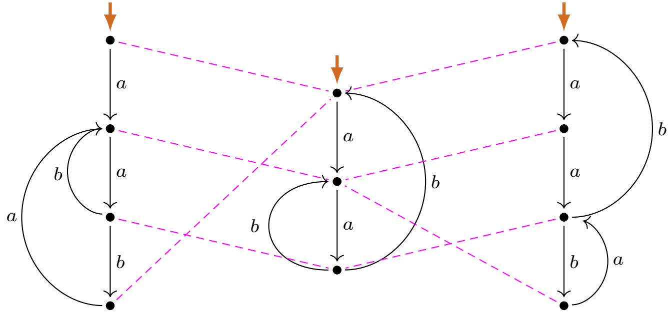


$\in im(\llbracket\cdot\rrbracket_{\boldsymbol{P}})$ $\notin im(\llbracket\cdot\rrbracket_{\boldsymbol{P}})$ $\in im(\llbracket\cdot\rrbracket_{\boldsymbol{P}})$

$\llbracket\cdot\rrbracket_{\boldsymbol{P}}$-expressible $\llbracket\cdot\rrbracket_{\boldsymbol{P}}$-expressible

Expressible process graphs (under bisimulation $\leftrightarrow$)



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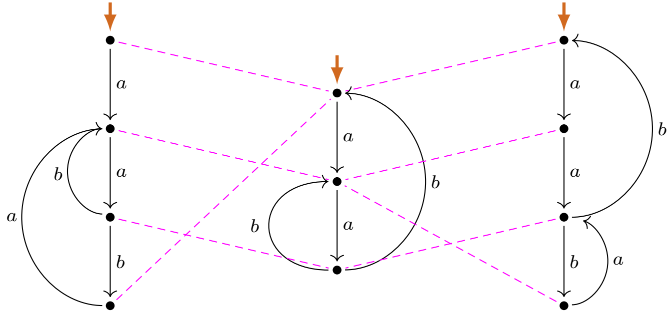


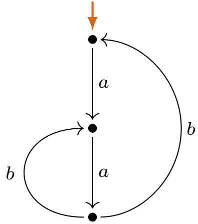
$\in im(\llbracket\cdot\rrbracket_{\boldsymbol{P}})$ $\llbracket\cdot\rrbracket_{\boldsymbol{P}}$-expressible

$\notin im(\llbracket\cdot\rrbracket_{\boldsymbol{P}})$ $\llbracket\cdot\rrbracket_{\boldsymbol{P}}$-expressible modulo $\underline{\leftrightarrow}$

$\in im(\llbracket\cdot\rrbracket_{\boldsymbol{P}})$ $\llbracket\cdot\rrbracket_{\boldsymbol{P}}$-expressible

Properties of $\boldsymbol{P}$

- Not every finite-state process is $[\![\cdot]\!]_{\boldsymbol{P}}$-expressible.



not $[\![\cdot]\!]_{\boldsymbol{P}}$-expressible

$[\![\cdot]\!]_{\boldsymbol{P}}$-expressible modulo $\underline{\leftrightarrow}$

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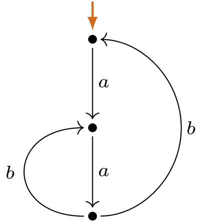

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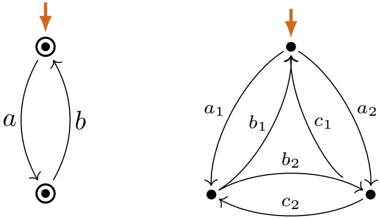

not $[\![\cdot]\!]_{\boldsymbol{P}}$-expressible

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- Not every finite-state process is $[\![\cdot]\!]_P$-expressible.
- Not every finite-state process is $[\![\cdot]\!]_P$-expressible modulo $\underline{\leftrightarrow}$.
- Fewer identities hold for $\underline{\leftrightarrow}_P$ than for $=_L$: $\underline{\leftrightarrow}_P \subsetneqq =_L$.

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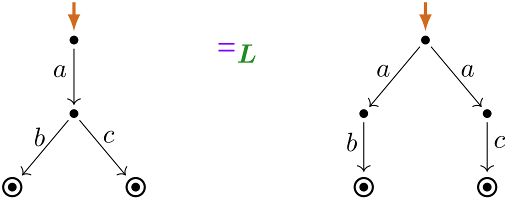

$$a \cdot (b + c) \quad =_{\boldsymbol{L}} \quad a \cdot b + a \cdot c$$

Properties of $\boldsymbol{P}$

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$$a \cdot (b + c) \quad \not\underline{\leftrightarrow}_{\boldsymbol{P}} \quad a \cdot b + a \cdot c$$

Complete axiomatization of $=_{\boldsymbol{L}}$ *(Aanderaa/Salomaa, 1965/66)*

Axioms :

(B1) $e + (f + g) = (e + f) + g$

(B2) $(e \cdot f) \cdot g = e \cdot (f \cdot g)$

(B3) $e + f = f + e$

(B4) $(e + f) \cdot g = e \cdot g + f \cdot g$

(B5) $e \cdot (f + g) = e \cdot f + e \cdot g$

(B6) $e + e = e$

(B7) $e \cdot 1 = e$

(B8) $e \cdot 0 = 0$

(B9) $e + 0 = e$

(B10) $e^* = 1 + e \cdot e^*$

(B11) $e^* = (1 + e)^*$

Inference rules : equational logic *plus*

$$\frac{e = f \cdot e + g}{e = f^* \cdot g}\ \text{FIX} \quad (\text{if } \underbrace{\{\epsilon\} \notin [\![f]\!]_{\boldsymbol{L}}}_{\textit{non-empty-word property}})$$

Sound and unsound axioms with respect to $\underline{\leftrightarrow}_P$

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(B1)	$e + (f + g) = (e + f) + g$	(B7)	$e \cdot 1 = e$
(B2)	$(e \cdot f) \cdot g = e \cdot (f \cdot g)$	(B8)	$e \cdot 0 = 0$
(B3)	$e + f = f + e$	(B9)	$e + 0 = e$
(B4)	$(e + f) \cdot g = e \cdot g + f \cdot g$	(B10)	$e^* = 1 + e \cdot e^*$
(B5)	$e \cdot (f + g) = e \cdot f + e \cdot g$	(B11)	$e^* = (1 + e)^*$
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Adaptation for $\underline{\leftrightarrow}_{P}$ *(Milner, 1984)* (Mil = Mil⁻ + RSP*)

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Inference rules : equational logic *plus*

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Adaptation for $\underline{\leftrightarrow}_{P}$ (Milner, 1984) (Mil = Mil$^-$ + RSP*)

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(B3)	$e + f = f + e$	(B9)	$e + 0 = e$
(B4)	$(e + f) \cdot g = e \cdot g + f \cdot g$	(B10)	$e^* = 1 + e \cdot e^*$
		(B11)	$e^* = (1 + e)^*$
(B6)	$e + e = e$		

Inference rules : equational logic *plus*

$$\frac{e = f \cdot e + g}{e = f^* \cdot g}\ \mathsf{RSP}^* \ \left(\text{if } \underbrace{\{\epsilon\} \notin [\![f]\!]_{L}}_{\textit{non-empty-word property}}\right)$$

Milner's questions

Q2. Complete axiomatization: Is Mil complete for $\underline{\leftrightarrow}_P$?

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Q1. Recognition: Which structural property of finite process graphs characterizes $[\![\cdot]\!]_P$-expressibility modulo $\underline{\leftrightarrow}$?

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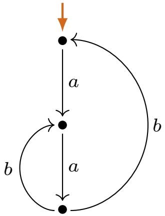
- Mil⁻ + one of two stronger rules (than RSP*) is complete *(G, 2006)*

New approach: Loop Existence and Elimination (LEE)



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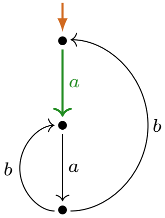

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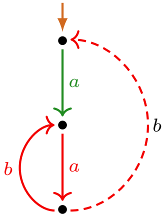

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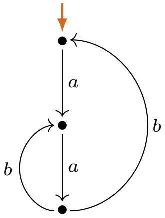

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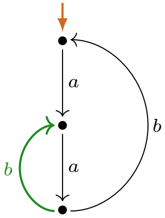

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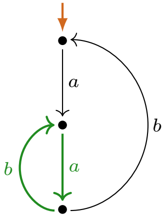

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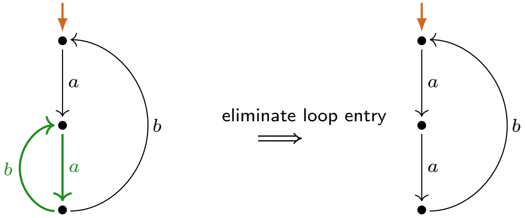

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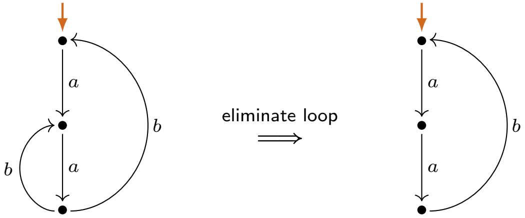

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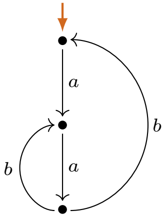
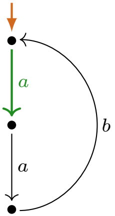


eliminate loop $\Longrightarrow$



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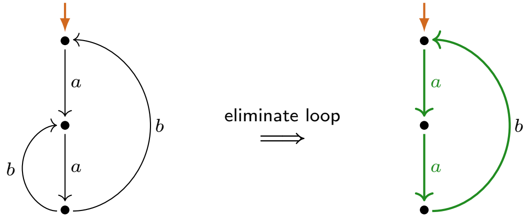

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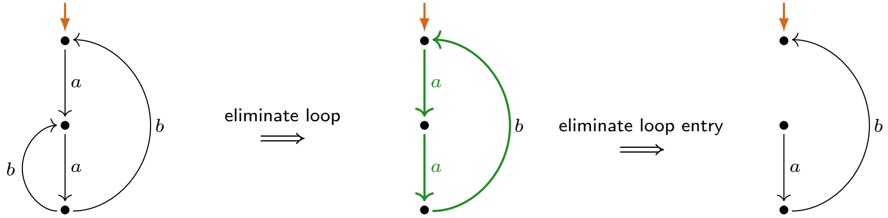

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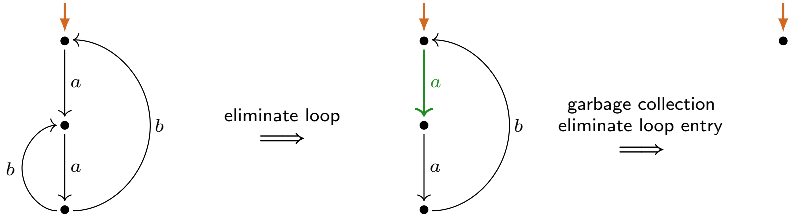

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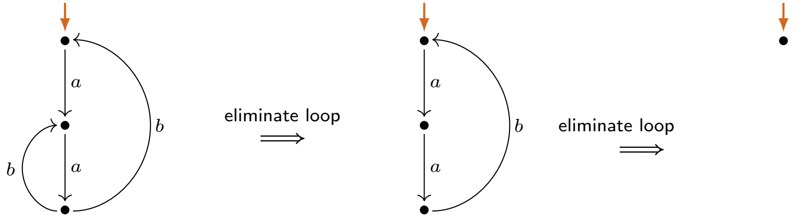

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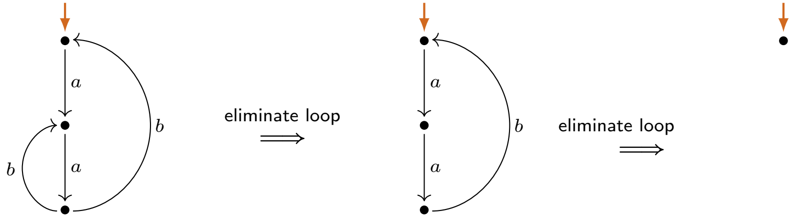

LEE:

repeated elimination of loops leads to a process graph without infinite trace

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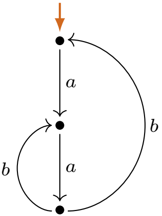

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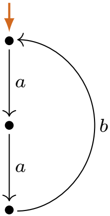

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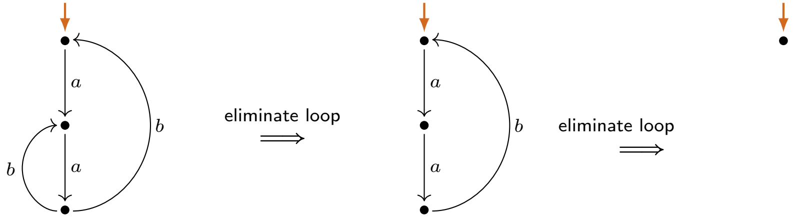

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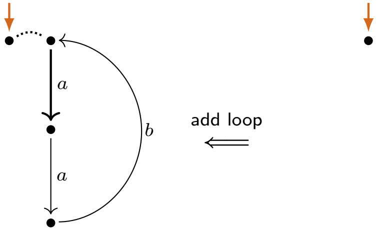

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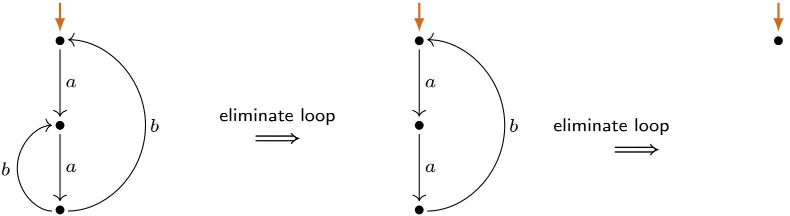

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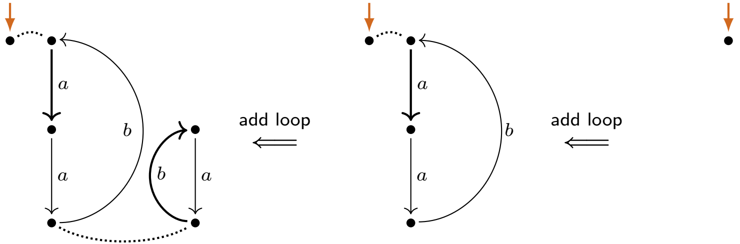
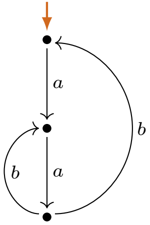
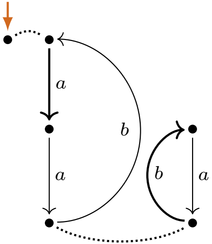


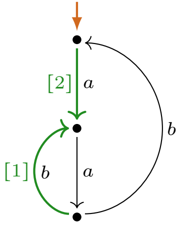
LEE-witness



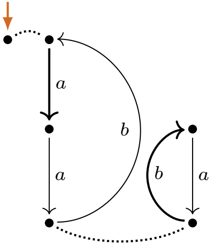


structured
LEE-witness

LEE-witness: structure constrained process graph

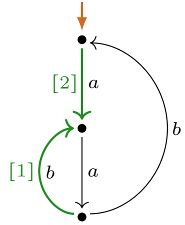


LEE-witness

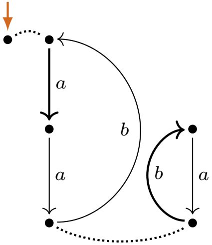


structured
LEE-witness

LEE-witness yields star expression



LEE-witness



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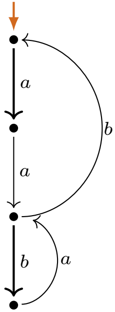

$[\![(aa(ba)^*b)^*0]\!]_{\boldsymbol{P}}$

structure constraints (L'Aquila)

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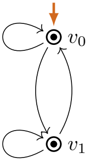
Loop elimination



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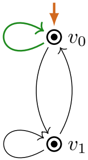

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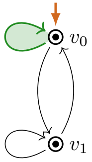

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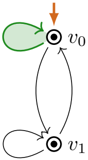
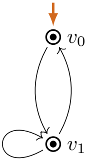


$\longrightarrow_{\text{elim}}$



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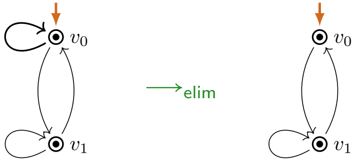

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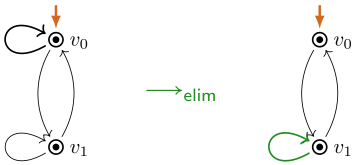

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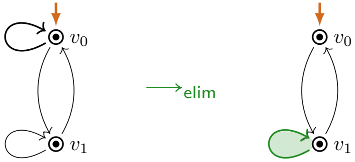

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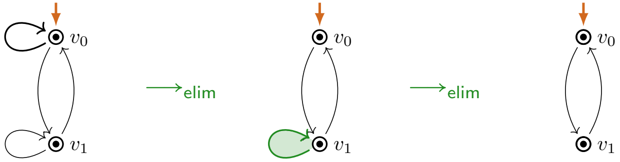

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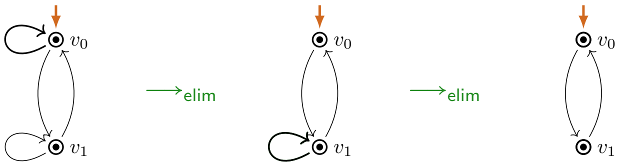

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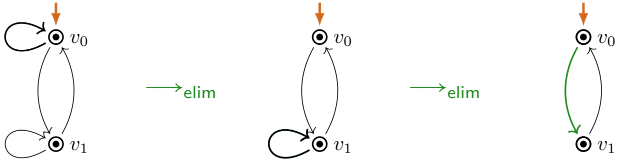

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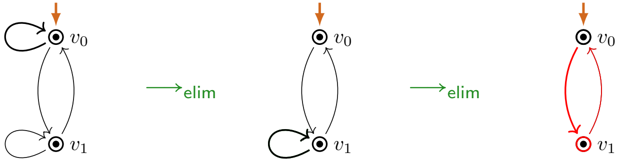

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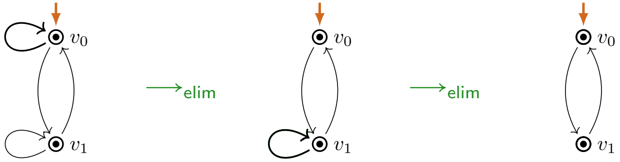

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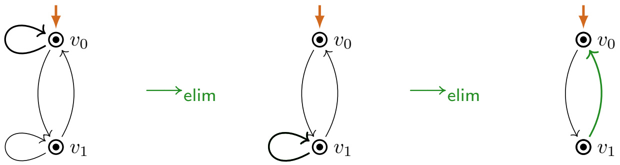

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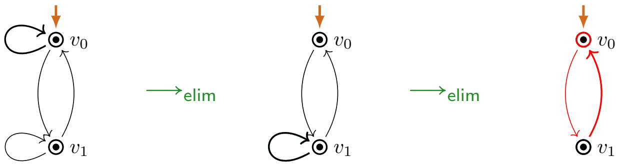

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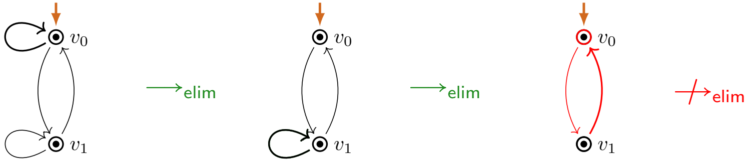

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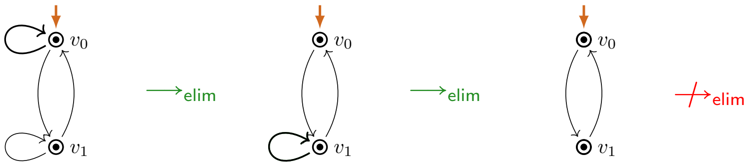

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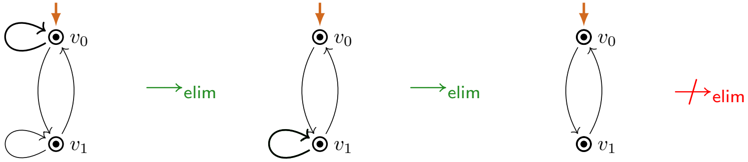

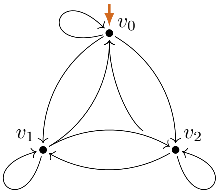

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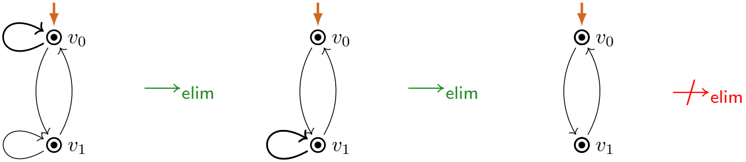

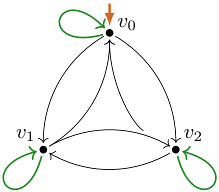

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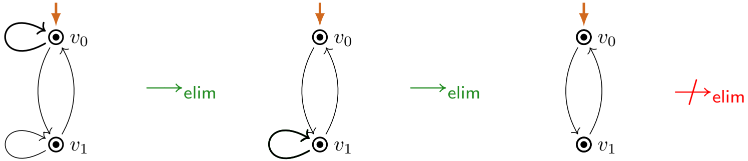

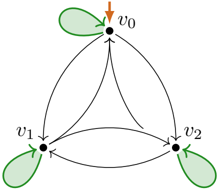

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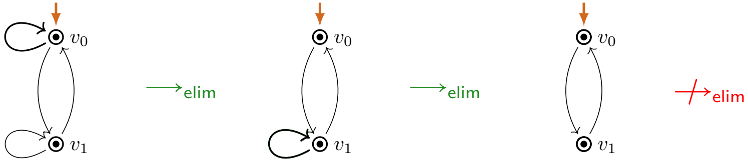

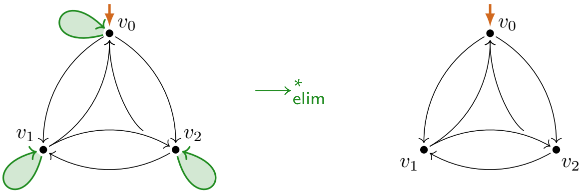

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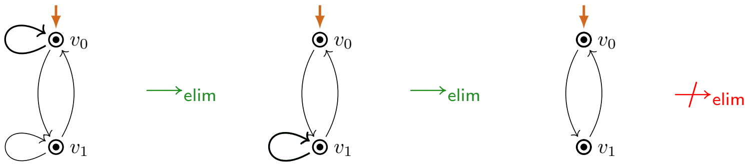

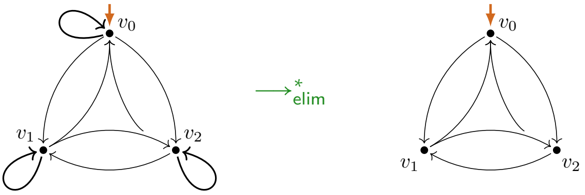

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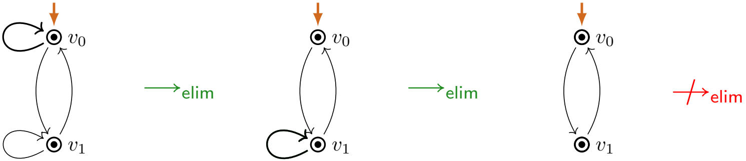

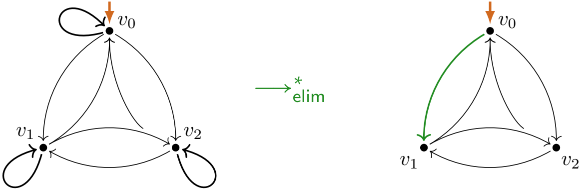

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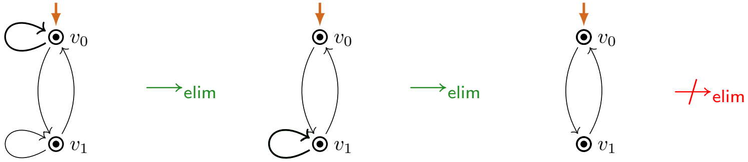

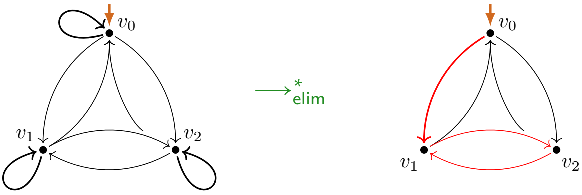

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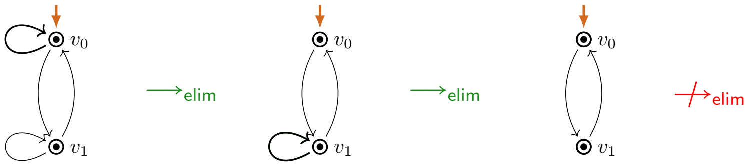
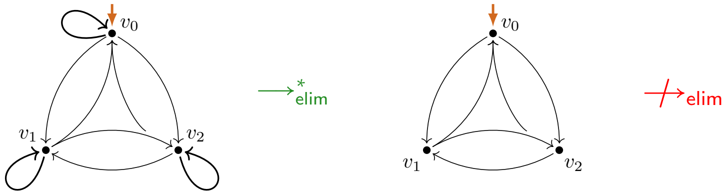
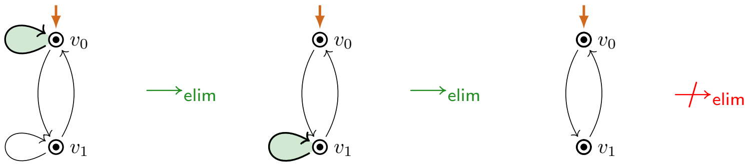


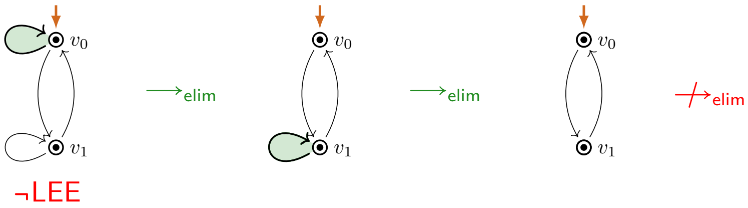


LEE fails



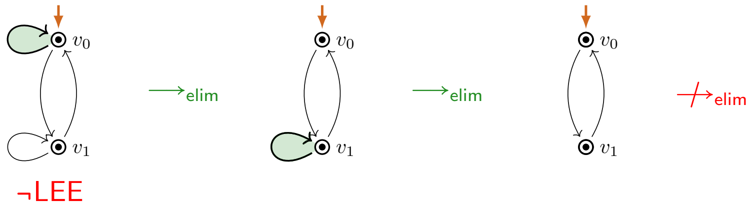
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\not\longrightarrow_{\text{elim}}$

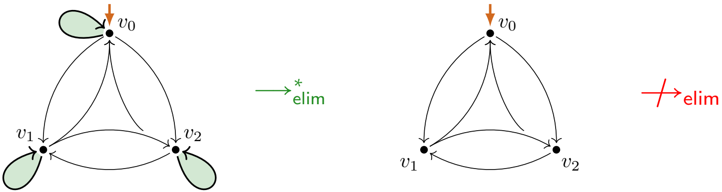
LEE fails



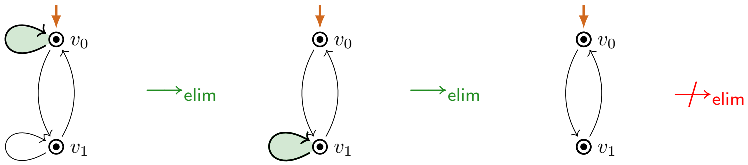
v_0
v_1
$\neg$LEE
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\not\longrightarrow_{\text{elim}}$

LEE fails

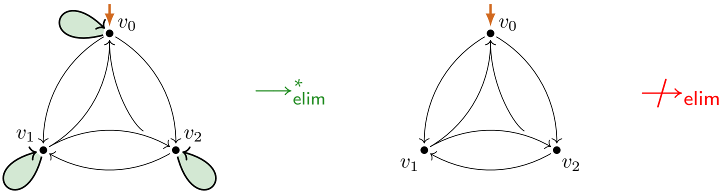

v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\longrightarrow_{\text{elim}}$
v_0
v_1
$\not\longrightarrow_{\text{elim}}$
$\neg$LEE


v_0
v_1
v_2
$\longrightarrow^{*}_{\text{elim}}$
v_0
v_1
v_2
$\not\longrightarrow_{\text{elim}}$

LEE fails



¬LEE

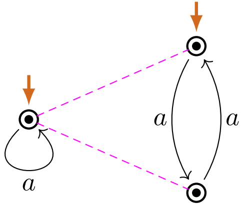


¬LEE

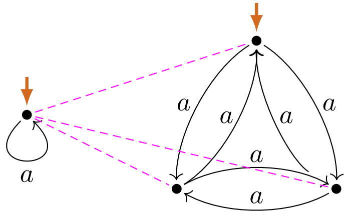
LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.
- LEE is not preserved by converse functional bisimulation.

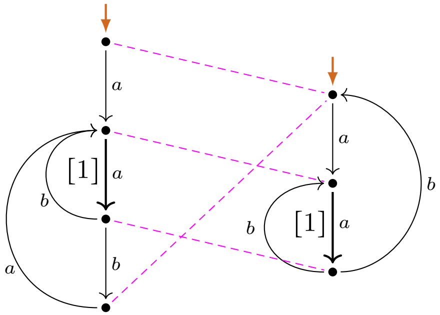


LEE ¬LEE

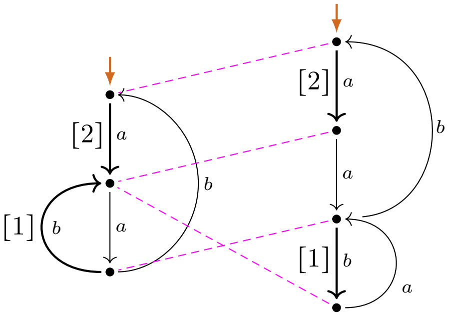


LEE ¬LEE

LEE is preserved under bisimulation collapse



$[\![a(a(b+ba))^*0]\!]_{\boldsymbol{P}}$



$[\![(aa(ba)^*b)^*0]\!]_{\boldsymbol{P}}$

Goals

- Completeness proof
  - is a large project: report (now ~ 250 pages, ~ 2 years)
  - writing up a crucial step:
    - pseudo-collapse LEE-witnesses with 1-transitions
- Interpretations of equational theories
  - Characterize interpretations with respect to what kinds of properties they permit to transfer between equational theories

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- Recognition problem
  - polynomial if no 1-transitions (testing for property LEE)
- Cost models for λ-calculus