A Coinductive Axiomatisation of Regular Expressions Under Bisimulation

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Abstract

Milner (1984) introduced a (non-standard) interpretation of regular expressions as finite-state processes. He asked whether a certain slight modification of Salomaa's axiomatisation for language equivalence of regular expressions is complete for bisimilarity of regular expressions under the process interpretation. We report on a recent observation and two results addressing this apparently still open problem: Antimirov's notion of "partial derivative" for regular expressions is closely linked to the process interpretation, and it can be used to build a proof system with a coinductive motivation that is complete for bisimilarity under the process interpretation. As a corollary it can be shown that Milner's adaptation of Salomaa's system is complete if in it the single-equation solving rule is replaced by a rule able to solve systems of equations.

In his paper [3] on a complete axiomatisation of bisimilarity between finite-state processes, Milner introduced an interpretation (here denoted by) P of regular expressions e as finite-state processes $P(e)$. Different from the standard language interpretation L, the interpretation P assigns the following meaning to the regular operators: $+$ and $.$ respectively denote *alternative composition* (also called *choice*) between processes, and *sequential composition* of processes; * denotes unbounded iteration of processes with the possibility of termination before each iteration; letters a stand for processes that are executions of *atomic actions*; and the constants 0 and 1 are interpreted as *deadlock*, and respectively, as the *empty process* (also called *skip*). The interpretation P gives rise to the equivalence relation "bisimilarity of process interpretations" on regular expressions that we denote by $\leftrightarrows_P$ here: for all regular expressions e and f, $e \leftrightarrows_P f$ is defined to hold if and only if $P(e) \leftrightarrows P(f)$ (where $\leftrightarrows$ stands for bisimilarity between processes).

As an axiomatisation of $\leftrightarrows_P$, Milner in [3] also suggested an adaptation of Salomaa's complete axiom system for language equivalence $=_L$ on regular expressions (for this purpose, in particular the left-distributivity law $x.(y+z) = x.y + x.z$, which is not valid in process theory, as well as the axiom $x.0 = 0$ have to be removed from Salomaa's system). Milner recognised that Salomaa's method of completeness proof cannot be applied, at least not directly, to yield also a completeness proof for the modified system with respect to $\leftrightarrows_P$, owing to the following fact: there exist finite-state processes that are not bisimilar to processes described by regular expressions, and hence that even linear systems of guarded recursive equations are not always solvable in process interpretations of regular expressions. Milner's question (Q1) of whether the modified version of Salomaa's system, or some extension of it with a finite number of other axioms, is complete for $\leftrightarrows_P$ has been frequently studied and a number of related results have been obtained (we mention work by Sewell (1997), Fokkink (1997), Corradini, De Nicola and Labella (1998)); but apparently (Q1) is yet unsolved. Due to the connection with Basic Process Algebra BPA, we denote Milner's system for $\leftrightarrows_P$ by $\mathbf{BPA^*_{0,1}}$+1-RSP*, where 1-RSP* is the rule that allows to solve the single equation $e = f.e + g$ by $e = f^*.e + g$ under the assumption that the empty word is not contained in $L(E)$.

Two related questions of Milner in [3] concerning the process interpretation P have already been solved: a question (Q3) concerning the minimal star-height of regular expressions with respect to $\leftrightarrows_P$ by Hirshfeld and Moller (1999), and more recently, a question (Q2) concerning a characterisation of finite-state processes that are expressible by a regular expression under P module $\leftrightarrows_P$ by Baeten and Corradini (2005), and by Baeten, Corradini, and the present author (2005).

Here we report on a recent observation and two results related to Milner's question (Q1): It turns out that there exists a direct link between the process interpretation P with work by Antimirov in [1] on "partial derivatives" of regular expressions. Exploiting this connection, a coinductively motivated proof system can be found that is sound and complete for $\leftrightarrows_P$. Finally, this result can be used to demonstrate the completeness with respect to $\leftrightarrows_P$ of a slight variant system of $\mathbf{BPA^*_{0,1}}$+1-RSP*.

Here we formulate a two coinductively motivated proof systems for regular expression equivalence under the process interpretation, and show that these systems are sound and complete. Finally we prove, using one of our systems, that Milner's adaptation of Salomaa's system is complete if the fixed-point rule of this system is replaced by a rule which expresses that systems of guarded linear equations are uniquely solvable.

References

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