Course Guide: An Introduction to Parameterized Complexity

Algorithmic and Formal-Methods Viewpoints

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Abstract This short introductory course on parameterized complexity in the advanced courses block of the GSSI's Ph.D. program 2023/24 will consist of 8 hours of lectures. The focus will be on fixed-parameter tractability results for graphs of bounded width (path-, tree-, clique-width) that can be obtained both by algorithmic techniques (dynamic programming) and by formal-method techniques (logical meta-theorems).

1 Course structure (overview)

In the introductory part (the first lecture), we will provide motivation, definition, and examples of fixed-parameter tractability (fpt) results. We will explain the method of kernelization for obtaining fpt results.

Then we will focus on concepts of graph width such as path-width, tree-width, and clique-width, and the interrelation of fpt-results with respect to different concepts of width. We will explain how fpt-results for problems on graphs of bounded width in one of these notions can be obtained by dynamic programming techniques (in the second lecture and in the guest lecture), and by Courcelle's theorem, a monadic second-order logic based metatheorem for obtaining complexity bounds (in the third regular lecture).

In the fourth lecture we will sketch the basics of fixed-parameter intractability results. These results state that certain problems are not fixed-parameter tractable.

The schedule consists of 8 hours of lectures. For more details about the course structure, please see Section 2, and the table overview in Section 3.

2 Course structure

Monday: Introduction

- motivations for parameterized complexity
- definitions/examples: parameterizations, fixed-parameter tractability (FPT)
- obtaining FPT results by:
  - ▷ kernelization
    - using the Crown Lemma
    - using the Sunflower Lemma

Tuesday: Path-width and tree-width

- path-width (Robertson and Seymour, 1983)
- tree-width (Bertelè and Brioschi, 1972, Halin 1976)
- clique-width (Courcelle, Engelfriet, Rozenberg, 1993)
- mention of spectrum of other notions of width
- interrelation of fpt-results for different versions of graph width

Wednesday: Algorithmic meta-theorems, and applications

- first-order logic, using it to express graph properties
- monadic second order logic, using it to express graph properties
- Courcelle's theorem for tree-width (Courcelle 1990), with applications
- Optimization version of Courcelle's Theorem, with applications
- Courcelle's theorem for clique-width, with applications

Friday: FPT-intractability

- motivation for fixed-parameter intractability results
- fixed-parameter tractable reductions
- XP: slicewise polynomial problems
- *W-Hierarchy and A-Hierarchy*
- locating problems we encountered on these hierarchies
- practice session: FPT-intractability results for the fire-fighting problem

3 Table overview

Monday, June 10 10.30 – 12.30	Tuesday, June 11	Wednesday, June 12 10.30 – 12.30	Thursday, June 13	Friday, June 14
Introduction & basic FPT results		**Algorithmic Meta-Theorems**		
motivation for FPT kernelization, Crown Lemma, Sunflower Lemma		1st-order logic, monadic 2nd-order logic, FPT-results by Courcelle's Theorems for tree and clique-width		
Algorithmic Techniques		*Formal-Method & Algorithmic Techniques*		
	14.30 – 16.30			14.30 – 16.30
	Notions of bounded graph width			**FPT-Intractability Classes & Hierarchies**
	path-, tree-, clique width, FPT-results by dynamic programming, transferring FPT results betw. widths			motivation for FP-intractability results, FPT-reductions, class XP (slicewise polynomial), W- and A-Hierarchies, placing problems on these hierarchies

4 Genesis of this course

Hugo Gilbert (http://hugogilbert.pythonanywhere.com/), postdoc-colleague (2018-2020) of mine at GSSI, suggested a course on parameterized complexity theory when we searched for a course topic that we could teach together. For the academic year 2019/20 we developed this advanced course as a two-weeks course (two hours lecture and an exercise session each day). We positioned it as an interpillar advanced course about both algorithmic and formal-methods viewpoints on parameterized complexity theory. In this way we aimed at strengthening the ties in our department between the groups of people who work in algorithmics, and in formal methods. Due to the length of this course, we were able to cover several additional topics. As the course took place in spring of the first year of the Covid-19 pandemics, it was taught entirely online.

Since Hugo Gilbert left GSSI in 2020 to return as an assistant professor to Paris, I teamed up with Alessandro Aloisio https://my.unint.eu/web/a.aloisio for the second edition of this course in the academic year 2020/21. Our cooperation had started in the previous year for preparing a practice session of the course, where I had summarized a fixed-parameter tractability result [2] by Alessandro and Alfredo Navarra on 'coverage in multi-interface networks' (it will be treated again in this year's guest lecture). We restricted the format of the course to a five-day short course (two hours lecture and an exercise session per day) on selected topics from parameterized complexity theory. Also that edition of the course was taught online.

In the previous academic years 2021/22 and 2022/23 I taught a 4-day version of the course and each with a day of an added guest lecture by Alessandro Aloisio. Since 2021/22 we had been able to return to in-presence teaching.

This year's (2023/24) edition of the course will be closely oriented at last year's edition.

5 Literature and resources

- Book *Parameterized Complexity Theory* by Flum and Grohe, [4], available in the GSSI library.
- Book *Parameterized Algorithms* by Cygin et al., [3], available online via the link `https://www.mimuw.edu.pl/~malcin/book/parameterized-algorithms.pdf` .
- Course materials from editions of the course at GSSI in previous years:
  - Course academic year 2019/20 (Hugo Gilbert and CG): see [5].
  - Course academic year 2020/21 (Alessandro Aloisio and CG): see[1].
  - Course academic year 2021/22 (CG): see [6].
  - Course academic year 2022/23 (CG): see [7].

References

1. Alessandro Aloisio and Clemens Grabmayer. A Short Introduction to Parameterized Complexity. Course material for the course with this title, in the Advanced Course's Period of the GSSI Ph.D. program, GSSI, L'Aquila, Italy, May 24–27, 2021. Accessible via: this abbreviated link `https://drive.google.com/drive/folders/...` on Google Drive.
2. Alessandro Aloisio and Alfredo Navarra. Constrained Connectivity in Bounded X-Width Multi-Interface Networks. *Algorithms*, 13(2), 2020.
3. Marek Cygan, Fedor V. Fomin, Lukasz Kowalik, Daniel Lokshtanov, Daniel Marx, Marcin Pilipczuk, Michal Pilipczuk, and Saket Saurabh. *Parameterized Algorithms*. Springer, 1st edition, 2015.
4. Jörg Flum and Martin Grohe. *Parameterized Complexity Theory*. Springer, 2006.
5. Hugo Gilbert and Clemens Grabmayer. Parameterized Complexity Theory (Algorithmic and Formal-Methods Viewpoints). Course material for the course with this title, given in the Advanced Course's Period of the GSSI Ph.D. program, GSSI, L'Aquila, Italy, April 1–9, 2020. Available on `schoology.com` with the following access link: NX65-NNMD-9VCP3 . A zip file of the slides also at: `https://clegra.github.io/lf/Parameterized-Complexity-Course-2020.zip`.
6. Clemens Grabmayer. An Introduction to Parameterized Complexity (Algorithmic and Formal-Methods Viewpoints). Course material for the course with this title, given in the Advanced Course's Period of the GSSI Ph.D. program, GSSI, L'Aquila, Italy, May 1–6, 2022. Accessible via the link: `https://drive.google.com/drive/folders/1v0MqSqscc5BZBiFbS0p3D_LWkX1W58cO?usp=share_link`.
7. Clemens Grabmayer. An Introduction to Parameterized Complexity (Algorithmic and Formal-Methods Viewpoints). Course material for the course with this title, given in the Advanced Course's Period of the GSSI Ph.D. program, GSSI, L'Aquila, Italy, June 19–23, 2023. Accessible via the link: `https://drive.google.com/drive/folders/1sxSyF9wq7g_BHO9Uh2iCWebzSFWwgnjU?usp=sharing`.