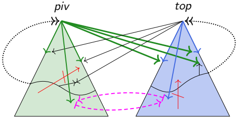
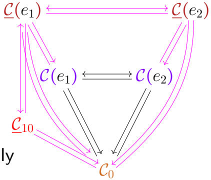
Milner's Proof System for Regular Expressions Modulo Bisimilarity is Complete

Crystallization: Near-Collapsing Process Graph Interpretations of Regular Expressions



Clemens Grabmayer

Department of Computer Science, GSSI, L'Aquila, Italy



Computer Science Seminar

GSSI

July 27, 2022

Process semantics of regular expressions $[\![\cdot]\!]_P$ *(Milner, 1984)*

$\mathbf{0} \xmapsto{[\![\cdot]\!]_P}$ deadlock δ, no termination

$\mathbf{1} \xmapsto{[\![\cdot]\!]_P}$ empty-step process ϵ, then terminate

$a \xmapsto{[\![\cdot]\!]_P}$ atomic action a, then terminate

$e + f \xmapsto{[\![\cdot]\!]_P}$ *(choice)* execute $[\![e]\!]_P$ or $[\![f]\!]_P$

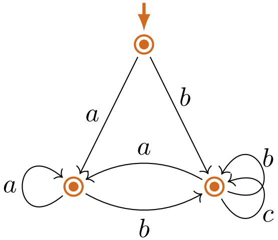
$e \cdot f \xmapsto{[\![\cdot]\!]_P}$ *(sequentialization)* execute $[\![e]\!]_P$, then $[\![f]\!]_P$

$e^* \xmapsto{[\![\cdot]\!]_P}$ *(iteration)* repeat (terminate or execute $[\![e]\!]_P$)

$[\![e]\!]_P \;:=\; [\mathcal{C}(e)]_{\underline{\leftrightarrow}}$ (bisimilarity equivalence class of chart $\mathcal{C}(e)$)

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

chart (Milner)

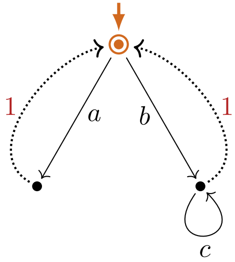


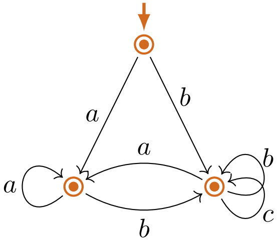
$\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

1-chart

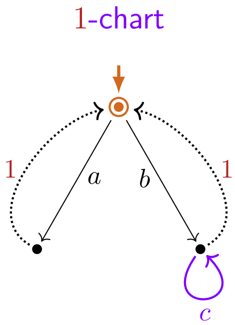
chart (Milner)

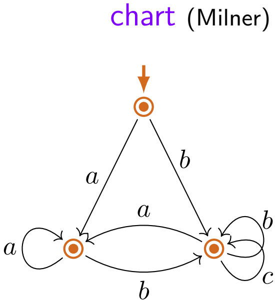




$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)





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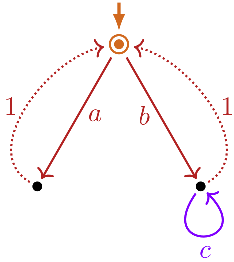
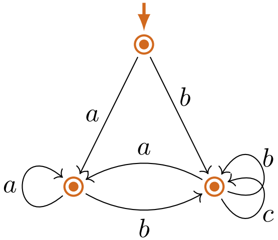


chart (Milner)



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

Process semantics $[\![\cdot]\!]_{P}$ (examples, bisimulation collapse)

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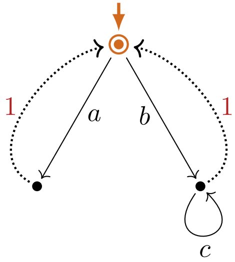
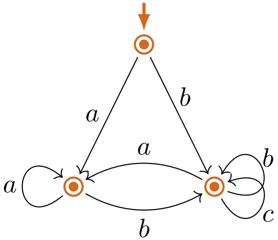
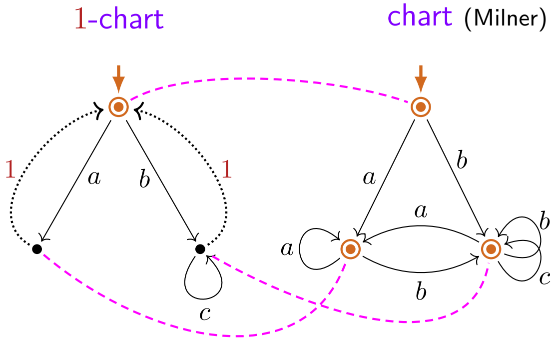


chart (Milner)



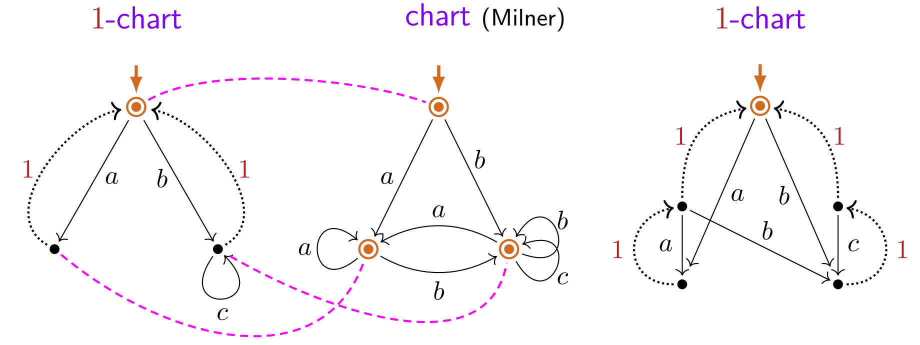
$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^{*} \cdot 1))^{*})$ $\mathcal{C}((a \cdot 1 + b \cdot (c^{*} \cdot 1))^{*})$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$ $\mathcal{C}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

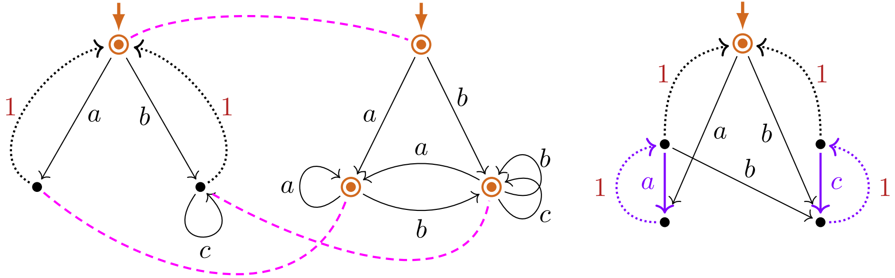


$\underline{\mathcal{C}}((a\cdot 1 + b\cdot(c^{*}\cdot 1))^{*})$ $\mathcal{C}((a\cdot 1 + b\cdot(c^{*}\cdot 1))^{*})$

$$\underline{\mathcal{C}}\Big(\big(a\cdot(1\cdot((a\cdot 1)^{*}\cdot 1)) \cdot b\cdot((1\cdot(c\cdot 1)^{*}))\cdot 1 + b\cdot((1\cdot(c\cdot 1)^{*}))\cdot 1\big)^{*}\Big)$$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

1-chart | chart (Milner) | 1-chart



$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

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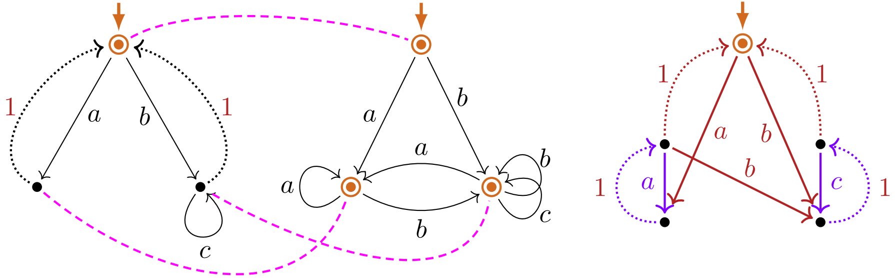
Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

1-chart chart (Milner) 1-chart



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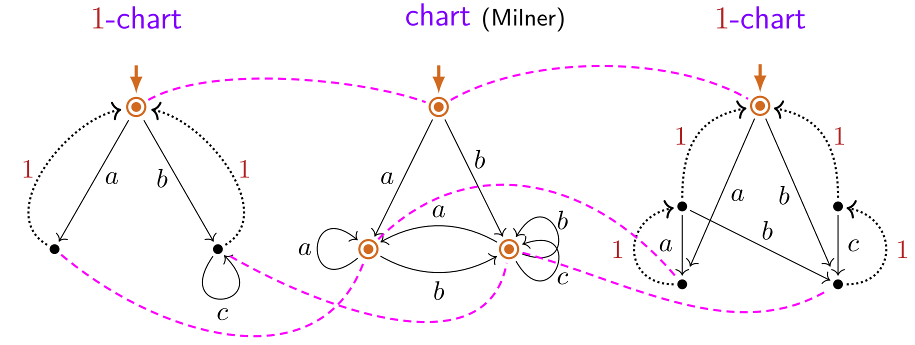
Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



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Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

1-chart

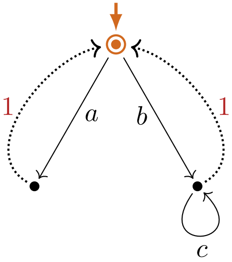
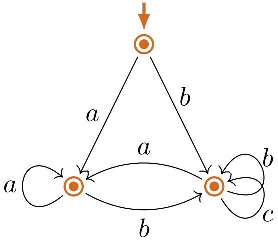
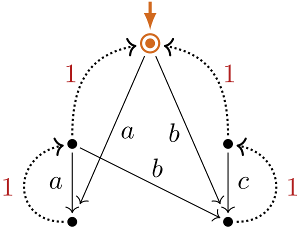


chart (Milner)



1-chart



$\underline{\mathcal{C}}((a\cdot 1 + b\cdot(c^*\cdot 1))^*)$

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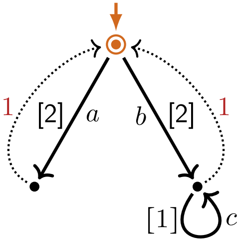
Process semantics $[\![\cdot]\!]_{\mathbf{P}}$ (examples, bisimulation collapse)

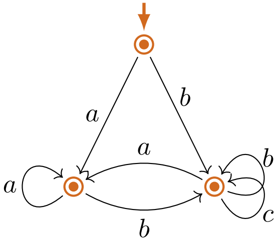
1-chart



LLEE

$\underline{\mathcal{C}}((a\cdot 1 + b\cdot(c^* \cdot 1))^*)$

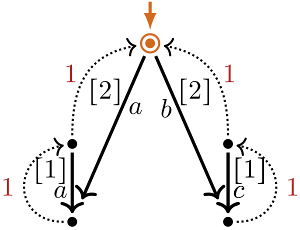
chart (Milner)



~~LLEE~~

$\mathcal{C}((a\cdot 1 + b\cdot(c^* \cdot 1))^*)$

1-chart

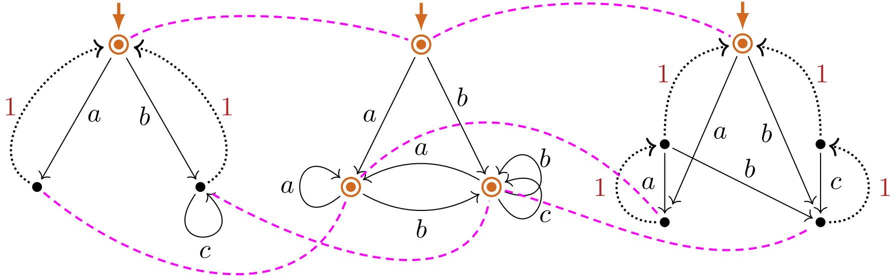


LLEE

$$\underline{\mathcal{C}}\Big(\big(a\cdot(1\cdot((a\cdot 1)^*\cdot 1)) \cdot b\cdot((1\cdot(c\cdot 1)^*))\cdot 1 + b\cdot((1\cdot(c\cdot 1)^*))\cdot 1\big)^*\Big)$$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)

1-chart | chart (Milner) | 1-chart



LLEE | ~~LLEE~~ | LLEE

$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*)$

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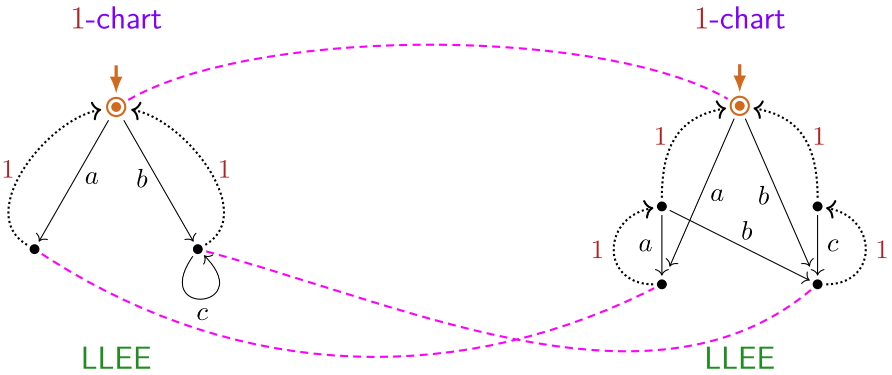
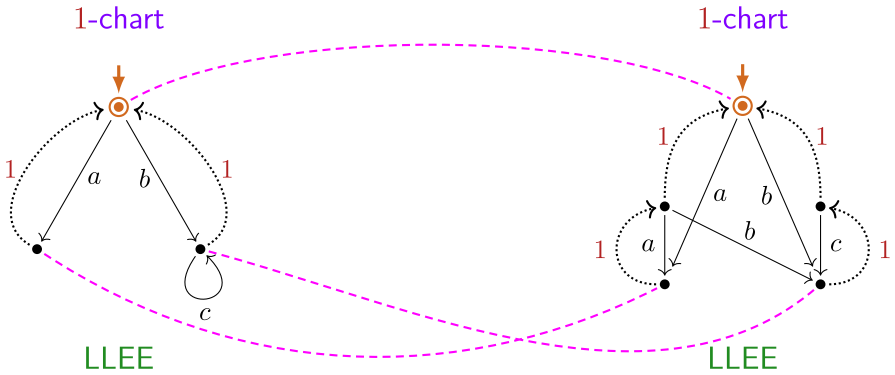
Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



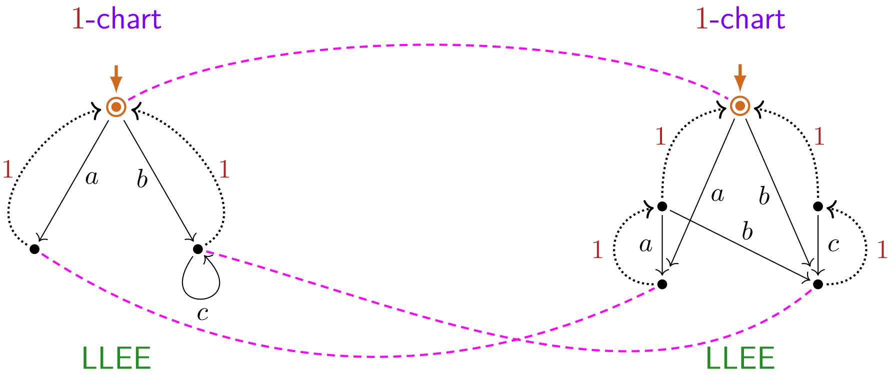
$$\underline{\mathcal{C}}((a \cdot 1 + b \cdot (c^* \cdot 1))^*) \quad \underline{\leftrightarrow} \quad \underline{\mathcal{C}}\Big(\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\Big)$$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$$[\![(a \cdot 1 + b \cdot (c^{*} \cdot 1))^{*}]\!]_P \quad = \quad [\![\big(a \cdot (1 \cdot ((a \cdot 1)^{*} \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^{*})) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^{*})) \cdot 1\big)^{*}]\!]_P$$

Process semantics $[\![\cdot]\!]_P$ (examples, bisimulation collapse)



$(a \cdot 1 + b \cdot (c^* \cdot 1))^*$ $\equiv_P$ $\big(a \cdot (1 \cdot ((a \cdot 1)^* \cdot 1)) \cdot b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1 + b \cdot ((1 \cdot (c \cdot 1)^*)) \cdot 1\big)^*\big)$

Milner's proof system Mil

Axioms :

(A1) $e + (f + g) = (e + f) + g$

(A2) $e + 0 = e$

(A3) $e + f = f + e$

(A4) $e + e = e$

(A5) $e \cdot (f \cdot g) = (e \cdot f) \cdot g$

(A6) $(e + f) \cdot g = e \cdot g + f \cdot g$

(A7) $e = 1 \cdot e$

(A8) $e = e \cdot 1$

(A9) $0 = 0 \cdot e$

(A10) $e^* = 1 + e \cdot e^*$

(A11) $e^* = (1 + e)^*$

But: $e \cdot (f + g) \neq e \cdot f + e \cdot g$

But: $e \cdot 0 \neq 0$

Inference rules : rules of equational logic *plus*

$$\frac{e = f \cdot e + g}{e = f^* \cdot g}\ \mathsf{RSP}^*$$ (if f does not terminate immediately)

Milner's question (1984)

Is Mil complete with respect to $\equiv_{\mathbf{P}}$? (Does $e \equiv_{\mathbf{P}} f \Longrightarrow e =_{\mathsf{Mil}} f$ hold?)

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution $\quad \mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2) \quad$ e_2 is solution

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution

$$\begin{tikzcd}
\mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] \\
& \mathcal{C}_0 &
\end{tikzcd}$$

e_2 is solution

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution LLEE LLEE e_2 is solution

$$\begin{tikzcd} \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] \\ & \mathcal{C}_0 & \end{tikzcd}$$

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

e_1 is solution

$$\begin{tikzcd}
\overset{\text{LLEE}}{\mathcal{C}(e_1)} \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \overset{\text{LLEE}}{\mathcal{C}(e_2)} \arrow[dl, Rightarrow] \\
& \underset{\text{LLEE}}{\mathcal{C}_0} &
\end{tikzcd}$$

e_2 is solution

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

$$
\begin{tikzcd}
e_1 \text{ is solution} & \begin{array}{c}\text{LLEE}\\ \mathcal{C}(e_1)\end{array} \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \begin{array}{c}\text{LLEE}\\ \mathcal{C}(e_2)\end{array} \arrow[dl, Rightarrow] & e_2 \text{ is solution} \\
& & \begin{array}{c}\mathcal{C}_0\\ \text{LLEE}\end{array} & \text{has solution } e_0 &
\end{tikzcd}
$$

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:

$\underbrace{\begin{array}{l} e_1 \text{ is solution} \\ e_0 \text{ is solution} \end{array}}$

LLEE
$\mathcal{C}(e_1)$

(assm)

LLEE
$\mathcal{C}(e_2)$

$\underbrace{\begin{array}{l} e_2 \text{ is solution} \\ e_0 \text{ is solution} \end{array}}$

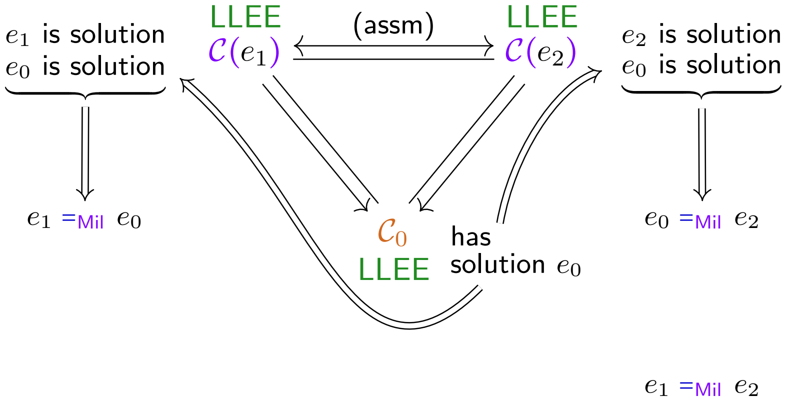
$\mathcal{C}_0$
LLEE

has
solution e_0

$e_1 =_{\mathsf{Mil}} e_2$

Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

For 1-free regular expressions e_1 and e_2:



Bisimulation collapse proof strategy (G/Fokkink, LICS'20)

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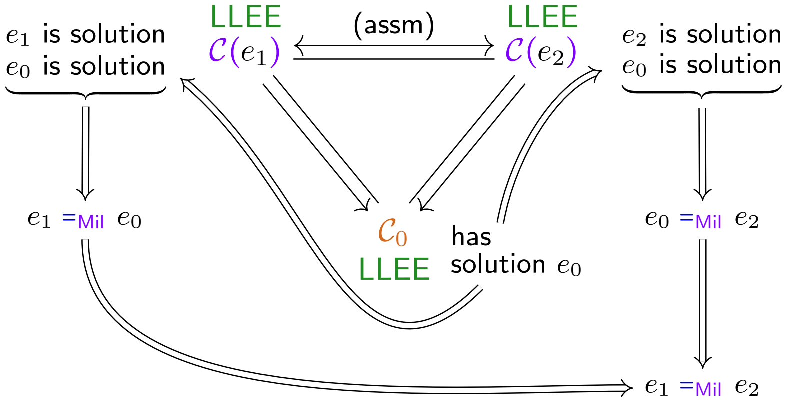


Bisimulation collapse proof strategy (general case)

Problem 1

$$
\begin{tikzcd}
\textit{chart interpretations} & \cancel{\text{LLEE}} & \mathcal{C}(e_1) \arrow[rr, leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & \cancel{\text{LLEE}} \\
\textit{bisimulation collapse} & & & \mathcal{C}_0 & &
\end{tikzcd}
$$

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

chart interpretations

$$
\begin{tikzcd}
\cancel{\text{LLEE}} & \mathcal{C}(e_1) \arrow[rr, leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & \cancel{\text{LLEE}} \\
 & & \mathcal{C}_0 & &
\end{tikzcd}
$$

bisimulation collapse

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

$$
\begin{tikzcd}
\textit{1-chart interpretations} & \text{LLEE}\ \underline{\mathcal{C}}(e_1) \arrow[d, Rightarrow, "\text{(IV)}"] & & \underline{\mathcal{C}}(e_2)\ \text{LLEE} \arrow[d, Rightarrow, "\text{(IV)}"'] \\
\textit{chart interpretations} & \cancel{\text{LLEE}}\ \mathcal{C}(e_1) \arrow[r, leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2)\ \cancel{\text{LLEE}} \arrow[dl, Rightarrow] \\
\textit{bisimulation collapse} & & \mathcal{C}_0 &
\end{tikzcd}
$$

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

1-*chart interpretations*

chart interpretations

bisimulation collapse

$$\begin{tikzcd} \text{LLEE} & \underline{\mathcal{C}}(e_1) \arrow[rr, Leftrightarrow] \arrow[d, Rightarrow, "\text{(IV)}"] & & \underline{\mathcal{C}}(e_2) \arrow[d, Rightarrow, "\text{(IV)}"'] & \text{LLEE} \\ \cancel{\text{LLEE}} & \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & \cancel{\text{LLEE}} \\ & & \mathcal{C}_0 & & \end{tikzcd}$$

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

1-*chart interpretations*

chart interpretations

bisimulation collapse

$$
\begin{tikzcd}
\text{LLEE}\ \underline{\mathcal{C}}(e_1) \arrow[rr, Leftrightarrow] \arrow[d, Rightarrow, "\text{(IV)}"] \arrow[ddr, Rightarrow, bend right] & & \underline{\mathcal{C}}(e_2)\ \text{LLEE} \arrow[d, Rightarrow, "\text{(IV)}"'] \arrow[ddl, Rightarrow, bend left] \\
\cancel{\text{LLEE}}\ \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2)\ \cancel{\text{LLEE}} \arrow[dl, Rightarrow] \\
& \mathcal{C}_0 &
\end{tikzcd}
$$

Bisimulation collapse proof strategy (general case)

Remedy for Problem 1 *(G, TERMGRAPH 2020)*

1-*chart interpretations*

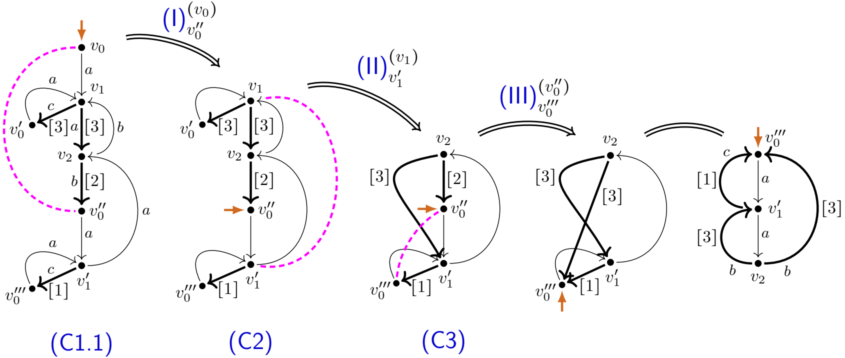
chart interpretations

bisimulation collapse

$$
\begin{tikzcd}
\text{LLEE} \;\; \underline{\mathcal{C}}(e_1) \arrow[rr, Leftrightarrow] \arrow[d, Rightarrow, "\text{(IV)}"] \arrow[ddr, Rightarrow, bend right] & & \underline{\mathcal{C}}(e_2) \;\; \text{LLEE} \arrow[d, Rightarrow, "\text{(IV)}"'] \arrow[ddl, Rightarrow, bend left] \\
\cancel{\text{LLEE}} \;\; \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \;\; \cancel{\text{LLEE}} \arrow[dl, Rightarrow] \\
& \mathcal{C}_0 \;\; \text{LLEE?} &
\end{tikzcd}
$$

LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)



Lemma

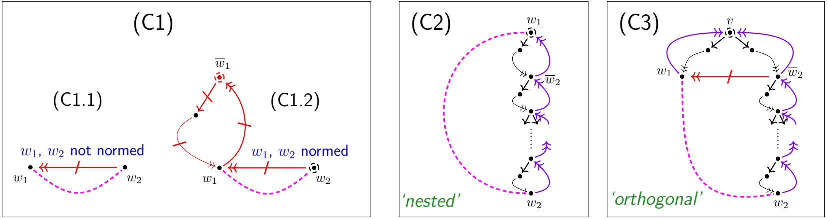
The bisimulation collapse of a LLEE-chart is again a LLEE-chart.

Reduced bisimilarity redundancies in LLEE-charts (no 1-trans.!)

(G/Fokkink, LICS'20)

w_1, w_2 in different scc's

w_1, w_2 in the same scc

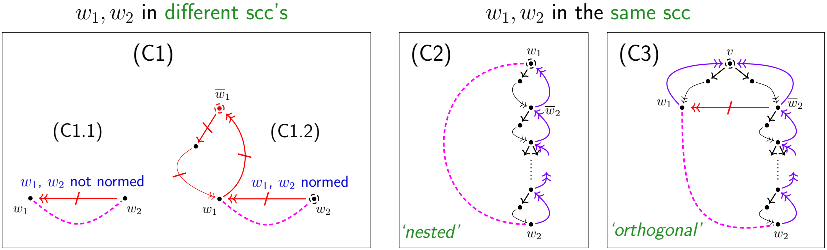


Lemma

Every not collapsed LLEE-chart contains bisimilar vertices $w_1 \neq w_2$ of kind (C1), (C2), or (C3) (a reduced bisimilarity redundancy $\langle w_1, w_2 \rangle$):

Reduced bisimilarity redundancies in LLEE-charts (no 1-trans.!)

(G/Fokkink, LICS'20)



Lemma

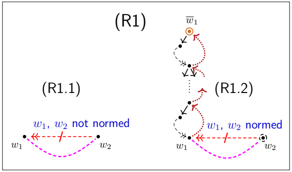
Every not collapsed LLEE-chart contains bisimilar vertices $w_1 \neq w_2$ of kind (C1), (C2), or (C3) (a reduced bisimilarity redundancy $\langle w_1, w_2 \rangle$):

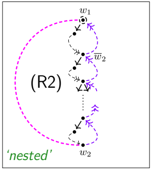
Lemma

Every reduced bisimilarity redundancy in a LLEE-chart can be eliminated LLEE-preservingly.

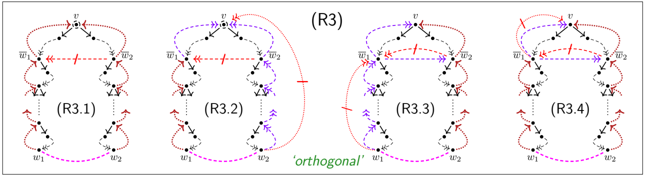
Reduced 1-bisimilarity redundancies in LLEE-1-charts

w_1, w_2 in different scc's





w_1, w_2 in the same scc

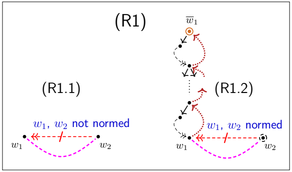


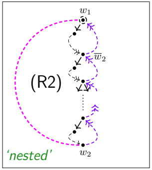
Lemma

Every not collapsed LLEE-1-chart contains 1-bisimilar vertices $w_1 \neq w_2$ (a reduced 1-bisimilarity redundancy $\langle w_1, w_2 \rangle$) of kind (R1), (R2), (R3).

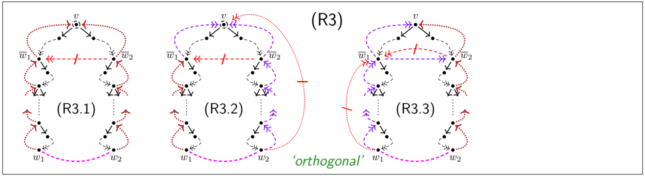
Reduced 1-bisimilarity redundancies in LLEE-1-charts

w_1, w_2 in different scc's





w_1, w_2 in the same scc



Lemma

Every not collapsed LLEE-1-chart contains 1-bisimilar vertices $w_1 \neq w_2$ (a reduced 1-bisimilarity redundancy $\langle w_1, w_2 \rangle$) of kind (R1), (R2), (R3).

Lemma

Every simple reduced 1-bisimilarity redundancies in a LLEE-1-chart can be eliminated LLEE-preservingly.

Reduced 1-bisimilarity redundancies in LLEE-1-charts

w_1, w_2 in different scc's

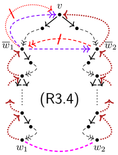
(R1)

(R2)

'nested'

w_1, w_2 in the same scc

(R3)



'orthogonal'

Lemma

Every not collapsed LLEE-1-chart contains 1-bisimilar vertices $w_1 \neq w_2$ *(a reduced 1-bisimilarity redundancy* $\langle w_1, w_2 \rangle$*) of kind* (R1), (R2), (R3).

Stumbling Block

How to LLEE-preservingly eliminate
reduced 1-bisimilarity redundancies of kind (R3.4)?

Reduced 1-bisimilarity redundancies in LLEE-1-charts

w_1, w_2 in different scc's

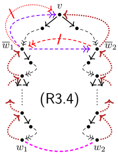
(R1)

(R2)

'nested'

w_1, w_2 in the same scc

(R3)



'orthogonal'

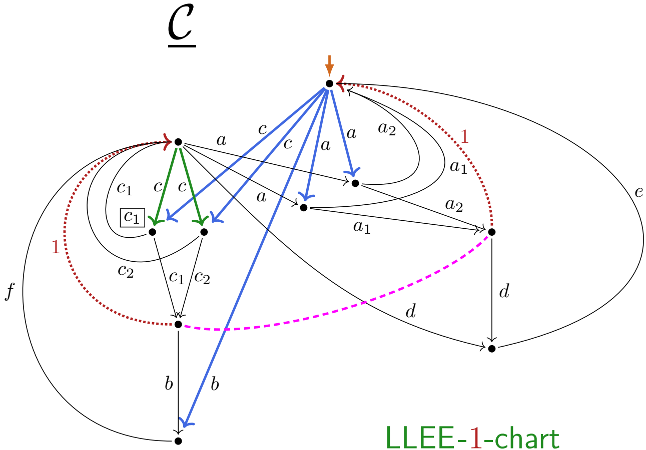
Lemma

Every not collapsed LLEE-1-chart contains 1-bisimilar vertices $w_1 \neq w_2$ (a reduced 1-bisimilarity redundancy $\langle w_1, w_2 \rangle$) of kind (R1), (R2), (R3).

Stumbling Block

How to LLEE-preservingly eliminate precrystalline reduced 1-bisimilarity redundancies?

Counterexample LLEE-preserving collapse

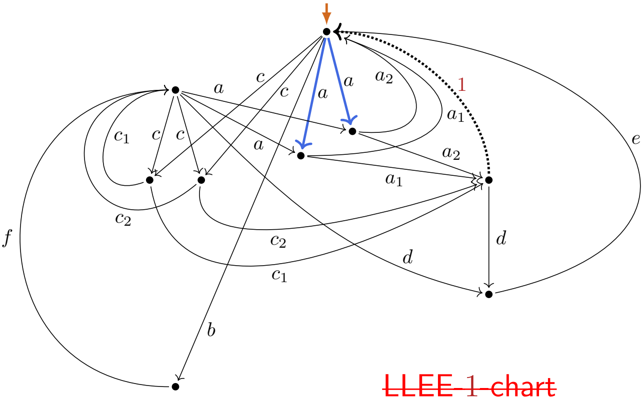


LLEE-1-chart

$\langle w_1, w_2 \rangle$ is a reduced
1-bisimilarity redundancy of kind (R3.4)

Counterexample LLEE-preserving collapse

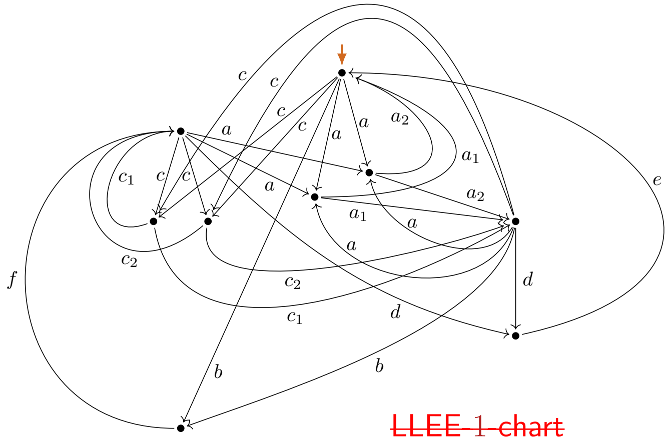
$\underline{\mathcal{C}}_1$



~~LLEE-1-chart~~

1-bisimulation collapse

Counterexample LLEE-preserving collapse



~~LLEE-1-chart~~
bisimulation collapse

Bisimulation collapse proof strategy (general case)

Problem 2: There are regular expressions e_1 and e_2 such that:

1-*chart interpretations*

chart interpretations

bisimulation collapse

$$
\begin{tikzcd}
\text{LLEE} \;\; \underline{\mathcal{C}}(e_1) \arrow[rr, Leftrightarrow] \arrow[dr, Rightarrow, "\text{(IV)}"] \arrow[ddr, Rightarrow, bend right] & & \underline{\mathcal{C}}(e_2) \;\; \text{LLEE} \arrow[dl, Rightarrow, "\text{(IV)}"'] \arrow[ddl, Rightarrow, bend left] \\
\cancel{\text{LLEE}} \;\; & \mathcal{C}(e_1) \arrow[r, Leftrightarrow, "\text{(assm)}"] \arrow[d, Rightarrow] & \mathcal{C}(e_2) \arrow[dl, Rightarrow] \;\; \cancel{\text{LLEE}} \\
& \mathcal{C}_0 \;\; \cancel{\text{LLEE}} &
\end{tikzcd}
$$

Bisimulation collapse proof strategy (general case)

Problem 2: There are regular expressions e_1 and e_2 such that:

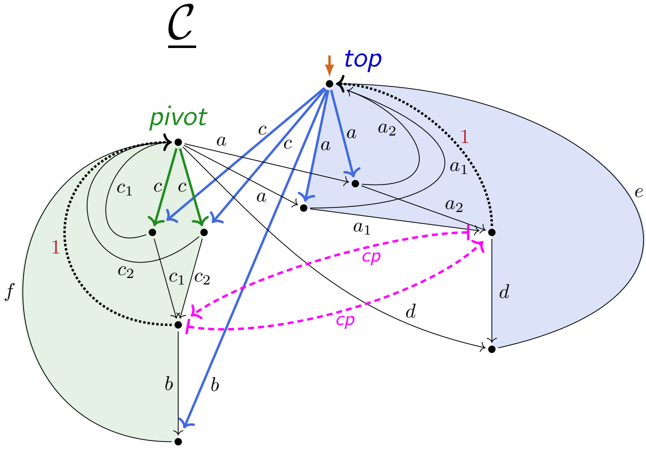
1-chart interpretations

chart interpretations

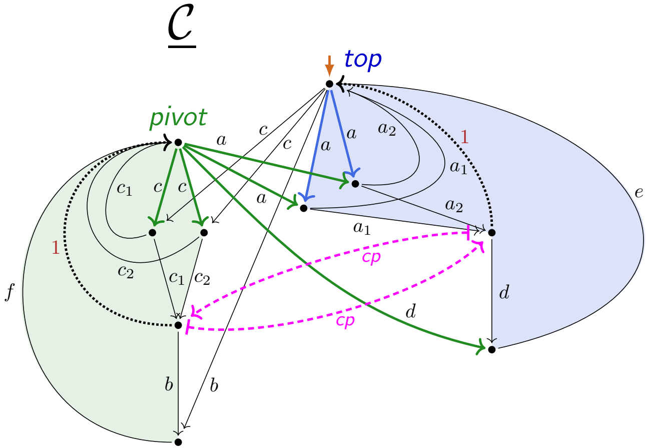
$$\begin{tikzcd}
\text{LLEE}\ \underline{\mathcal{C}}(e_1) \arrow[rrrr, Leftrightarrow] \arrow[dr, Rightarrow, "\text{(IV)}"] \arrow[ddrr, Rightarrow, bend right] & & & & \underline{\mathcal{C}}(e_2)\ \text{LLEE} \arrow[dl, Rightarrow, "\text{(IV)}"'] \arrow[ddll, Rightarrow, bend left] \\
\cancel{\text{LLEE}} & \mathcal{C}(e_1) \arrow[rr, Leftrightarrow, "\text{(assm)}"] \arrow[dr, Rightarrow] & & \mathcal{C}(e_2) \arrow[dl, Rightarrow] & \cancel{\text{LLEE}} \\
& & \text{for all } \mathcal{C}_0\text{:}\ \cancel{\text{LLEE}} & &
\end{tikzcd}$$

$\mathcal{C}(e_1)$, $\mathcal{C}(e_2)$, $\underline{\mathcal{C}}(e_1)$ and $\underline{\mathcal{C}}(e_2)$ are **not** LLEE-preservingly jointly minimizable under bisimilarity.

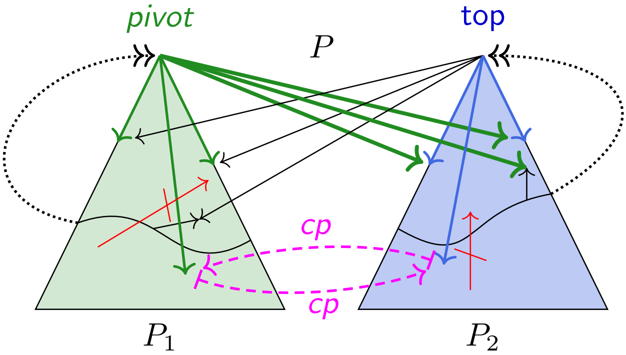
Twin-Crystal



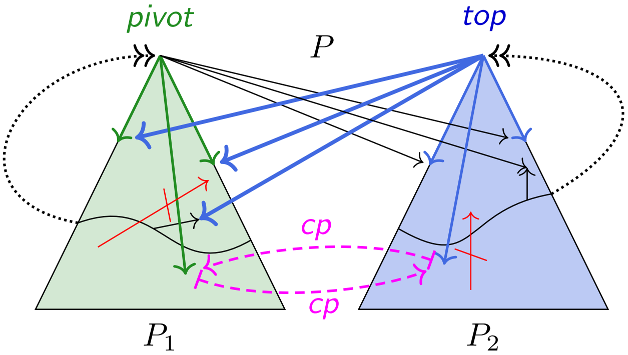
Twin-Crystal



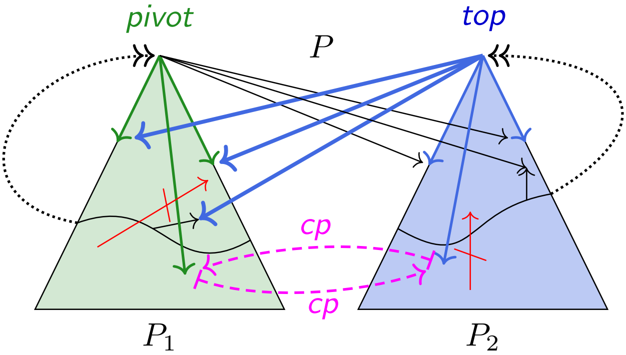
Twin-Crystal



Twin-Crystal

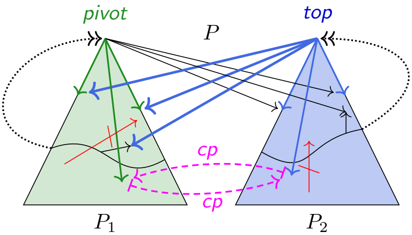


Twin-Crystal



- Provable solutions of twin-crystals are complete:
  they can be transferred to their bisimulation collapses

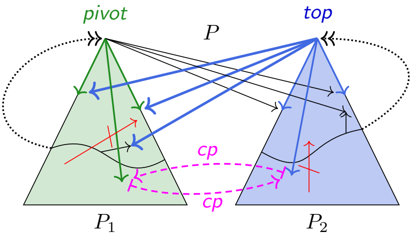
Crystallization



twin-crystal

Crystallized 1-charts = LLEE-1-charts that are collapsed apart from some scc's that are twin-crystals.

Crystallization

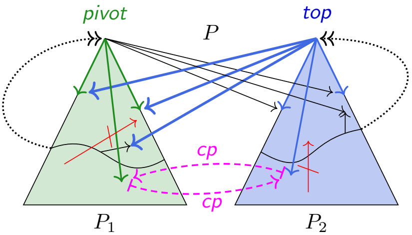


twin-crystal

Crystallized 1-charts = LLEE-1-charts that are collapsed apart from some scc's that are twin-crystals.

(CR) Crystallization: Every LLEE-1-chart can be reduced under bisimilarity to a bisimilar crystallized 1-chart.

Crystallization



twin-crystal

Crystallized 1-charts = LLEE-1-charts that are collapsed apart from some scc's that are twin-crystals.

(CR) Crystallization: Every LLEE-1-chart can be reduced under bisimilarity to a bisimilar crystallized 1-chart.

(CC) Every provable solution of a crystallized 1-chart gives rise to provable solution on the bisimulation collapse.

Completeness proof of Mil (structure)

chart interpretations $\mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2)$

$\overset{?}{\Longrightarrow}$ $e_1 =_{\mathsf{Mil}} e_2$

Completeness proof of Mil (structure)

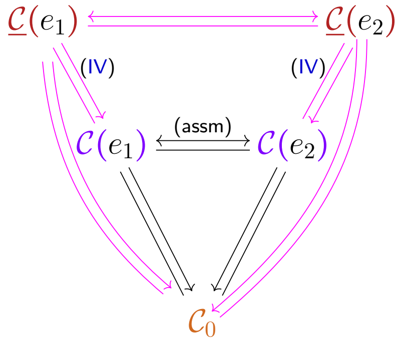
chart interpretations

$$\mathcal{C}(e_1) \xleftrightarrow{\text{(assm)}} \mathcal{C}(e_2)$$

Completeness proof of Mil (structure)

1-chart interpretations

LLEE
e_1 is solution

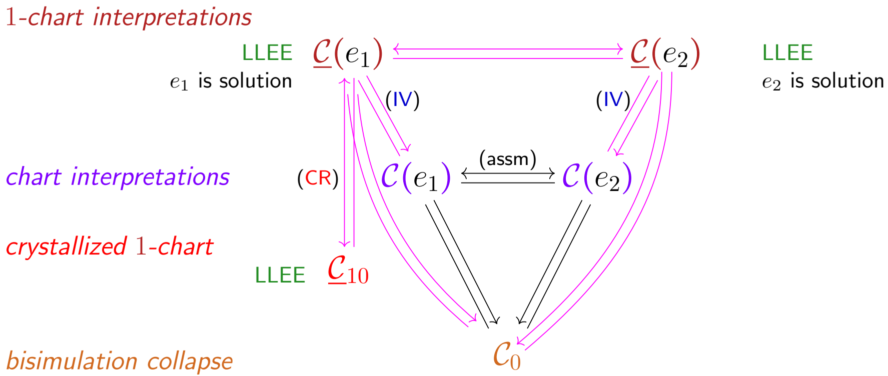


LLEE
e_2 is solution

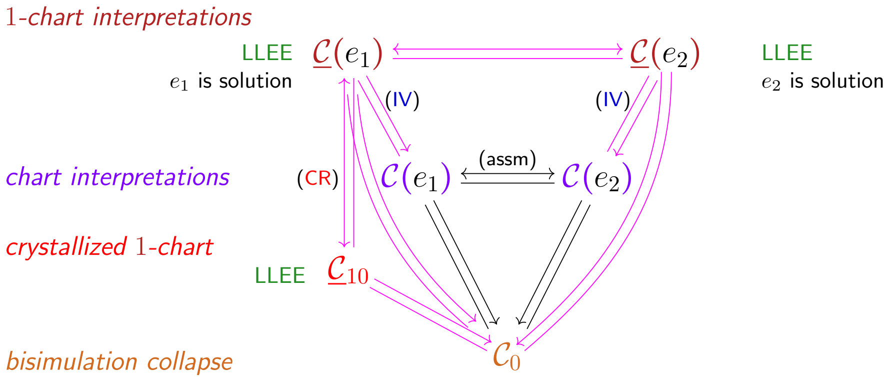
chart interpretations

bisimulation collapse

Completeness proof of Mil (structure)



Completeness proof of Mil (structure)



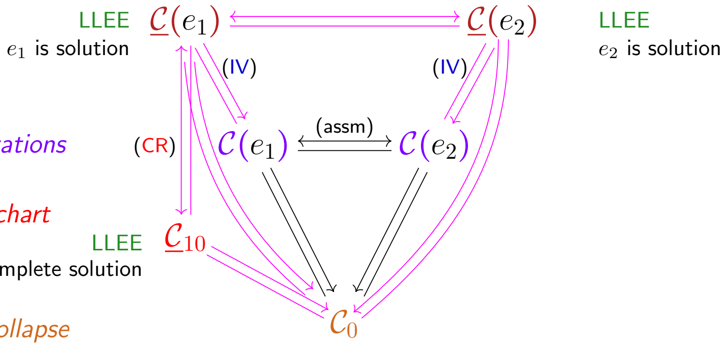
Completeness proof of Mil (structure)

1-chart interpretations

chart interpretations

crystallized 1-chart

bisimulation collapse



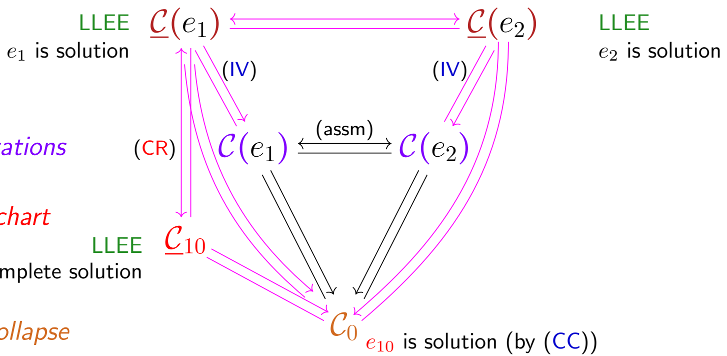
Completeness proof of Mil (structure)

1-chart interpretations

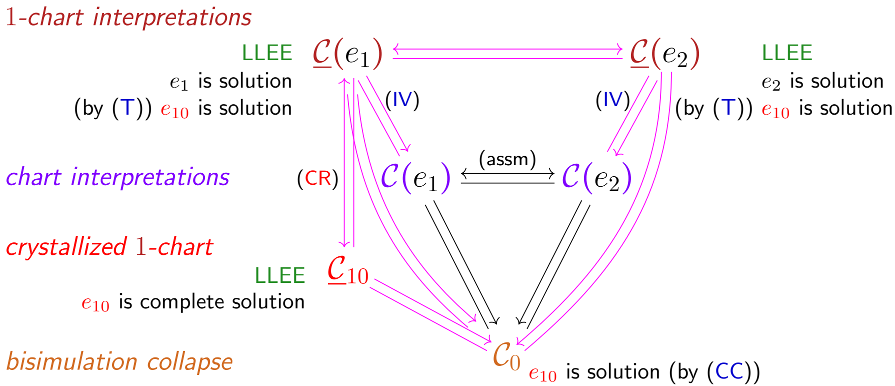
chart interpretations

crystallized 1-chart

bisimulation collapse



Completeness proof of Mil (structure)



Completeness proof of Mil (structure)

1-chart interpretations

LLEE
e_1 is solution
(by (T)) e_{10} is solution

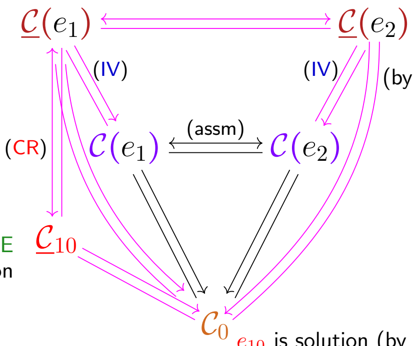
LLEE
e_2 is solution
(by (T)) e_{10} is solution

chart interpretations

crystallized 1-chart

LLEE
e_{10} is complete solution

bisimulation collapse

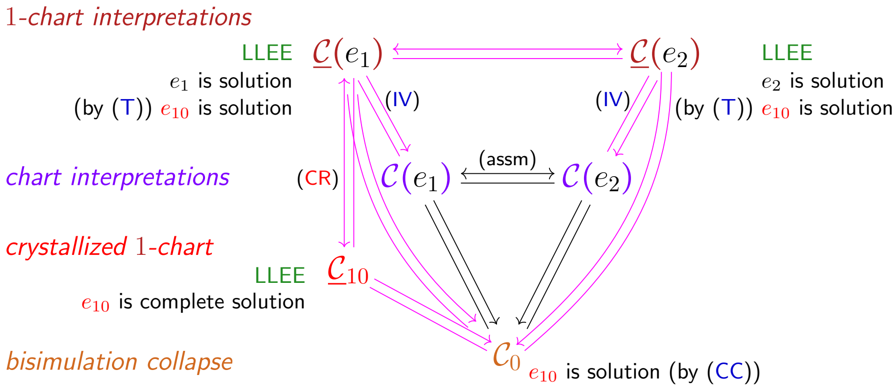


e_{10} is solution (by (CC))

$\mathcal{C}(e_1)$ LLEE
e_1 is solution
e_{10} is solution

$\mathcal{C}(e_2)$ LLEE
e_2 is solution
e_{10} is solution

Completeness proof of Mil (structure)



$\left.\begin{array}{r} \mathcal{C}(e_1)\ \text{LLEE} \\ e_1\ \text{is solution} \\ e_{10}\ \text{is solution} \end{array}\right\} \xRightarrow[\text{(SE)}]{} e_1 =_{\text{Mil}} e_{10}$

$\mathcal{C}(e_2)$ LLEE
e_2 is solution
e_{10} is solution

Completeness proof of Mil (structure)

1-chart interpretations

LLEE
e_1 is solution
(by (T)) e_{10} is solution

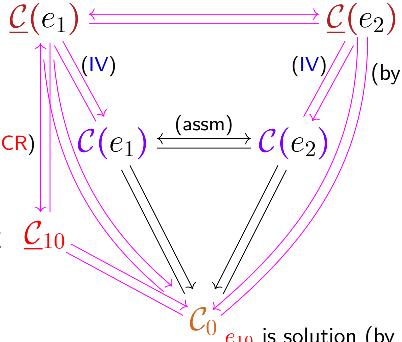
LLEE
e_2 is solution
(by (T)) e_{10} is solution

chart interpretations

crystallized 1-chart

LLEE
e_{10} is complete solution

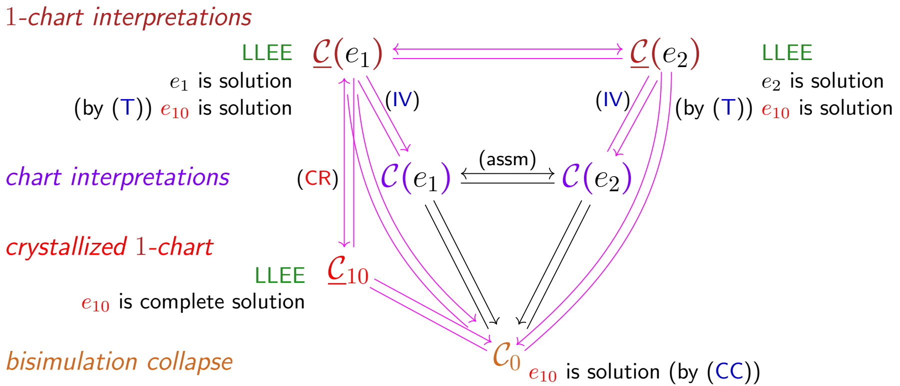
bisimulation collapse



e_{10} is solution (by (CC))

$$\left.\begin{array}{r} \mathcal{C}(e_1) \text{ LLEE} \\ e_1 \text{ is solution} \\ e_{10} \text{ is solution} \end{array}\right\} \underset{\text{(SE)}}{\Longrightarrow} e_1 =_{\mathsf{Mil}} e_{10} \qquad e_{10} =_{\mathsf{Mil}} e_2 \underset{\text{(SE)}}{\Longleftarrow} \left\{\begin{array}{l} \mathcal{C}(e_2) \text{ LLEE} \\ e_2 \text{ is solution} \\ e_{10} \text{ is solution} \end{array}\right.$$

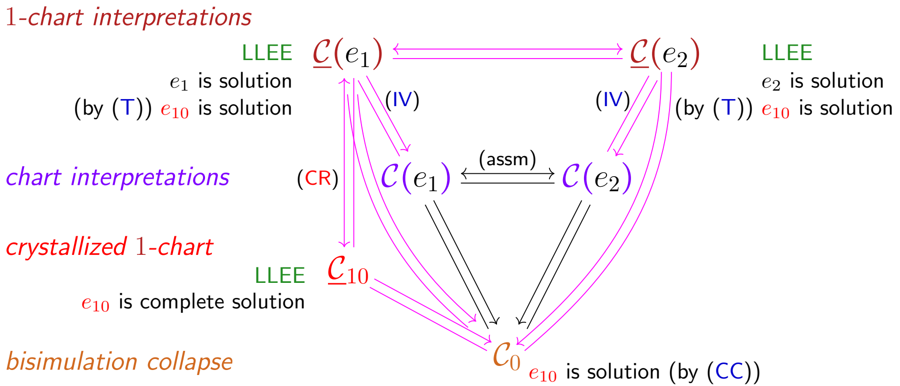
Completeness proof of Mil (structure)



$$\left.\begin{array}{r}\mathcal{C}(e_1)\ \text{LLEE}\\ e_1\ \text{is solution}\\ e_{10}\ \text{is solution}\end{array}\right\}\underset{\text{(SE)}}{\Longrightarrow} e_1 =_{\mathsf{Mil}} e_{10} \qquad e_{10} =_{\mathsf{Mil}} e_2 \underset{\text{(SE)}}{\Longleftarrow}\left\{\begin{array}{l}\mathcal{C}(e_2)\ \text{LLEE}\\ e_2\ \text{is solution}\\ e_{10}\ \text{is solution}\end{array}\right.$$

$$\Longrightarrow \qquad e_1 =_{\mathsf{Mil}} e_2$$

Completeness proof of Mil (structure)



Theorem

Milner's proof system Mil is complete for process semantics equivalence $\equiv_{\mathbf{P}}$ of regular expressions.

Since: $e_1 \equiv_{\mathbf{P}} e_2 \implies [\![e_1]\!]_{\mathbf{P}} = [\![e_2]\!]_{\mathbf{P}} \implies \mathcal{C}(e_1) \leftrightarrow \mathcal{C}(e_1) \implies e_1 =_{\mathsf{Mil}} e_2$.

Outlook

poster presentation

- tomorrow, 10–10.30

next steps and projects

- monograph project: proof in fine-grained detail
- computation/animation tool for crystallization
- use crystallization for recognition problem

resources on Github:

- `https://github.com/clegra/crystallization/blob/main`
  - article (after rebuttal): `/cryst-article.pdf`
  - poster: `/poster-lics2022.pdf`
  - presentation: `/presentation-lics2022.pdf`

acknowledgment & thanks to:

- Wan Fokkink (for long collaboration)

Thank you for your attention!