A Complete Proof System for 1-Free Regular Expressions Modulo Bisimilarity

Clemens Grabmayer and Wan Fokkink

Departments of Computer Science

GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

VRIJE UNIVERSITEIT AMSTERDAM
Faculty of Science

L'Aquila, Italy, and Amsterdam, The Netherlands

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Overview

- 1-free regular (star) expressions
- Milner's process interpretation
  - axiomatization question (1984) for system Mil
- proof system BBP (Bergstra–Bethke–Ponse) for 1-free star expr's

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- new approach: structure-constrained process graphs
  - (layered) loop existence and elimination (LLEE)
- lemmas
  - preservation of LLEE under bisimulation collapse
- completeness proof

Regular Expressions

Definition (*Kleene, 1951*)

Regular expressions over alphabet A with binary Kleene star:

$$e, f \; ::= \; \mathbf{0} \mid \mathbf{1} \mid \boldsymbol{a} \mid e + f \mid e \cdot f \mid e^{\circledast} f \qquad \text{(for } a \in A\text{)}.$$

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1-free regular (*star*) expressions over alphabet A:

$$e, f \;::=\; \mathbf{0} \mid \boldsymbol{a} \mid e + f \mid e \cdot f \mid e^{\circledast} f \qquad \text{(for } a \in A\text{)}.$$

Process semantics $[\![\cdot]\!]_P$ *(Milner, 1984)*

$\mathbf{0} \xmapsto{[\![\cdot]\!]_P}$ deadlock δ, no termination

$\mathbf{1} \xmapsto{[\![\cdot]\!]_P}$ empty process ϵ, then terminate

$a \xmapsto{[\![\cdot]\!]_P}$ atomic action a, then terminate

$e + f \xmapsto{[\![\cdot]\!]_P}$ alternative composition of $[\![e]\!]_P$ and $[\![f]\!]_P$

$e \cdot f \xmapsto{[\![\cdot]\!]_P}$ sequential composition of $[\![e]\!]_P$ and $[\![f]\!]_P$

$e^* \xmapsto{[\![\cdot]\!]_P}$ unbounded iteration of $[\![e]\!]_P$, option to terminate

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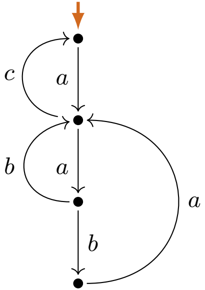
Process semantics $[\![\cdot]\!]_P$ (examples)



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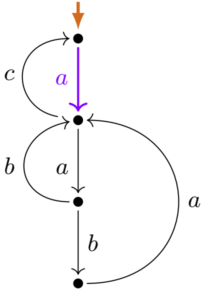

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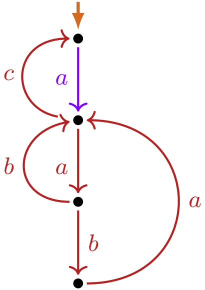

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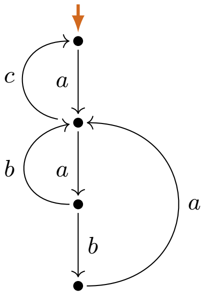

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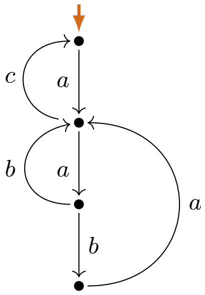
Process chart semantics $[\![\cdot]\!]_P = \mathcal{C}(\cdot)$ (examples)



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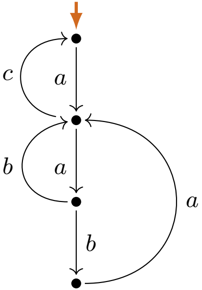

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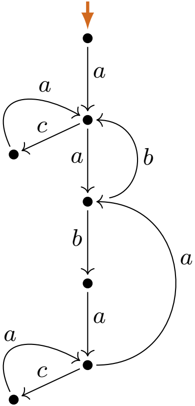

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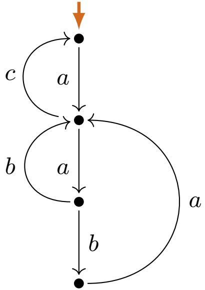

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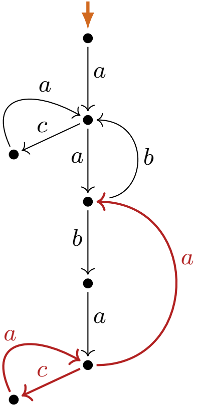

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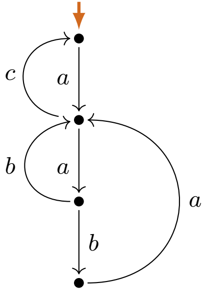

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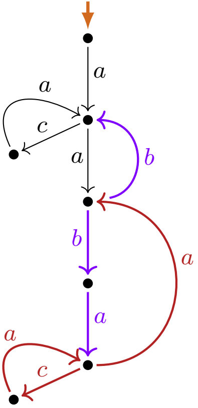

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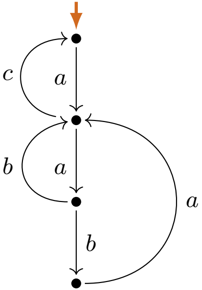

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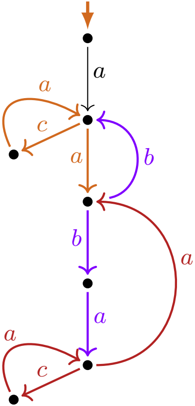

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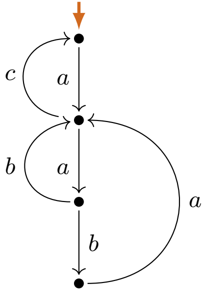

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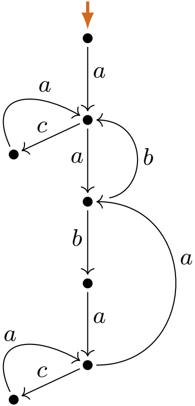

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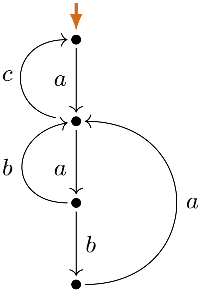

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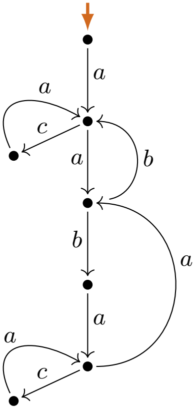

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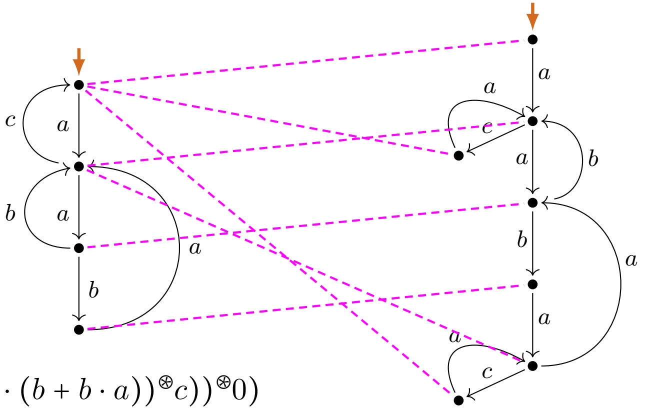

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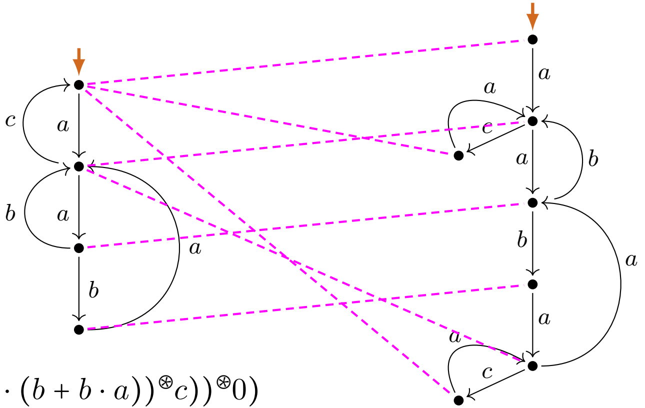

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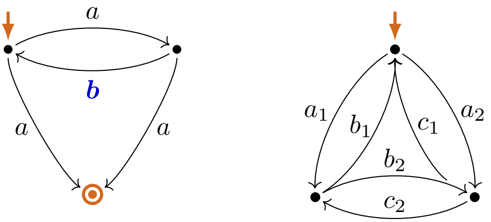
$\underline{\leftrightarrow}_P \; a \cdot ((c \cdot a + a \cdot (b \cdot a \cdot ((c \cdot a)^{\circledast} a))^{\circledast} b)^{\circledast} 0)$

Properties of $[\![\cdot]\!]_P$

- Not every finite-state process is $[\![\cdot]\!]_P$-expressible modulo $\underline{\leftrightarrow}$.



Properties of $[\![\cdot]\!]_P$

- Not every finite-state process is $[\![\cdot]\!]_P$-expressible modulo $\underline{\leftrightarrow}$.
- Fewer identities hold for $\underline{\leftrightarrow}_P$ than for $=_L$: $\underline{\leftrightarrow}_P \subsetneqq =_L$.

Complete axiomatization $\mathbf{F}_1$ of $=_L$ *(Aanderaa/Salomaa, 1965/66)*

Axioms:

(A1) $e + (f + g) = (e + f) + g$

(A2) $(e \cdot f) \cdot g = e \cdot (f \cdot g)$

(A3) $e + f = f + e$

(A4) $(e + f) \cdot g = e \cdot g + f \cdot g$

(A5) $e \cdot (f + g) = e \cdot f + e \cdot g$

(A6) $e + e = e$

(A7) $e \cdot 1 = e$

(A8) $e \cdot 0 = 0$

(A9) $e + 0 = e$

(UKS1) $e^* = 1 + e \cdot e^*$

(UKS2) $e^* = (1 + e)^*$

Inference rules: equational logic *plus*

$$\frac{e = f \cdot e + g}{e = f^* \cdot g}\ \text{FIX} \quad (\text{if } \underbrace{\{\epsilon\} \notin [\![f]\!]_L}_{\text{non-empty-word property}})$$

non-empty-word property

Sound and unsound axioms with respect to $\underline{\leftrightarrow}_P$

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(A1)	$e + (f + g) = (e + f) + g$	(A7)	$e \cdot 1 = e$
(A2)	$(e \cdot f) \cdot g = e \cdot (f \cdot g)$	(A8)	$e \cdot 0 = 0$
(A3)	$e + f = f + e$	(A9)	$e + 0 = e$
(A4)	$(e + f) \cdot g = e \cdot g + f \cdot g$	(UKS1)	$e^* = 1 + e \cdot e^*$
(A5)	$e \cdot (f + g) = e \cdot f + e \cdot g$	(UKS2)	$e^* = (1 + e)^*$
(A6)	$e + e = e$		

Inference rules: equational logic *plus*

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Adaptation Mil for $\underline{\leftrightarrow}_P$ *(Milner, 1984)*

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Adaptation BBP for $\underline{\leftrightarrow}_P$ on 1-free star expr's *(Bergstra, Bethke, Ponse)*

Axioms :

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(A3) $e + f = f + e$

(A4) $(e + f) \cdot g = e \cdot g + f \cdot g$

(A6) $e + e = e$

(A8)′ $0 \cdot e = 0$

(A9) $e + 0 = e$

(BKS1) $e^{\circledast} f = e \cdot (e^{\circledast} f) + f$

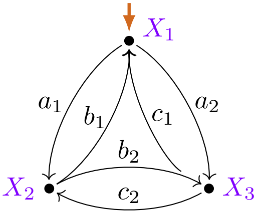
(BKS2) $(e^{\circledast} f) \cdot g = e^{\circledast}(f \cdot g)$

Inference rules : equational logic *plus*

$$\frac{e = f \cdot e + g}{e = f^{\circledast} g}\ \text{RSP}^{\circledast}$$

Not expressible ⇒ not solvable

chart



not expressible modulo $\leftrightarrows$

equational specification

$$X_1 = a_1 \cdot X_2 + a_2 \cdot X_3$$

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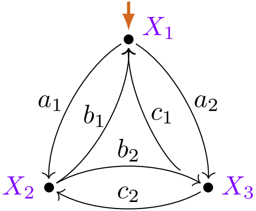

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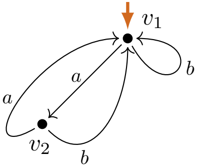
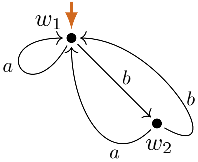
not solvable in BBP nor in Mil

Why Salomaa's proof approach does not work for BBP



$\underline{\leftrightarrow}$



$[\![(a \cdot (a+b) + b)^{\circledast} 0]\!]_{P}$ $\quad$ $[\![(a + b \cdot (a+b))^{\circledast} 0]\!]_{P}$

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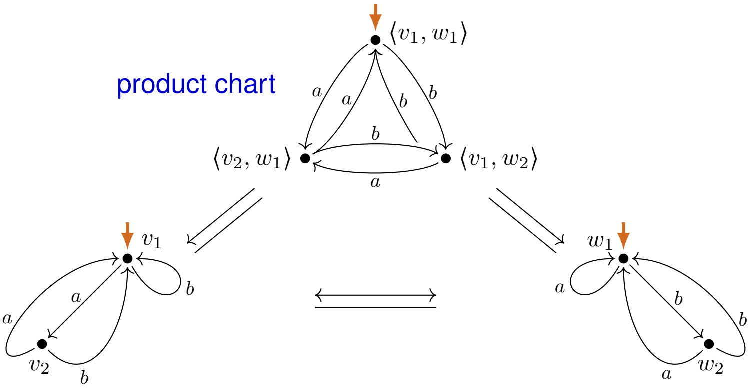

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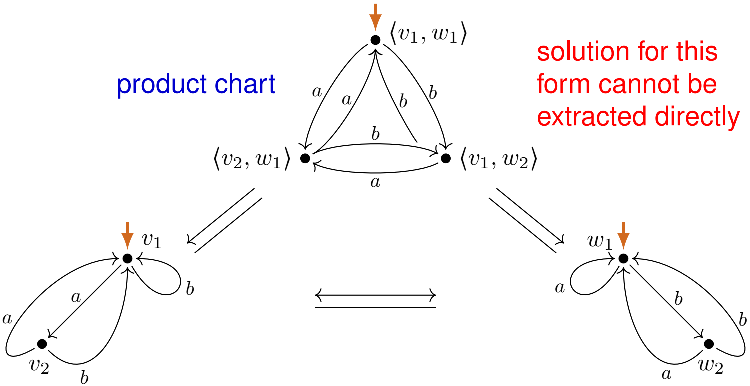

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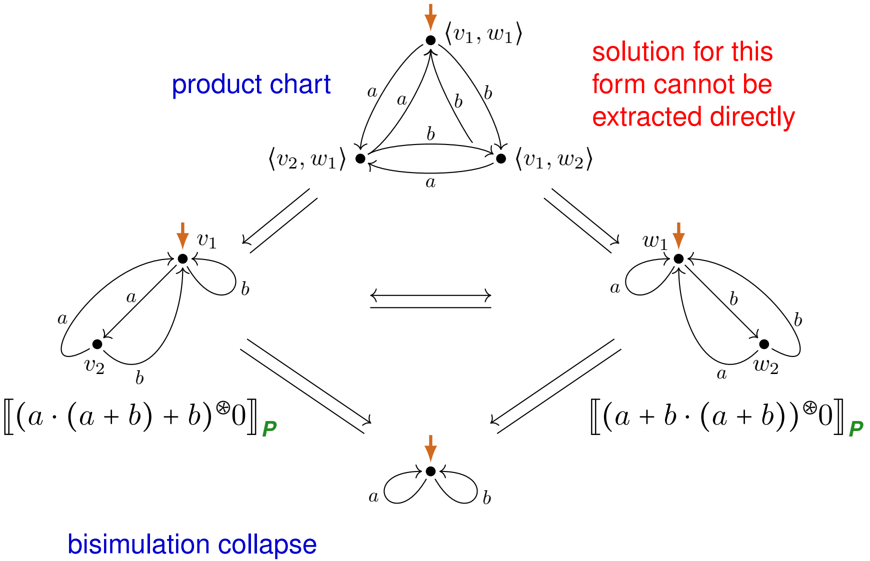
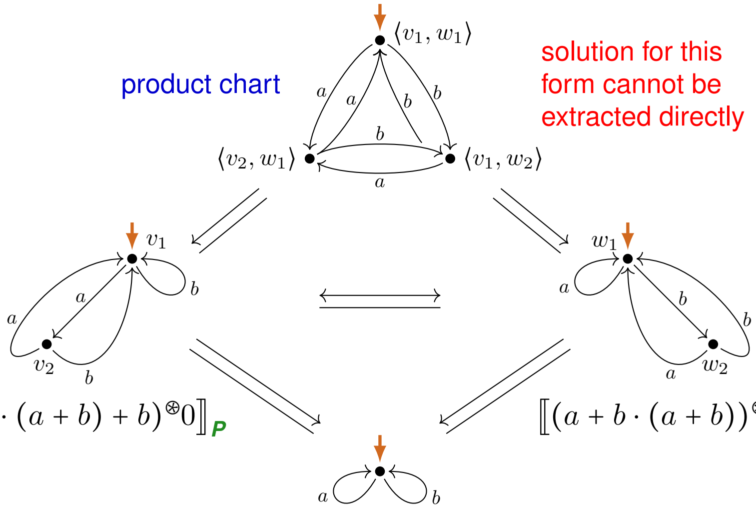


Why Salomaa's proof approach does not work for BBP

product chart

solution for this form cannot be extracted directly



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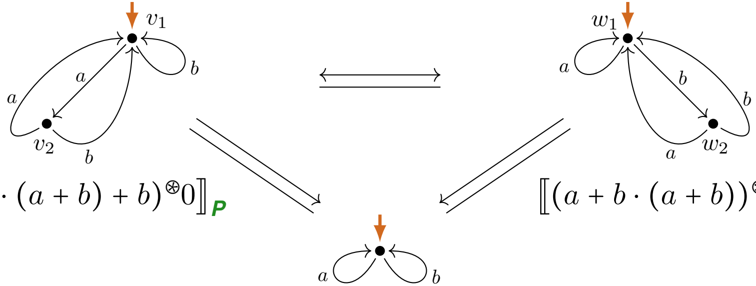
bisimulation collapse

$[\![(a + b)^{\circledast} 0]\!]_{P}$

solution can be extracted directly

New proof idea



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bisimulation collapse $[\![(a + b)^{\circledast} 0]\!]_{P}$

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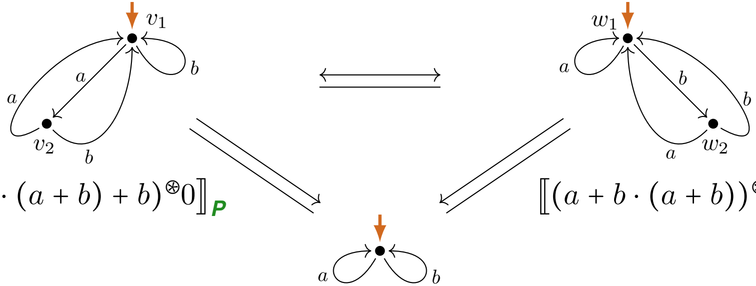

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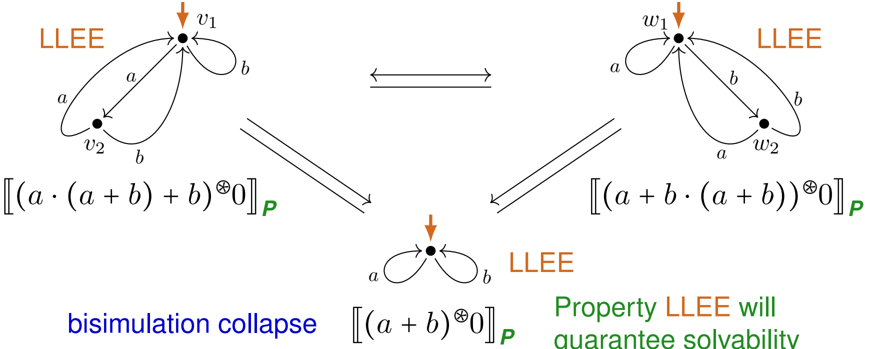
bisimulation collapse $[\![(a+b)^{\circledast} 0]\!]_{\mathit{P}}$

Can a solution always be extracted directly?

New proof idea



bisimulation collapse

Property LLEE will guarantee solvability

Loop chart

Definition

A chart is a loop chart if:

(L1) There is an infinite path from the start vertex.

(L2) Every infinite path from the start vertex returns to it.

(L3) Termination is not possible.

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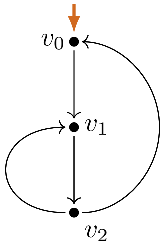

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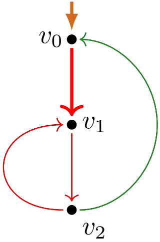

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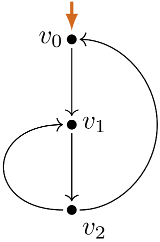

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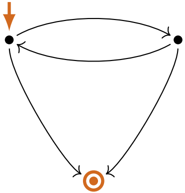

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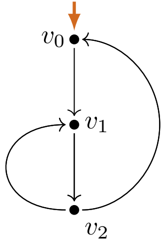

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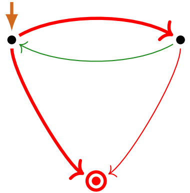

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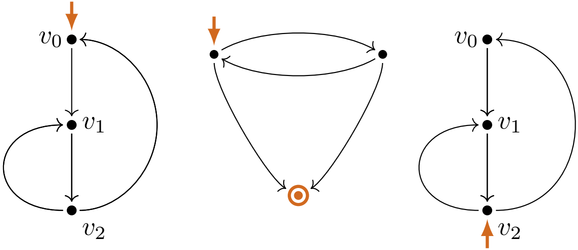

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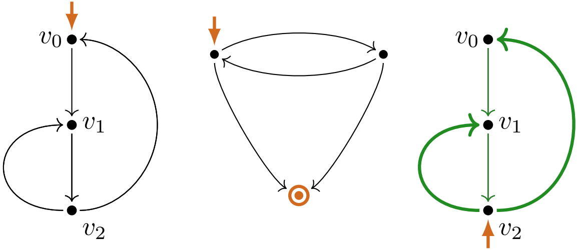

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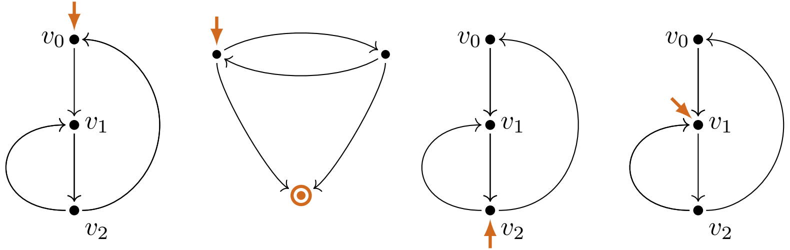

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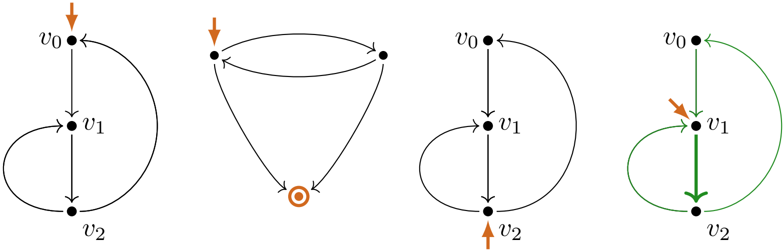

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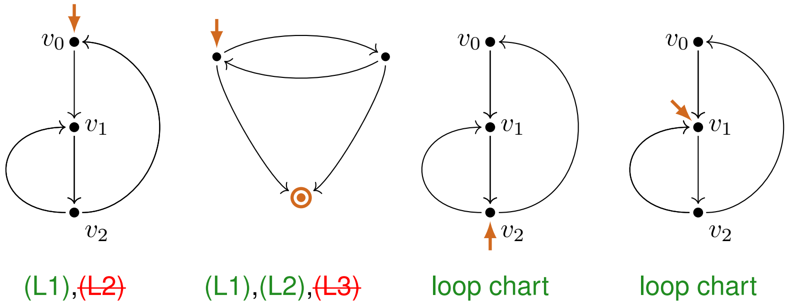

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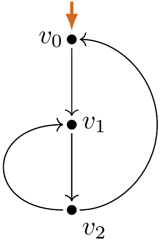

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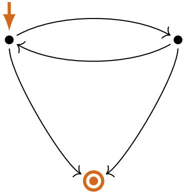

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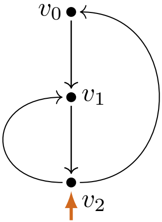
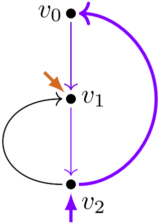
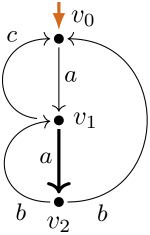


loop chart



loop subchart

Loop existence and elimination



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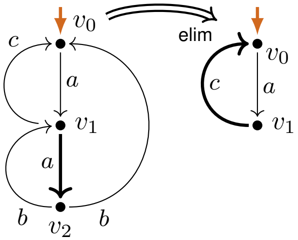

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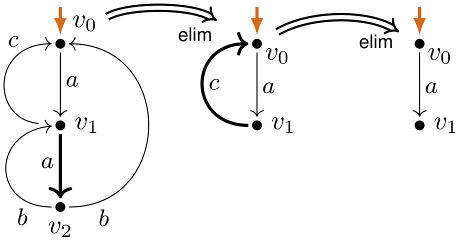

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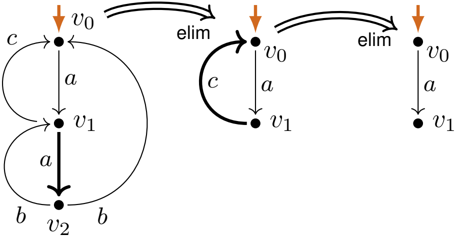

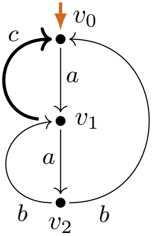

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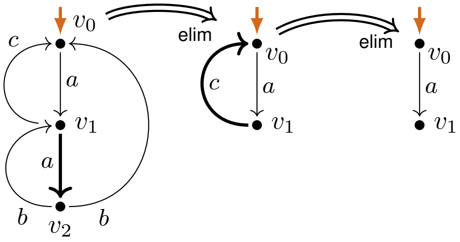

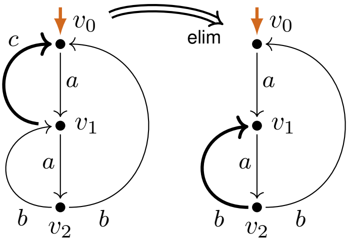

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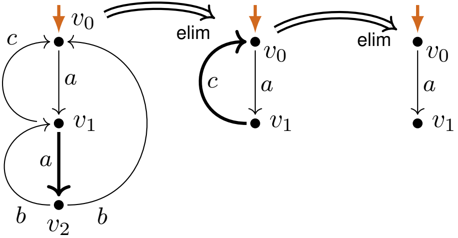

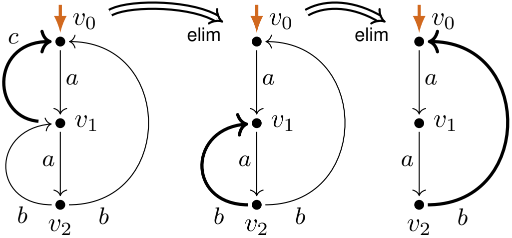

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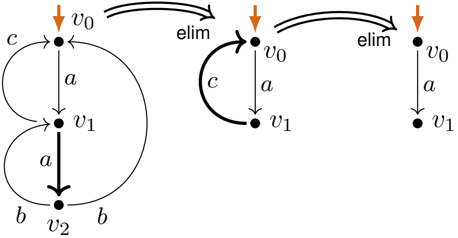

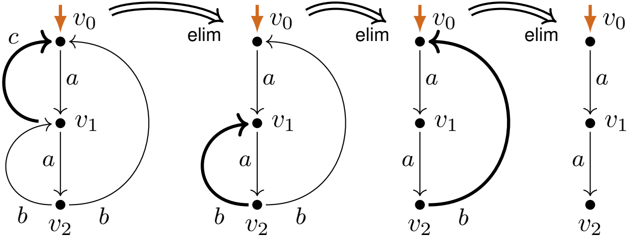

LEE

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A chart $\mathcal{C}$ satisfies LEE (*loop existence and elimination*) if:

$$\exists \mathcal{C}_0 \left(\mathcal{C} \Longrightarrow^{*}_{\text{elim}} \mathcal{C}_0 \not\Longrightarrow_{\text{elim}} \wedge \ \mathcal{C}_0 \text{ permits no infinite path} \right).$$

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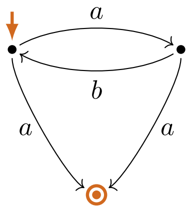
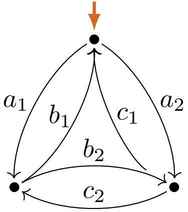
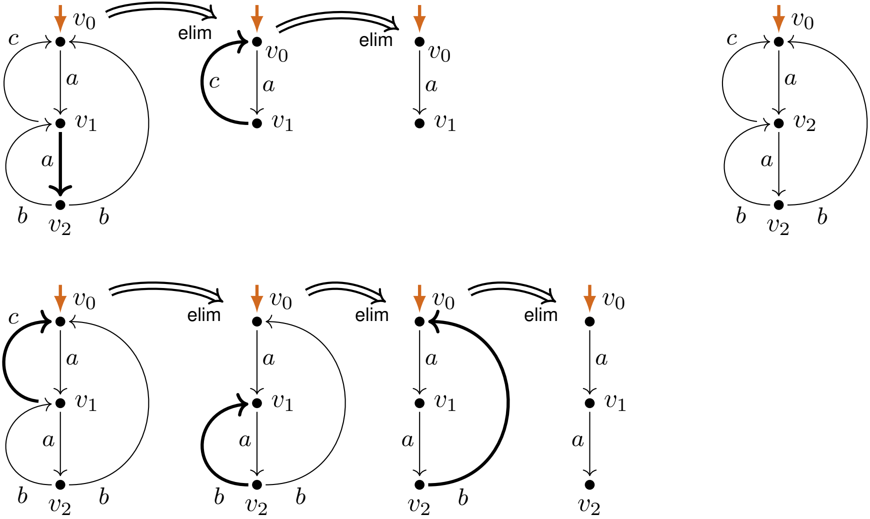


¬LEE



¬LEE

LEE



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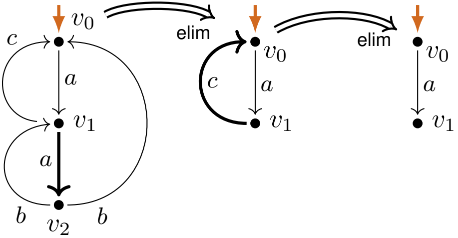

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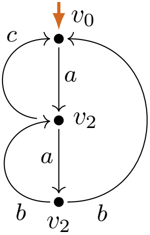

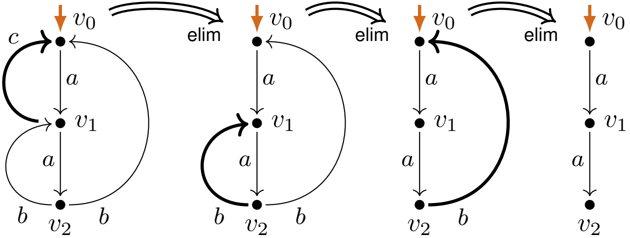

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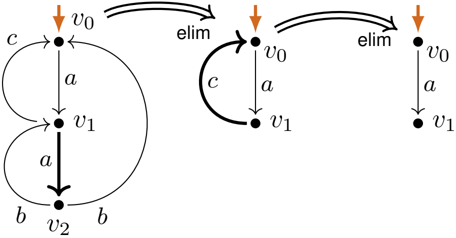
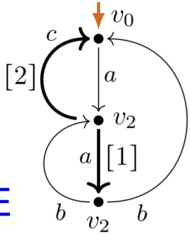
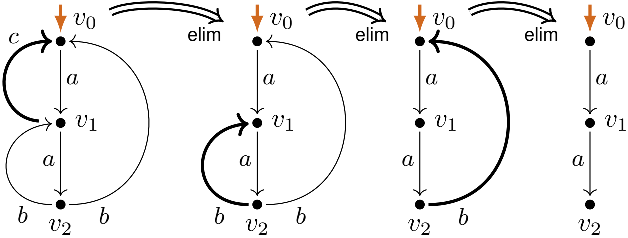

v_0
c
a
v_1
a
b
v_2
b
elim
c
v_0
a
v_1
elim
v_0
a
v_1

LEE


v_0
c
[2]
a
v_2
a
[1]
b
v_2
b


v_0
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v_1
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v_0
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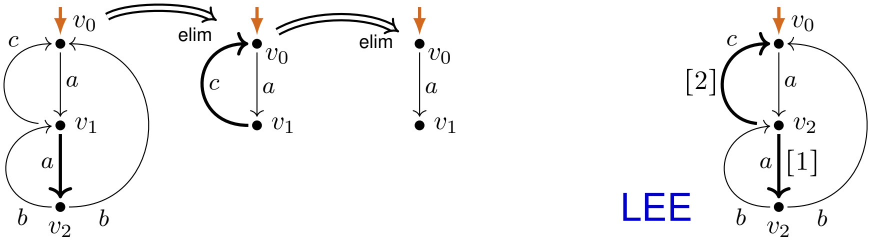
v_2

LEE



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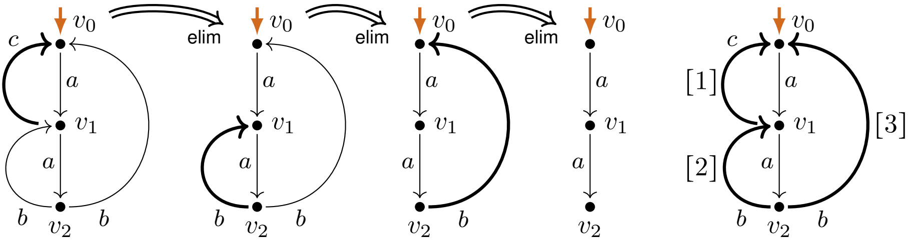

v_0
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v_0
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v_1
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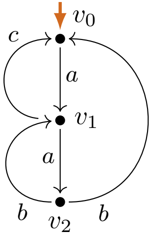
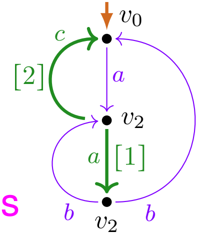
[2]
a
b
v_2
b

Layered LEE witness and LLEE-charts



LLEE-witness



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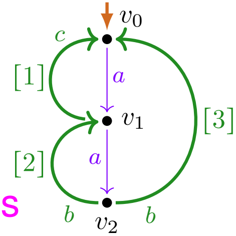

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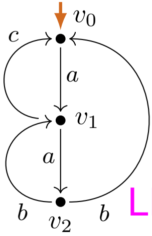
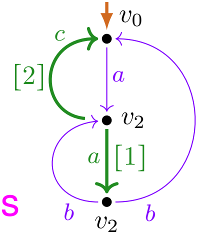
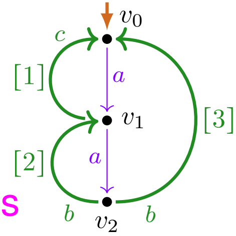


LLEE-chart

LLEE-witness



LLEE-witness



Lemmas

(C) *The bisimulation collapse of a LLEE-chart is again a LLEE-chart.*

(I, SI) *The chart interpretation $\mathcal{C}(e)$ of a 1-free star expression e*

- *is a LLEE-chart,*
- *has a provable solution with start value e.*

(E) *From every LLEE-chart $\mathcal{C}$ a provable solution can be extracted.*

(SE) *All provable solutions of a LLEE-chart are provably equal.*

(P) *If $\mathcal{C}_1 \mathrel{\underline{\to}} \mathcal{C}_2$, then every provable solution of $\mathcal{C}_2$ can be pulled back to obtain a provable solution of $\mathcal{C}_1$ with the same start value.*

Completeness of BBP

Theorem

BBP is sound and complete for $\underline{\leftrightarrow}_{P}$ of 1-free star expressions:

$$\textit{For all } 1\textit{-free star expr.'s } e_1, e_2 \; \left[\, e_1 =_{\mathsf{BBP}} e_2 \iff e_1 \underline{\leftrightarrow}_{P}\, e_2 \,\right].$$

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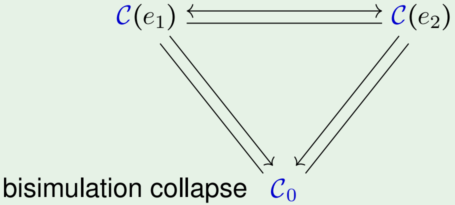

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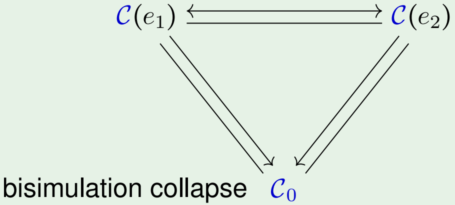

(I) *For every* 1*-free star expression* e*,*
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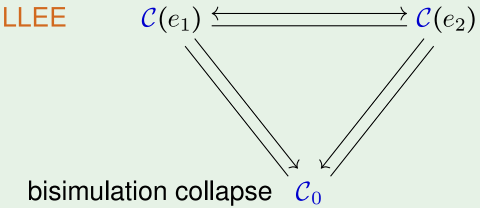

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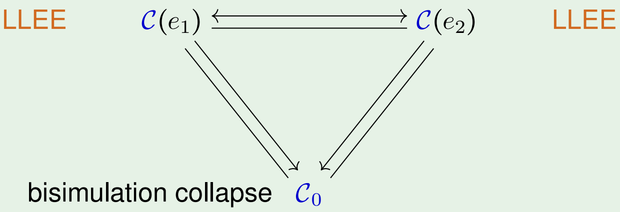

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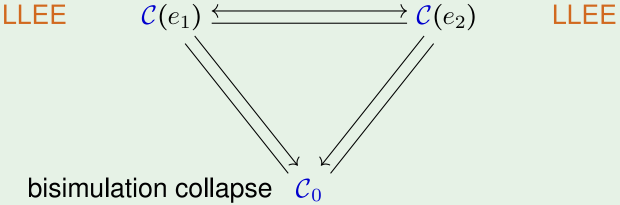

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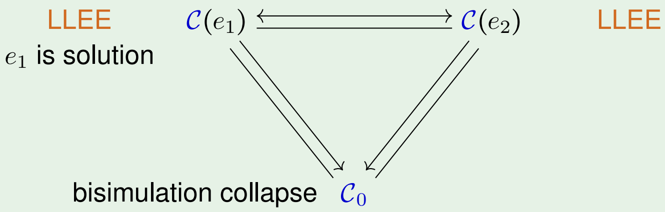

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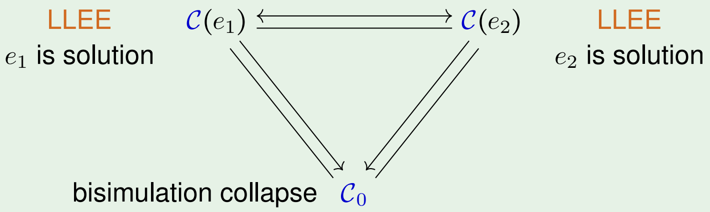

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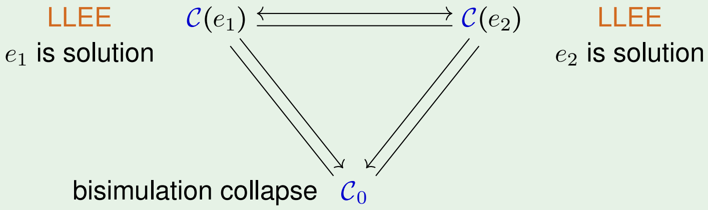

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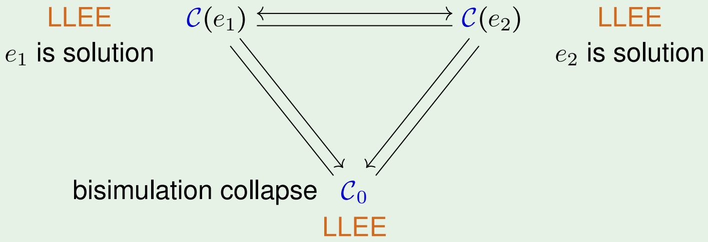

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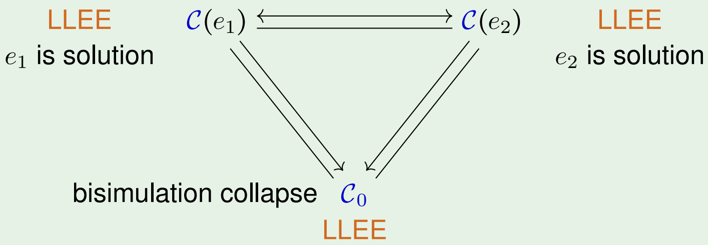

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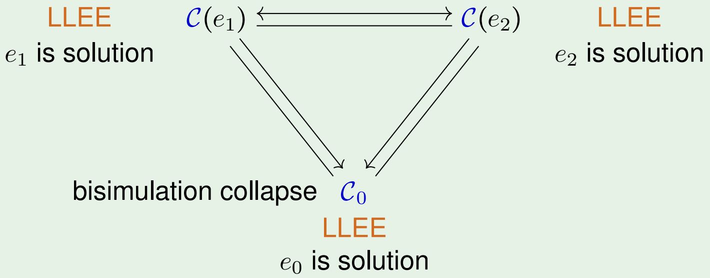

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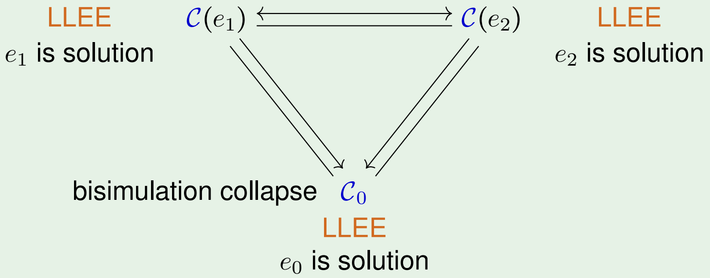

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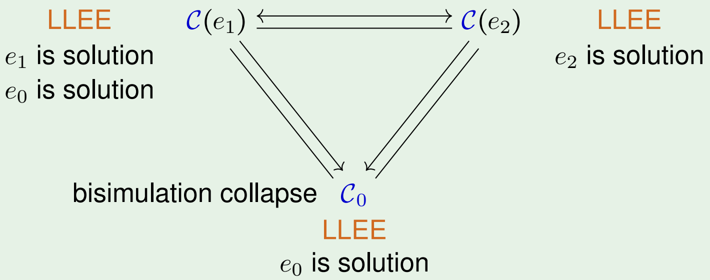

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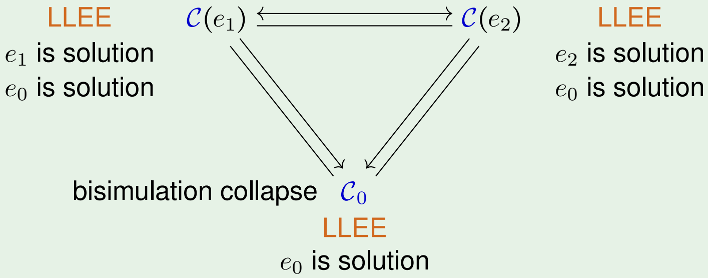

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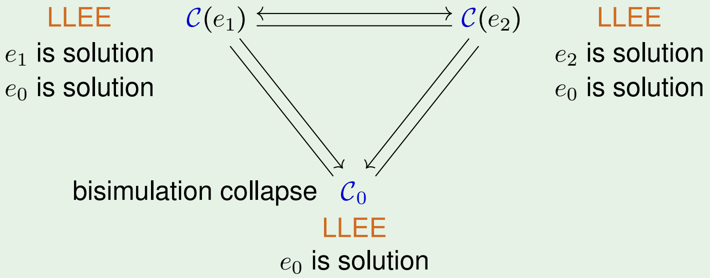

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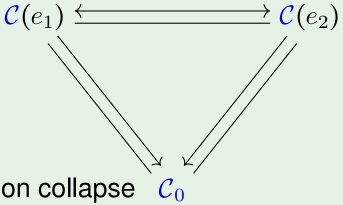
e_1 is solution
e_0 is solution
$\Longrightarrow e_1 =_{\text{BBP}} e_0$



LLEE
e_2 is solution
e_0 is solution

bisimulation collapse $\mathcal{C}_0$
LLEE
e_0 is solution

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$\mathcal{C}(e_1)$ $\Longleftrightarrow$ $\mathcal{C}(e_2)$

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$\mathcal{C}(e_1)$ ⟷ $\mathcal{C}(e_2)$

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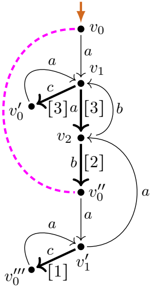
LLEE-preserving collapse (example, corollary)

Lemma (C)

The bisimulation collapse of a LLEE-chart is again a LLEE-chart.



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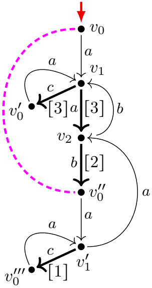

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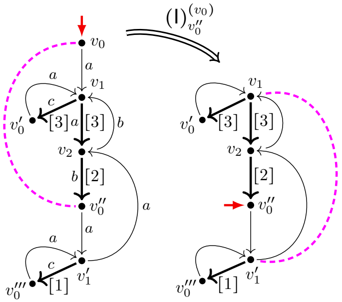

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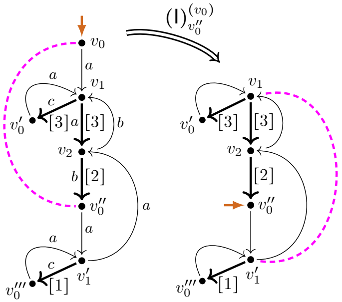

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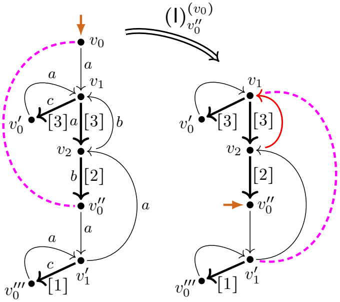

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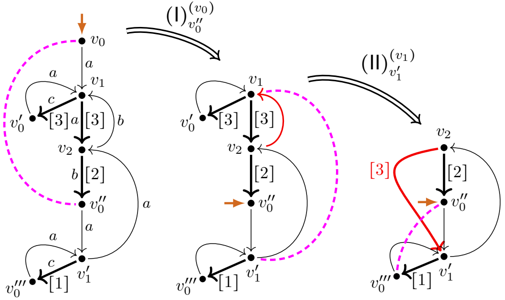

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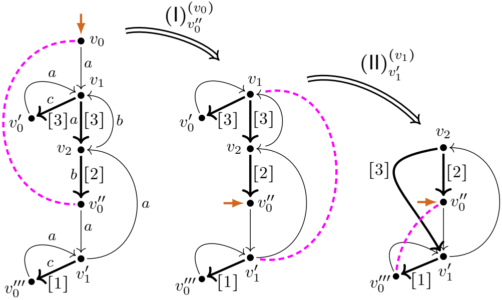

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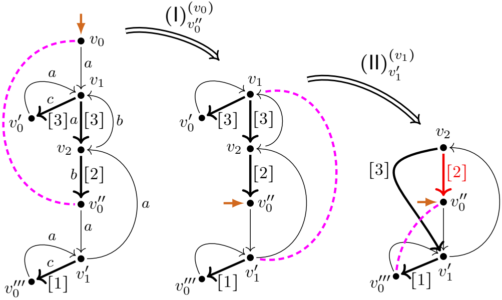

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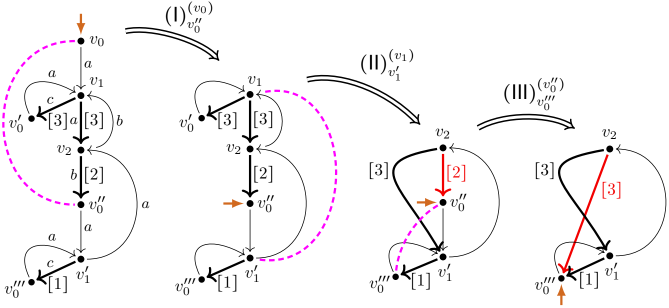

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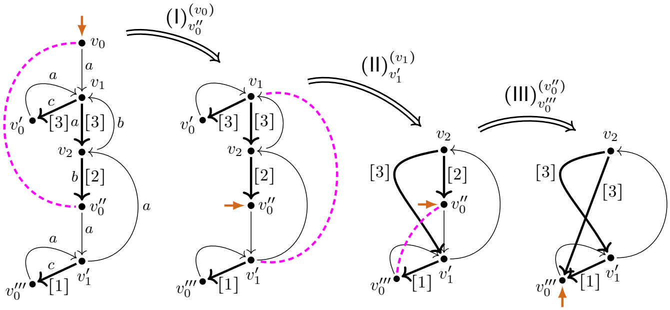

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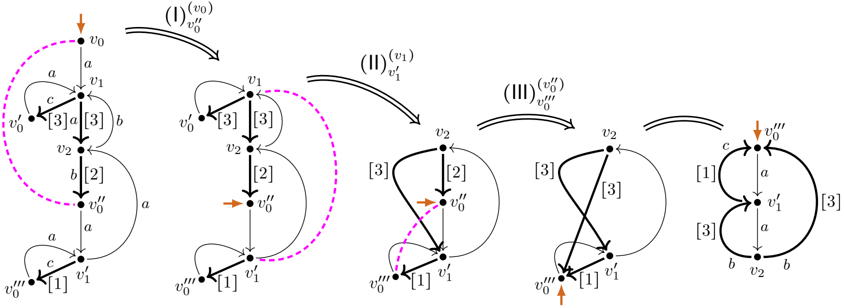

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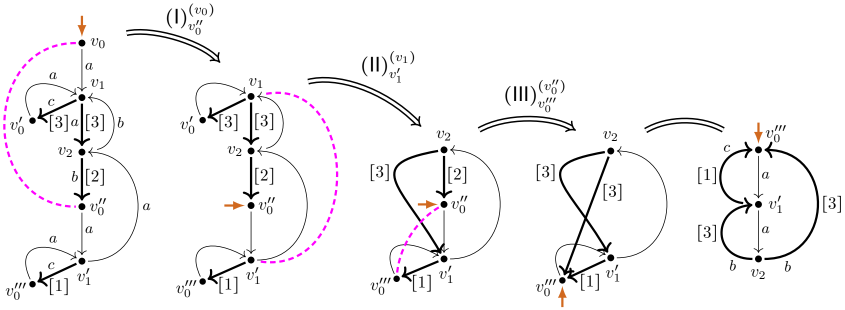

Corollary

A chart is expressible by a 1-free star expression modulo $\leftrightarrow$
if and only if its bisimulation collapse is a LLEE-chart.

Summary and outlook

We have obtained a partial solution for Milner's problem:

- BBP: adaptation of Milner's system Mil to 1-free star expr's
- graph property: loop existence and elimination (LLEE)
  - guarantees solvability via extraction **(E)**
  - holds for chart interpretations of 1-free star expressions **(I)**
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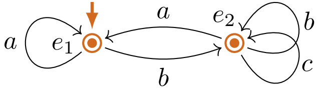
- properties **(I)** and **(C)** do not hold for all star expressions:



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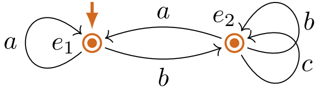

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- Possible workaround: use 1-charts (with explicit 1-transitions)

Resources

report version of article

- CG & Wan Fokkink: A Complete Proof System for 1-Free Regular Expressions Modulo Bisimilarity, arXiv:2004.12740, May 2020.

extended abstract (1-charts)

- CG: Structure-Constrained Process Graphs for the Process Interpretation of Regular Expressions, TERMGRAPH 2020, July 5, 2020. http://www.termgraph.org.uk/2020/.

Resources

report version of article

- CG & Wan Fokkink: A Complete Proof System for 1-Free Regular Expressions Modulo Bisimilarity, arXiv:2004.12740, May 2020.

extended abstract (1-charts)

- CG: Structure-Constrained Process Graphs for the Process Interpretation of Regular Expressions, TERMGRAPH 2020, July 5, 2020. http://www.termgraph.org.uk/2020/.

Thank you for your attention!