Towards a Characterisation of the Graph Structure of Process Interpretations of Regular Expressions

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Overview

- Regular expressions (unary/binary star, 1-free/under-star-1-free $(*/\not{1})$)
- Milner's process interpretation P/semantics $[\![\cdot]\!]_P$
  - P-/$[\![\cdot]\!]_P$-expressible graphs ($\rightsquigarrow$ expressibility question)
- Layered Loop existence and elimination (LLEE/1-LLEE)
  - interpretation/extraction correspondences with $(*/\not{1})$/all reg. expr.
  - LLEE is preserved by bisimulation collapse (1-LLEE is not)
- TERMGRAPH 2024: compact process interpretation $P^\bullet$
  - image of $P^\bullet$ is closed under bisimulation collapse (image of P is not)
- characterisation of image of $P^\bullet$ restricted to $(*/\not{1})$ expressions
- characterization of image of $P^\bullet$
- Relationship with co-reducibility

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Process interpretation P of regular expressions *(Milner, 1984)*

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Regular Expressions

Definition (*~ Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary Kleene star:

$$e, e_1, e_2 \ ::= \ \mathbf{0} \mid \ a \mid e_1 + e_2 \mid e_1 \cdot e_2 \mid e^* \qquad (\text{for } a \in A).$$

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Regular Expressions

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$$e, e_1, e_2 ::= \mathbf{0} \mid \mathbf{1} \mid \boldsymbol{a} \mid e_1 + e_2 \mid e_1 \cdot e_2 \mid e_1 \circledast e_2 \qquad (\text{for } \boldsymbol{a} \in A).$$

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Definition (for process interpretation)

1-free regular expressions over alphabet A with binary Kleene star:

$$f, f_1, f_2 \;::=\; \mathbf{0} \mid \boldsymbol{a} \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1 \circledast f_2 \qquad (\text{for } \boldsymbol{a} \in A).$$

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Regular Expressions (under-star-/1-free)

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The set $\mathit{RExp}^{(1)}(A)$ of 1-free regular expressions over A is defined by:

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Regular Expressions $((*/\not{1})/1\text{-free})$

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The set $\mathit{RExp}^{(\not{1})}(A)$ of 1-free regular expressions over A is defined by:

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the set $\mathit{RExp}^{(*/\not{1})}(A)$ of under-star-1-free $(*/\not{1})$ reg. expr. over A by:

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Process semantics $[\![\cdot]\!]_P$ of regular expressions *(Milner, 1984)*

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$$[\![e]\!]_P := [P(e)]_{\leftrightarrow} \quad \text{(bisimilarity equivalence class of process } P(e))$$

Process interpretation P (formally)

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'} \ (i \in \{1,2\})$$

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Definition

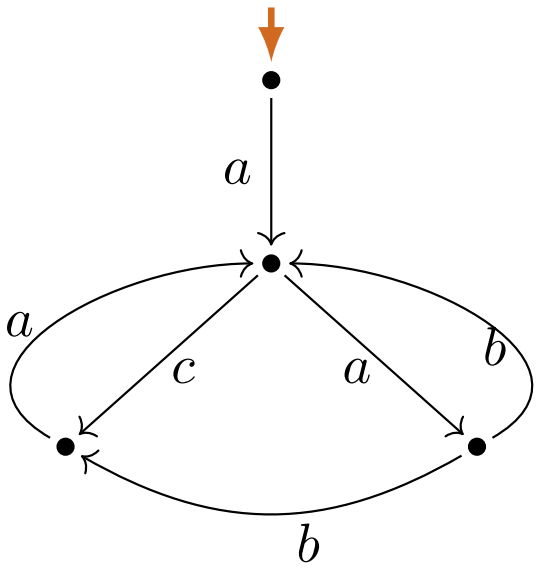
The process interpretation $P(e)$ of a regular expression e:

$P(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}$,

The process semantics $[\![e]\!]_P$ of e is: $[\![e]\!]_P := [P(e)]_{\underline{\leftrightarrow}}$.

P-expressibility and $[\![\cdot]\!]_P$-expressibility

G_4



$$P(\overbrace{(a \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0}^{f})$$

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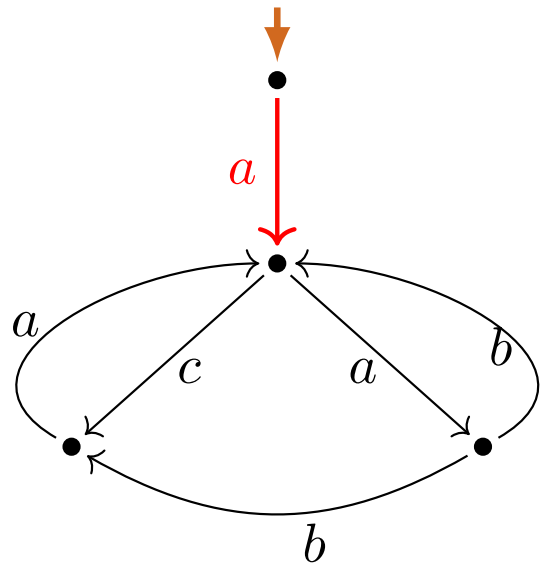

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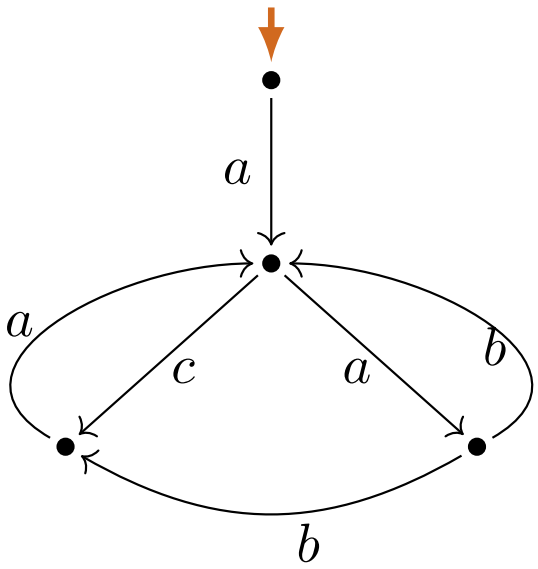

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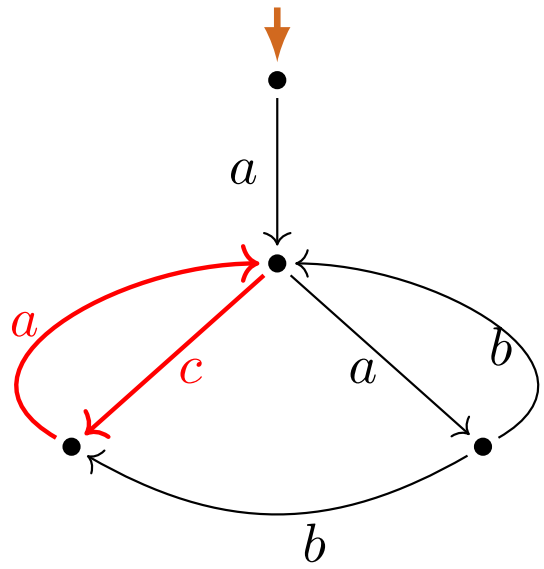

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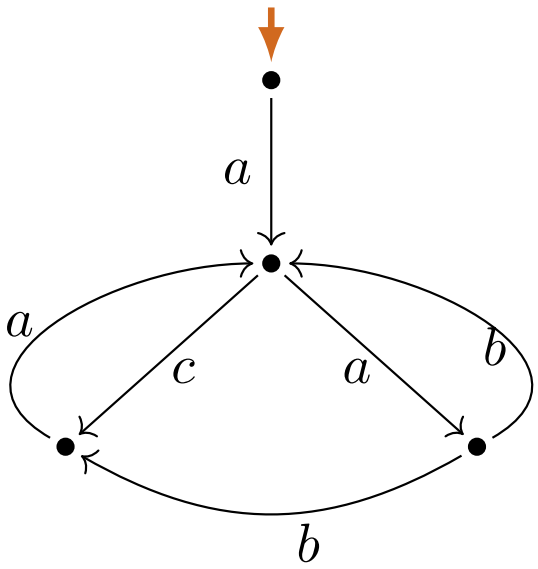

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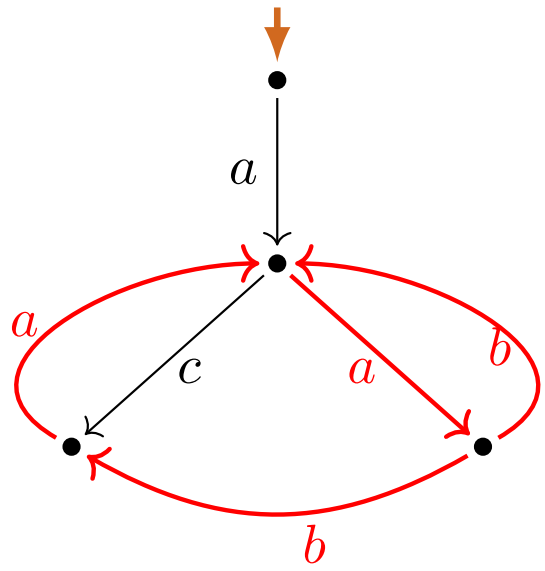

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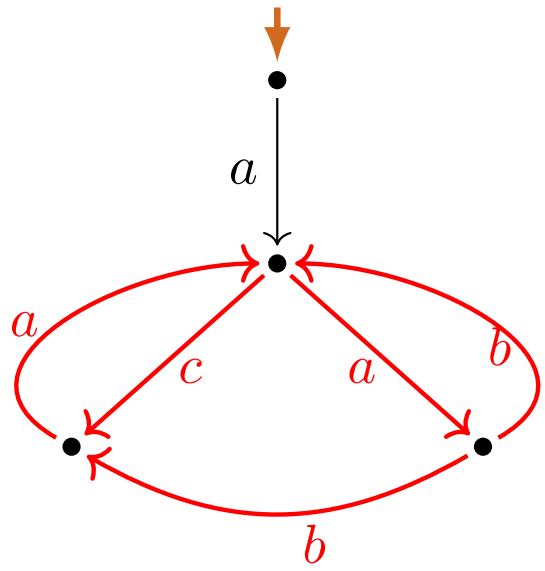

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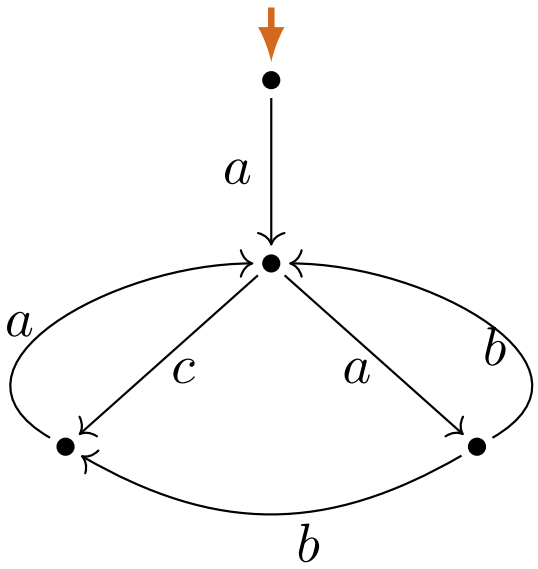

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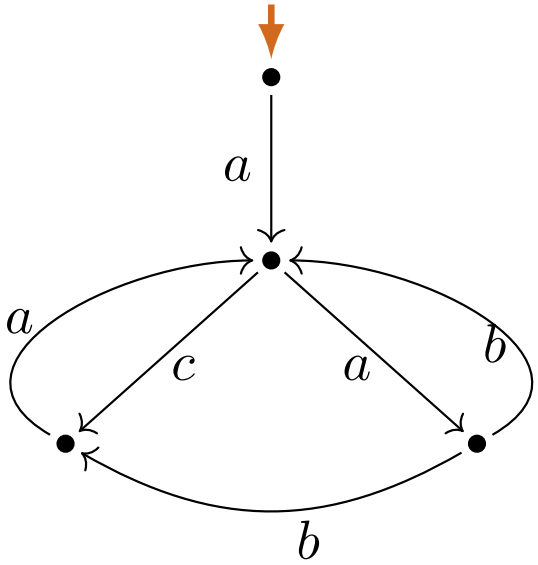
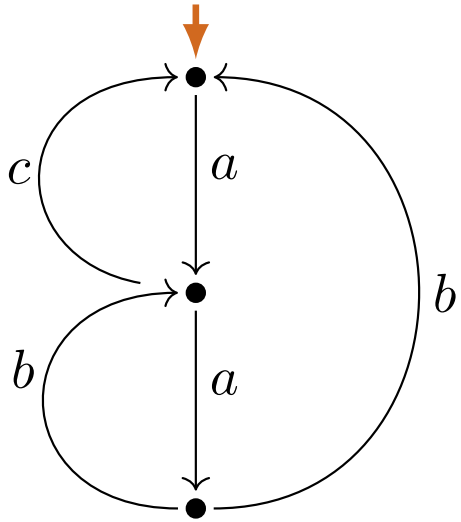


G_5



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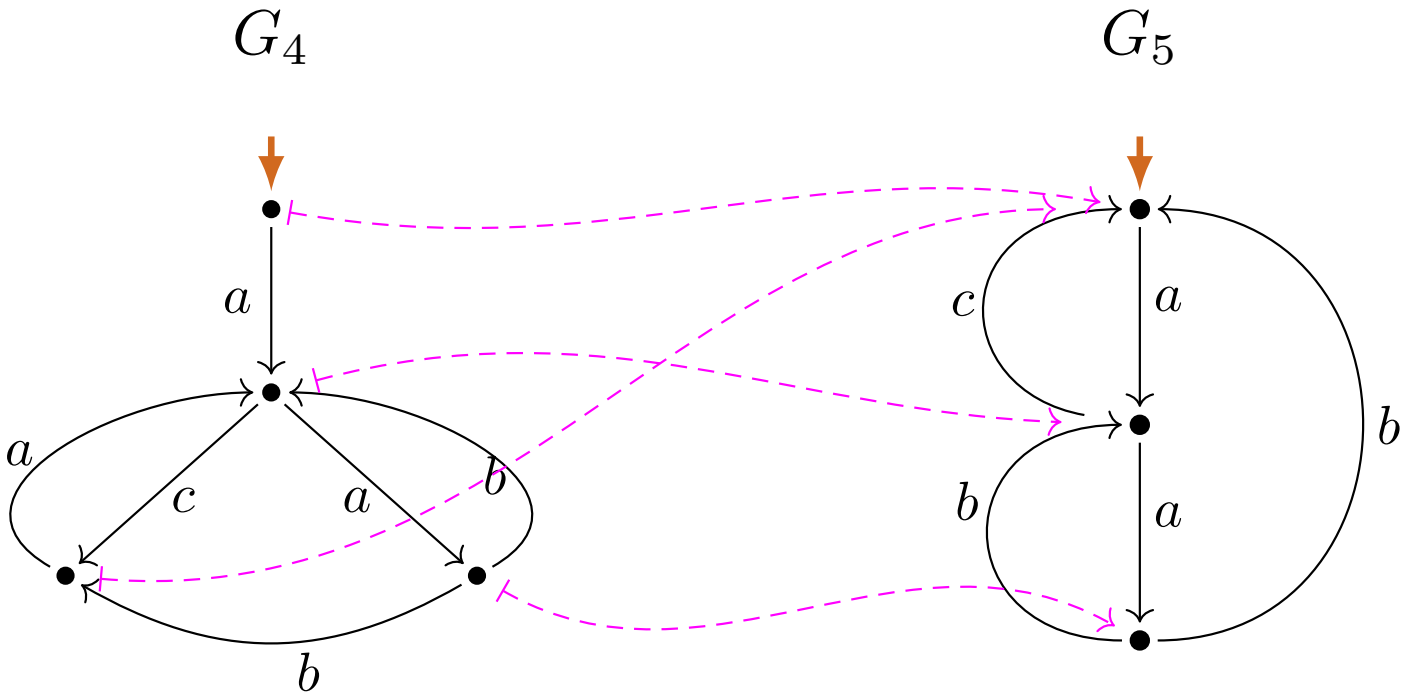

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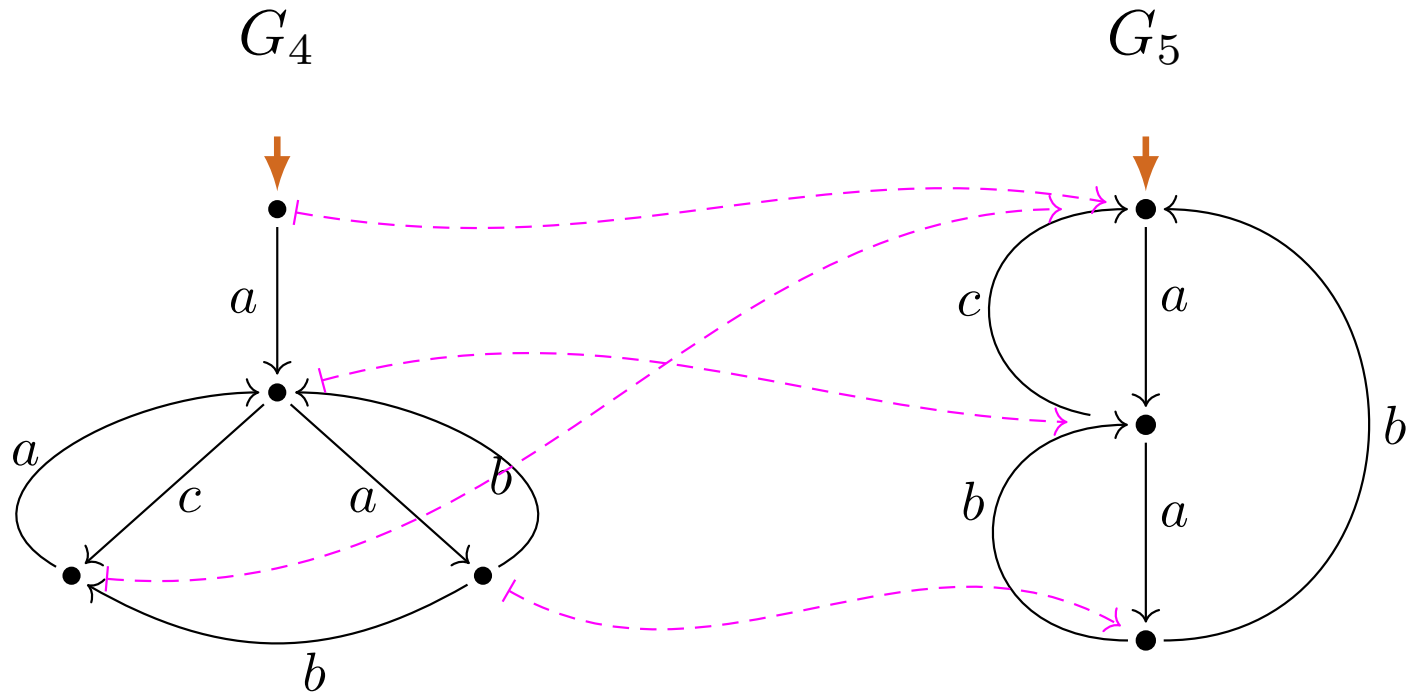

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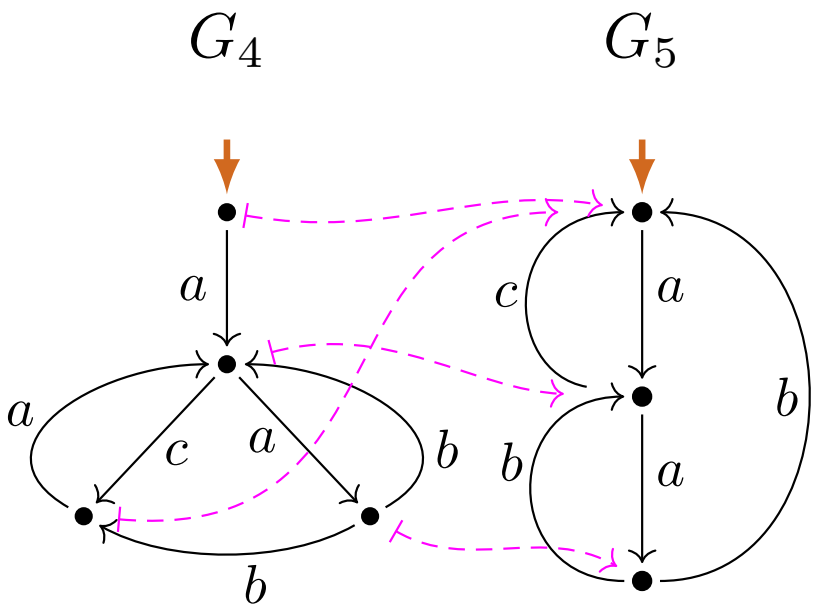
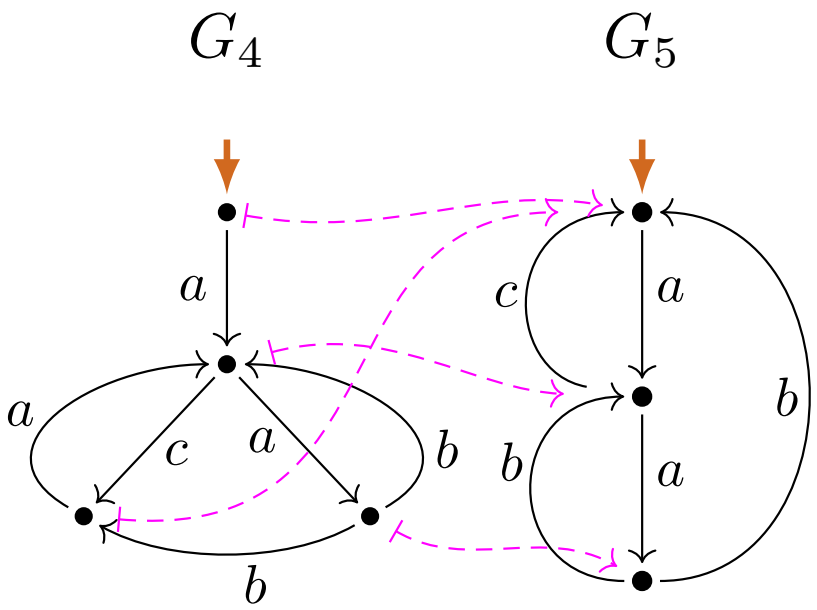
$G_4 \in [\![f]\!]_P$ $\qquad$ $G_5 \in [\![f]\!]_P$

P-expressibility and $[\![\cdot]\!]_P$-expressibility/ (examples)



$$P((a \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0)$$

P-expressibility and $[\![\cdot]\!]_P$-expressibility/ (examples)



P-expressible

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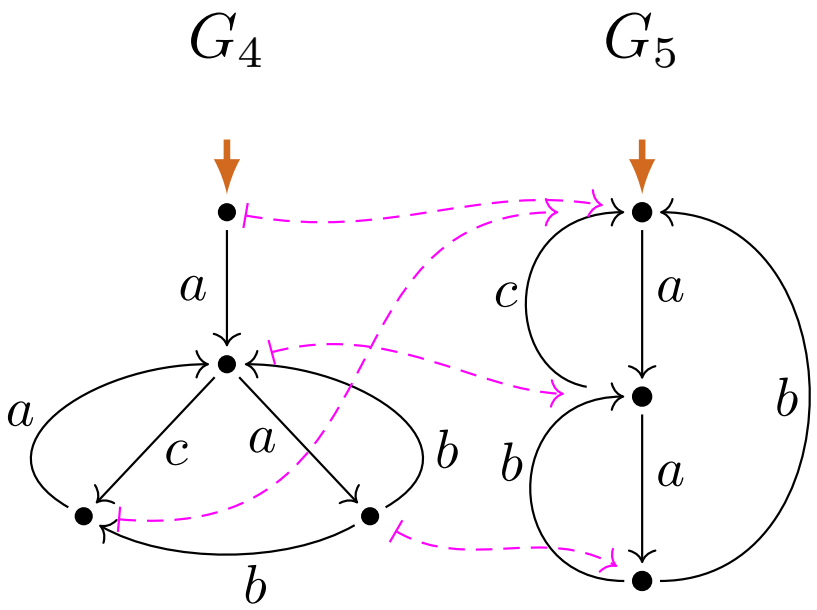
P-expressibility and $[\![\cdot]\!]_P$-expressibility/ (examples)

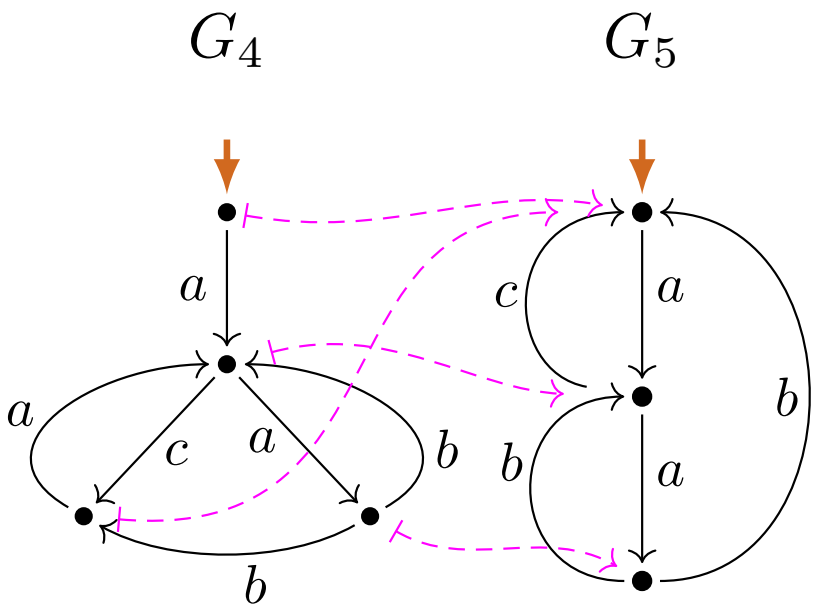


P-expressible

$[\![\cdot]\!]_P$-expressible $[\![\cdot]\!]_P$-expressible

$$P((a \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0)$$

P-expressibility and $[\![\cdot]\!]_P$-expressibility/ (examples)



P-expressible
$[\![\cdot]\!]_P$-expressible

P-expressible ?
$[\![\cdot]\!]_P$-expressible

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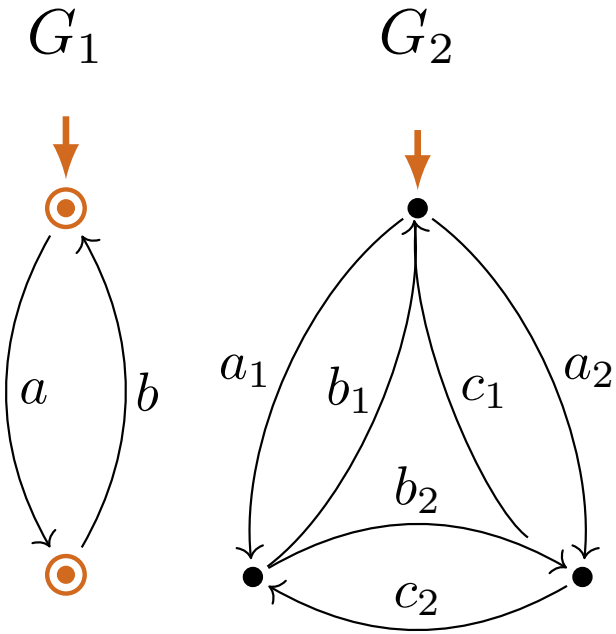
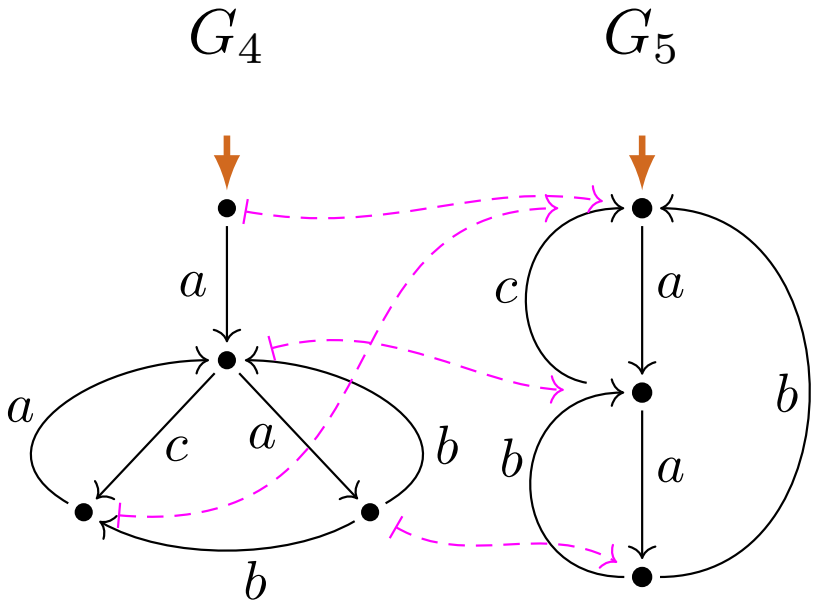
P-expressibility and $[\![\cdot]\!]_P$-expressibility/ (examples)





P-expressible
$[\![\cdot]\!]_P$-expressible

P-expressible ?
$[\![\cdot]\!]_P$-expressible

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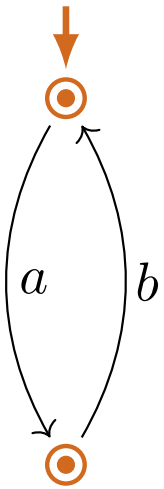
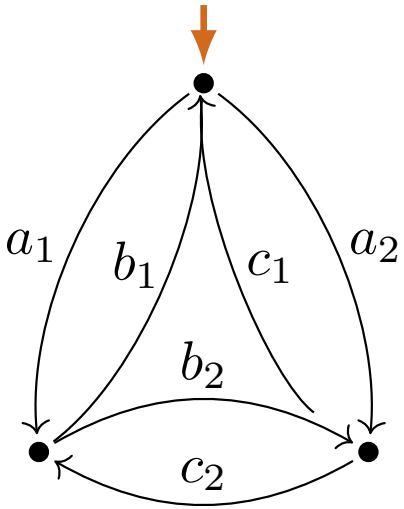
P-expressibility and $[\![\cdot]\!]_P$-expressibility/ (examples)

G_1

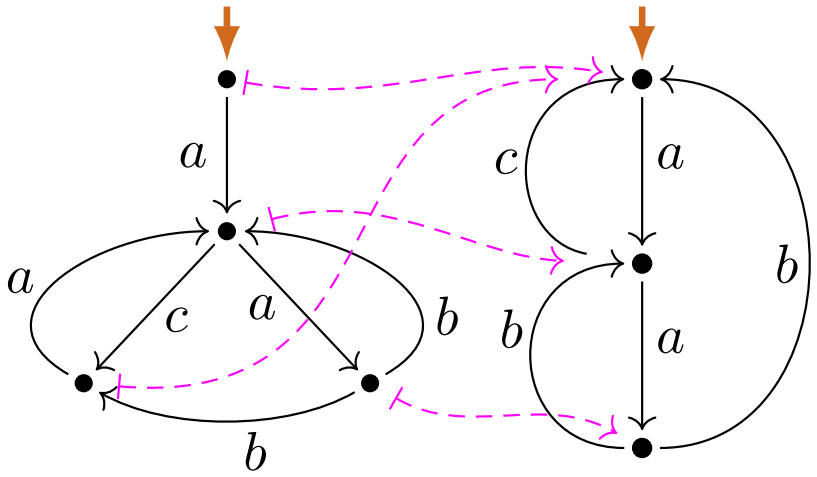


G_2



not P-expressible

G_4



P-expressible
$[\![\cdot]\!]_P$-expressible

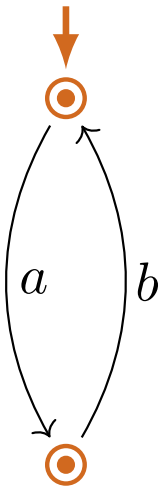
G_5

P-expressible ?
$[\![\cdot]\!]_P$-expressible

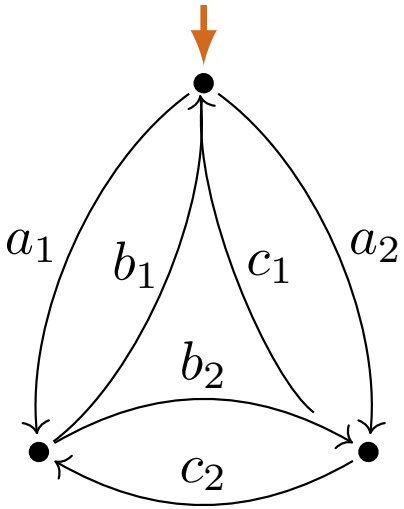
$$P((a \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0)$$

P-expressibility and $[\![\cdot]\!]_P$-expressibility/ (examples)

G_1

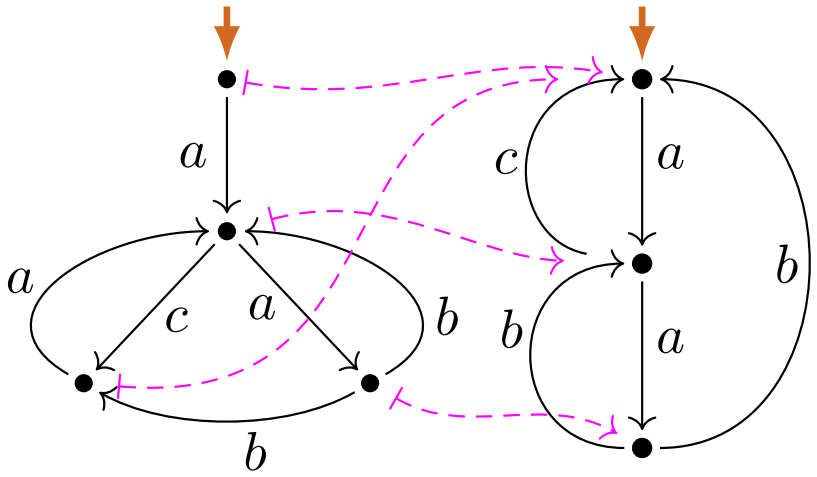


G_2



not P-expressible
not $[\![\cdot]\!]_P$-expressible

G_4



P-expressible
$[\![\cdot]\!]_P$-expressible

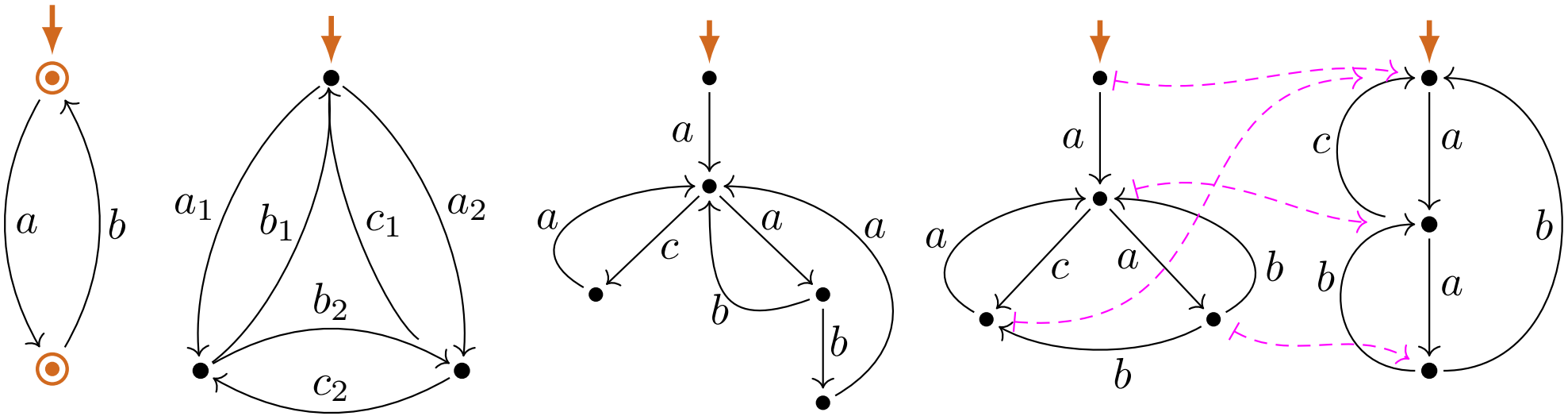
G_5

P-expressible **?**
$[\![\cdot]\!]_P$-expressible

$$P((a \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0)$$

P-expressibility and $[\![\cdot]\!]_P$-expressibility/ (examples)

G_1 G_2 G_3 G_4 G_5



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not $[\![\cdot]\!]_P$-expressible

P-expressible
$[\![\cdot]\!]_P$-expressible

P-expressible **?**
$[\![\cdot]\!]_P$-expressible

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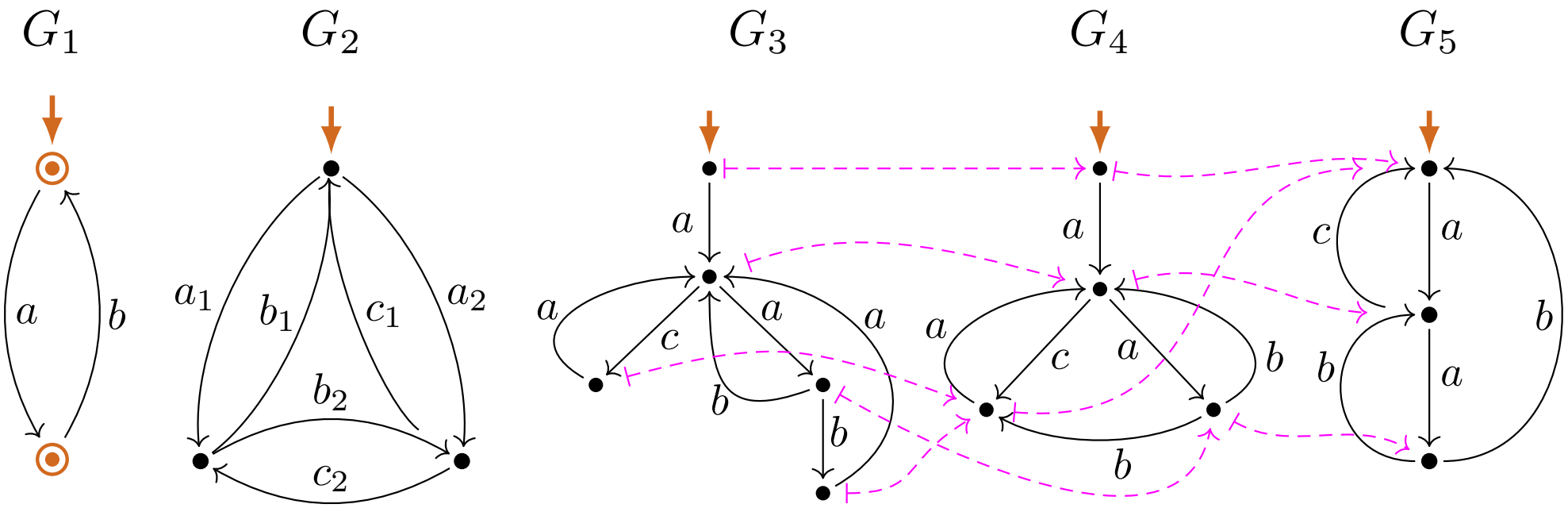

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$[\![\cdot]\!]_P$-expressible

P-expressible **?**
$[\![\cdot]\!]_P$-expressible

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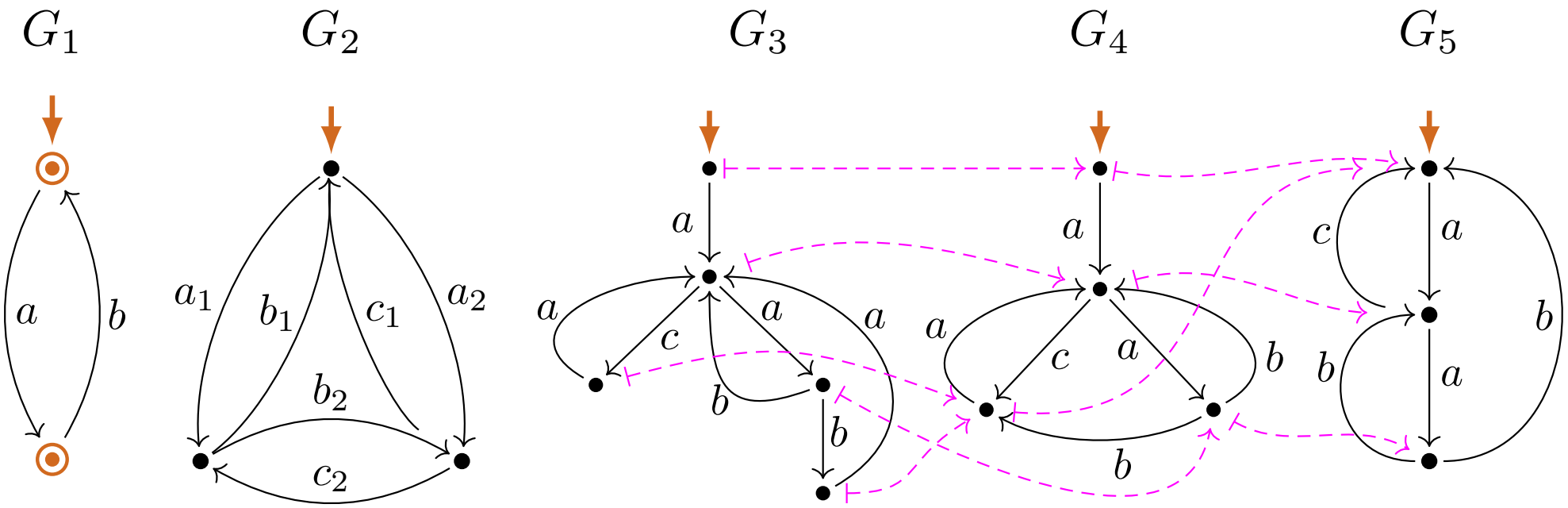

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not $[\![\cdot]\!]_P$-expressible

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$[\![\cdot]\!]_P$-expressible

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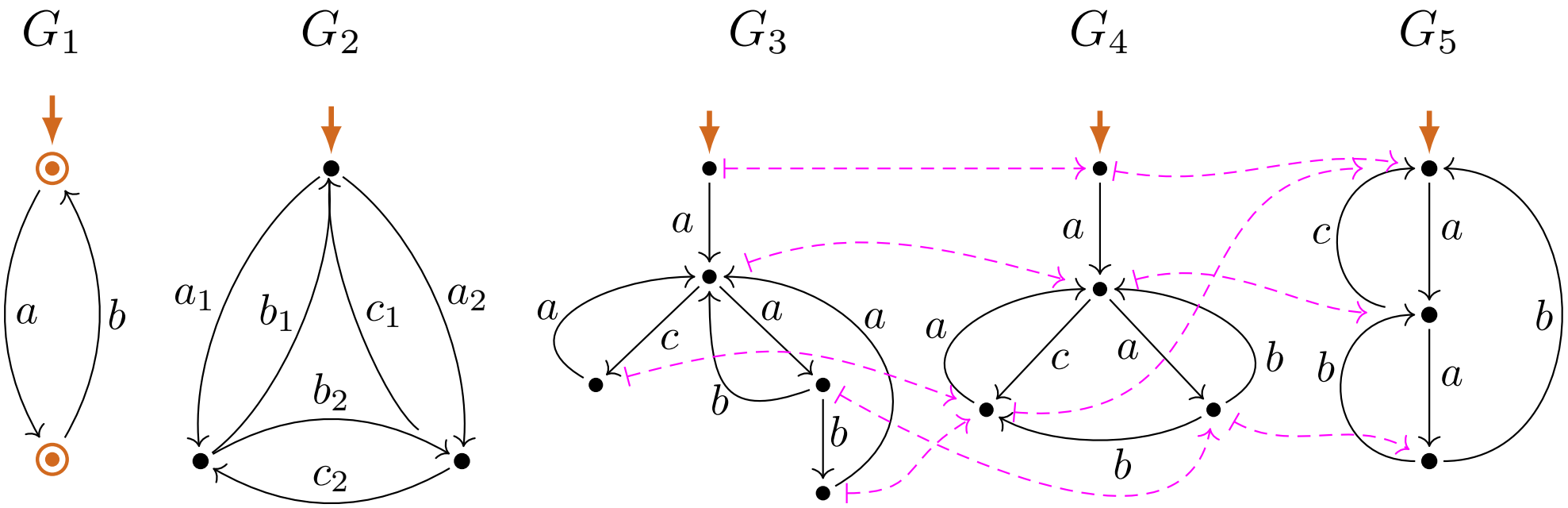

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P-expressible ?
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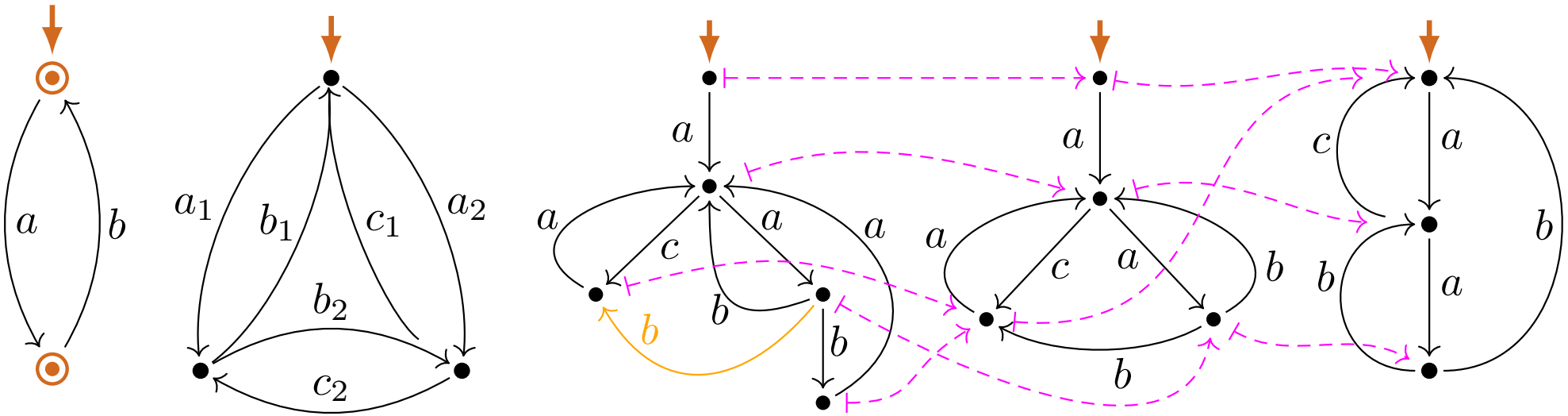
P-expressibility and $[\![\cdot]\!]_P$-expressibility/ (examples)

G_1 G_2 G_3^+ G_3 G_4 G_5



not P-expressible
not $[\![\cdot]\!]_P$-expressible

$[\![\cdot]\!]_P$-expressible

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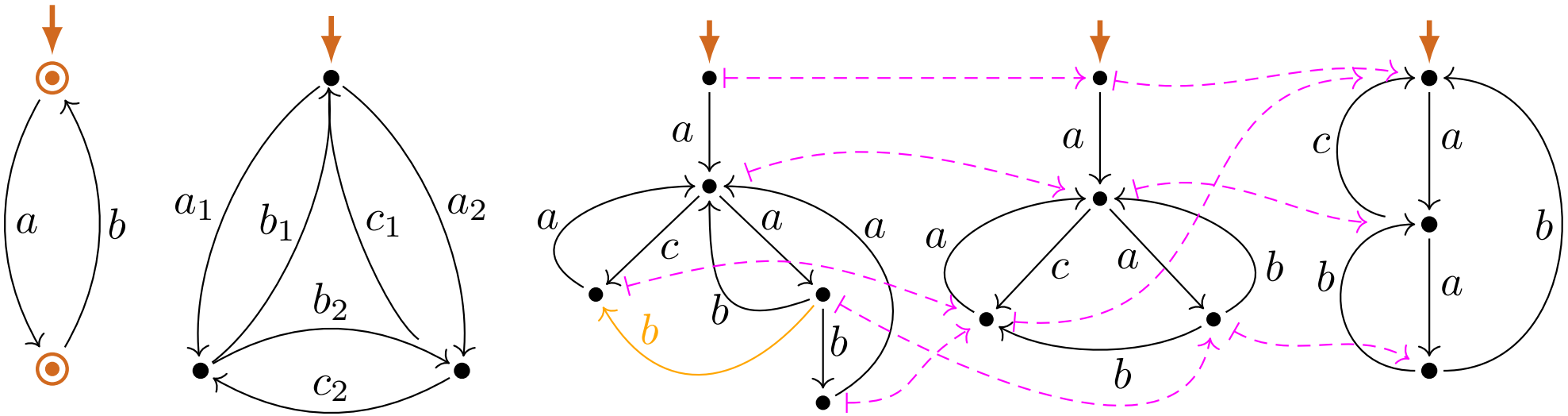
G_1 G_2 G_3^+ G_3 G_4 G_5



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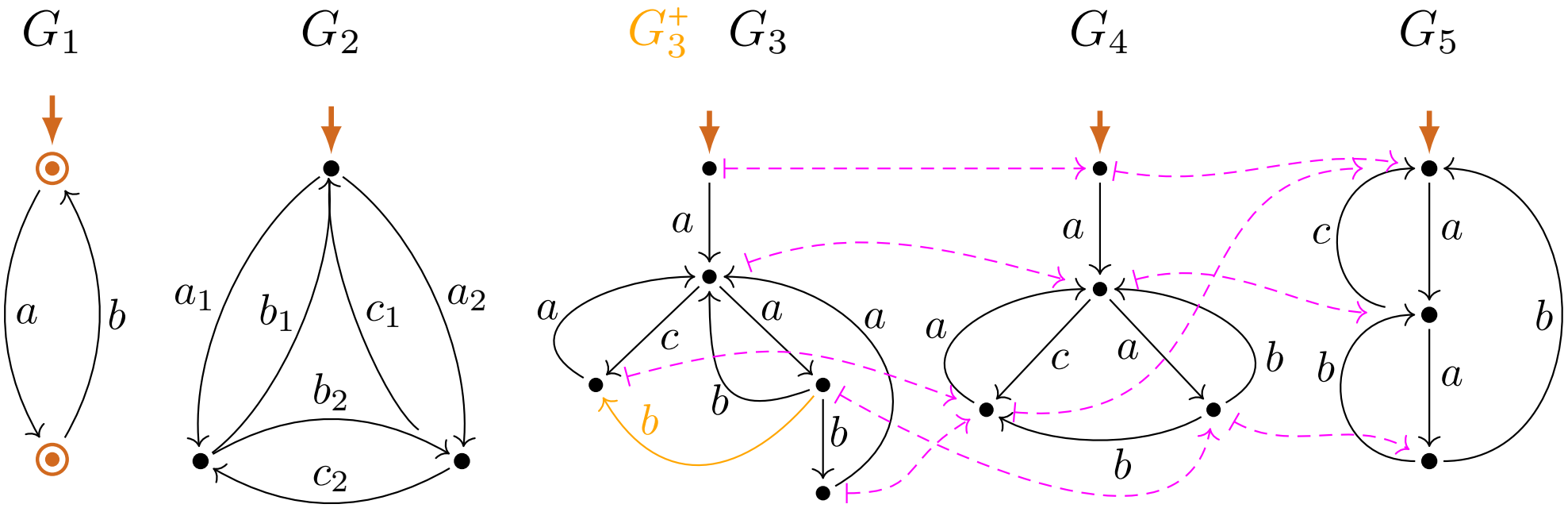

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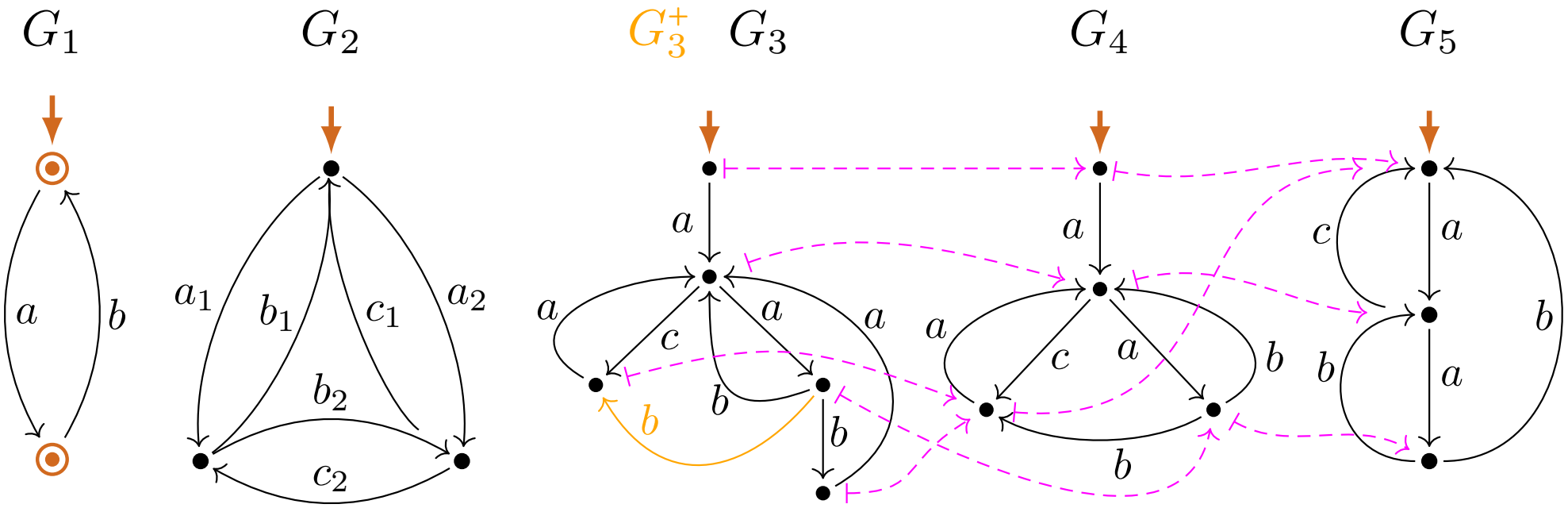

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P-expressible
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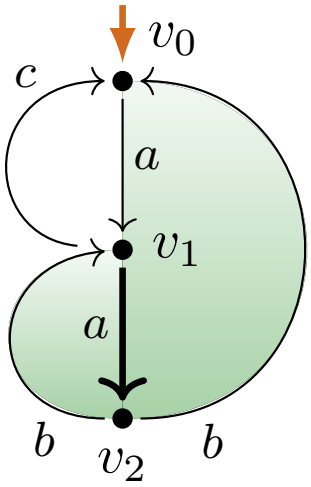
P-expressible ?
$[\![\cdot]\!]_P$-expressible

$$P((a \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0)$$

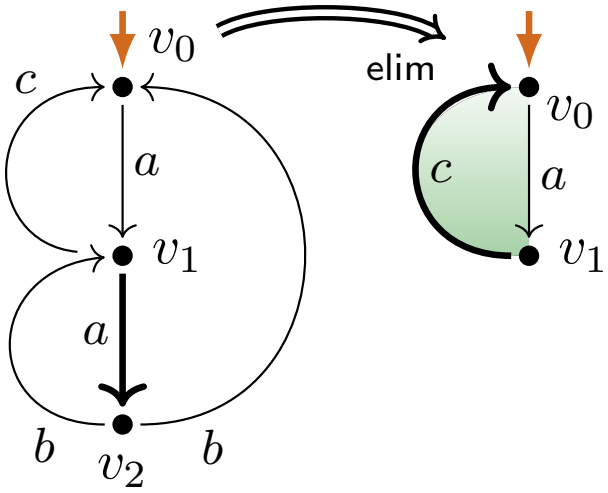
LLEE and 1-LLEE

- Regular expressions (unary/binary star, 1-free/under-star-1-free $(*/\underline{1})$)
- Milner's process interpretation P/semantics $[\![\cdot]\!]_P$
  - P-/$[\![\cdot]\!]_P$-expressible graphs ($\rightsquigarrow$ expressibility question)
- Layered Loop existence and elimination (LLEE/1-LLEE)
  - interpretation/extraction correspondences with $(*/\underline{1})$/all reg. expr.
  - LLEE is preserved by bisimulation collapse (1-LLEE is not)
- TERMGRAPH 2024: compact process interpretation $P^\bullet$
  - image of $P^\bullet$ is closed under bisimulation collapse (image of P is not)
- characterisation of image of $P^\bullet$ restricted to $(*/\underline{1})$ expressions
- characterization of image of $P^\bullet$
- Relationship with co-reducibility

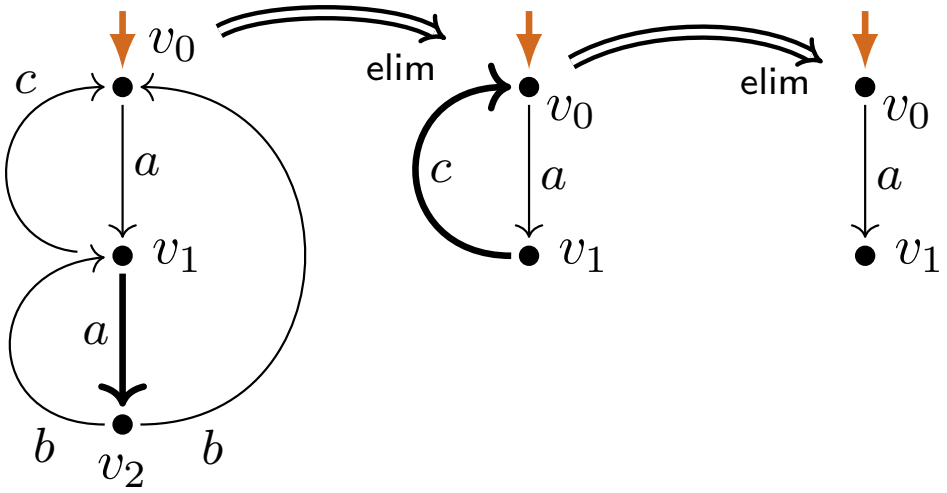
Loop existence and elimination



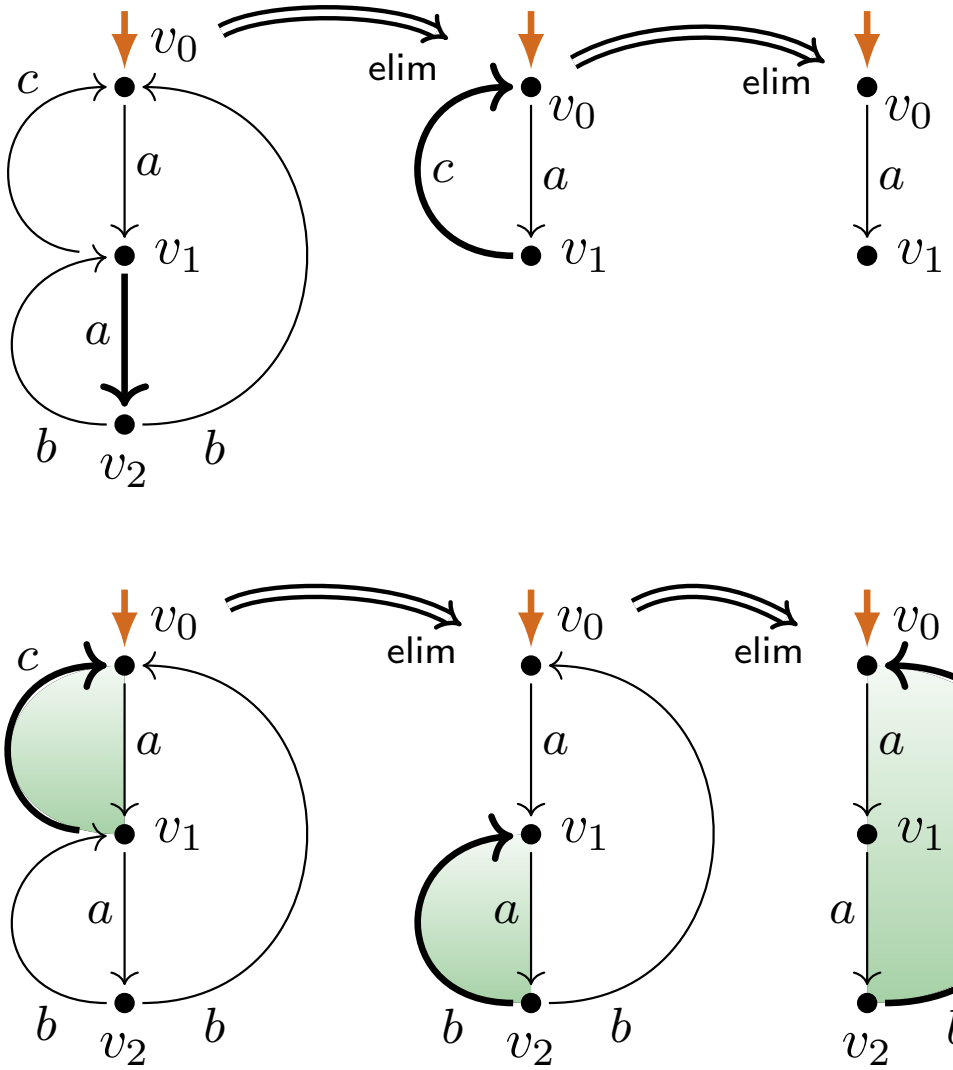
Loop existence and elimination



Loop existence and elimination



Loop existence and elimination



LEE

Definition

A graph G satisfies LEE (*loop existence and elimination*) if:

$$\exists G_0 \left(G \longrightarrow^{*}_{\mathsf{elim}} G_0 \not\longrightarrow_{\mathsf{elim}} \right.$$
$$\left. \wedge\ G_0 \text{ permits no infinite path} \right).$$

LEE

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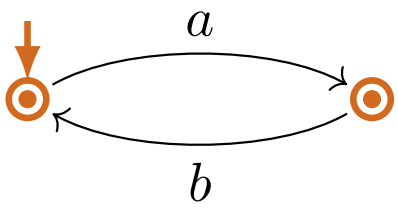

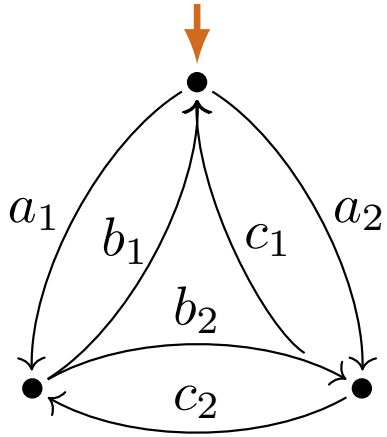

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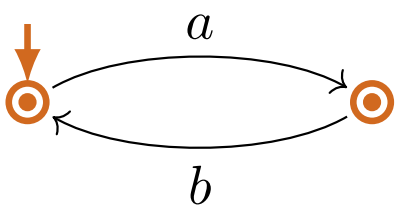
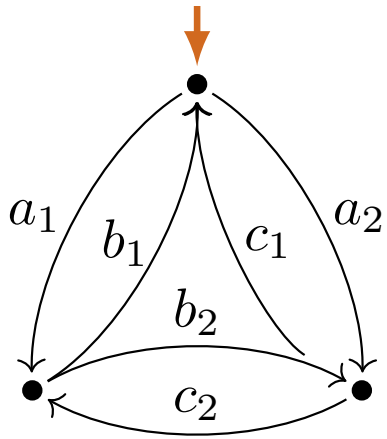
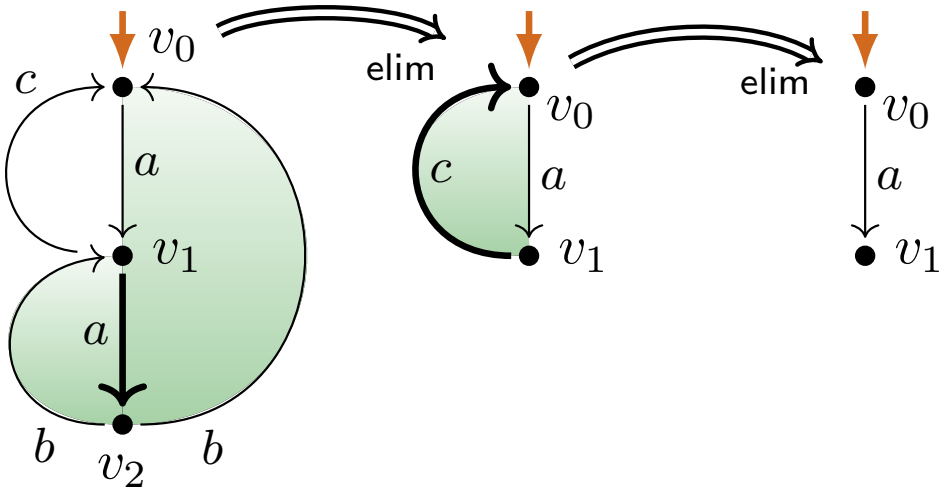


not LEE

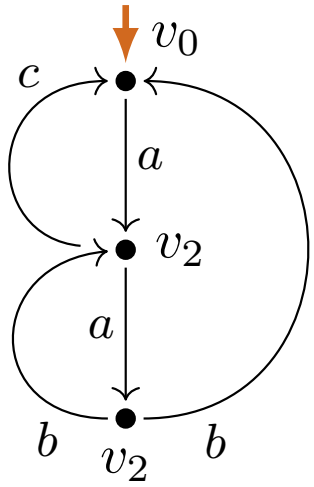


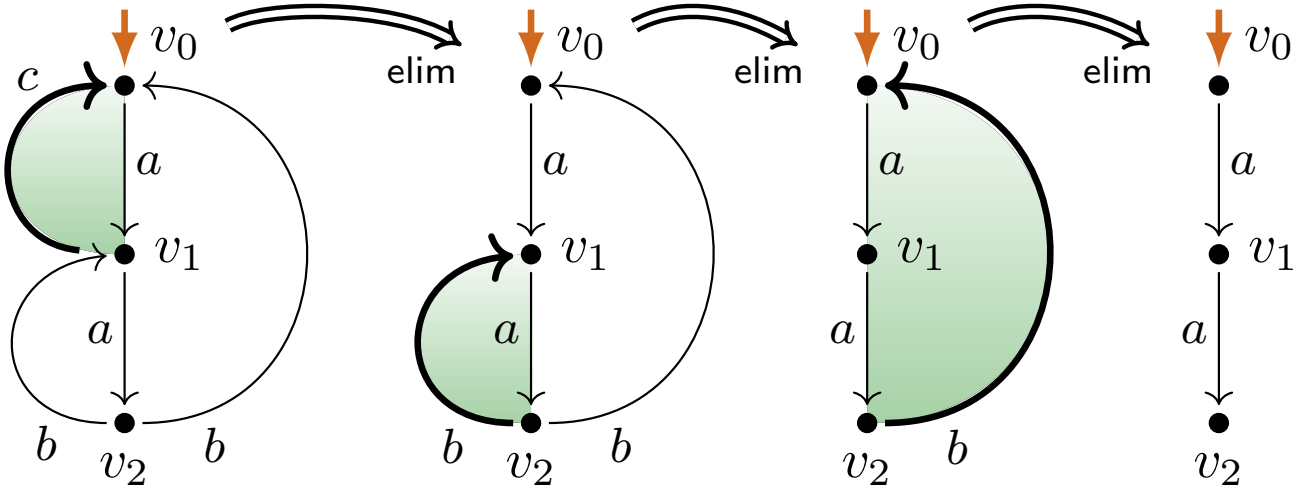
not LEE

LEE

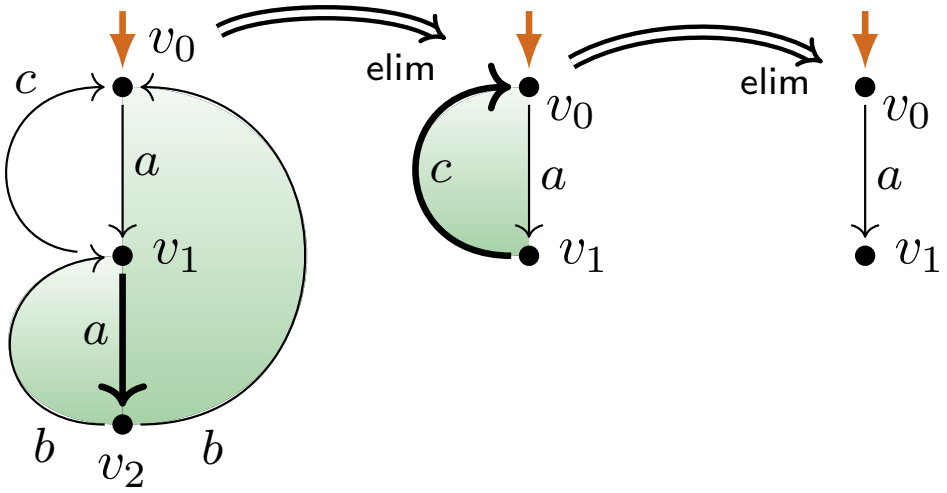

v_0
c
a
v_1
a
b
v_2
b
elim
v_0
c
a
v_1
elim
v_0
a
v_1

LEE

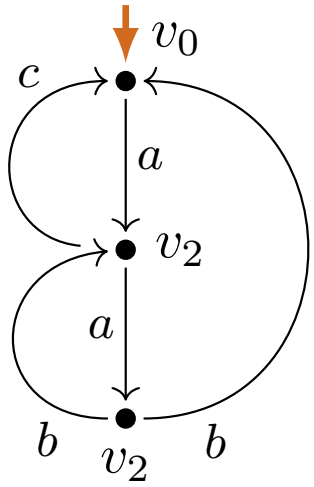

v_0
c
a
v_2
a
b
v_2
b

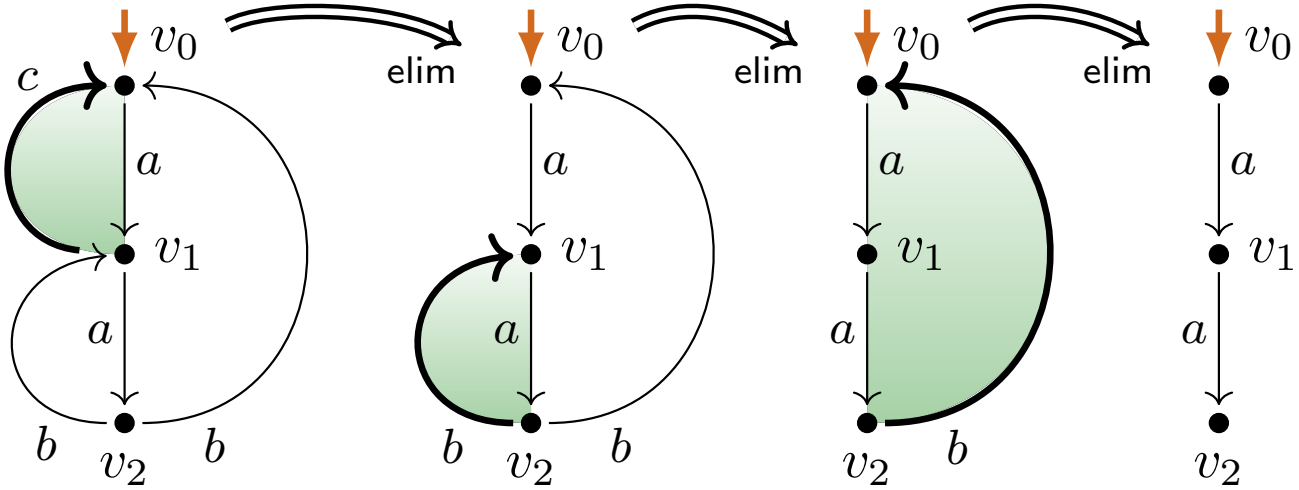

v_0
c
a
v_1
a
b
v_2
b
elim
v_0
a
v_1
a
b
v_2
b
elim
v_0
a
v_1
a
v_2
b
elim
v_0
a
v_1
a
v_2

Layered LEE

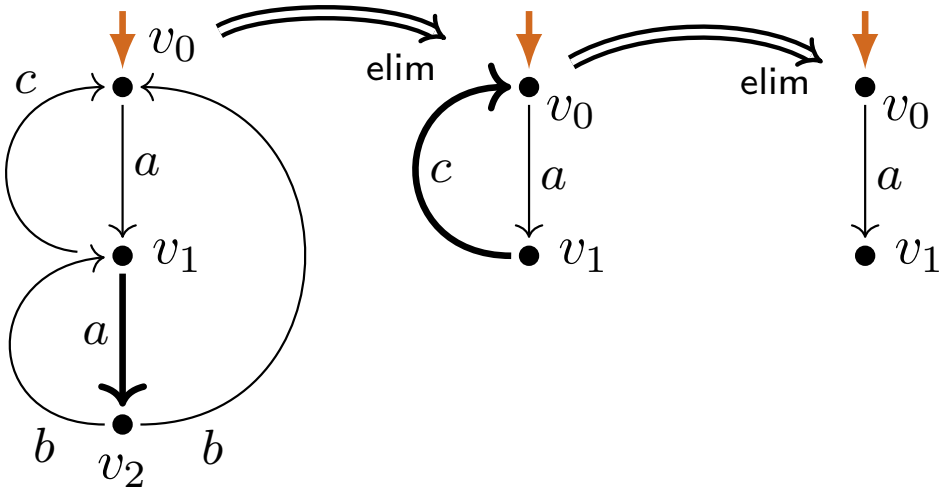


layered LEE
LLEE

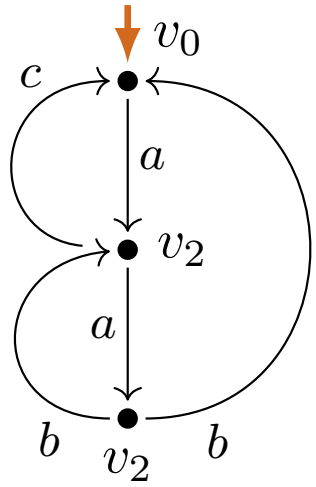


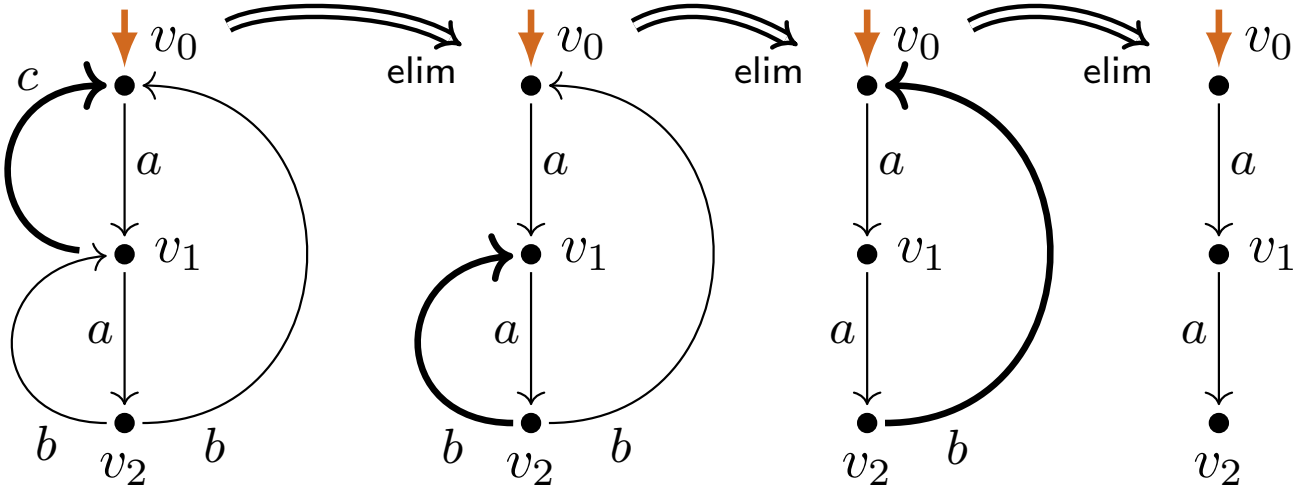


Layered LEE

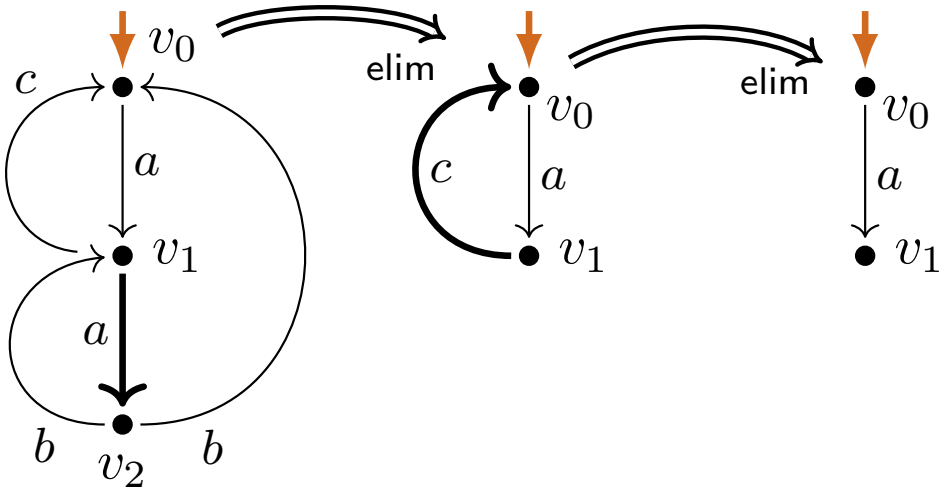


layered LEE
LLEE



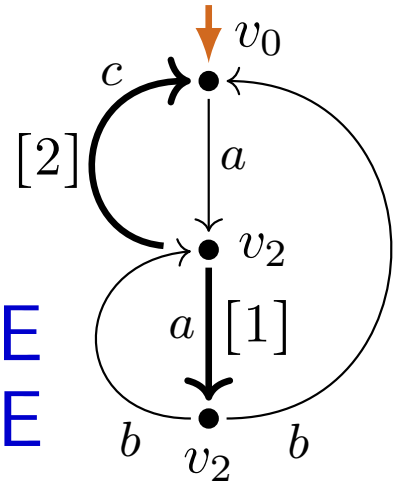


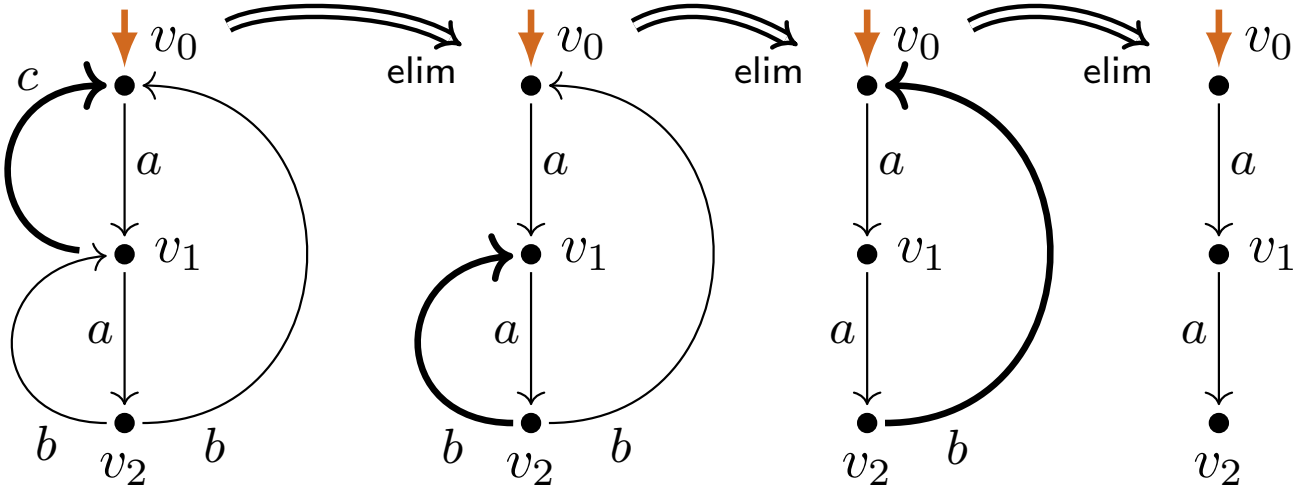
Layered LEE



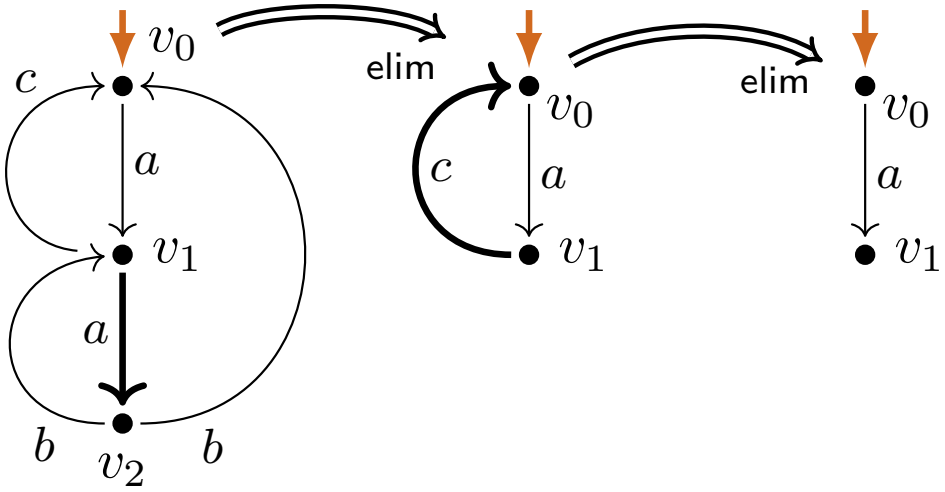
layered LEE

LLEE



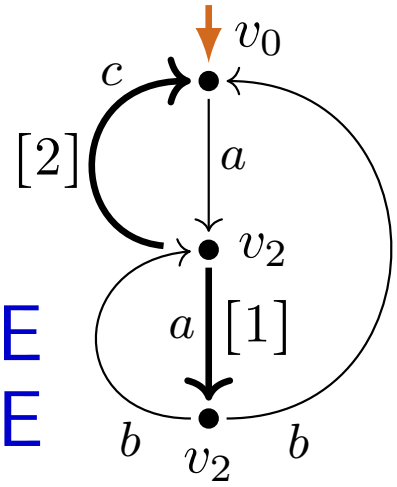


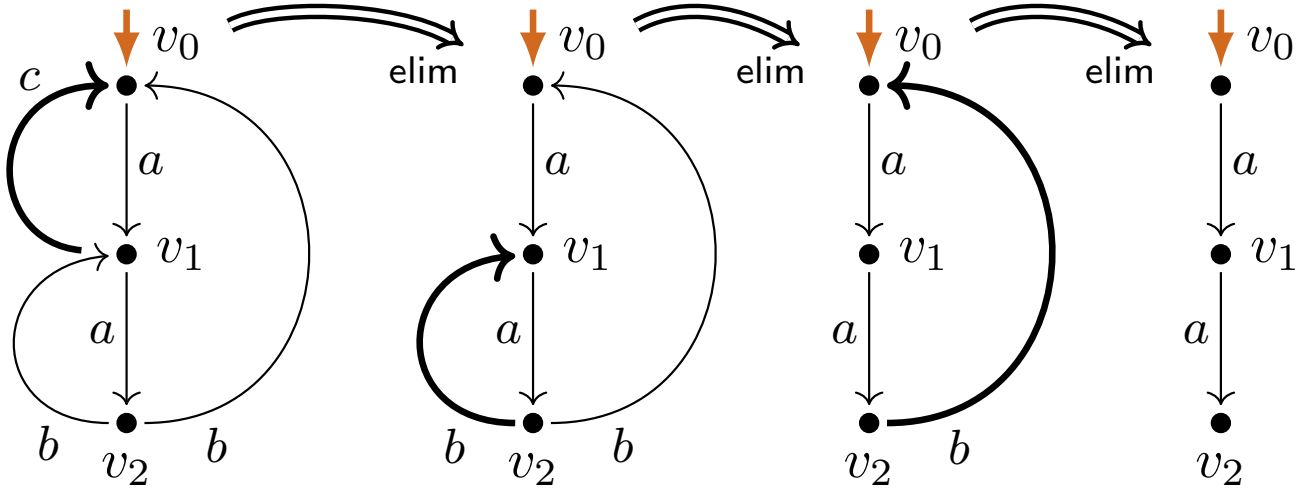
LLEE-witness and LLEE-graph

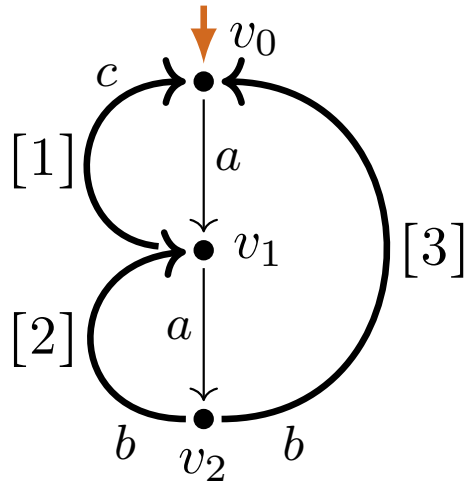


layered LEE

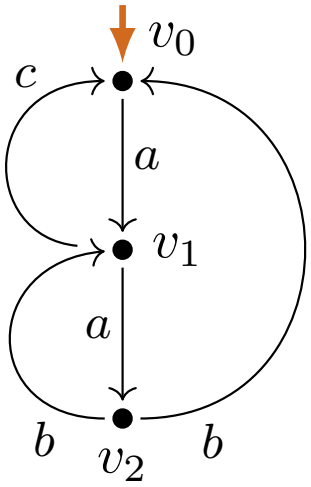
LLEE



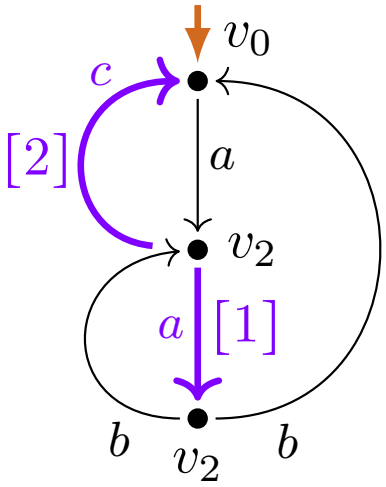




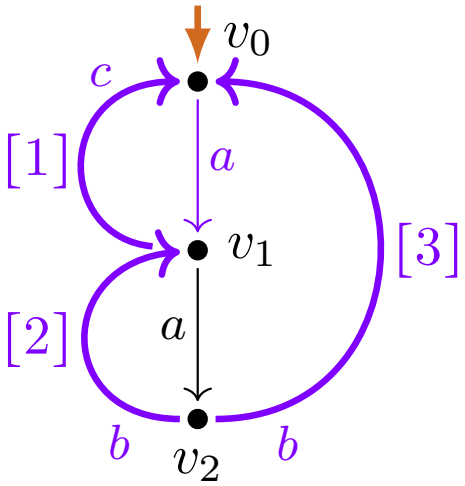
LLEE-witness and LLEE-graph



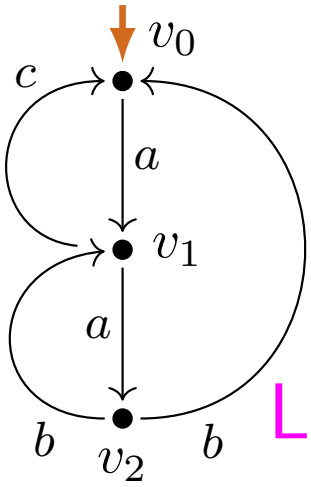
LLEE-witness



LLEE-witness

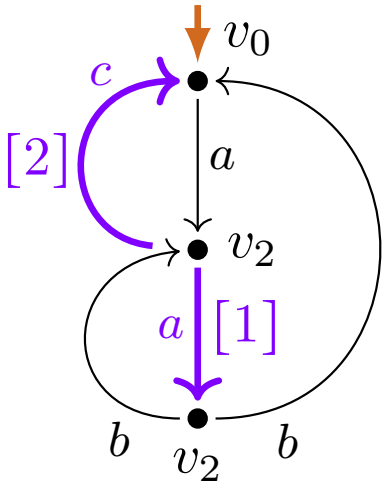


LLEE-witness and LLEE-graph

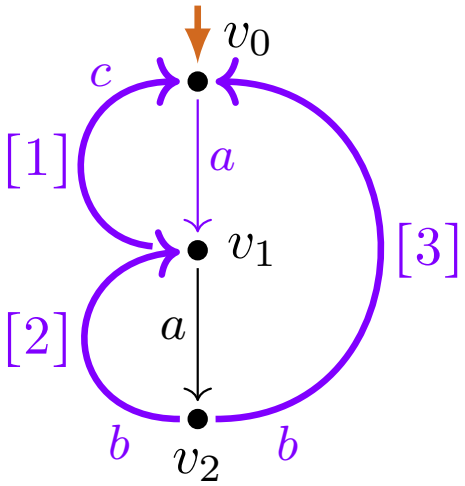


LLEE-graph

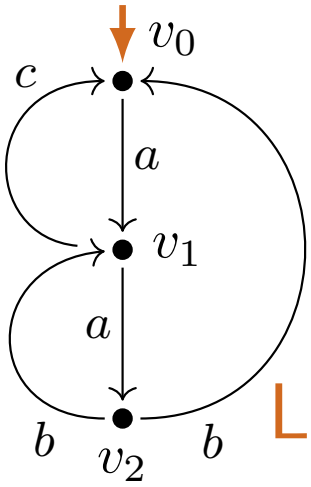
LLEE-witness



LLEE-witness

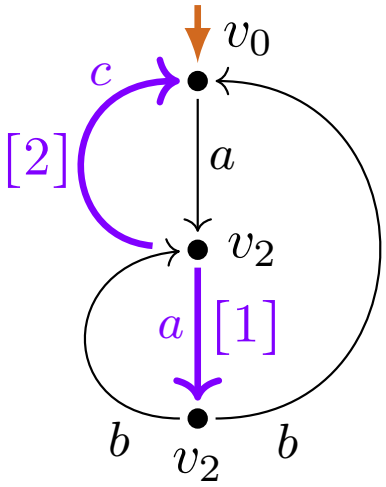


LLEE-witness and LLEE-graph

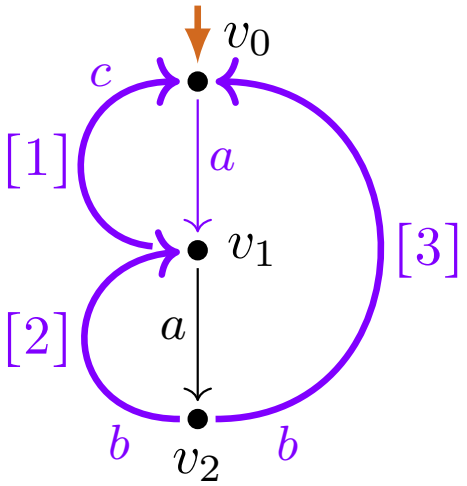


LLEE-graph

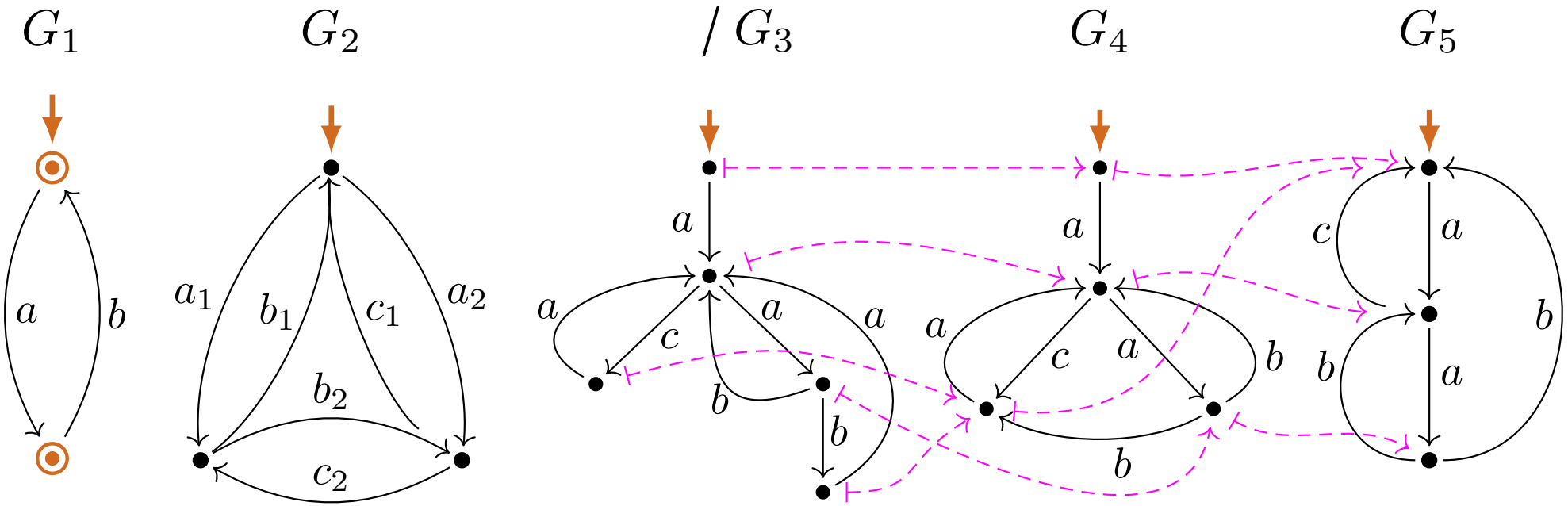
layered LLEE-witness



layered LLEE-witness



LLEE versus P-/$[\![\cdot]\!]_P$-expressibility (examples)



not P-expressible
not $[\![\cdot]\!]_P$-expressible

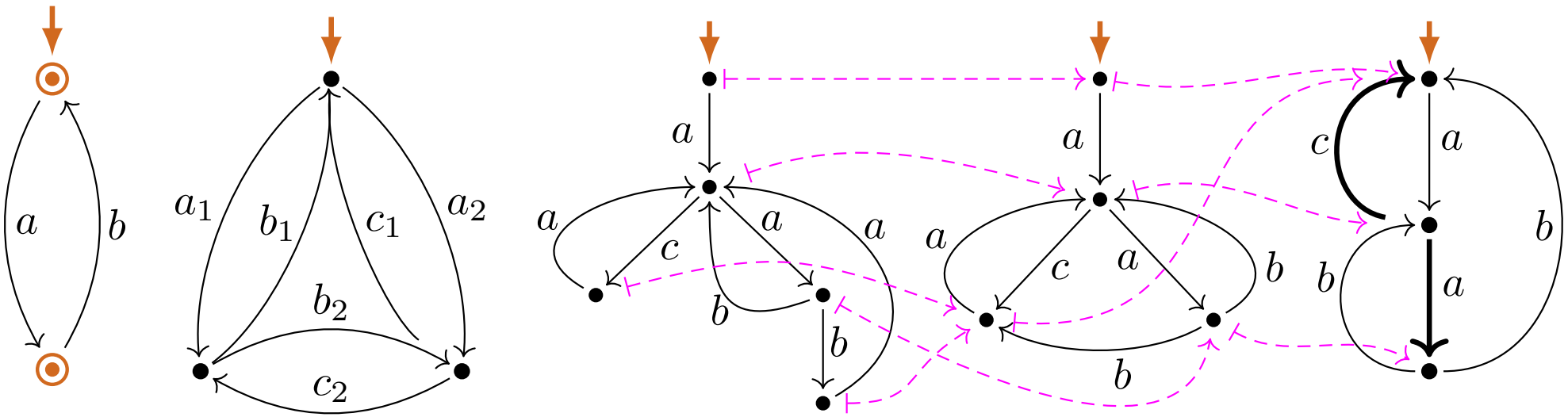
P-expressible ?
$[\![\cdot]\!]_P$-expressible

P-expressible
$[\![\cdot]\!]_P$-expressible

P-expressible ?
$[\![\cdot]\!]_P$-expressible

LLEE versus P-/$[\![\cdot]\!]_P$-expressibility (examples)

G_1 G_2 / G_3 G_4 G_5



not P-expressible
not $[\![\cdot]\!]_P$-expressible

$[\![\cdot]\!]_P$-expressible

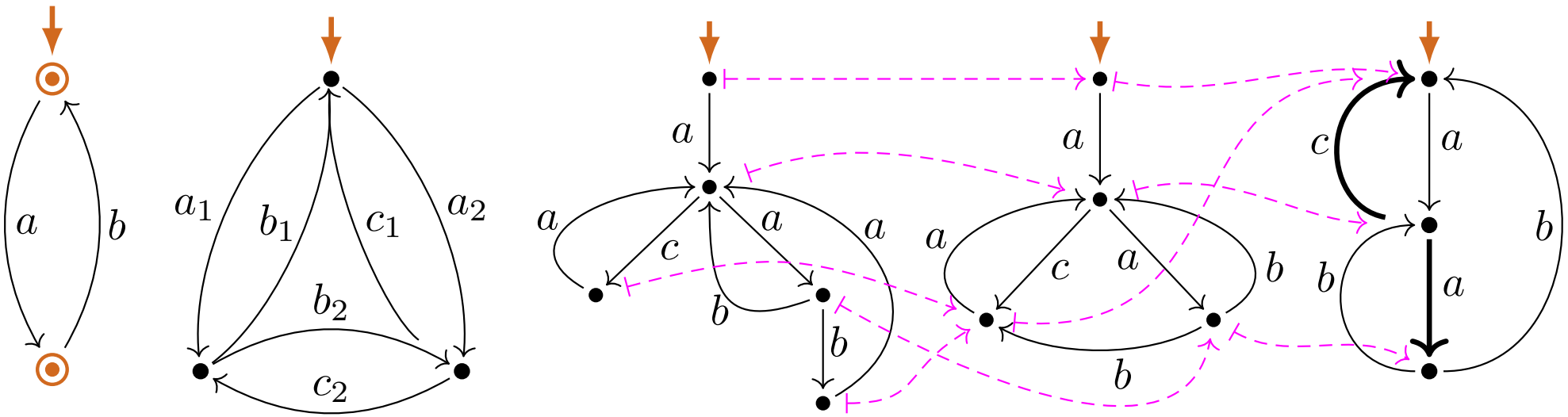
P-expressible
$[\![\cdot]\!]_P$-expressible

P-expressible **?**
$[\![\cdot]\!]_P$-expressible

LLEE

LLEE versus P-/$[\![\cdot]\!]_P$-expressibility (examples)

G_1 G_2 / G_3 G_4 G_5



not P-expressible
not $[\![\cdot]\!]_P$-expressible

$[\![\cdot]\!]_P$-expressible

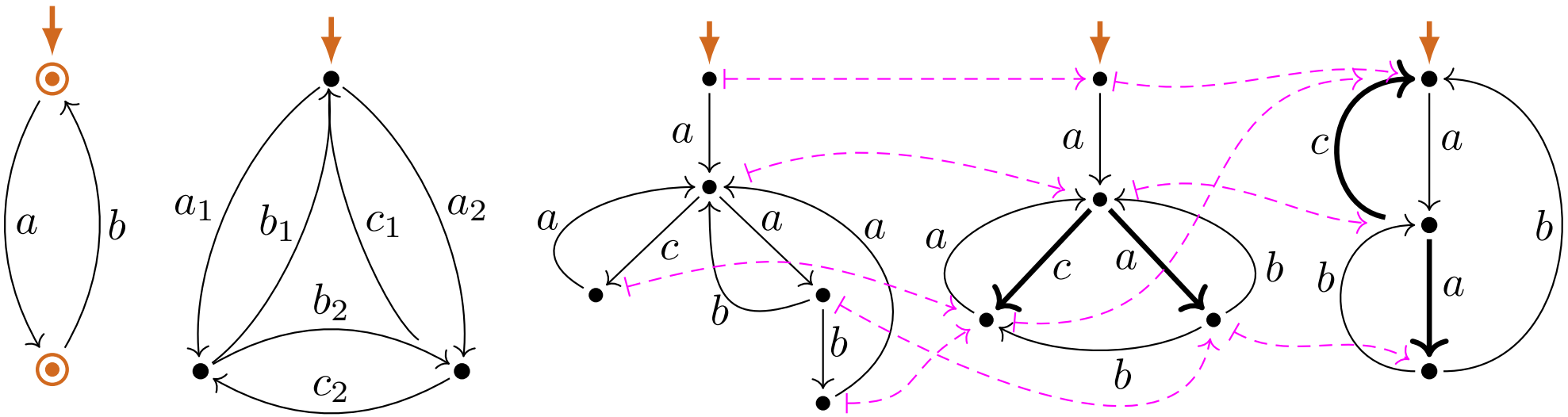
P-expressible
$[\![\cdot]\!]_P$-expressible

P-expressible **?**
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

LLEE versus P-/$[\![\cdot]\!]_P$-expressibility (examples)

G_1 G_2 / G_3 G_4 G_5



not P-expressible
not $[\![\cdot]\!]_P$-expressible

$[\![\cdot]\!]_P$-expressible

P-expressible
$[\![\cdot]\!]_P$-expressible

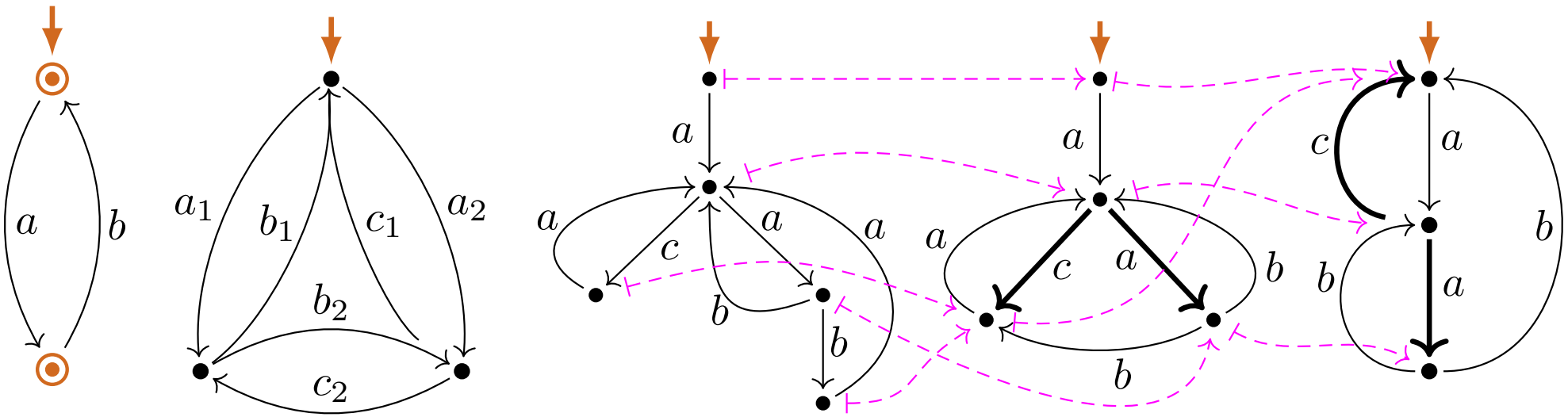
LLEE

P-expressible **?**
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

LLEE versus P-/$[\![\cdot]\!]_P$-expressibility (examples)

G_1 G_2 / G_3 G_4 G_5



not P-expressible
not $[\![\cdot]\!]_P$-expressible

$[\![\cdot]\!]_P$-expressible

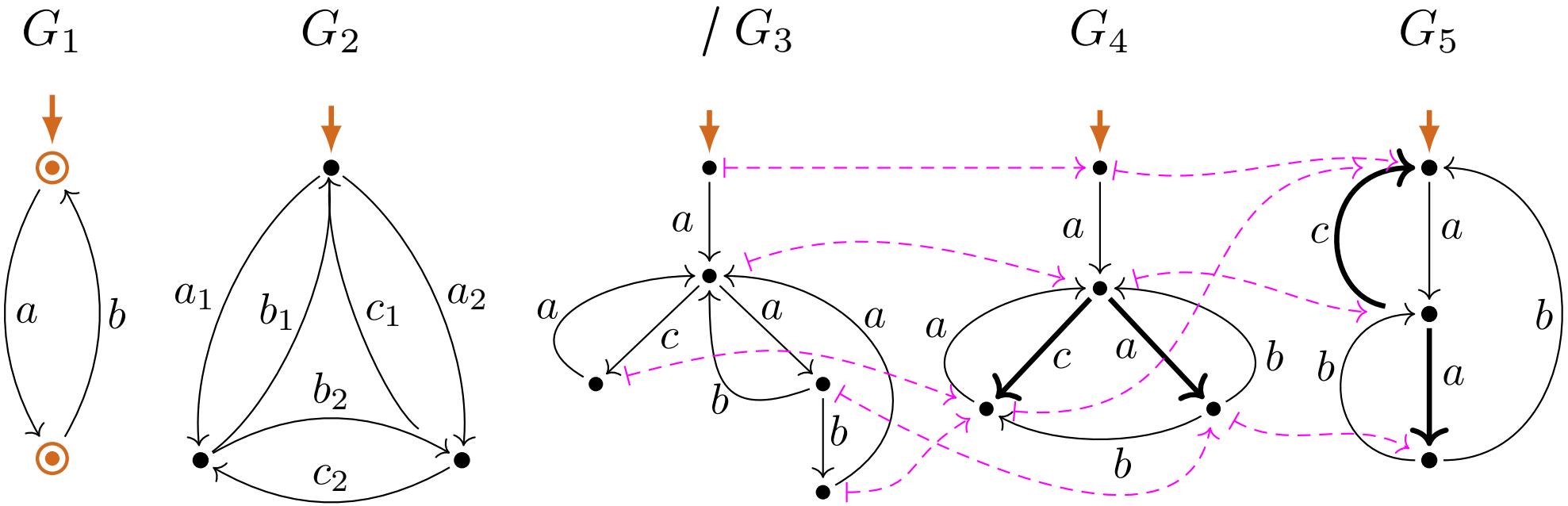
P-expressible
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

P-expressible **?**
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

LLEE versus P-/$[\![\cdot]\!]_P$-expressibility (examples)



not P-expressible
not $[\![\cdot]\!]_P$-expressible

not LLEE

$[\![\cdot]\!]_P$-expressible

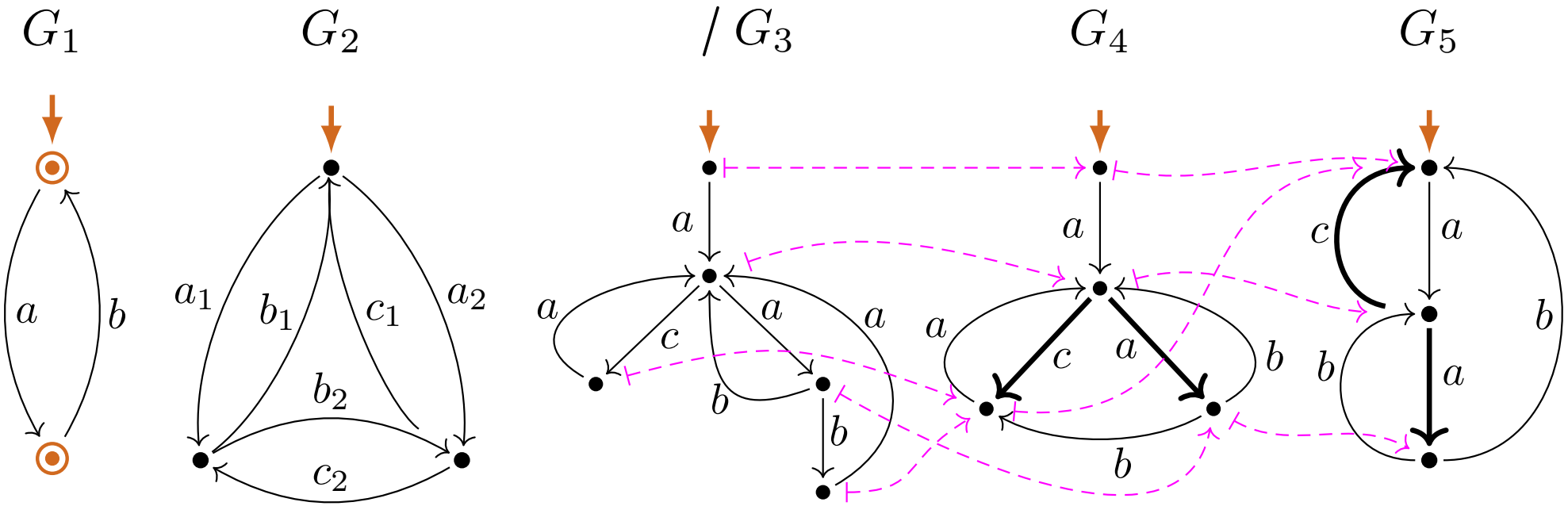
P-expressible
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

P-expressible **?**
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

LLEE versus P-/$[\![\cdot]\!]_P$-expressibility (examples)



not P-expressible
not $[\![\cdot]\!]_P$-expressible

not LLEE
not 1-LLEE

$[\![\cdot]\!]_P$-expressible

P-expressible
$[\![\cdot]\!]_P$-expressible

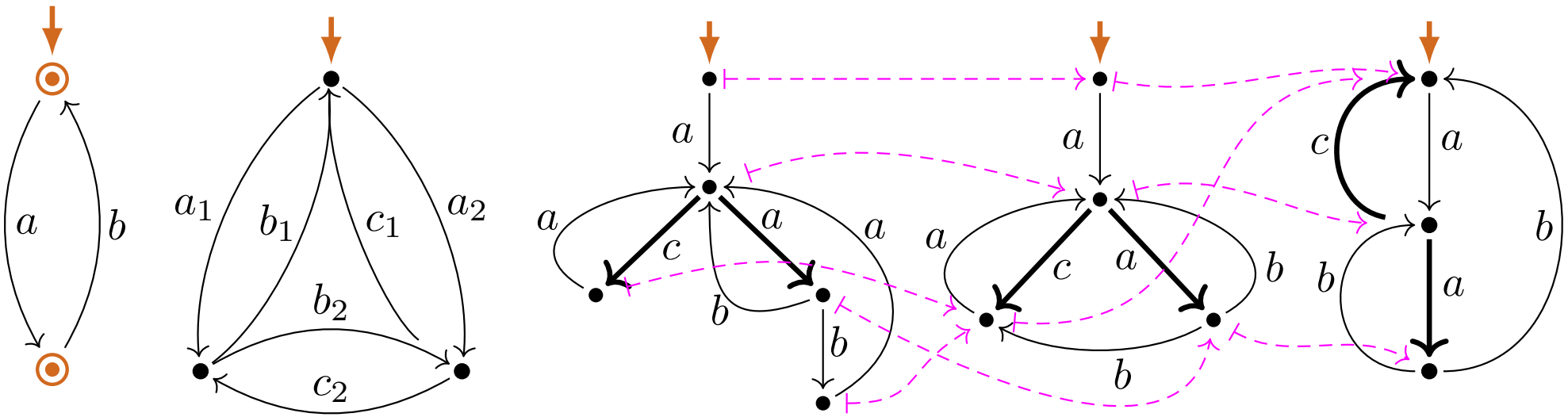
LLEE
1-LLEE

P-expressible ?
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

LLEE versus P-/$[\![\cdot]\!]_P$-expressibility (examples)

G_1 G_2 / G_3 G_4 G_5



not P-expressible
not $[\![\cdot]\!]_P$-expressible

not LLEE
not 1-LLEE

$[\![\cdot]\!]_P$-expressible

LLEE

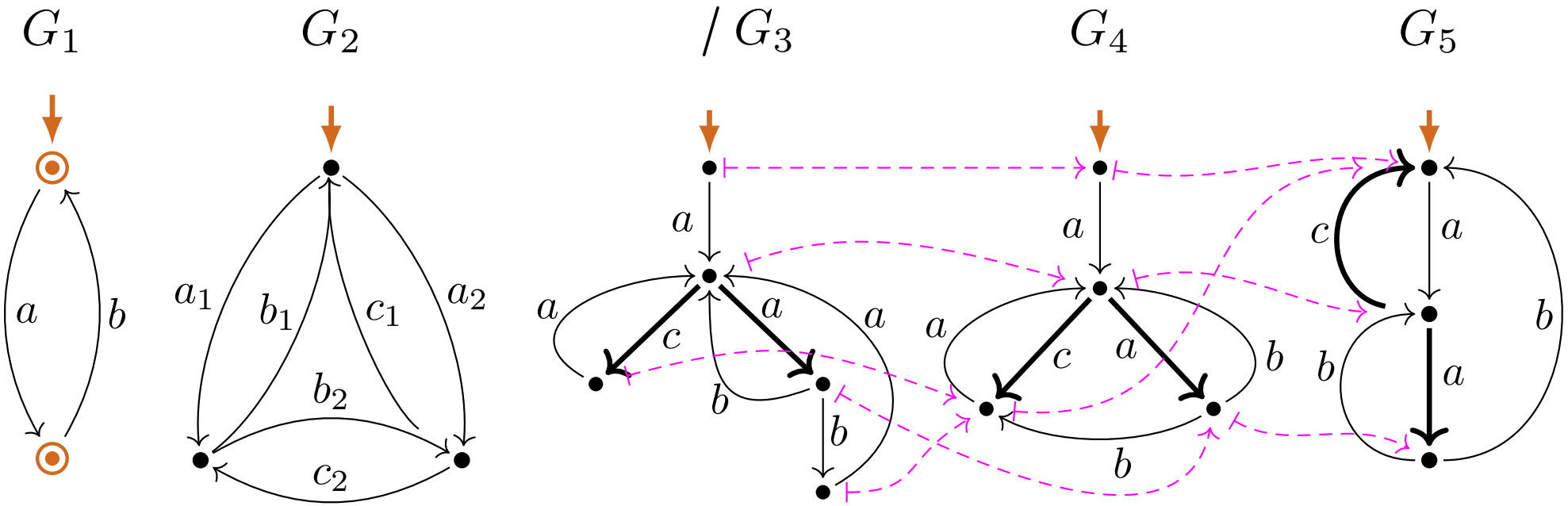
P-expressible
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

P-expressible **?**
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

LLEE versus P-/$[\![\cdot]\!]_P$-expressibility (examples)



not P-expressible
not $[\![\cdot]\!]_P$-expressible

not LLEE
not 1-LLEE

$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

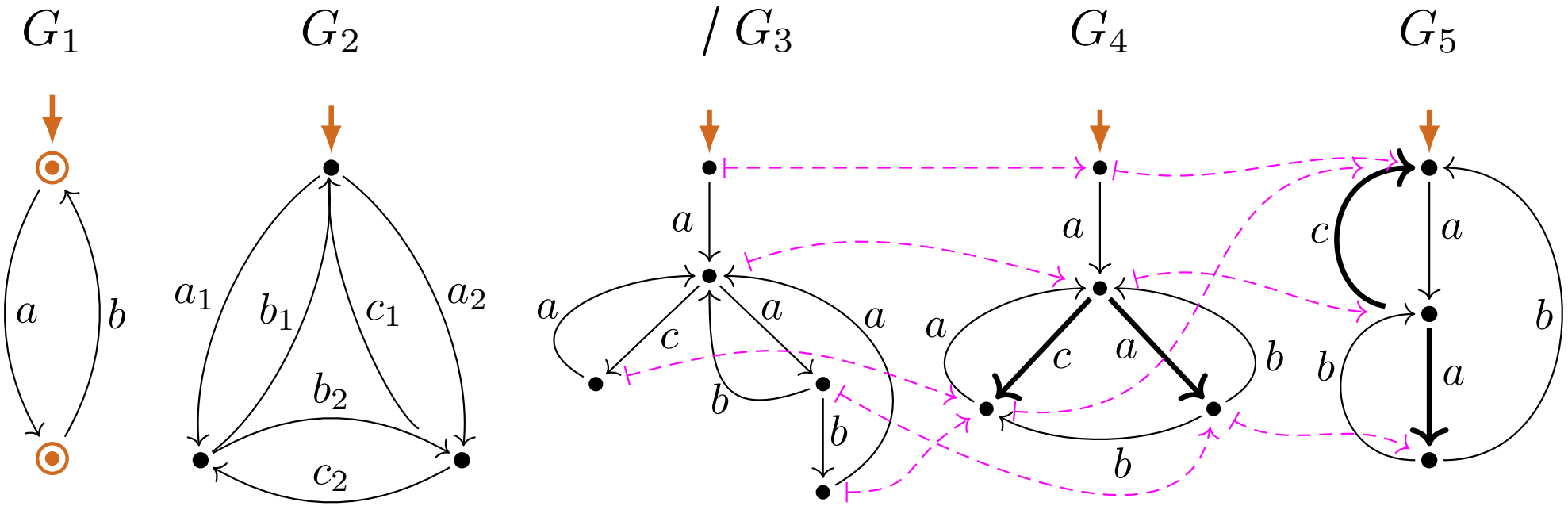
P-expressible
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

P-expressible **?**
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

LLEE versus P-/$[\![\cdot]\!]_P$-expressibility (examples)



not P-expressible
not $[\![\cdot]\!]_P$-expressible

not LLEE
not 1-LLEE

$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

P-expressible
$[\![\cdot]\!]_P$-expressible

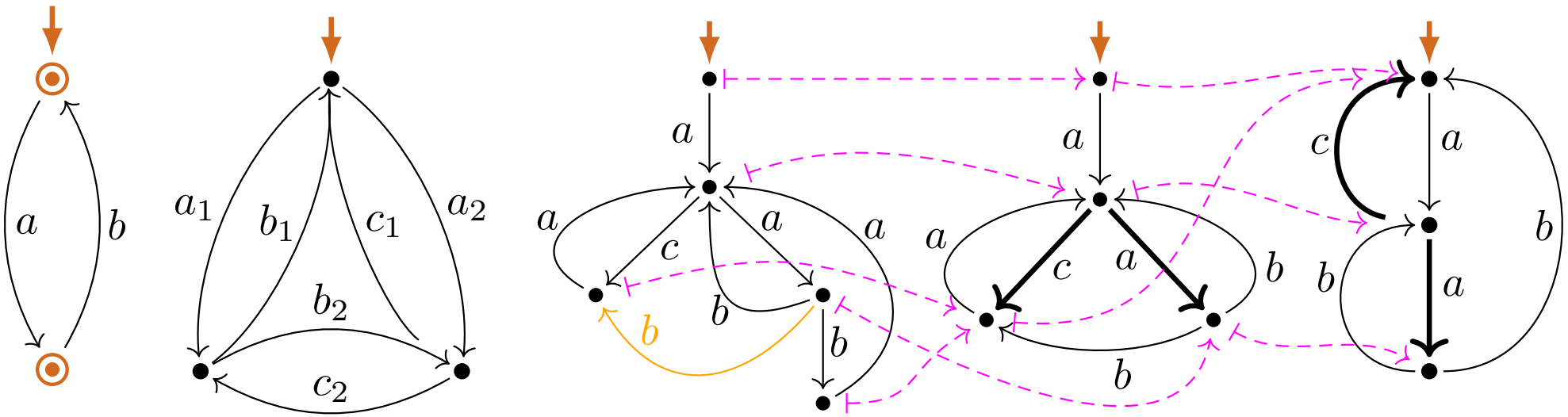
LLEE
1-LLEE

P-expressible **?**
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

LLEE versus P-/$[\![\cdot]\!]_P$-expressibility (examples)

G_1 G_2 G_3^+ / G_3 G_4 G_5



not P-expressible
not $[\![\cdot]\!]_P$-expressible

not LLEE
not 1-LLEE

P-expressible **?**
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

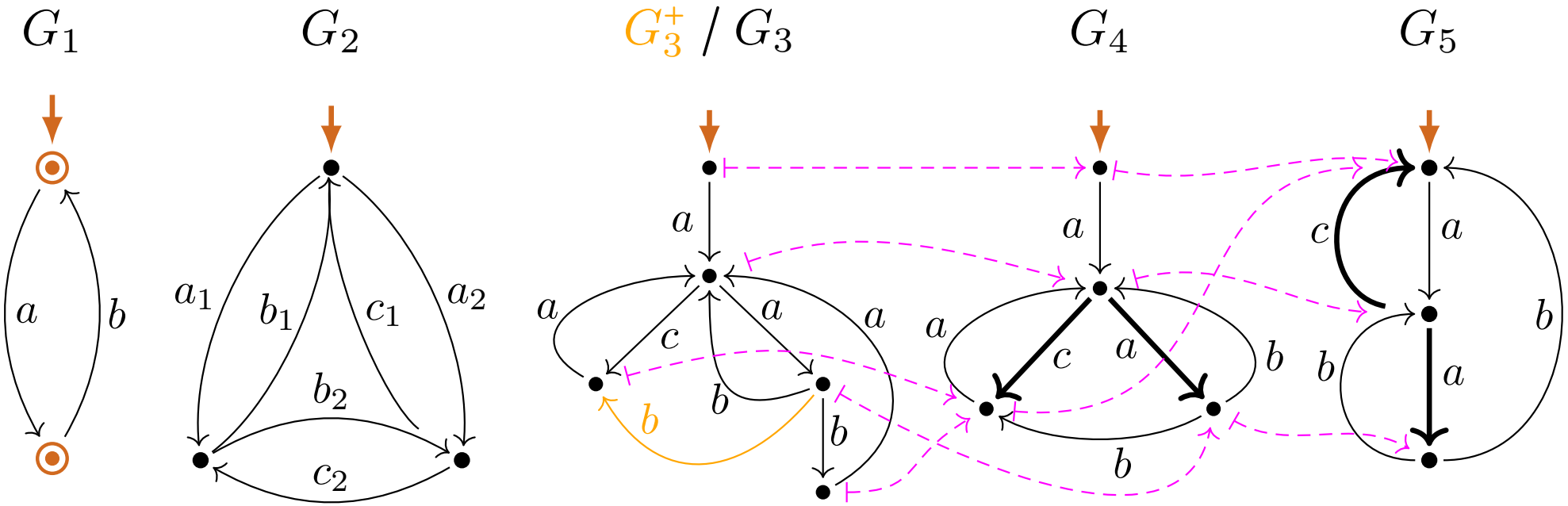
P-expressible
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

P-expressible **?**
$[\![\cdot]\!]_P$-expressible

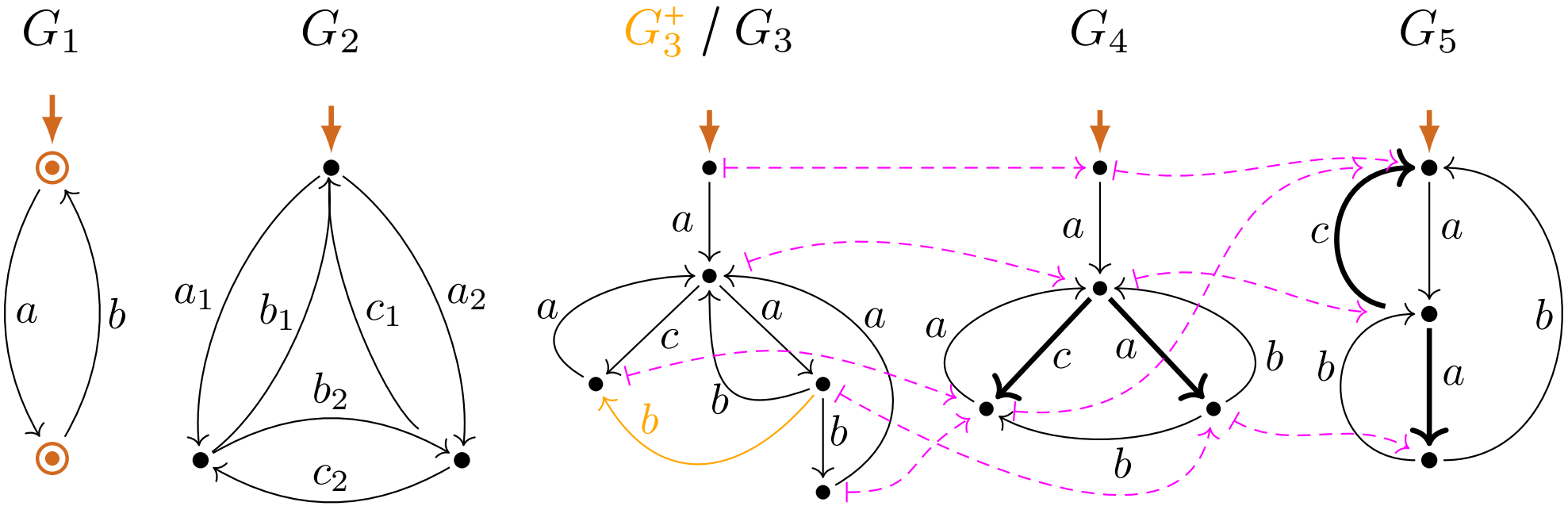
LLEE
1-LLEE

LLEE versus P-/$[\![\cdot]\!]_P$-expressibility (examples)



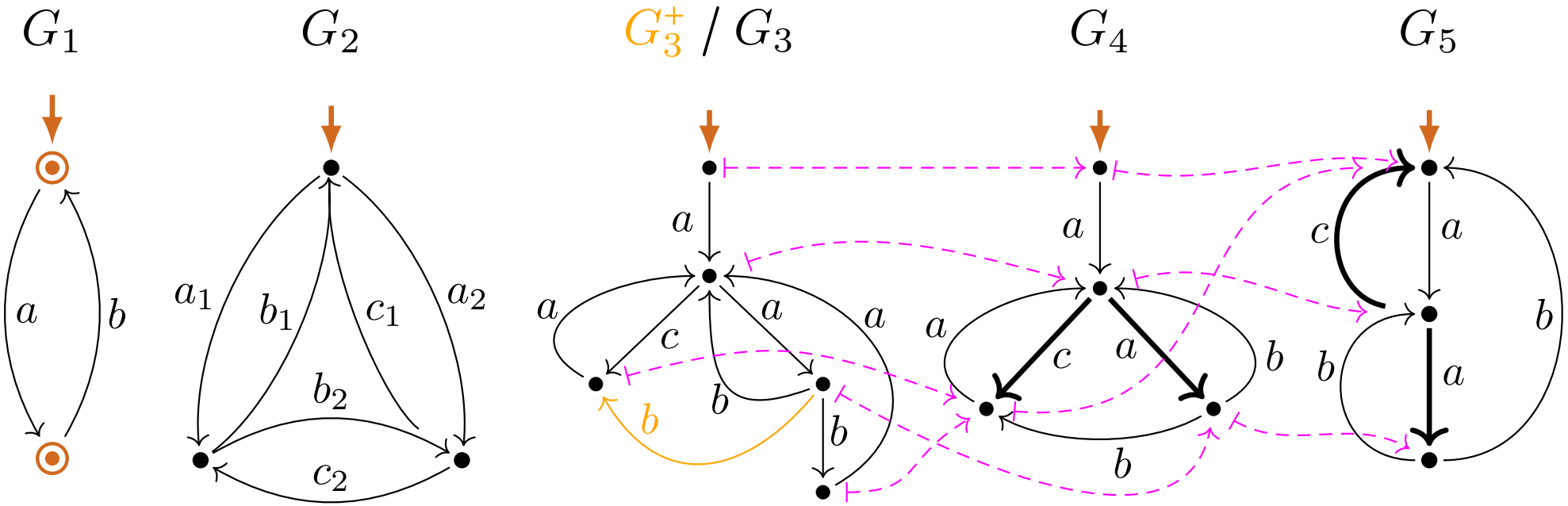
G_1	G_2	G_3^+ / G_3	G_4	G_5
not P-expressible		P-expressible **?**	P-expressible	P-expressible **?**
not $[\![\cdot]\!]_P$-expressible		$[\![\cdot]\!]_P$-expressible	$[\![\cdot]\!]_P$-expressible	$[\![\cdot]\!]_P$-expressible
not LLEE		**not** LLEE / LLEE	LLEE	LLEE
not 1-LLEE		1-LLEE	1-LLEE	1-LLEE

LLEE versus P-/$[\![\cdot]\!]_P$-expressibility (examples)



G_1	G_2	G_3^+ / G_3	G_4	G_5
not P-expressible		P-expressible **?**	P-expressible	P-expressible **?**
not $[\![\cdot]\!]_P$-expressible		$[\![\cdot]\!]_P$-expressible	$[\![\cdot]\!]_P$-expressible	$[\![\cdot]\!]_P$-expressible
not LLEE		**not** LLEE / LLEE	LLEE	LLEE
not 1-LLEE		**not** 1-LLEE/ 1-LLEE	1-LLEE	1-LLEE

LLEE versus P-/$[\![\cdot]\!]_P$-expressibility (examples)



not P-expressible
not $[\![\cdot]\!]_P$-expressible

not LLEE
not 1-LLEE

$P(\cdot)$-expressible **?**
$[\![\cdot]\!]_P$-expressible

not LLEE / LLEE
not 1-LLEE/ 1-LLEE

P-expressible
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

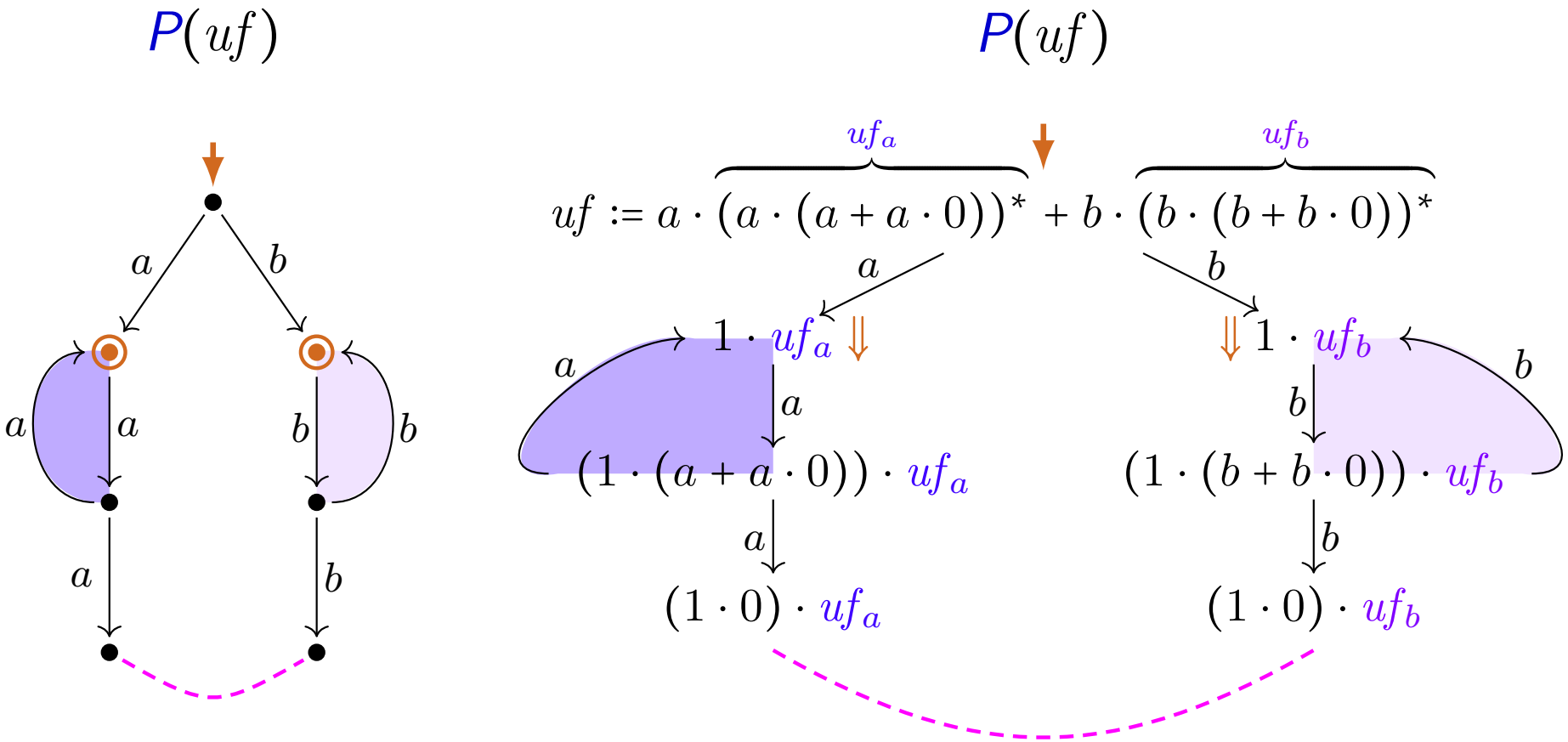
P-expressible **?**
$[\![\cdot]\!]_P$-expressible

LLEE
1-LLEE

TERMGRAPH '24

- Regular expressions (unary/binary star, 1-free/under-star-1-free $(*/\not{1})$)
- Milner's process interpretation P/semantics $[\![\cdot]\!]_P$
  - P-/$[\![\cdot]\!]_P$-expressible graphs ($\rightsquigarrow$ expressibility question)
- Layered Loop existence and elimination (LLEE/1-LLEE)
  - interpretation/extraction correspondences with $(*/\not{1})$/all reg. expr.
  - LLEE is preserved by bisimulation collapse (1-LLEE is not)
- TERMGRAPH 2024: compact process interpretation $P^\bullet$
  - image of $P^\bullet$ is closed under bisimulation collapse (image of P is not)
- characterisation of image of $P^\bullet$ restricted to $(*/\not{1})$ expressions
- characterization of image of $P^\bullet$
- Relationship with co-reducibility

Image of P is **not** closed under bisimulation collapse



Process interpretation P (formally)

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{\mathbf{1}\Downarrow} \qquad \frac{e_i\Downarrow}{(e_1 + e_2)\Downarrow}\ (i \in \{1,2\}) \qquad \frac{e_1\Downarrow \qquad e_2\Downarrow}{(e_1 \cdot e_2)\Downarrow} \qquad \frac{}{(e^*)\Downarrow}$$

$$\frac{}{a \xrightarrow{a} \mathbf{1}} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'}\ (i \in \{1,2\})$$

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \qquad \frac{e_1\Downarrow \qquad e_2 \xrightarrow{a} e_2'}{e_1 \cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

Responsible rule

Definition (Transition system specification $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2}$$

Problem: if e_1' is not normed, $e_1' \cdot e_2$ has redundant subexpression e_2

Compact process interpretation $P^\bullet$

Definition (Transition system specification $\mathcal{T}^\bullet$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

Compact process interpretation $P^\bullet$

Definition (Transition system specification $\mathcal{T}^\bullet$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

Compact process interpretation $P^\bullet$

Definition (Transition system specification $\mathcal{T}^\bullet$, changed rules w.r.t. $\mathcal{T}$)

$$\pi(e) := \begin{cases} e_1 \cdot e_2 & \text{if } e = e_1 \cdot e_2 \text{ and } e_1 \text{ is normed,} \\ \pi(e_1) & \text{if } e = e_1 \cdot e_2 \text{ and } e_1 \text{ is not normed,} \\ e & \text{otherwise.} \end{cases}$$

$$\frac{e_1 \xrightarrow{a} ed}{e_1 \cdot e_2 \xrightarrow{a} \pi(ed \cdot e_2)}$$

Compact process interpretation $P^\bullet$

Definition (Transition system specification $\mathcal{T}^\bullet$, changed rules w.r.t. $\mathcal{T}$)

$$\pi(e) := \begin{cases} e_1 \cdot e_2 & \text{if } e = e_1 \cdot e_2 \text{ and } e_1 \text{ is normed,} \\ \pi(e_1) & \text{if } e = e_1 \cdot e_2 \text{ and } e_1 \text{ is not normed,} \\ e & \text{otherwise.} \end{cases}$$

$$\frac{e_1 \xrightarrow{a} ed}{e_1 \cdot e_2 \xrightarrow{a} \pi(ed \cdot e_2)} \qquad \frac{e \xrightarrow{a} ed}{e^* \xrightarrow{a} \pi(ed \cdot e^*)}$$

Compact process interpretation $P^\bullet$

Definition (Transition system specification $\mathcal{T}^\bullet$, changed rules w.r.t. $\mathcal{T}$)

$$\pi(e) := \begin{cases} e_1 \cdot e_2 & \text{if } e = e_1 \cdot e_2 \text{ and } e_1 \text{ is normed,} \\ \pi(e_1) & \text{if } e = e_1 \cdot e_2 \text{ and } e_1 \text{ is not normed,} \\ e & \text{otherwise.} \end{cases}$$

$$\frac{e_1 \xrightarrow{a} ed}{e_1 \cdot e_2 \xrightarrow{a} \pi(ed \cdot e_2)} \qquad \frac{e \xrightarrow{a} ed}{e^* \xrightarrow{a} \pi(ed \cdot e^*)}$$

Definition

The compact process (graph) interpretation $P^\bullet(e)$ of a reg. expr's e:
$P^\bullet(e)$:= labeled transition graph generated by $\pi(e)$ by derivations in $\mathcal{T}^\bullet$.

Compact process interpretation $P^\bullet$

Definition (Transition system specification $\mathcal{T}^\bullet$, changed rules w.r.t. $\mathcal{T}$)

$$\pi(e) := \begin{cases} e_1 \cdot e_2 & \text{if } e = e_1 \cdot e_2 \text{ and } e_1 \text{ is normed,} \\ \pi(e_1) & \text{if } e = e_1 \cdot e_2 \text{ and } e_1 \text{ is not normed,} \\ e & \text{otherwise.} \end{cases}$$

$$\frac{e_1 \xrightarrow{a} ed}{e_1 \cdot e_2 \xrightarrow{a} \pi(ed \cdot e_2)} \qquad \frac{e \xrightarrow{a} ed}{e^* \xrightarrow{a} \pi(ed \cdot e^*)}$$

Definition

The compact process (graph) interpretation $P^\bullet(e)$ of a reg. expr's e:
$P^\bullet(e)$:= labeled transition graph generated by $\pi(e)$ by derivations in $\mathcal{T}^\bullet$.

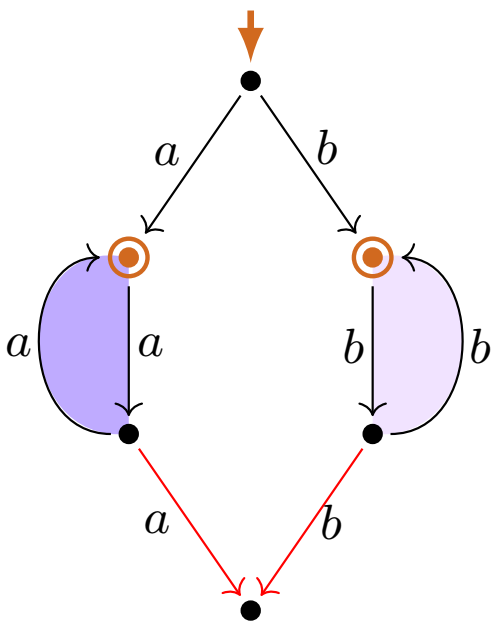
Lemma ($P^\bullet$ increases sharing; $P^\bullet$, P have same bisimulation semantics)

(i) $P(e) \mathrel{\underline{\leftrightarrow}} P^\bullet(\pi(e))$ *for all regular expressions* e.

(ii) (G *is* $[\![\cdot]\!]_{P^\bullet}$*-expressible* $\iff$ G *is* $[\![\cdot]\!]_P$*-expressible) for all graphs* G.

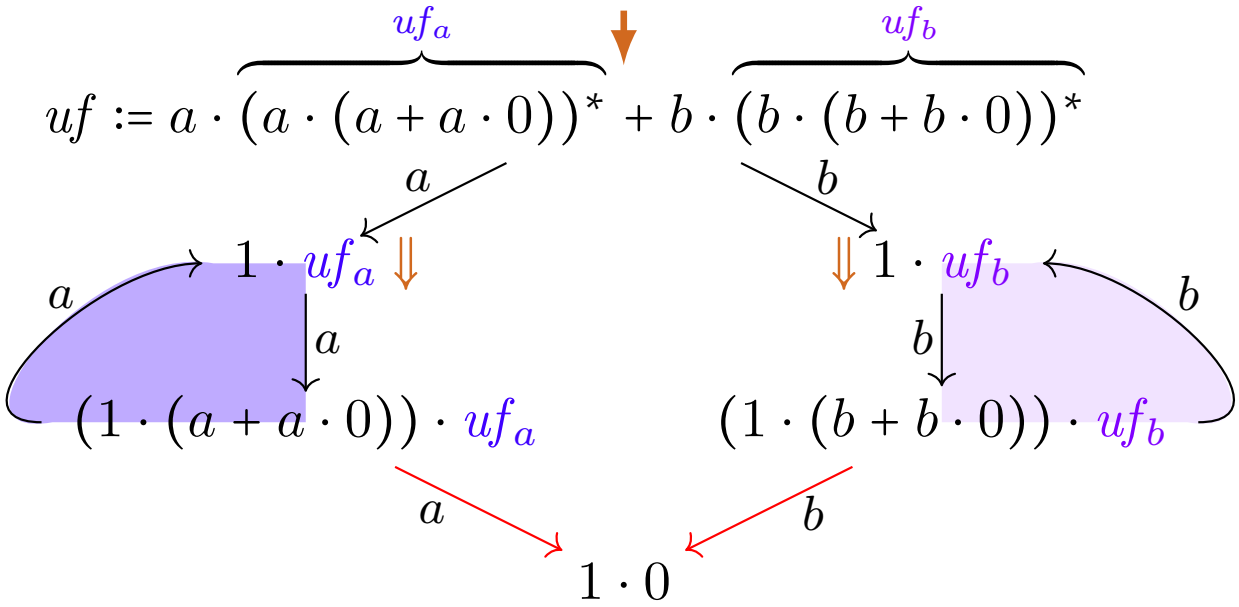
Image of $P^\bullet$ under bisimulation collapse . . .

$P^\bullet(uf)$



$P^\bullet(uf)$

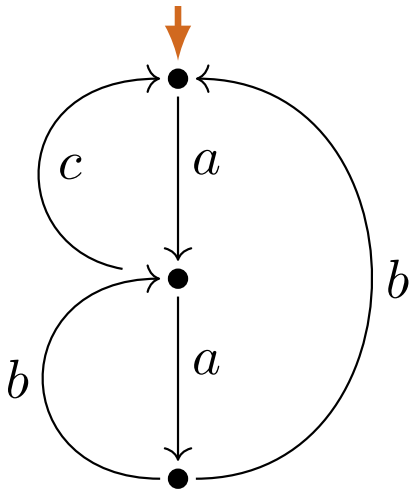
$$uf := \overbrace{a \cdot (a \cdot (a + a \cdot 0))^*}^{uf_a} + \overbrace{b \cdot (b \cdot (b + b \cdot 0))^*}^{uf_b}$$



Refined extraction expression (example)

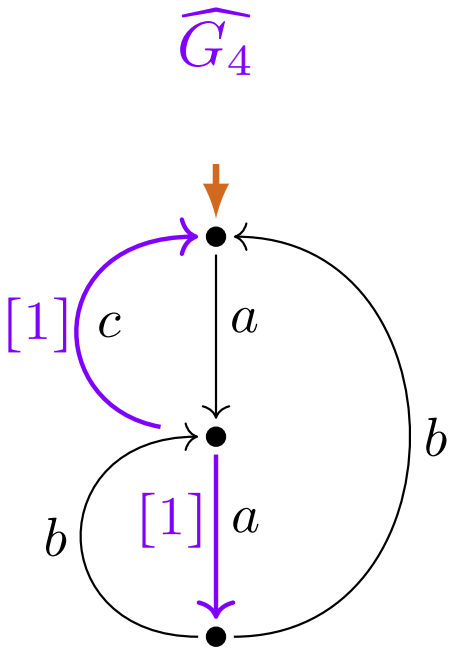
Define readback steps to precisely reverse $P^\bullet$-defined transitions!

G_4



Refined extraction expression (example)

Define readback steps to precisely reverse $P^\bullet$-defined transitions!



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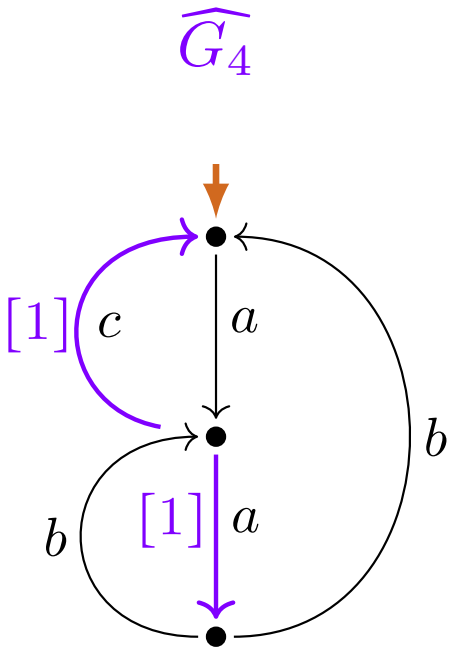

$$(1 \cdot (\qquad\qquad\qquad)^*) \cdot 0$$

Refined extraction expression (example)

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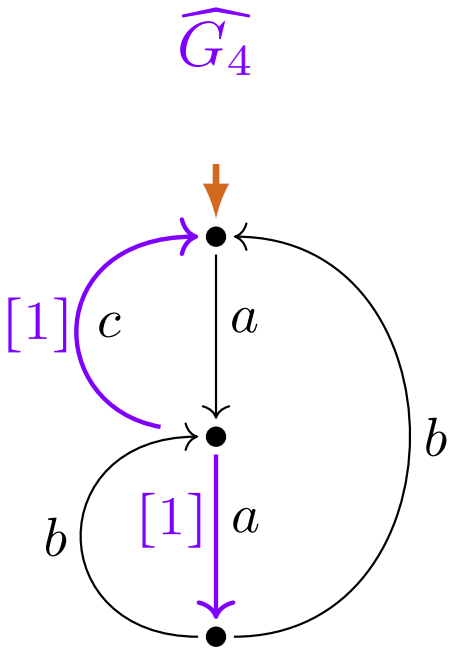
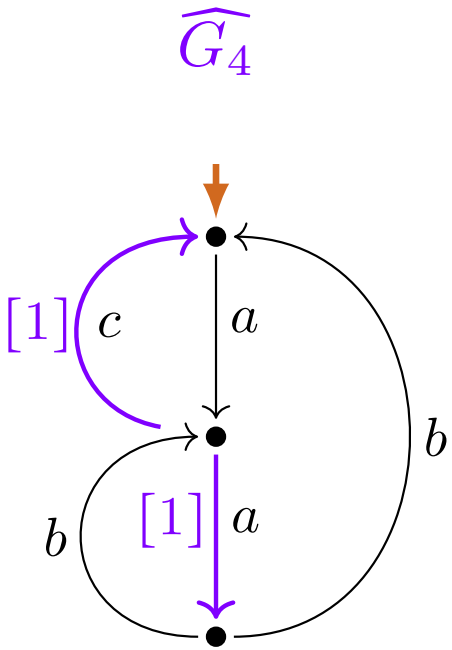


$$(1 \cdot (\quad \overset{a}{\vdots} \quad)^*) \cdot 0$$

Refined extraction expression (example)

Define readback steps to precisely reverse $P^\bullet$-defined transitions!



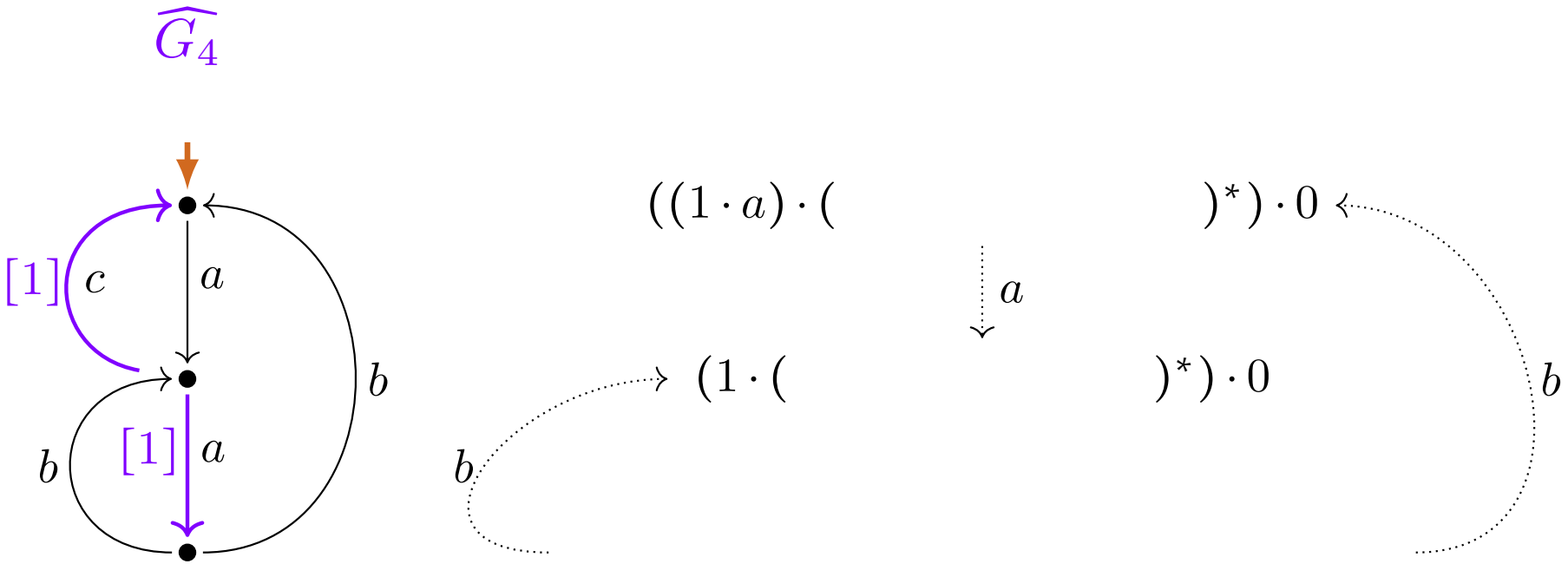
$$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$$

$$\vdots\ a$$

$$(1 \cdot (\qquad)^*) \cdot 0$$

Refined extraction expression (example)

Define readback steps to precisely reverse $P^\bullet$-defined transitions!



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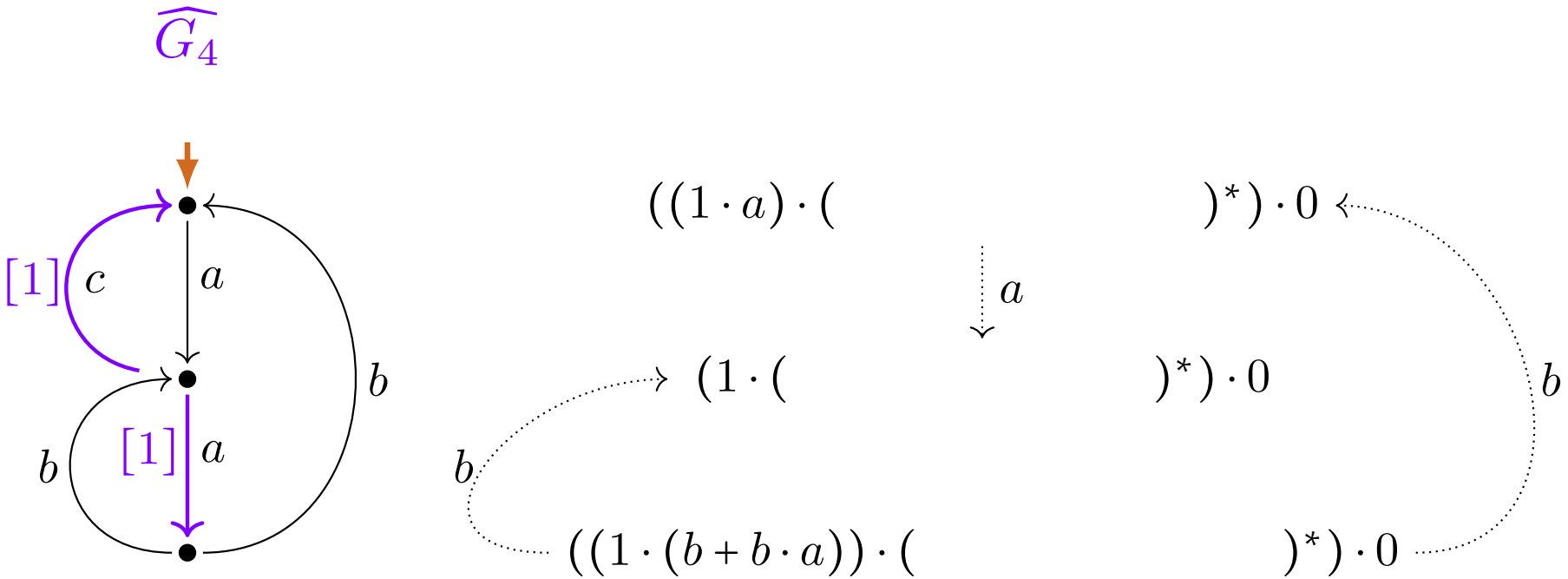
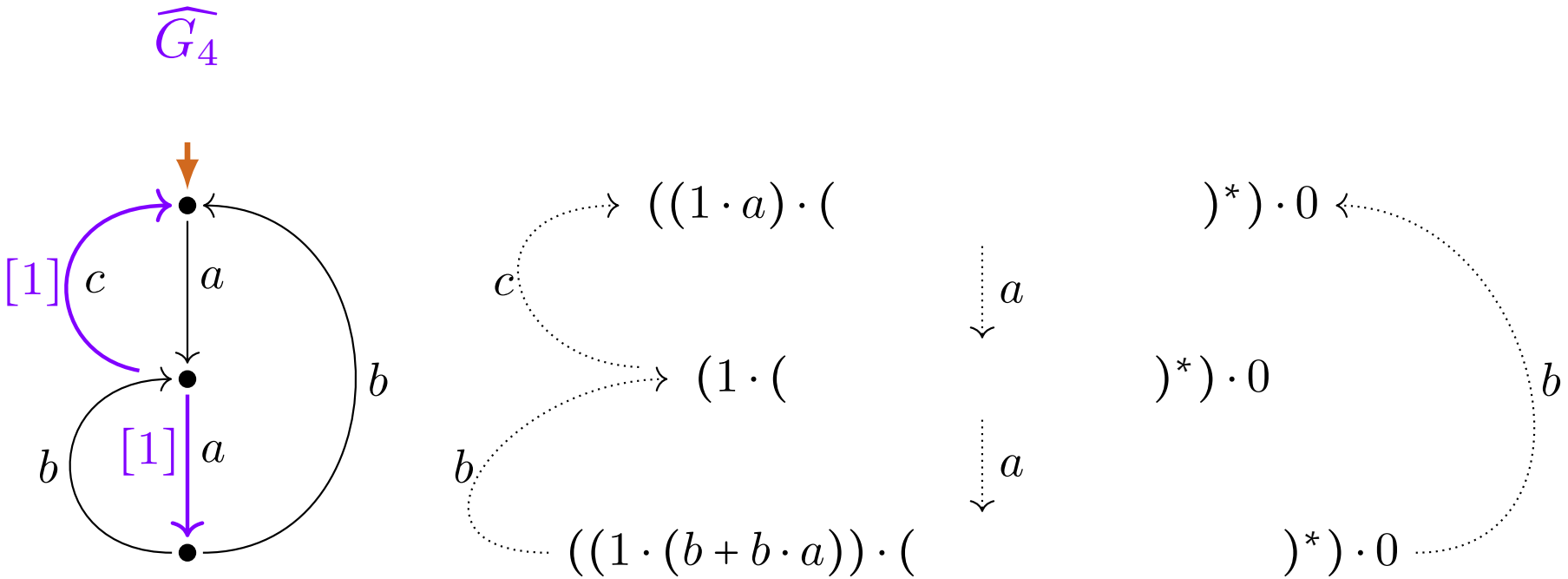


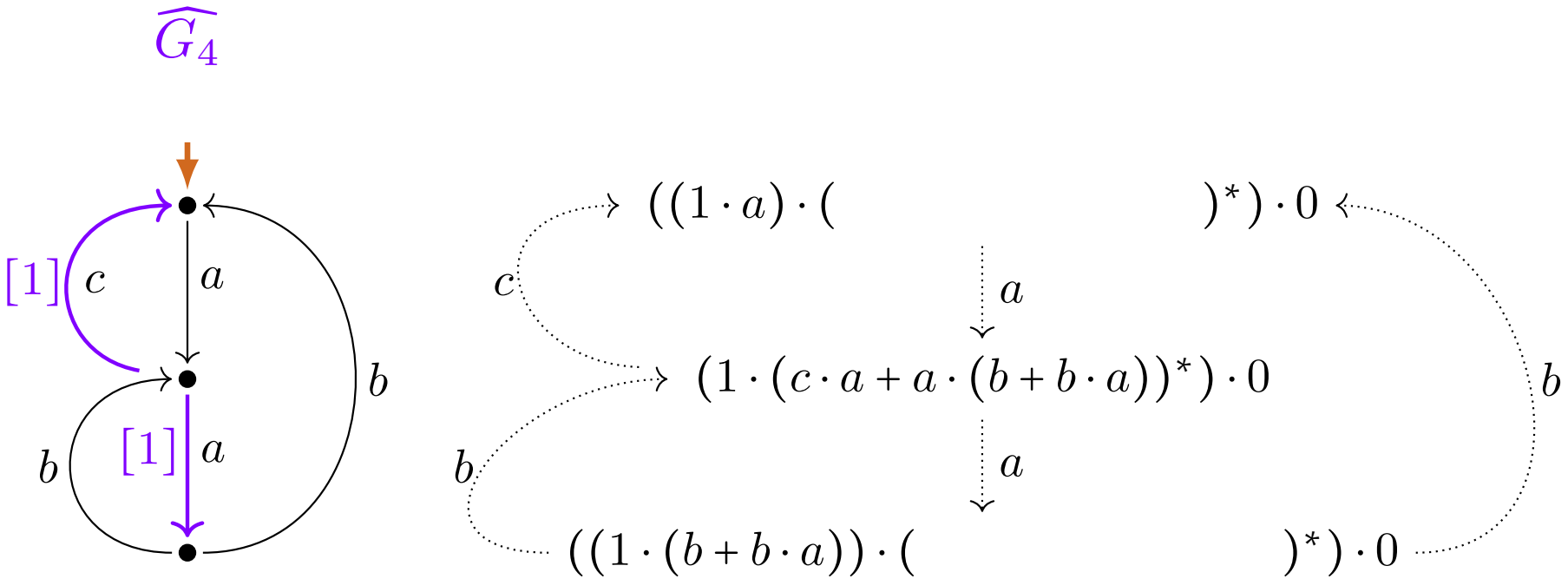
Refined extraction expression (example)

Define readback steps to precisely reverse $P^\bullet$-defined transitions!



Refined extraction expression (example)

Define readback steps to precisely reverse $P^\bullet$-defined transitions!



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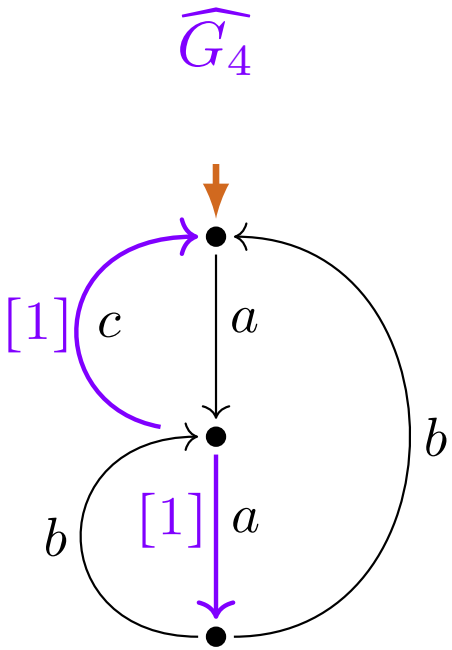
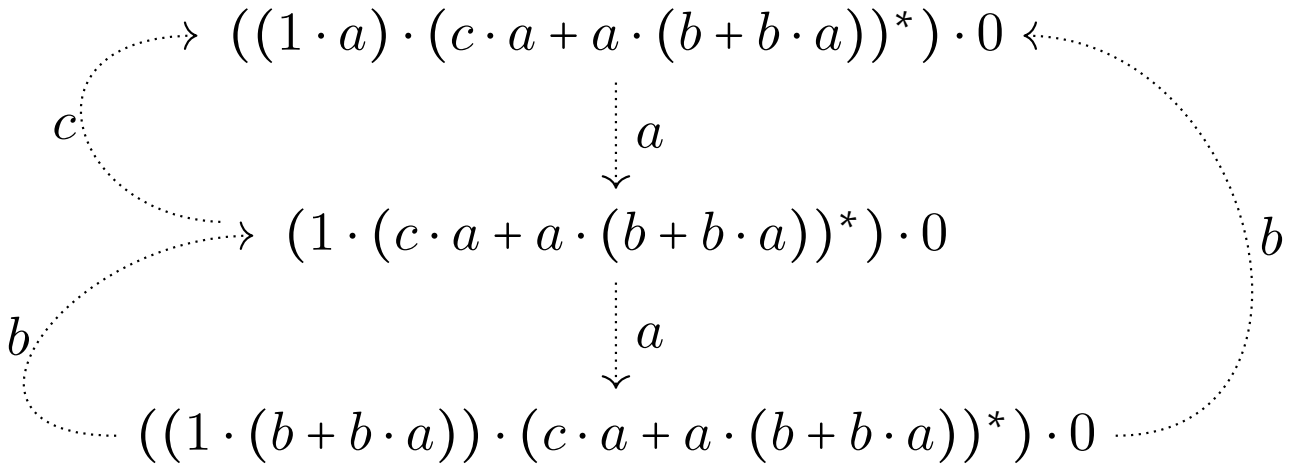
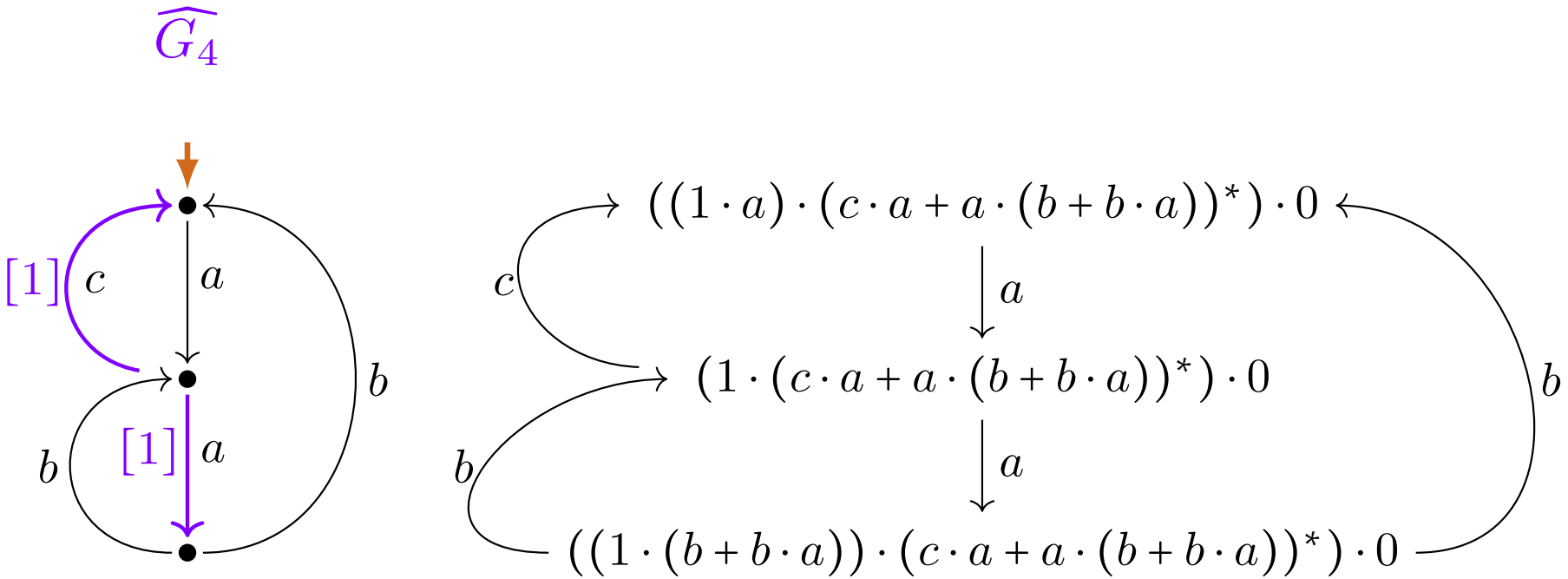




Refined extraction expression (example)

Define readback steps to precisely reverse $P^\bullet$-defined transitions!

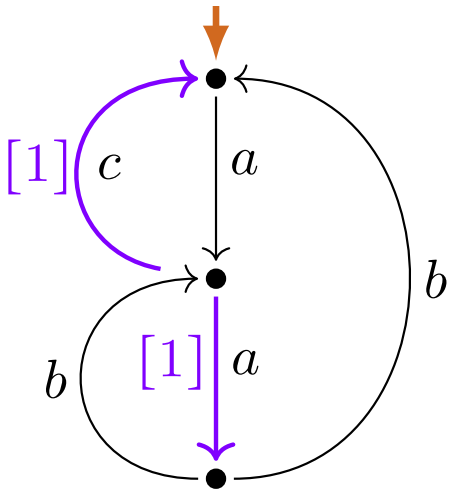


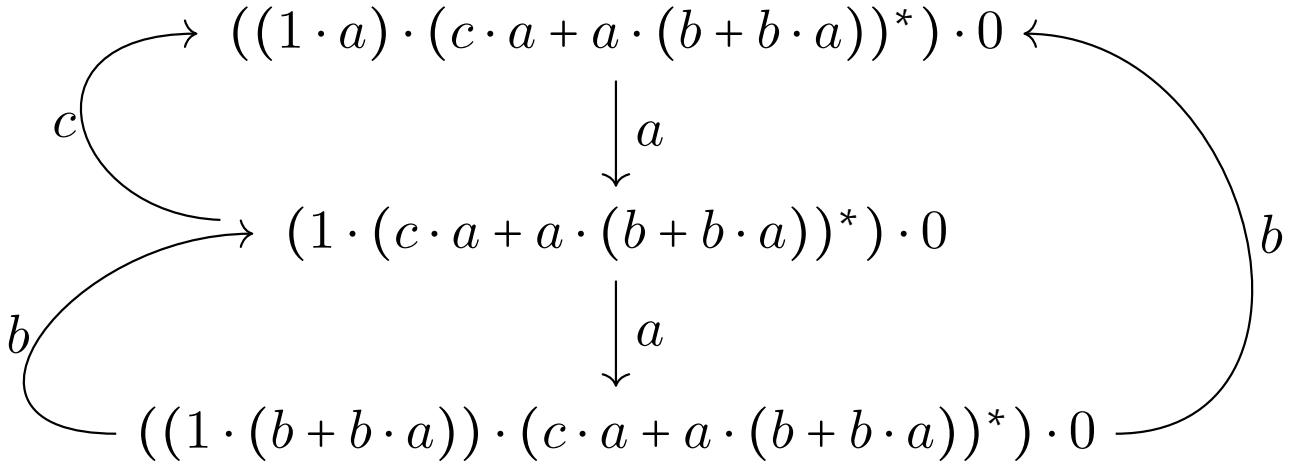
Refined extraction expression (example)

Define readback steps to precisely reverse $P^\bullet$-defined transitions!

$\widehat{G_4}$

$P^\bullet(uf) = P(uf) \simeq G_4$

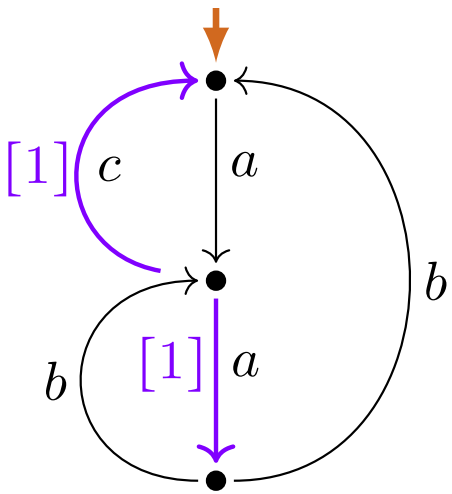




Refined extraction expression (example)

Define readback steps to precisely reverse $P^\bullet$-defined transitions!

$\widehat{G_4}$



$$P^\bullet(uf) = P(uf) \simeq G_4$$

$$\overbrace{((1\cdot a)\cdot(c\cdot a + a\cdot(b+b\cdot a))^*)\cdot 0}^{uf}$$

$$\downarrow a$$

$$(1\cdot(c\cdot a + a\cdot(b+b\cdot a))^*)\cdot 0$$

$$\downarrow a$$

$$((1\cdot(b+b\cdot a))\cdot(c\cdot a + a\cdot(b+b\cdot a))^*)\cdot 0$$

Refined extraction expression (example)

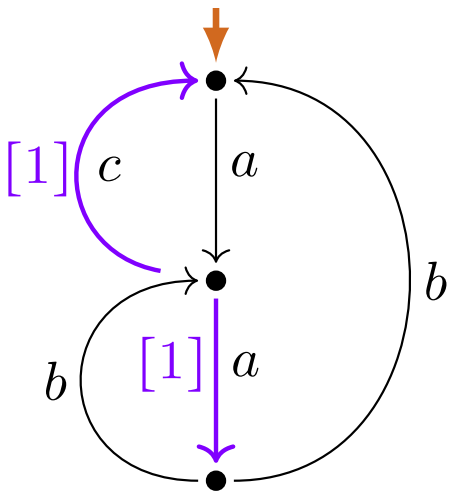
Lemma (*TERMGRAPH '24*)

Process graphs with LLEE are $P^\bullet$-expressible by $(*/\not{1})$ reg. expressions:

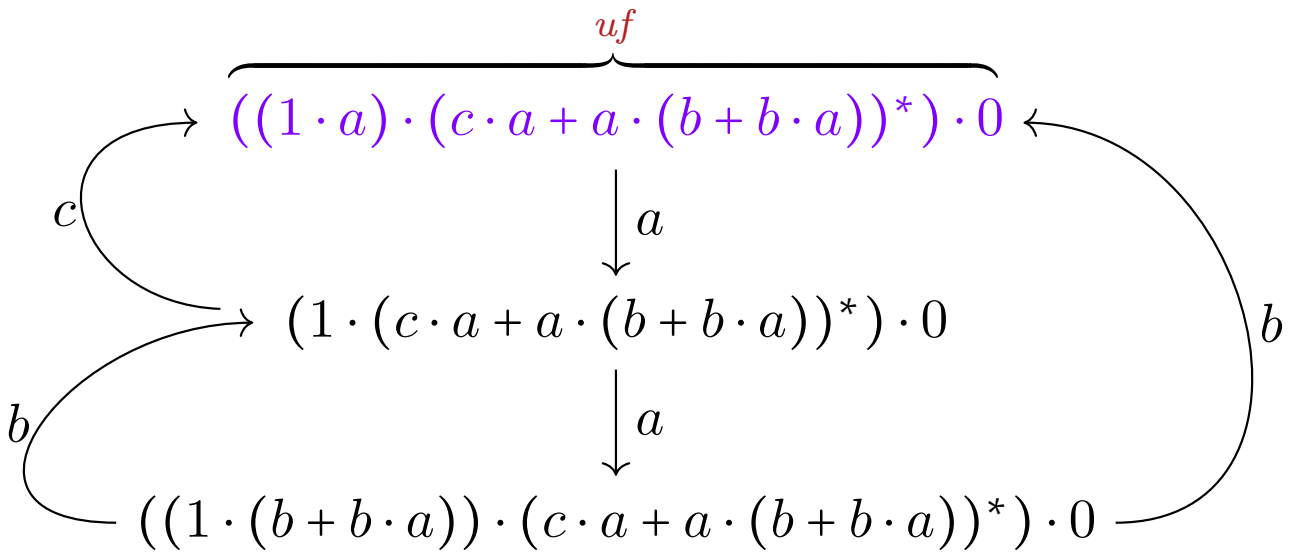
$$\mathsf{LLEE}(G) \implies \exists\, uf \in \mathit{RExp}^{(*/\not{1})} \big(G \twoheadrightarrow P^\bullet(uf) \big).$$

Proof: Define readback steps to precisely reverse $P^\bullet$-defined transitions!

$\widehat{G_4}$



$$P^\bullet(uf) = P(uf) \simeq G_4$$



Characterising the image of $P^\bullet$ for $(*/\not{1})$ expressions

- Regular expressions (unary/binary star, 1-free/under-star-1-free $(*/\not{1})$)
- Milner's process interpretation P/semantics $[\![\cdot]\!]_P$
  - P-/$[\![\cdot]\!]_P$-expressible graphs ($\rightsquigarrow$ expressibility question)
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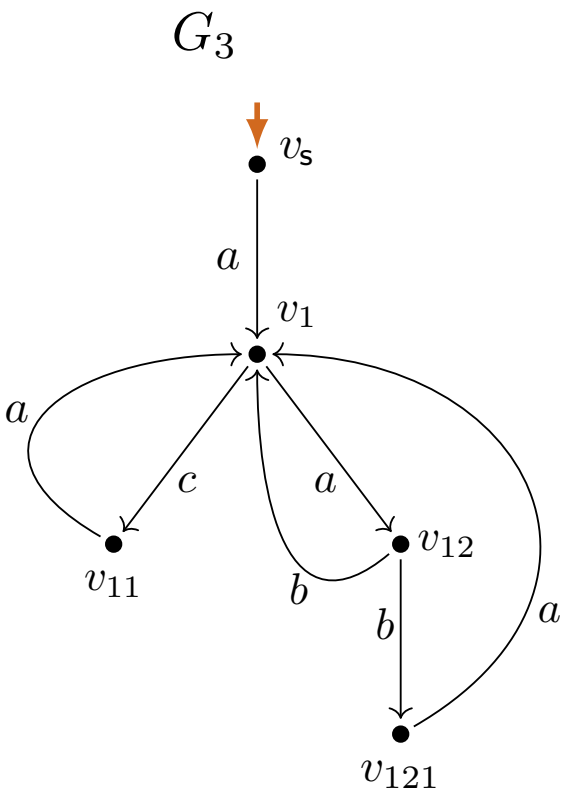
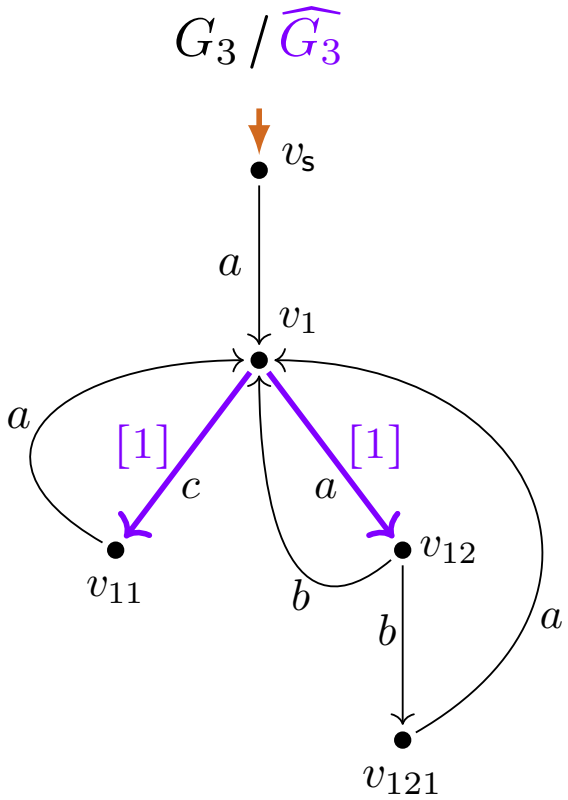
- characterization of image of $P^\bullet$
- Relationship with co-reducibility

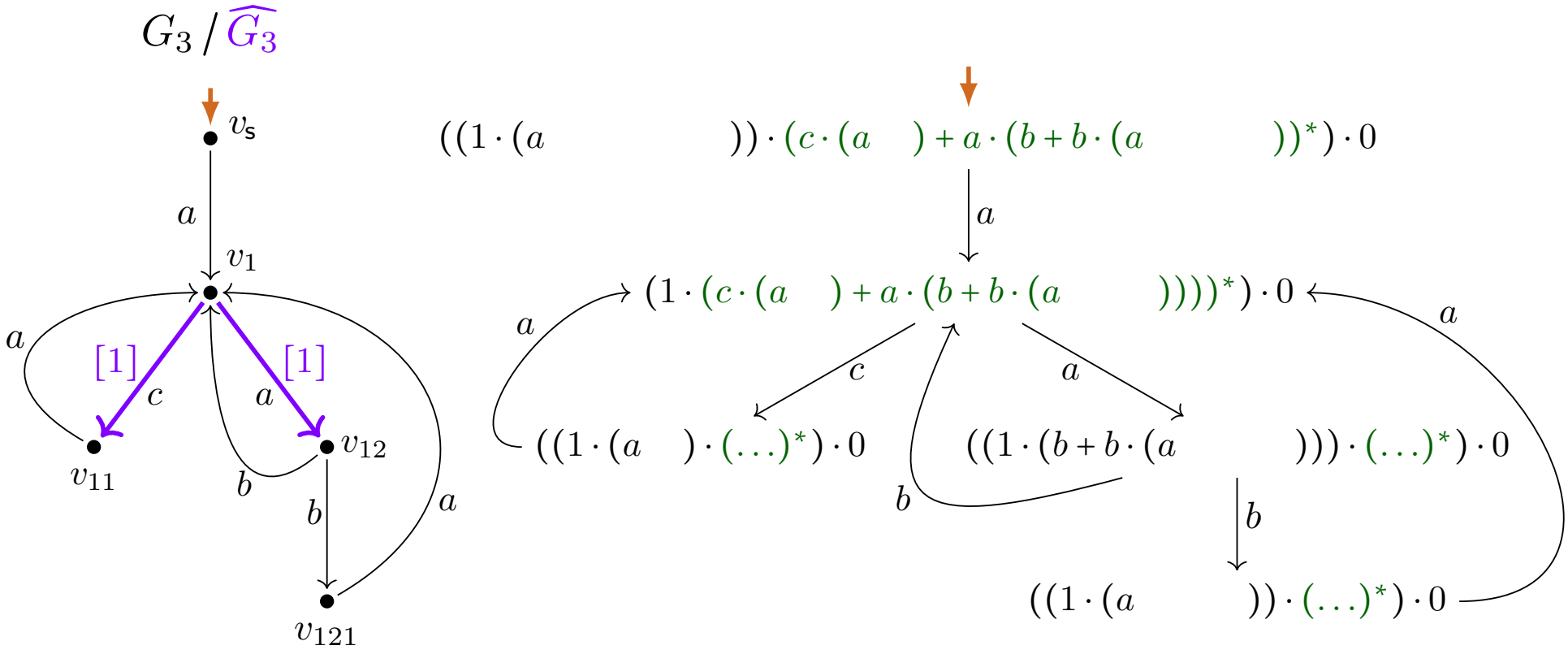
Refined extraction



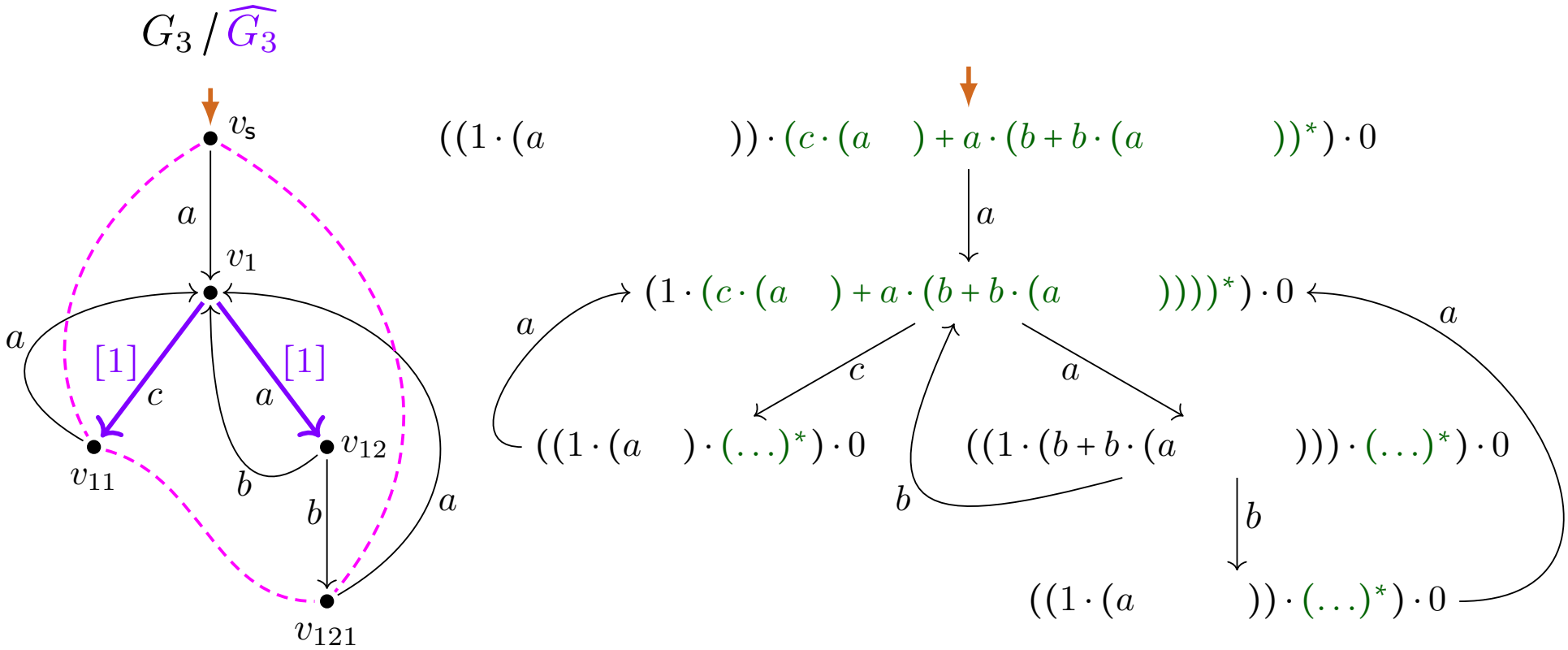
Refined extraction



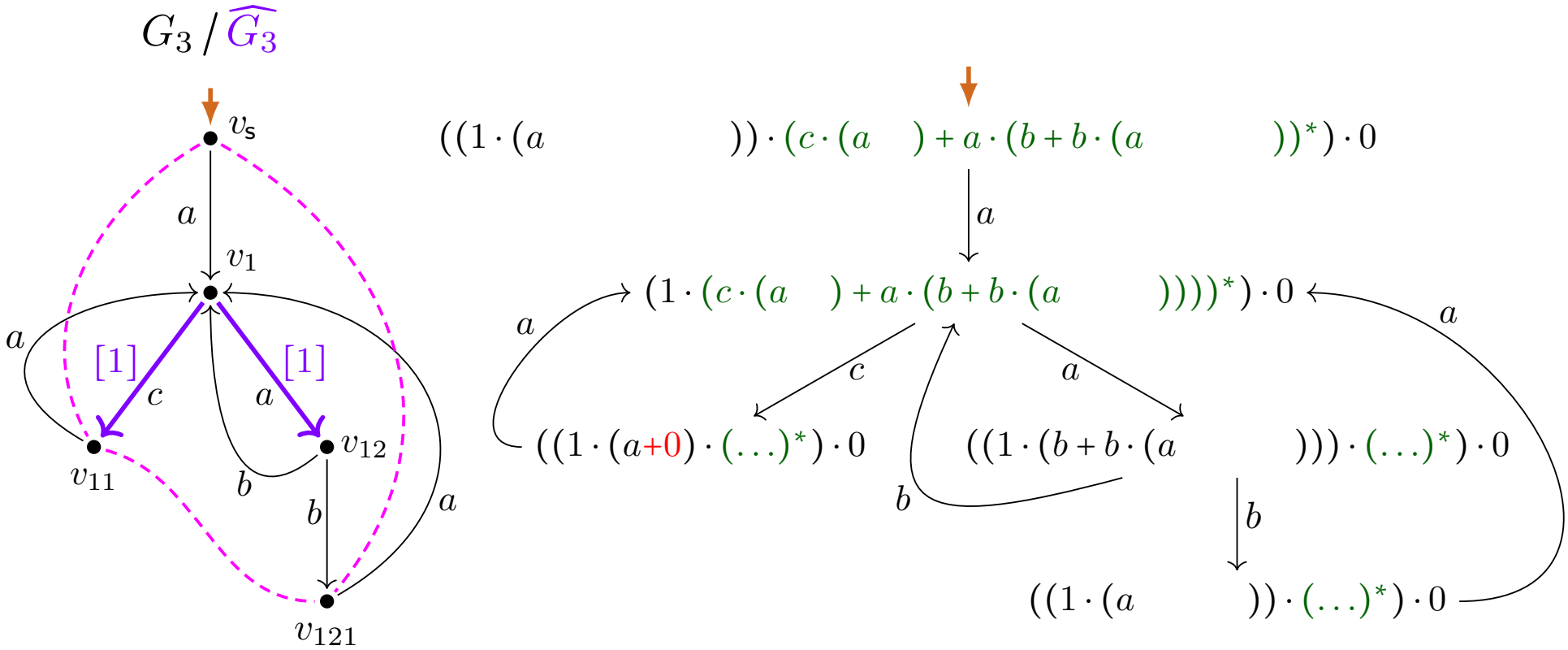
Refined extraction



Refined extraction



Refined extraction, adapted to prevent sharing



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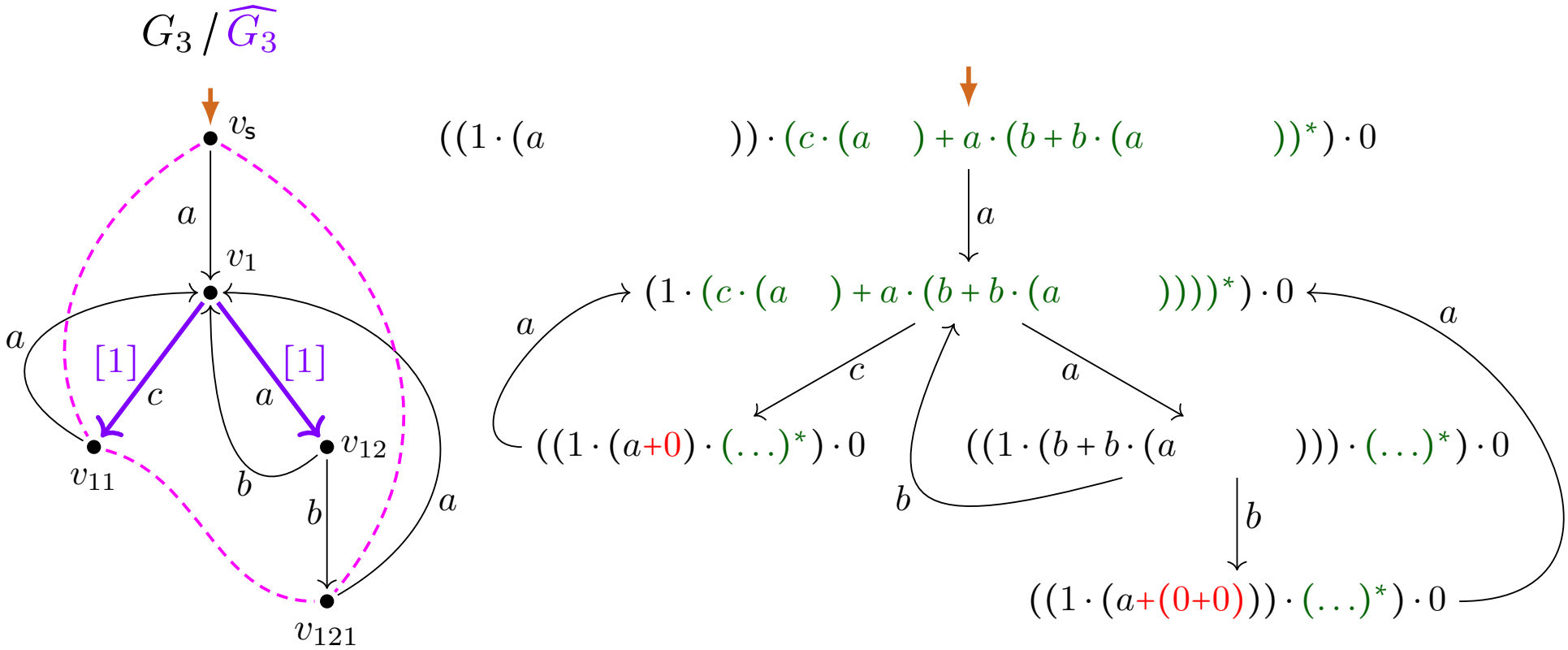

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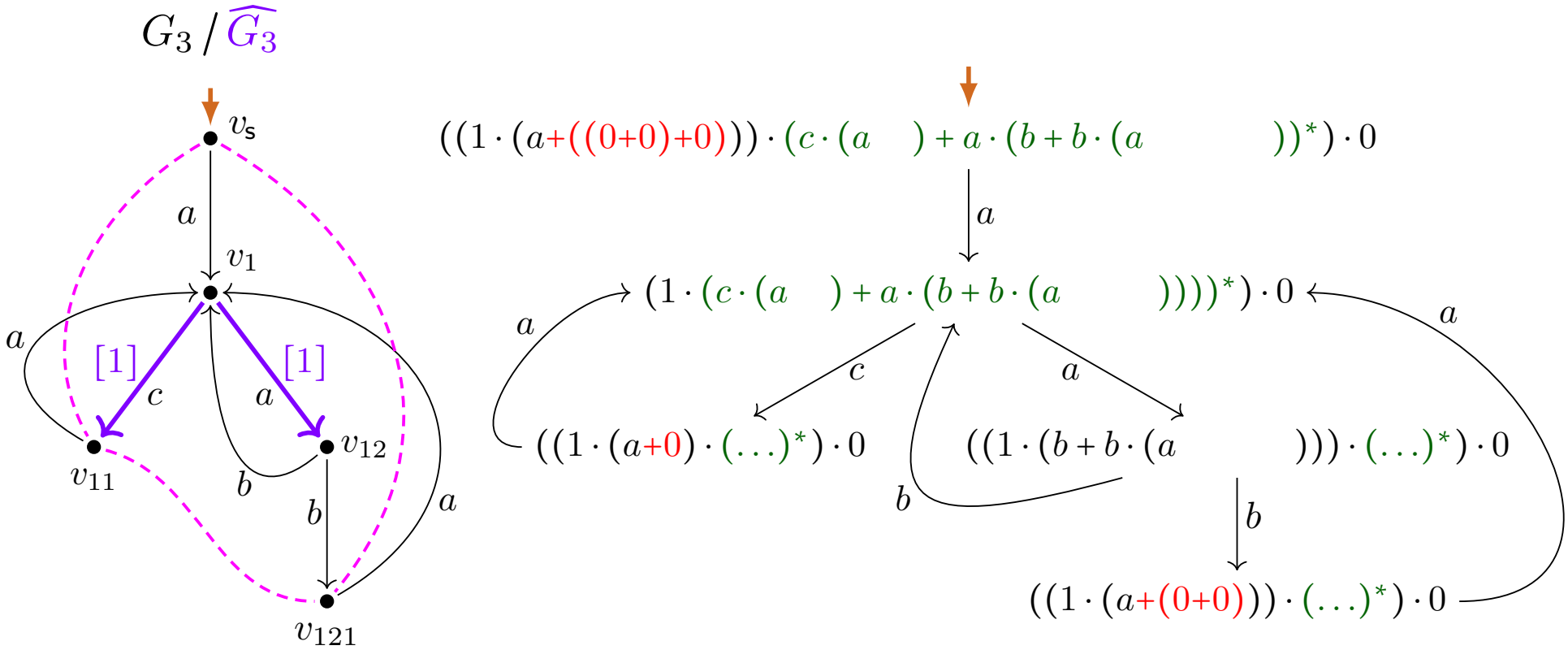

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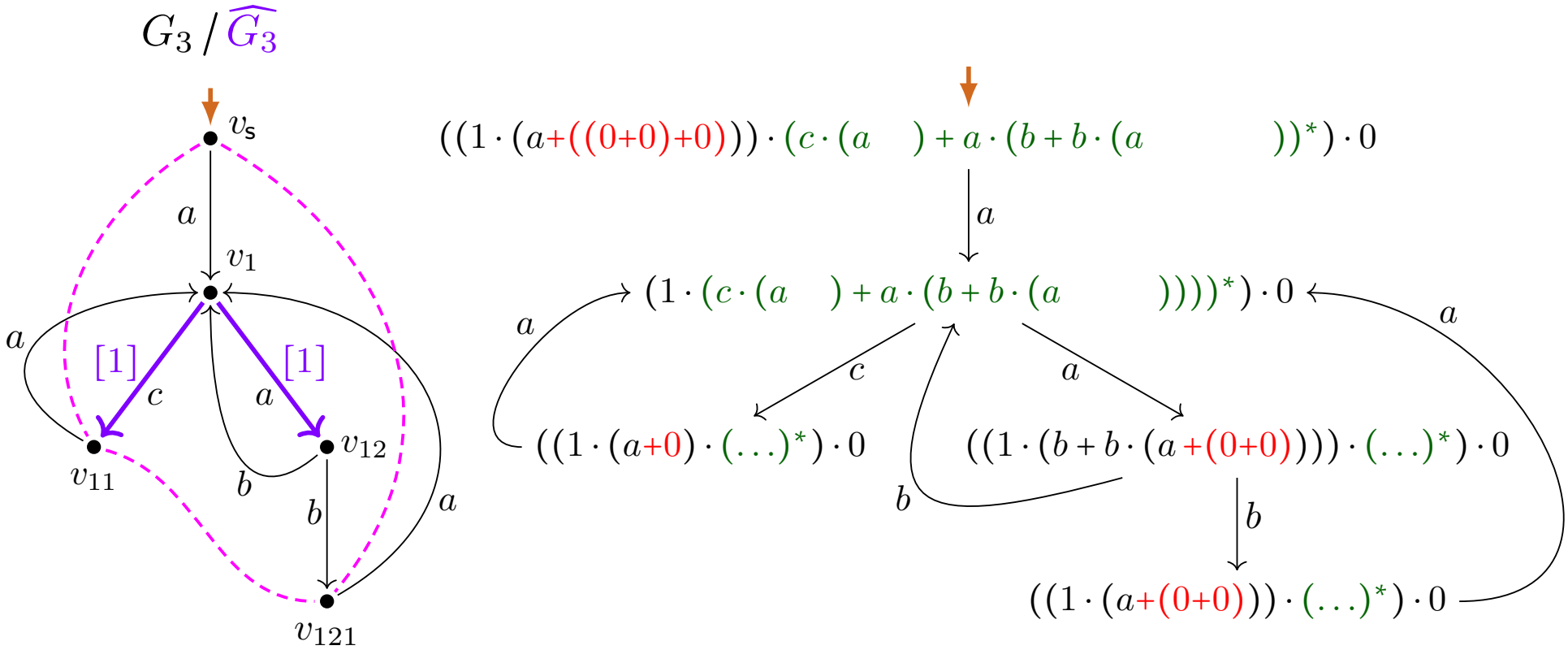

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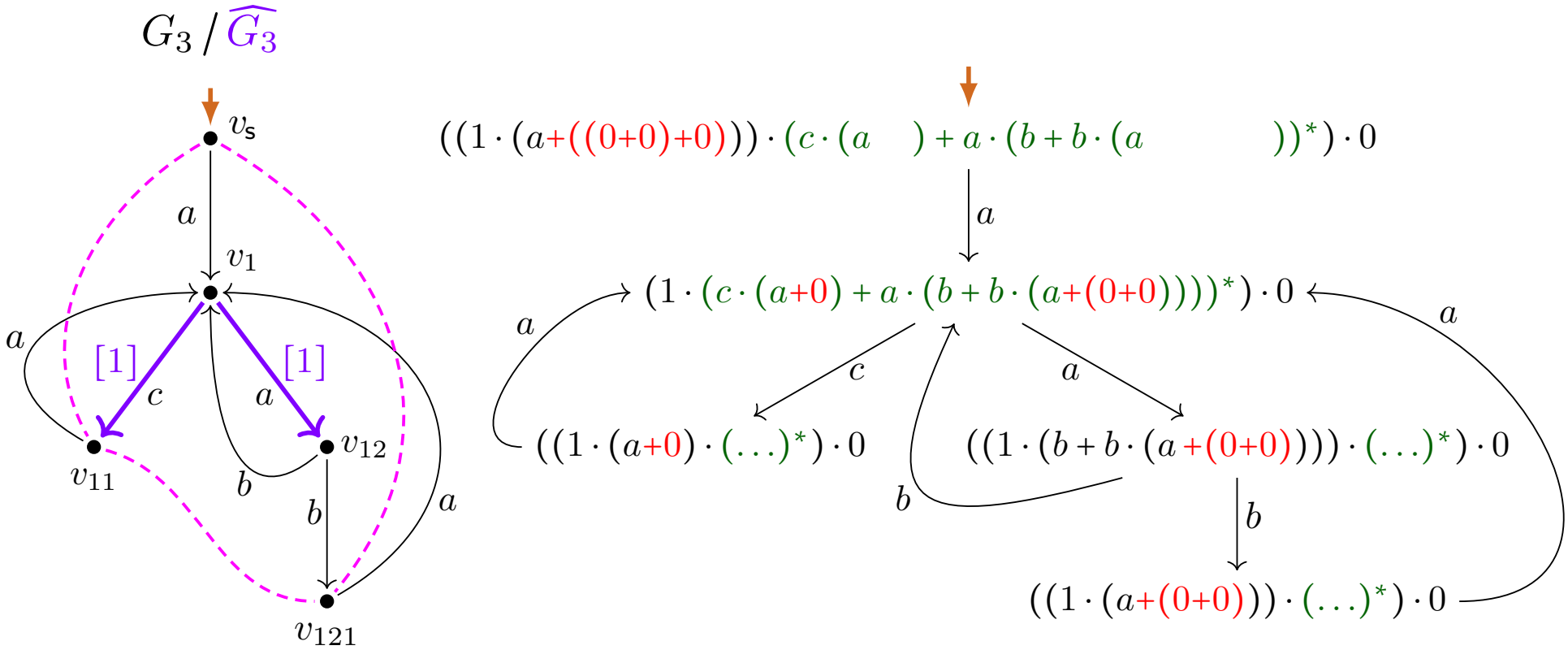

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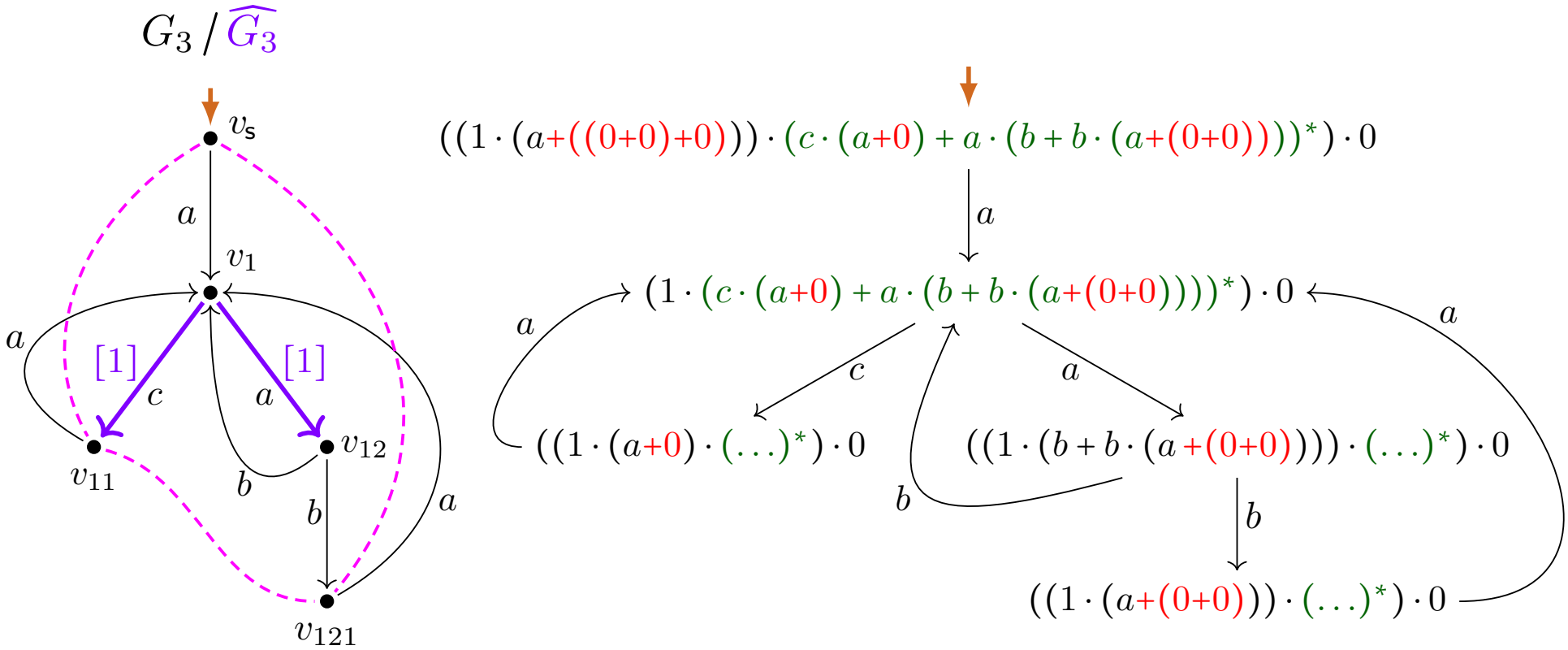

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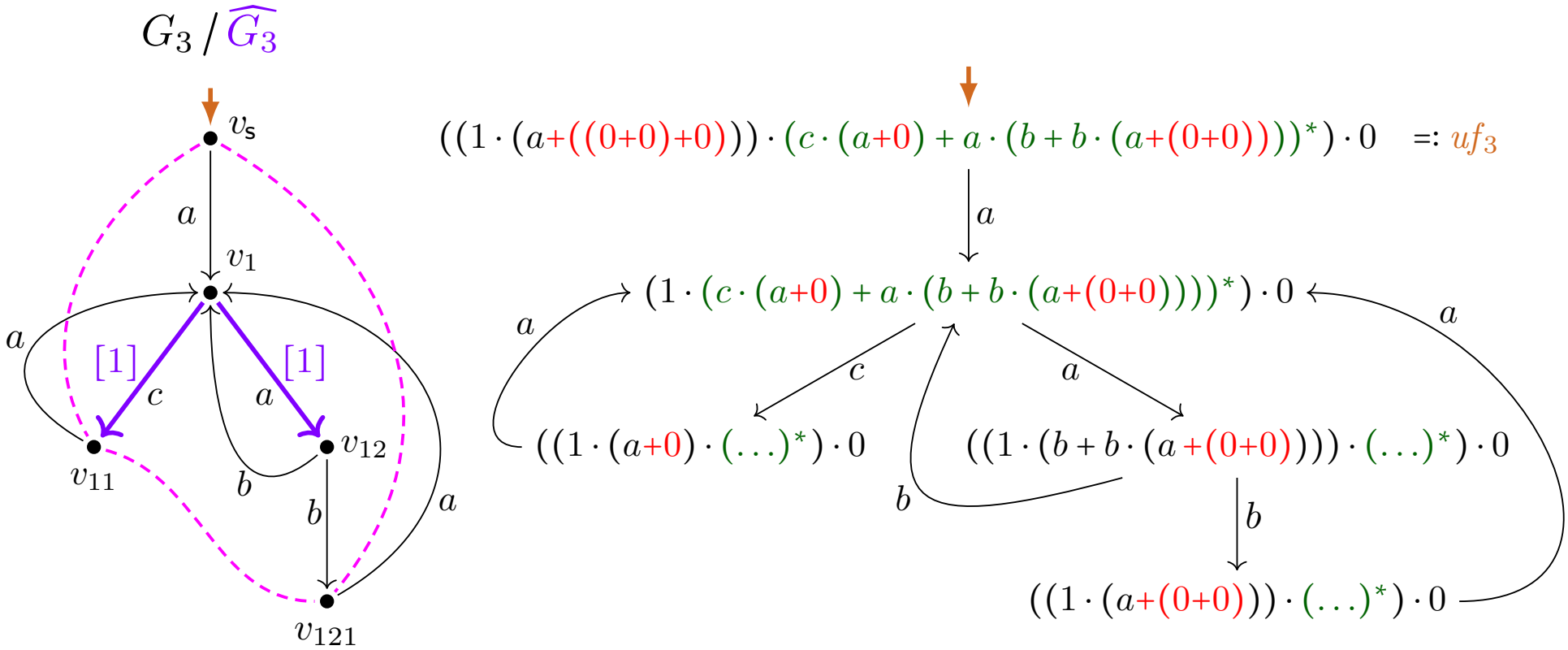

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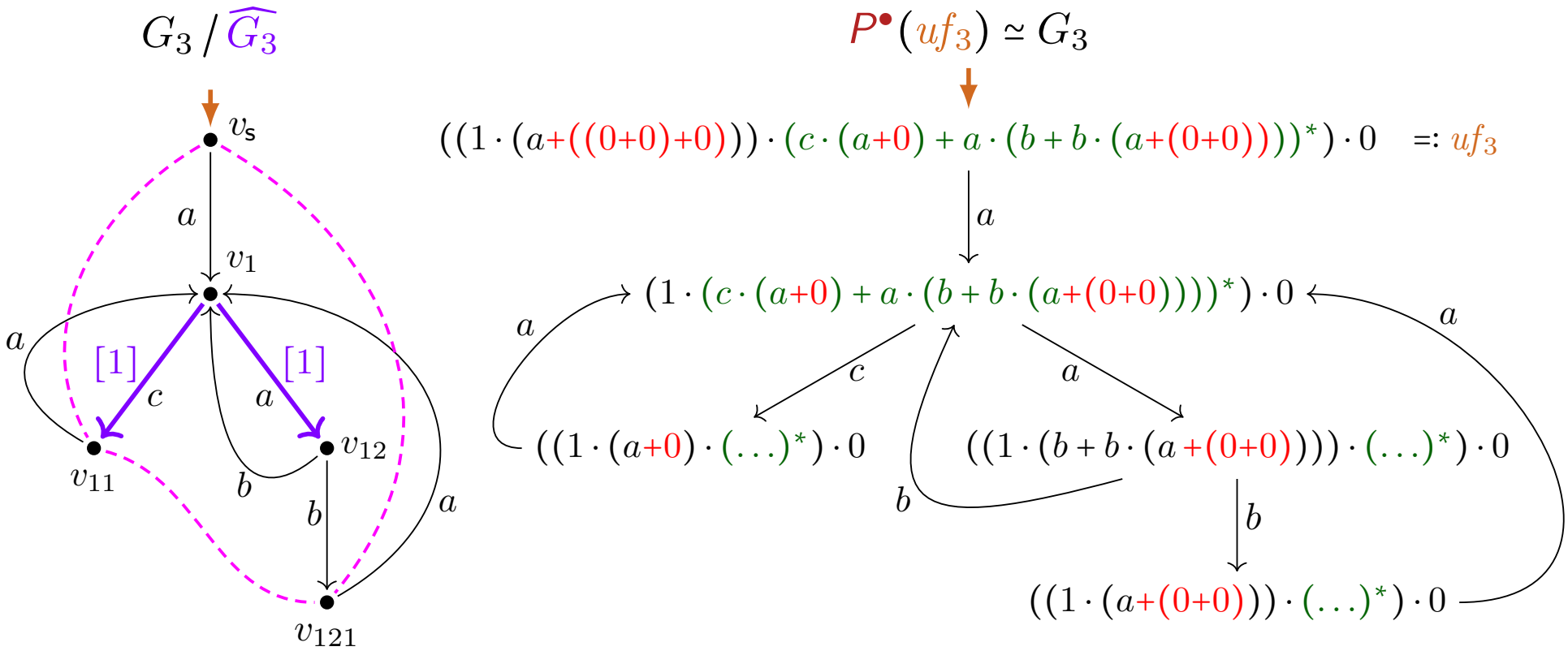
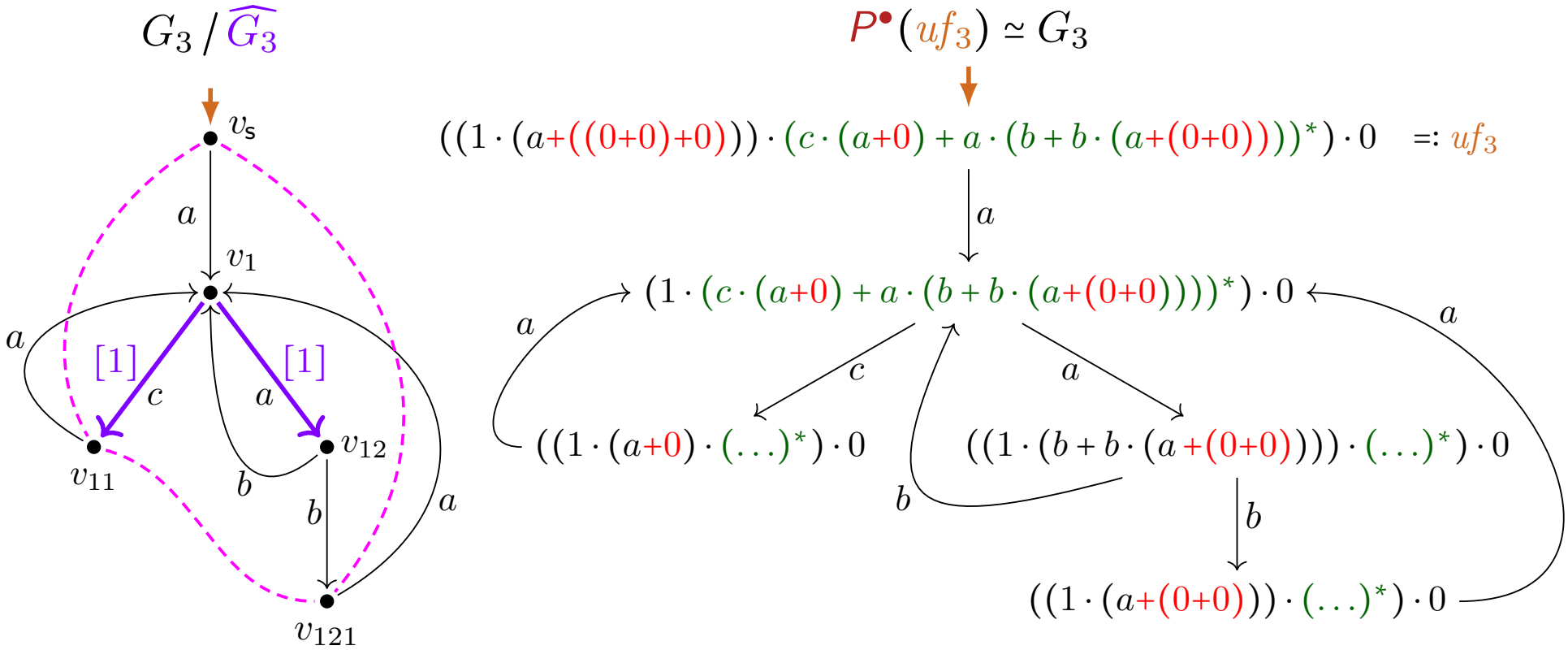


Refined extraction, adapted to prevent sharing

Lemma [Refined extraction, prevent sharing]

From every graph G with LLEE
a $(*/\perp)$ regular expression uf can be extracted such that $G \simeq P^\bullet(uf)$.



Characterizing the image of $P^\bullet$ for $(*/\dot{1})$ expressions

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Characterizing the image of $P^\bullet$ for $(*/\pm)$ expressions

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Statement

A process graph G is $P^\bullet$-expressible by an $(*/\pm)$ regular expression
$\iff$ G is finite, and G satisfies LLEE.

Characterizing the image of $P^\bullet$ for $(*/\pm)$ expressions

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Statement

A process graph G is $P^\bullet$-expressible by an $(*/\pm)$ regular expression
$\iff$ G is finite, and G satisfies LLEE.

That is: $\mathrm{im}(P^\bullet)\,|_{RExp^{(*/\pm)}} = \{G \mid G \text{ is finite and satisfies LLEE}\}$.

Consequence

For every process graph G, TFAE:

(i) G is $[\![\cdot]\!]_P$-expressible by an $(*/\pm)$ regular expression.

(ii) The bisimulation collapse of G is finite, and satisfies LLEE.

(iii) The bisimulation collapse of G
is $P^\bullet$-expressible by a $(*/\pm)$ regular expression.

Characterising the image of $P^\bullet$

- Regular expressions (unary/binary star, 1-free/under-star-1-free $(*/\not{1})$)
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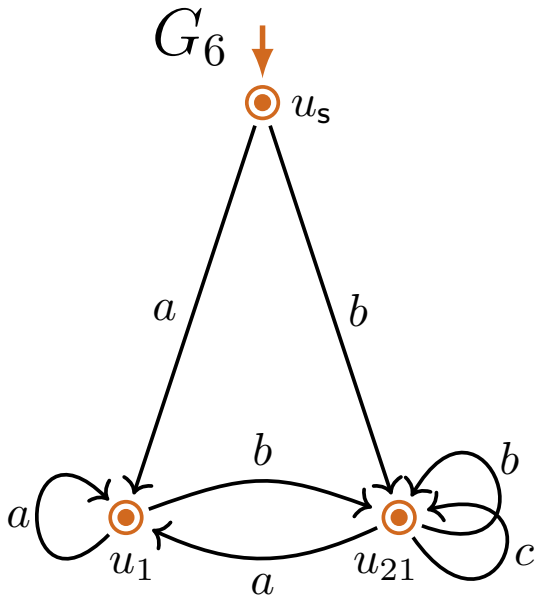
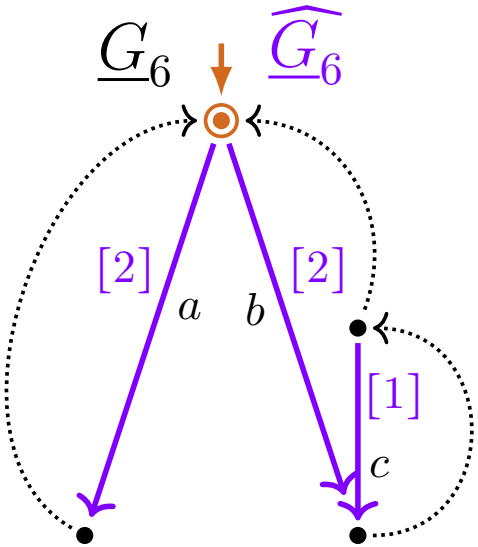
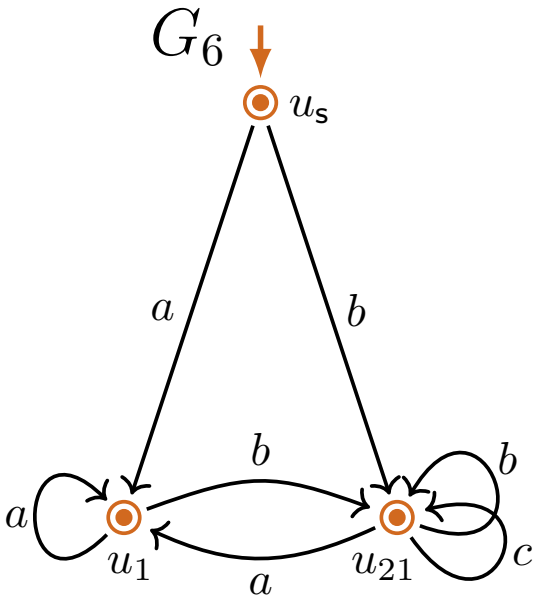
Refined extraction from 1-LLEE graph



$\neg\mathsf{LLEE}((\underline{G_6}])$

Refined extraction from 1-LLEE graph

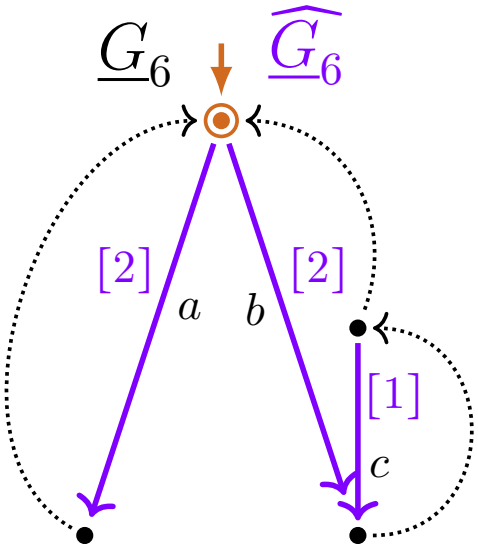


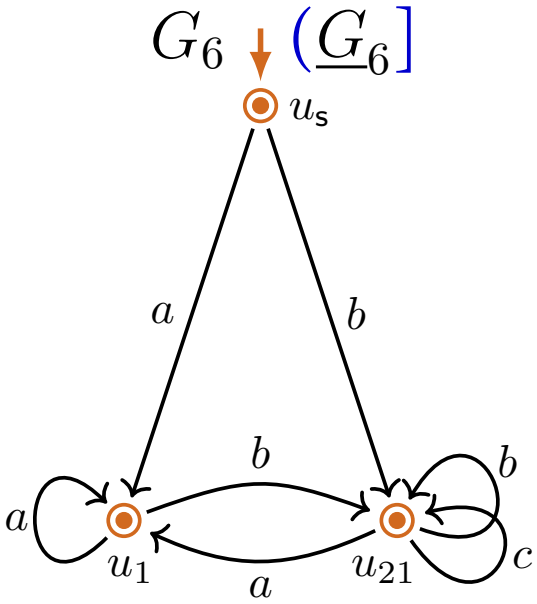


LLEE($\underline{G}_6$)

¬LLEE(($\underline{G}_6$])

Refined extraction from 1-LLEE graph

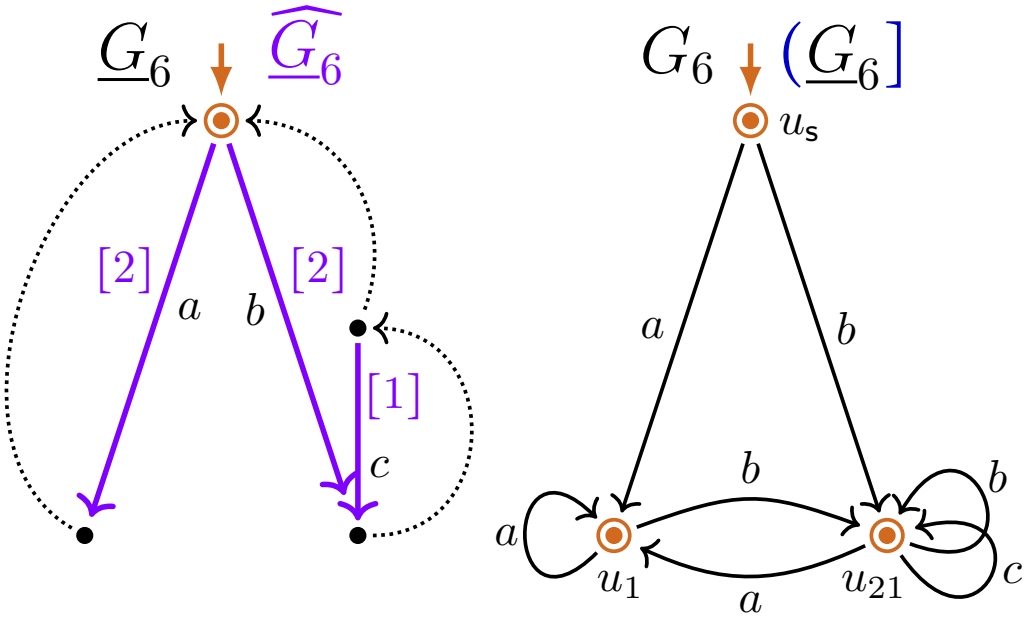




LLEE($\underline{G}_6$)

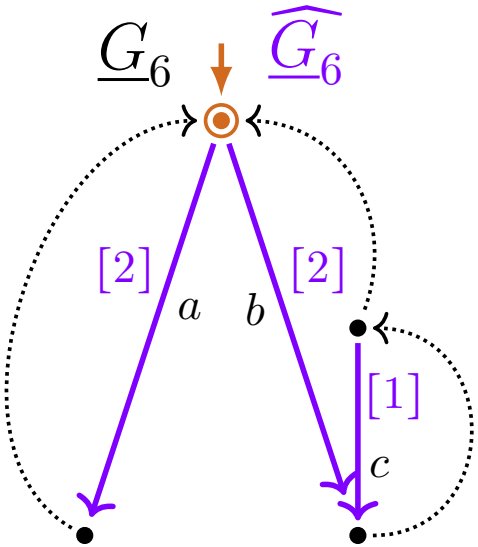
¬LLEE(($\underline{G}_6$])

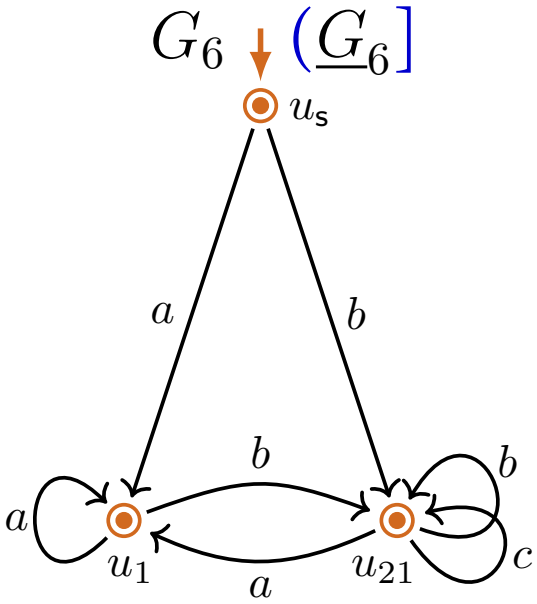
Refined extraction from 1-LLEE graph

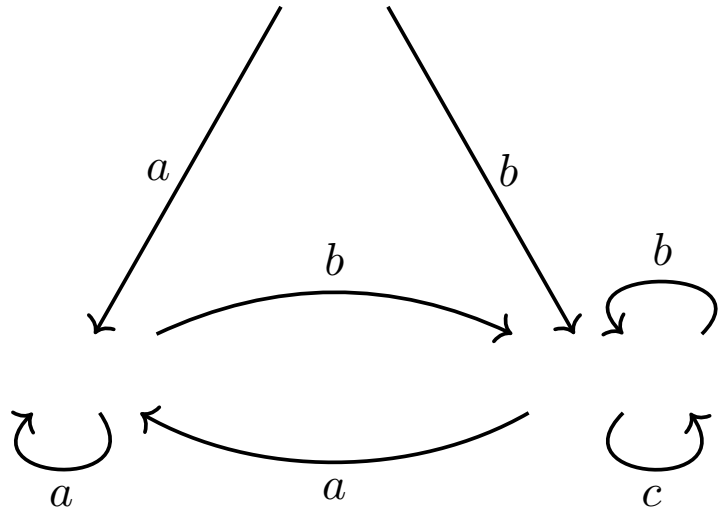


LLEE($\underline{G}_6$) 1-LLEE(($\underline{G}_6$]), ¬LLEE(($\underline{G}_6$])

Refined extraction from 1-LLEE graph

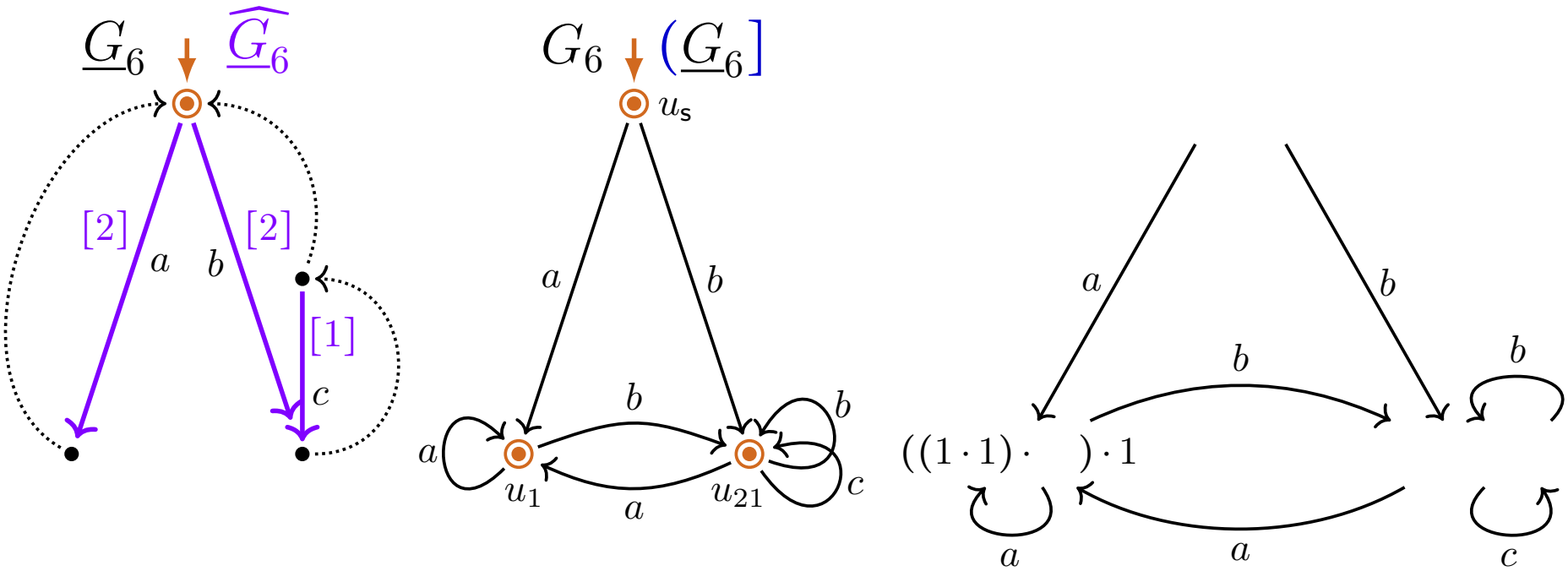






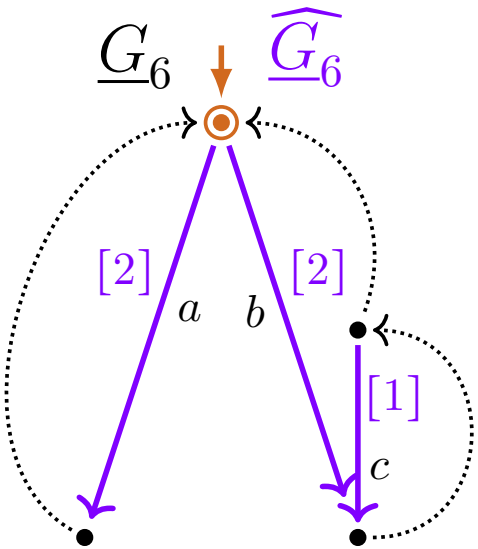
LLEE($\underline{G}_6$) 1-LLEE($(\underline{G}_6]$), ¬LLEE($(\underline{G}_6]$)

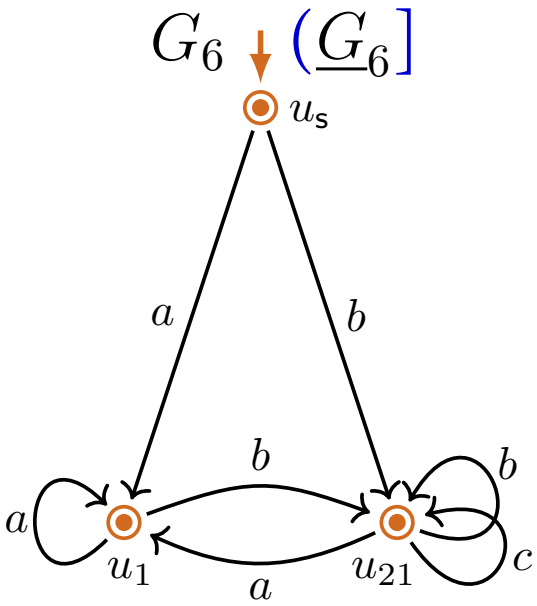
Refined extraction from 1-LLEE graph

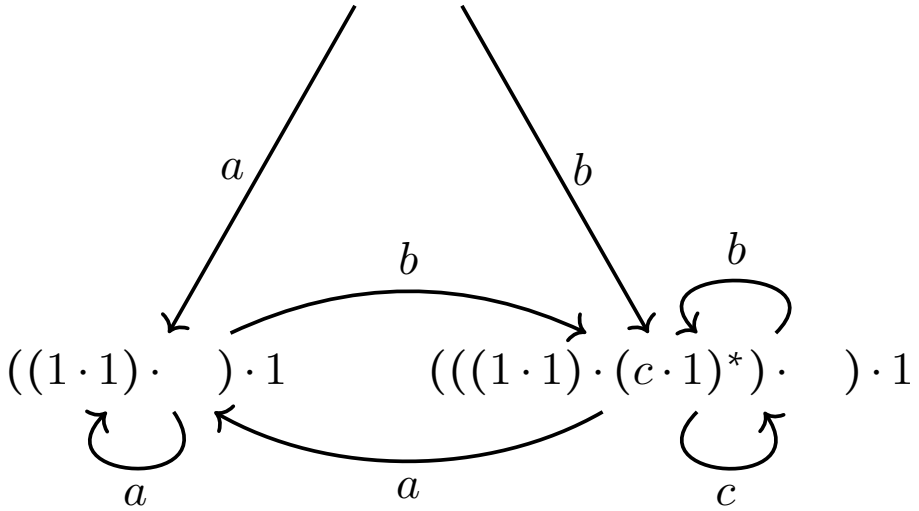


LLEE($\underline{G}_6$) 1-LLEE($(\underline{G}_6]$), ¬LLEE($(\underline{G}_6]$)

Refined extraction from 1-LLEE graph

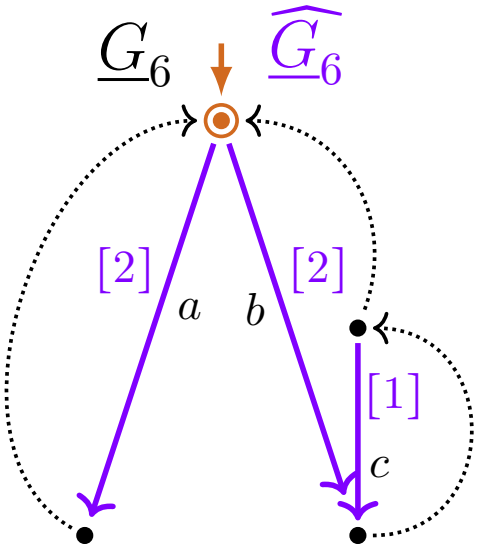


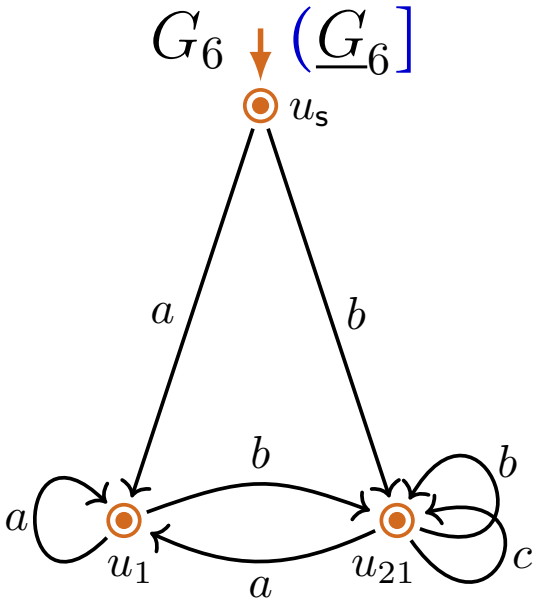


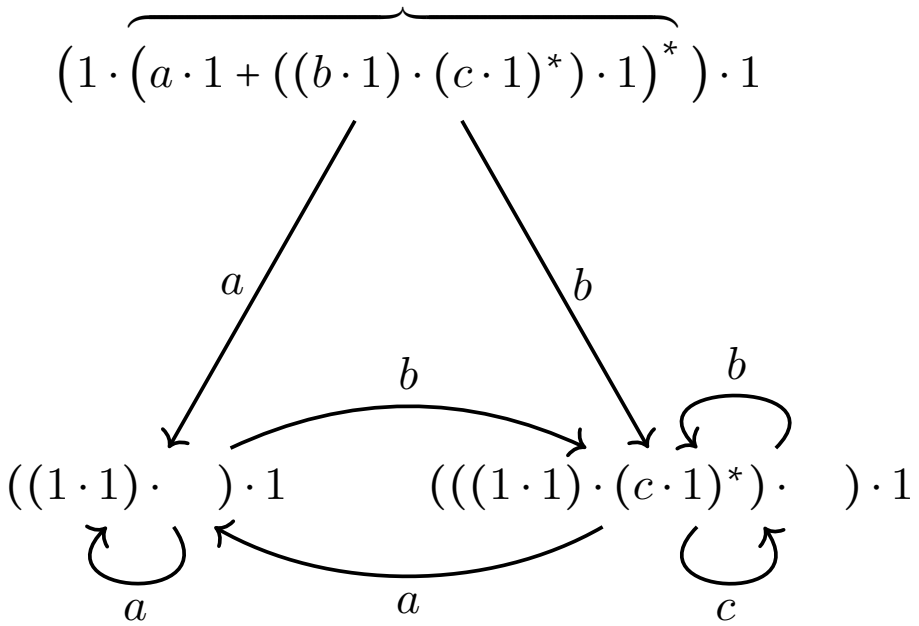


LLEE($\underline{G}_6$) 1-LLEE(($\underline{G}_6$]), ¬LLEE(($\underline{G}_6$])

Refined extraction from 1-LLEE graph

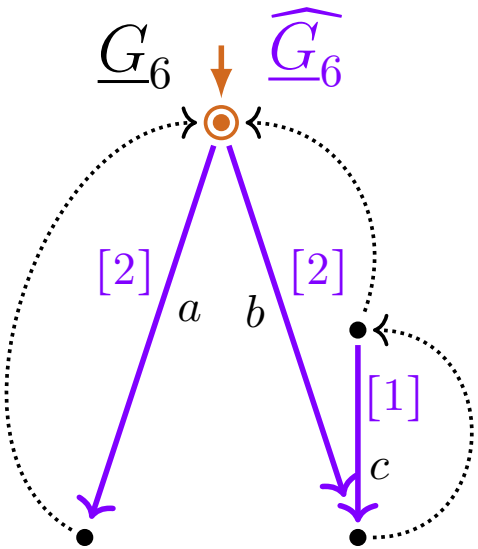


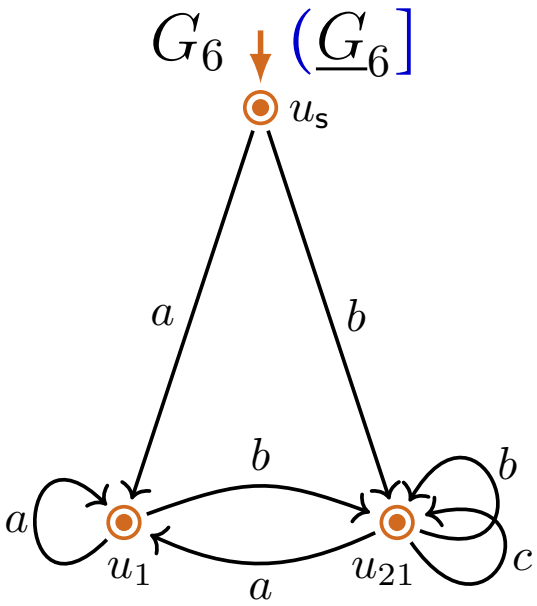


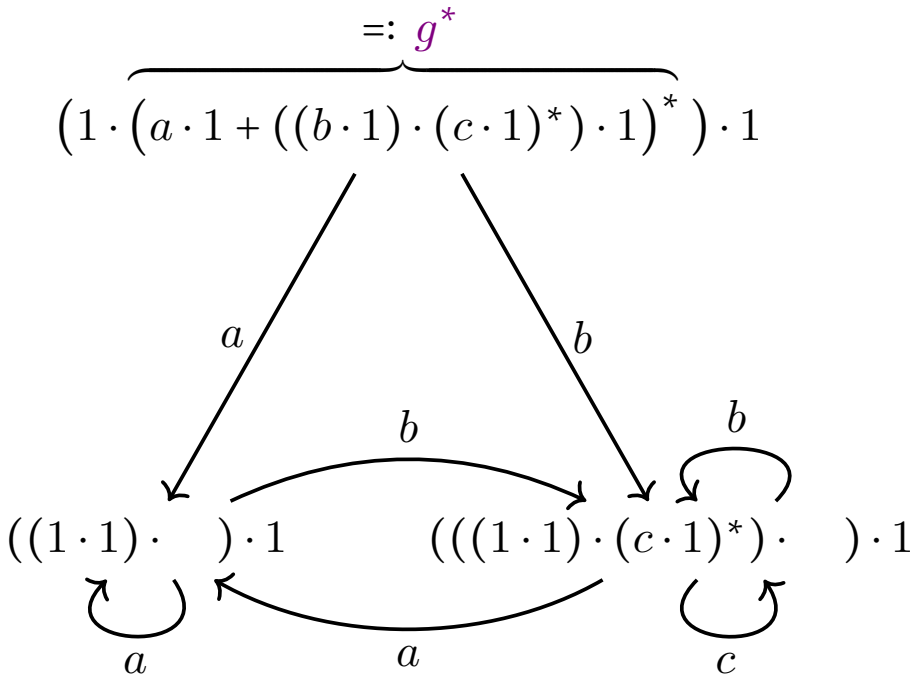


LLEE($\underline{G}_6$) 1-LLEE($(\underline{G}_6]$), ¬LLEE($(\underline{G}_6]$)

Refined extraction from 1-LLEE graph

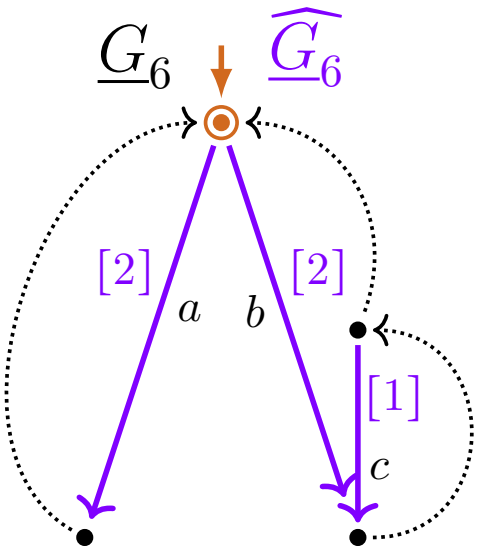


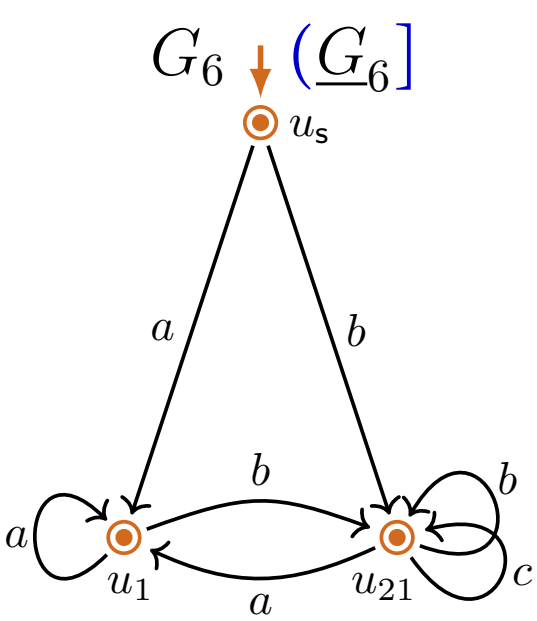


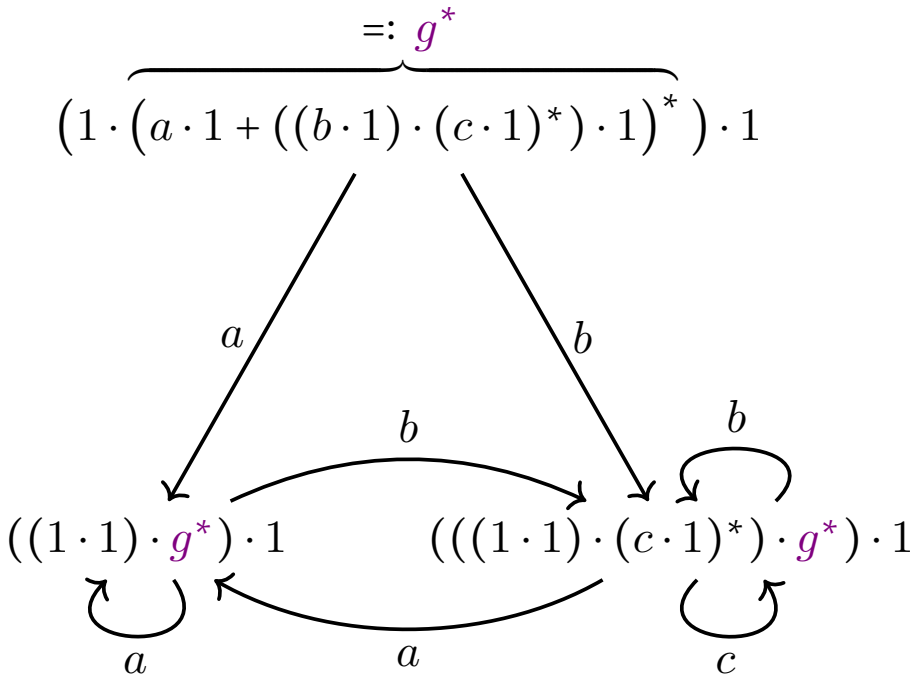


LLEE($\underline{G_6}$) 1-LLEE($(\underline{G_6}]$), ¬LLEE($(\underline{G_6}]$)

Refined extraction from 1-LLEE graph







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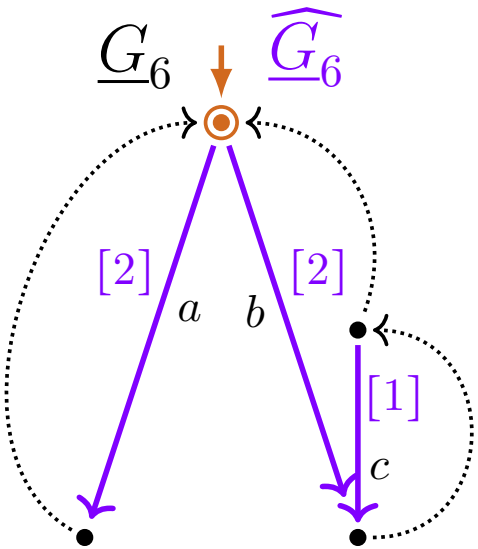

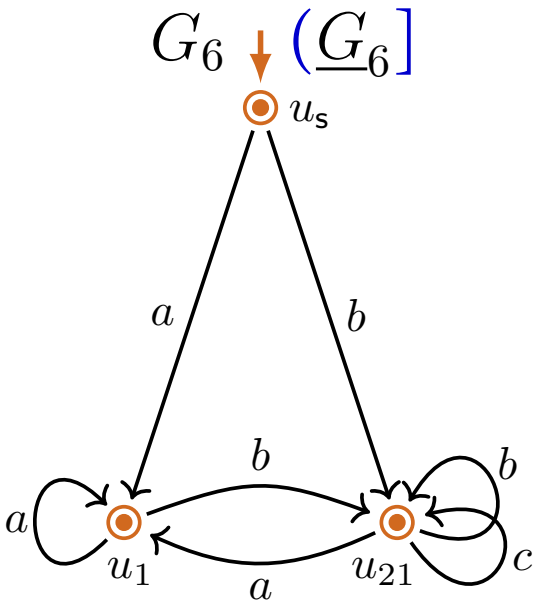

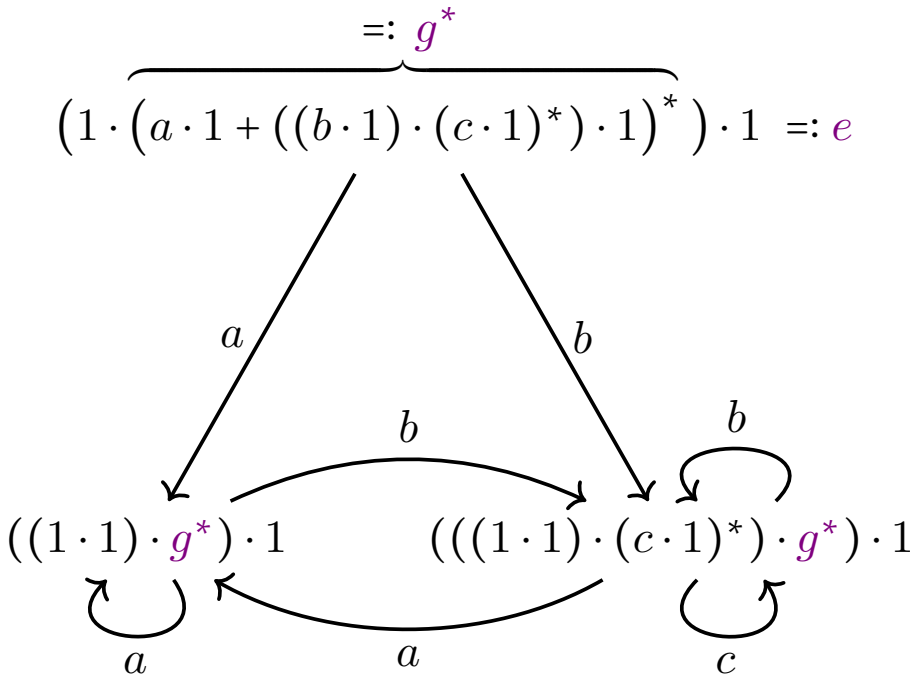

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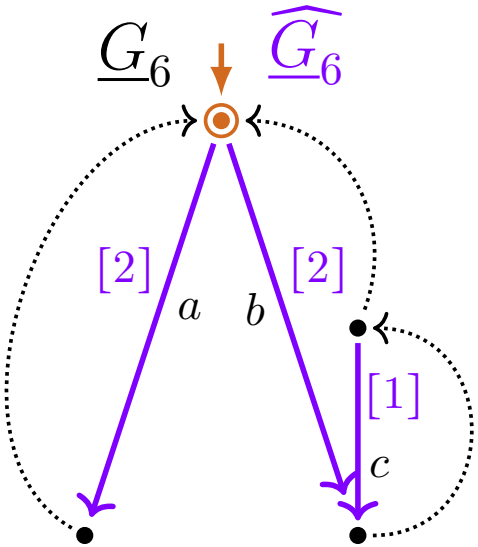
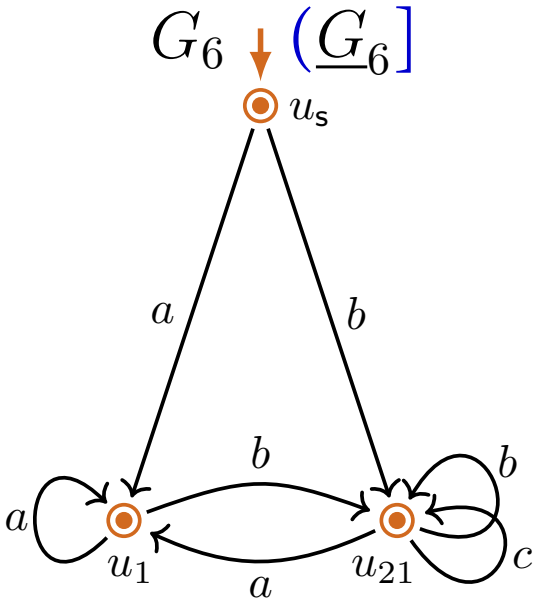
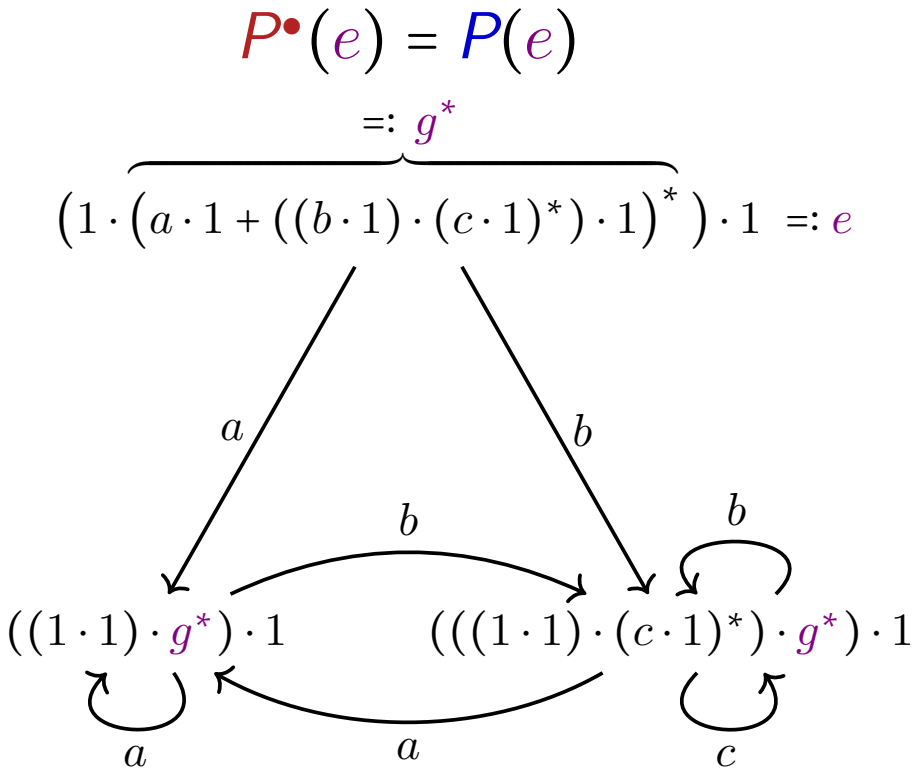
Refined extraction from 1-LLEE graph







LLEE(G_6) 1-LLEE($(G_6]$), ¬LLEE($(G_6]$)

Characterizing the image of $P^\bullet$

Lemma [Refined extraction, prevent sharing]

From every graph G with 1-LLEE
a regular expression e can be extracted such that $G \simeq P^\bullet(e)$.

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$\iff$ G is finite, and G satisfies 1-LLEE.

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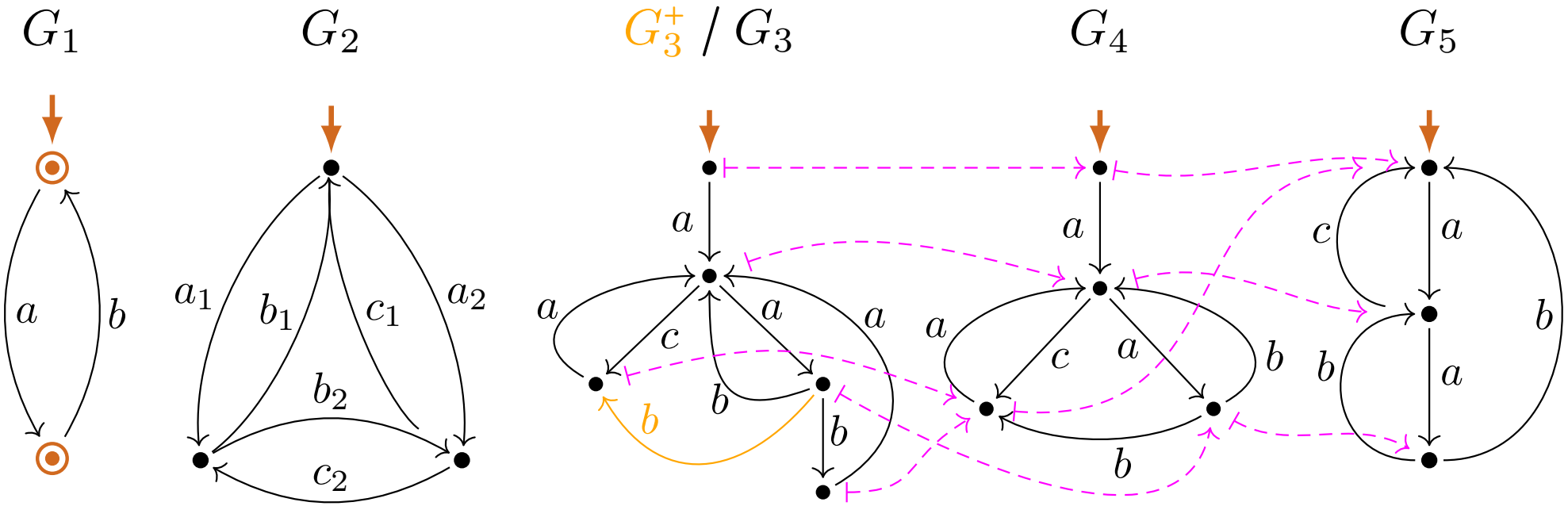
$\iff$ G is finite, and G satisfies 1-LLEE.

That is: $\mathrm{im}(P^\bullet) = \{G \mid G \text{ is finite and satisfies } 1\text{-LLEE}\}$
$= \{(\underline{G}] \mid 1\text{-graph } \underline{G} \text{ is finite and satisfies LLEE}\}$.

$P^\bullet$-expressibility (final results)



G_1	G_2	G_3^+ / G_3	G_4	G_5
not $P^\bullet$-expressible	**not** $P^\bullet$-expressible	**not** $P^\bullet$-expressible / $P^\bullet((*/\perp))$-expr.	$P^\bullet((*/\perp))$-expr.	$P^\bullet((*/\perp))$-expr.
not $[\![\cdot]\!]_P$-expressible	**not** $[\![\cdot]\!]_P$-expressible	$[\![\cdot]\!]_P$-expressible	$[\![\cdot]\!]_P$-expressible	$[\![\cdot]\!]_P$-expressible
not LLEE	**not** LLEE	**not** LLEE / LLEE	LLEE	LLEE
not 1-LLEE	**not** 1-LLEE	**not** 1-LLEE/ 1-LLEE	1-LLEE	1-LLEE

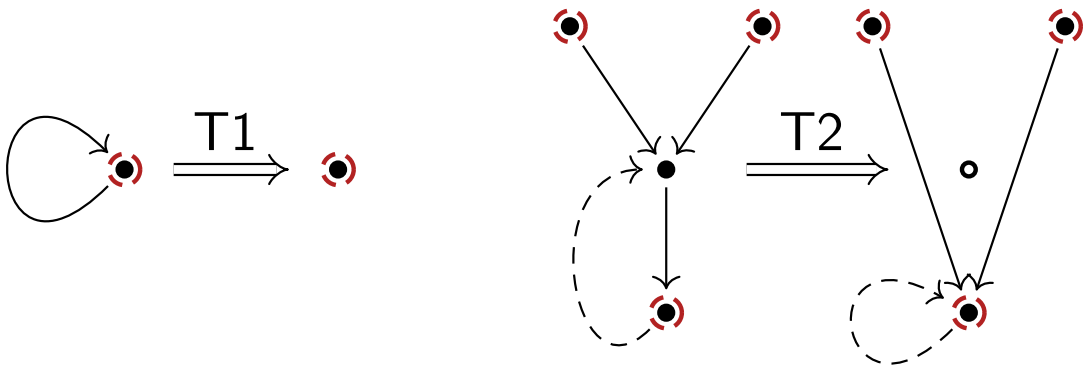
Relationship with Co-Reducibility

- Regular expressions (unary/binary star, 1-free/under-star-1-free $(*/\not{1})$)
- Milner's process interpretation P/semantics $[\![\cdot]\!]_P$
  - P-/$[\![\cdot]\!]_P$-expressible graphs ($\rightsquigarrow$ expressibility question)
- Layered Loop existence and elimination (LLEE/1-LLEE)
  - interpretation/extraction correspondences with $(*/\not{1})$/all reg. expr.
  - LLEE is preserved by bisimulation collapse (1-LLEE is not)
- TERMGRAPH 2024: compact process interpretation $P^\bullet$
  - image of $P^\bullet$ is closed under bisimulation collapse (image of P is not)
- characterisation of image of $P^\bullet$ restricted to $(*/\not{1})$ expressions
- characterization of image of $P^\bullet$
- Relationship with co-reducibility

LLEE = co-reducible$^+$ to acyclic graph

Connection to work by Laarakker and Kappé (TERMGRAPH 2026) on:

- co-reducibility of state graphs,
- based on flow graph reducibility by Hecht and Ullman (1972).



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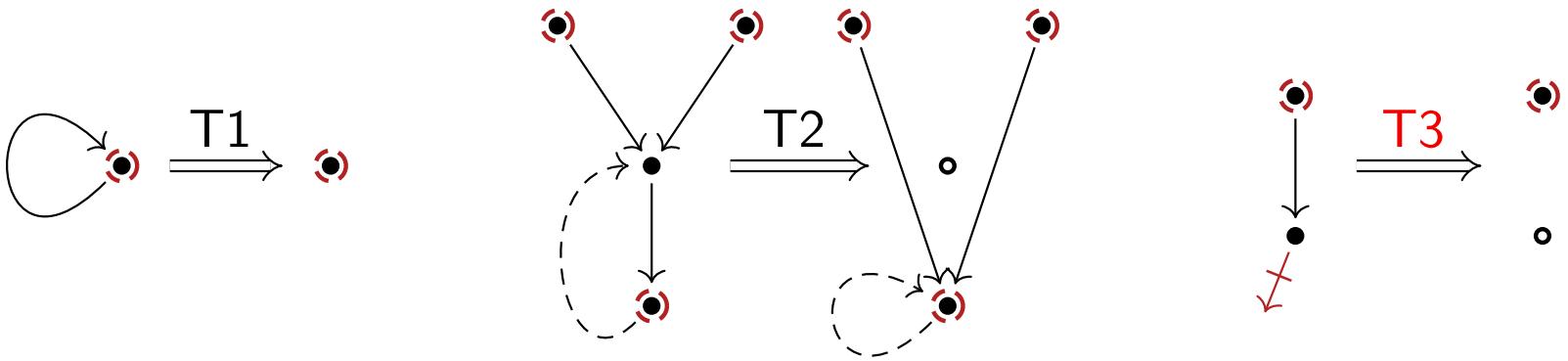

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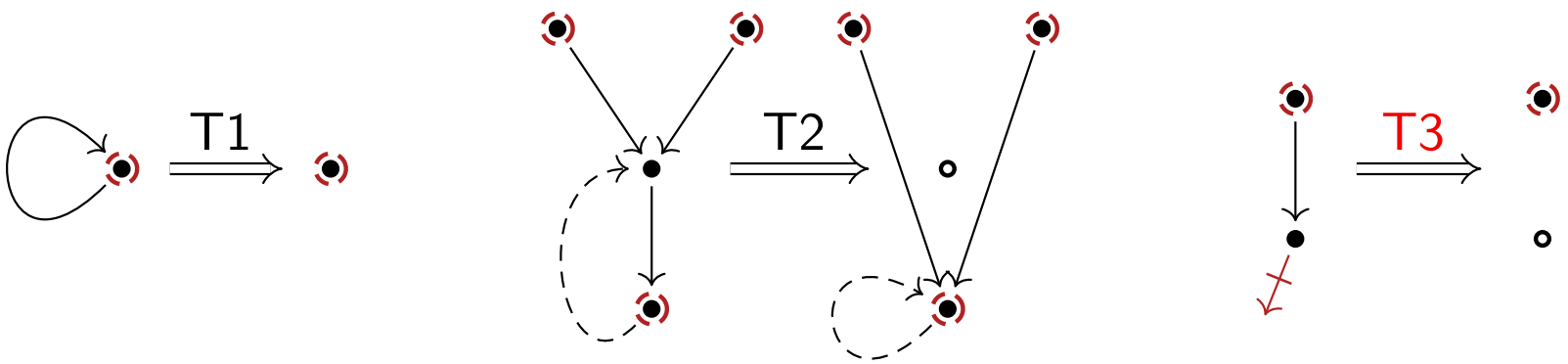

Statement

A process graph G has the property LLEE

$\iff$ the state graph of G is co-reducible$^+$ to an acyclic graph.

Summary

- Regular expressions (unary/binary star, 1-free/under-star-1-free $(*/\dot{1})$)
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IWC and IFIP-1.6 Talks

IWC (Friday, 24 July)

CG: *Loop Elimination in Process Graphs is Confluent When Pruning Steps are Added*

Ext. Abstract https://clegra.github.io/lf/le-conf-IWC-26.pdf

- critical-pair like analysis of loop elimination steps $\longrightarrow_{\text{elim}}$
- $\longrightarrow_{\text{elim}}$ forks are joinable by $\longrightarrow_{\text{elim}}$ steps and prune steps $\longrightarrow_{\text{prune}}$

IFIP-1.6 meeting (yesterday)

CG: *Finding Small Steps for Hard Problems, in Work on the Process Interpretation of Regular Expressions*

Slides https://clegra.github.io/lf/IFIP-1_6-2026.pdf

- Reducing for Graphs Representing Regular Expressions under Bisimilarity

Resources

- Slides/extended abstract on clegra.github.io
  - slides: .../lf/TG-26.pdf
  - extended abstract: .../lf/pi-struc-TG-26.pdf
- CG: Closing the Image of the Process Interpretation of 1-Free Regular Expressions Under Bisimulation Collapse
  - TERMGRAPH 2024, extended abstract
- CG: The Image of the Process Interpretation of Regular Expressions is Not Closed under Bisimulation Collapse
  - arXiv:2303.08553, 2021/2023.
- CG: Milner's Proof System for Regular Expressions Modulo Bisimilarity is Complete
  - LICS 2022, arXiv:2209.12188, poster.
- CG, Wan Fokkink: A Complete Proof System for 1-Free Regular Expressions Modulo Bisimilarity,
  - LICS 2020, arXiv:2004.12740, video on youtube.
- CG: Modeling Terms by Graphs with Structure Constraints
  - TERMGRAPH 2018, EPTCS 288, arXiv:1902.02010.