# Avoiding Repetitive Reduction Patterns in Lambda Calculus with letrec 

(Work In Progress)

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#### Abstract




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## In this talk

We report on:

- an optimising transformation for $\lambda$-calculus with letrec
- by which i.p. the cyclic passing on of unchanged arguments during evaluation can often be prevented

Examples:

- Haskell functions repeat, replicate, ++ , map, until
- a specification of the Thue-Morse sequence


## In this talk

We report on:

- an optimising transformation for $\lambda$-calculus with letrec
- by which i.p. the cyclic passing on of unchanged arguments during evaluation can often be prevented

Examples:

- Haskell functions repeat, replicate, ++ , map, until
- a specification of the Thue-Morse sequence

Concepts used:

- visible/concealed redexes
- generalised $\beta$-reduction
- domination in digraphs
- static analysis of cyclically reappearing redexes
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## $\lambda$-Terms and $\lambda$-Trees

| $T:$ | $:=$ | $V$ |
| :---: | :---: | :---: |
| $\mid$ | $T T$ |  |
|  |  | $\lambda V . T$ |

(variable)
(application)
(abstraction)
$(\lambda x . g(f x)) 3$

$\beta$－Reduction
$(\lambda x . M) N \rightarrow_{\beta} \quad M[x:=N]$

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\begin{array}{ccc}
(\lambda x . M) N & \rightarrow_{\beta} & M[x:=N] \\
& \overbrace{x}^{@} \_{3}
\end{array}
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$$
(\lambda x . g(f x)) 3 \rightarrow_{\beta} \quad g(f 3)
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## letrec-Terms and $\lambda$-Graphs

| T : $:=$ | v | (variable) |
| :---: | :---: | :---: |
|  | $T$ T | (application) |
|  | $\lambda V . T$ | (abstraction) |
|  | $f(T, \ldots, T)$ | (primitive functions) |
|  | let Defs in $T$ | (letrec) |
| Defs : : $=$ | $v_{1}=T \ldots v_{n}=T$ | (equations) |

let repeat $=\lambda x . x$ : repeat $x$ in repeat

## letrec-Terms and $\lambda$-Graphs


(variable)
(application)
(abstraction)
(primitive functions)
(letrec)
(equations)
let repeat $=\lambda x . x$ : repeat $x$ in repeat



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 let repeat $=\lambda x \cdot x:$ repeat $x$
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let repeat $=\lambda x \cdot x:$ repeat $x \quad$ in repeat
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## $x_{x}:$ repeat $x$




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## Visible and concealed redexes <br> 路

Common practice in existing compilers:

- Exhaustive reduction of visible redexes






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Exhaustive reducion of vibe redex




## Visible and concealed redexes



Common practice in existing compilers:

- Exhaustive reduction of visible redexes
- This is in general not possible for concealed redexes
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## repeat 3

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\begin{aligned}
& \text { let repeat }=\lambda x \cdot x: \text { repeat } x \\
& \text { in repeat } 3
\end{aligned}
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## Optimising repeat

let repeat $=\lambda x$. let $x s=x: x s$ in $x s$
in repeat
let repeat $=\lambda x$. let $x s=x: x s$ in $x s$
in repeat
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$\underset{\substack{\text { Iet repeat } \\ \text { in repeat }}}{\text { lox．let } x s=x: x s \text { in } x s}$

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$\begin{aligned} & \text { replicate } 0 \\ & \text { replicate } n x=[]\end{aligned}$
repreplicaticate $n x=1) x$
ret rec $0=[]$
rec $n=x:$ rec $(n-1)$
in rec $n$


Avoiding Repetitive Reduction Patterns in $\lambda_{l e t r e c}$
replicate
$\begin{aligned} & \text { replicate } 0 x= \\ & \text { replicate } n x= x: r e p l i c a t e(n-1) x \\ & \text { replicate } n x= \text { let } \operatorname{rec} 0=[] \\ & \quad \text { in ec } n=x: \operatorname{rec}(n-1) \\ & \\ &\end{aligned}$




Avoiding Repetitive Reduction Patterns in $\lambda_{l e t r e c}$
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## replicate - generalised $\beta$-reduction

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## Generalised $\beta$－Reduction

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Generalised $\beta$－Reduction









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replicate - duplication of the function body


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Avoiding Repetitive Reduction Patterns in $\lambda_{\text {letrec }}$ .


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## replicate - duplication of the function body



## replicate - duplication of the function body


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## replicate - header trick



## replicate - header trick



## replicate - header trick


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## Rewrite Rule Formulation <br> Rewrite Rule Formulation <br> $\qquad$

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\begin{align*}
& f=\lambda x_{1} \ldots \lambda x_{n} \cdot \lambda y \cdot C\left[f t_{1} \ldots t_{n} y\right] \\
& \quad \rightarrow \\
& f=\lambda x_{1} \ldots \lambda x_{n} \cdot \lambda y . \\
& \quad \begin{array}{l}
\text { let } f^{\prime}=\lambda x_{1} \ldots \lambda x_{n} \cdot C\left[f^{\prime} t_{1} \ldots t_{n}\right] \\
\\
\text { in } f^{\prime} x_{1} \ldots x_{n}
\end{array}
\end{align*}
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## Rewriting repeat號

let repeat $=\lambda x . x$ ：repeat $x$
let repeat $=\lambda x$ ．let $x s=x: x s$ in $x s$ let repeat
in repeat

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in repeat
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$t=\lambda x \cdot x:$ repeat $x$

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& \begin{aligned}
\text { replicate } 0 x= & {[] } \\
\text { replicate } n x= & x: \text { replicate }(n-1) x
\end{aligned} \\
& \text { replicate } n x= \text { let } \operatorname{rec} 0=[] \\
& \operatorname{rec} n=x: \operatorname{rec}(n-1)
\end{aligned}
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\operatorname{map} f(x: x s) & =f x: \text { map } x s
\end{aligned} \\
& \begin{aligned}
\operatorname{map}_{-}[] & =[] \\
\operatorname{map} f(x: x s) & =f x: \operatorname{map} f x s
\end{aligned} \\
& \begin{aligned}
\operatorname{map} f=\operatorname{let} \operatorname{rec}[] & =[] \\
\operatorname{rec}(x: x s) & =f x: \operatorname{rec} x s
\end{aligned} \\
& \begin{aligned}
\text { map } f=\text { let } \operatorname{rec}[] & =[] \\
\operatorname{rec}(x: x s) & =f x: \operatorname{rec} x s
\end{aligned} \\
& \text { in } \mathrm{rec} \\
& \rightarrow \\
& \text { } \\
& \rightarrow \\
& (x: x s)=f x: r e c x s
\end{aligned}
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\begin{aligned}
& \text { = } 1
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## Rewriting until

until $p f x=$ if $p x$ then $x$ else until $p f(f x)$
until pf $x=$ let $r e c x=$ if $p x$ then $x$ else rec ( $f x$ ) in $\operatorname{rec} x$

## Rewriting the Thue-Morse Sequence

let $x a b=b: z i p(x a b)(y a b)$ $y s t=s: z i p(y s t)(x s t)$ $z i p(x: x s)(y: y s)=x: y: z i p x s y s$ in $x 01$
let $x a b=$ let $x^{\prime}=b: z i p x^{\prime}(y a b)$ in $x^{\prime}$
$y s t=$ let $y^{\prime}=s:$ zip $y^{\prime}(x s t)$ in $y^{\prime}$
$z i p(x: x s)(y: y s)=x: y: z i p x s y s$
in $x 01$

## Binding-Graph Method

$$
\begin{aligned}
& \text { let } x a b=b: z i p(x a b)(y a b) \\
& y s t=s: z i p(y s t)(x s t) \\
& z i p(x: x s)(y: y s)=x: y: z i p x s y s \\
& \text { in } x 01
\end{aligned}
$$

Binding relation: $a \_\subseteq S \times S$


## Binding-Graph Method

$$
\begin{aligned}
& \text { let } x a b=b: z i p(x a b)(y a b) \\
& y s t=s: z i p(y s t)(x s t) \\
& z i p(x: x s)(y: y s)=x: y: z i p x s y s \\
& \text { in } x 01
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& z i p(x: x s)(y: y s)=x: y: z i p x s y s \\
& \text { in } x 01
\end{aligned}
$$

Binding relation: $\circ \subseteq \subseteq S \times S$


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\begin{aligned}
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& y s t=s: z i p(y s t)(x s t) \\
& z i p(x: x s)(y: y s)=x: y: z i p x s y s \\
& \text { in } x 01
\end{aligned}
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Binding relation: $\circ \subseteq \subseteq S \times S$


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\end{aligned}
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Binding relation: $a \_\subseteq S \times S$


## Binding-Graph Method

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\begin{aligned}
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& y s t=s: z i p(y s t)(x s t) \\
& z i p(x: x s)(y: y s)=x: y: z i p x s y s \\
& \text { in } x 01
\end{aligned}
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Binding relation: $a \_\subseteq S \times S$


## Binding-Graph Method

$$
\begin{aligned}
& \text { let } x a b=b: z i p(x a b)(y a b) \\
& y s t=s: z i p(y s t)(x s t) \\
& z i p(x: x s)(y: y s)=x: y: z i p x s y s \\
& \text { in } x 01
\end{aligned}
$$

Binding relation: $a \_\subseteq S \times S$


## Binding-Graph Method

$$
\begin{aligned}
& \text { let } x a b=b: z i p(x a b)\left(\begin{array}{ll}
y & a b
\end{array}\right) \\
& \quad y s t=s: z i p(y s t)(x s t) \\
& \quad \text { zip }(x: x s)(y: y s)=x: y: z i p x s y s \\
& \text { in } x 01
\end{aligned}
$$

Binding relation: $a \_\subseteq S \times S$


## Binding-Graph Method

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\begin{aligned}
& \text { let } x a b=b: z i p(x a b)\left(\begin{array}{ll}
y a b
\end{array}\right) \\
& \quad y s t=s: z i p(y s t)(x s t) \\
& \quad \text { zip }(x: x s)(y: y s)=x: y: z i p x s y s \\
& \text { in } x 0
\end{aligned}
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Binding relation: $a \_\subseteq S \times S$


## Strong domination

## Strong domination:

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\operatorname{sdom}_{G}(d, w):=
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$$
\forall p_{0} \longmapsto \ldots \longmapsto p_{n}=v \quad n \geq 0
$$



## Strong domination

## Strong domination:

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\operatorname{sdom}_{G}(d, w):=
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$$
\forall p_{0} \longmapsto \ldots \longmapsto p_{n}=v: d \in\left\{p_{0}, \ldots, p_{n}\right\}
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$$
n \geq 0
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## Strong domination

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\begin{gathered}
\operatorname{sdom}_{G}(d, w):= \\
\forall p_{0} \mapsto \ldots \mapsto p_{n}=v: d \in\left\{p_{0}, \ldots, p_{n}\right\} \vee d \mapsto^{+} p_{0} \wedge p_{0} \not \mapsto^{+} d \quad n \geq 0
\end{gathered}
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## Optimising the Thue-Morse Sequence

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let \(x a b=b: z i p(x a b)(y a b)\)
    yst \(=s: z i p(y s t)(x s t)\)
    \(z i p(x: x s)(y: y s)=x: y: z i p x s y s\)
in \(x 01\)
```



## Optimising the Thue-Morse Sequence

```
let \(x=1\) :zip \(x y\)
\[
y=0: z i p y x
\]
\[
z i p(x: x s)(y: y s)=x: y: z i p x s y s
\]
```

in $x$

－practical aspects
－implementation
－repetitive reduction patters in the wild：population census
－benchmarks
.
－analysis of effects for different run－time systems
－theoretical aspects
－HRS formulation
－domination after unfolding
－efficiency measure for comparing different results of optimisation
－interactions between optimisation of different parameter cycles
－correctness proof
－full paper



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## Thanks

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#### Abstract




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and for inspiration，and many discussions，to：
－Doaitse Swierstra
－Vincent van Oostrom
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－Vincent van Oostrom

[^6]－Doaitse Swierstra
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[^1]:    let repeat $=\lambda x \cdot x:$ repeat $x$
    let repeat $=\lambda x \cdot x:$ repeat $x$
    in $3:$ repeat 3

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[^2]:    $\square$

[^3]:    eles,

    Rochel, Grabmayer

[^4]:    $\square$

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    oaitse Swierstra
    $\square$
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    Doaitse Swierstra

