Bisimulation Slices and Transfer Functions

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Overview

- fragments of bisimulations
  - motivation 1: nearly collapsed LTSs (labeled transition systems) for the process semantics of regular expressions
  - bisimulating slices and grounded bisimulation slices
- functional bisimulation fragments
  - motivation 2: transfer of LTS spec's via functional bisimulations
  - transfer functions and local transfer functions
  - transfer functions via elevation from local-transfer functions
- application aimed at
  - using transfer to prove specifications equal on nearly collapsed 1-LTSs

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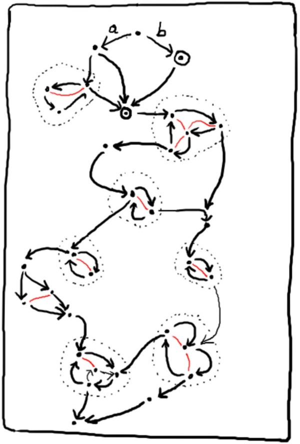
Motivation 1: nearly collapsed LTSs

strongly connected components (scc's)

termination permitted



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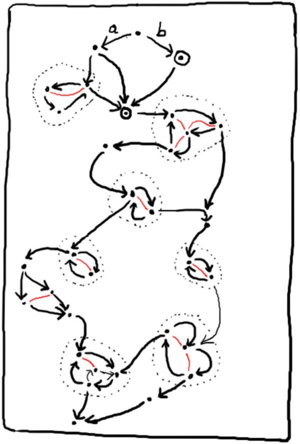

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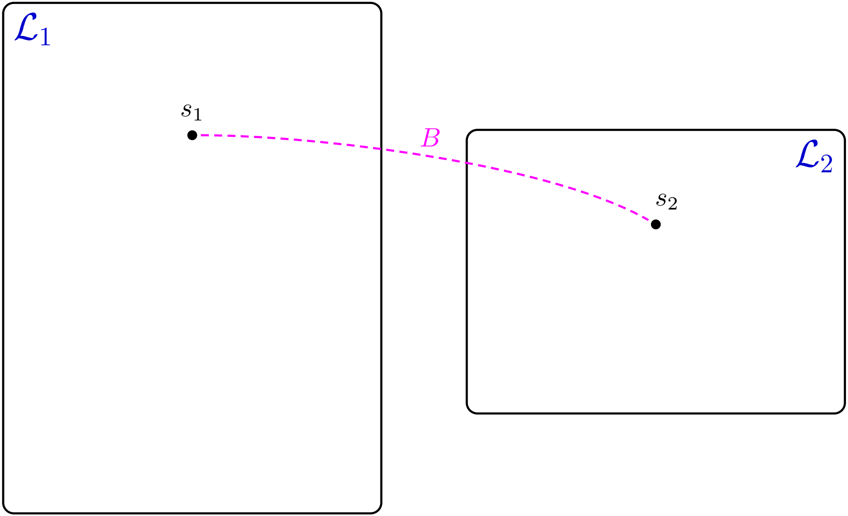
Definition

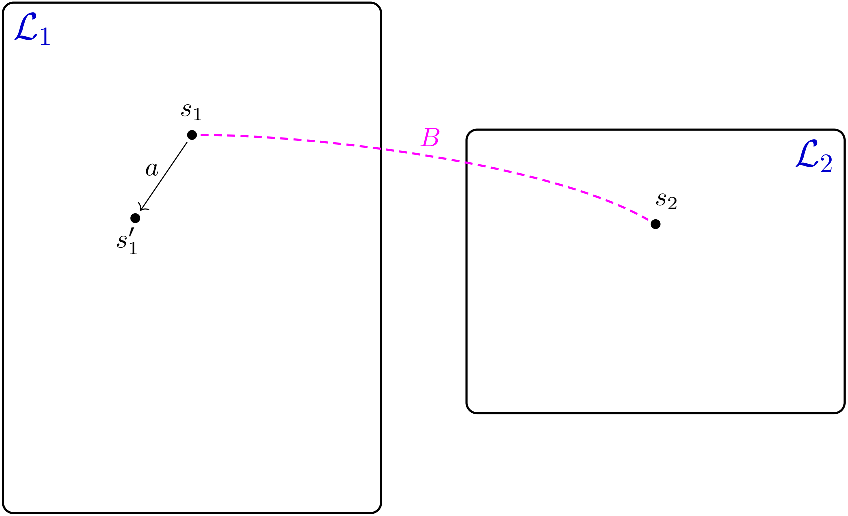
A *labeled transition system* is a tuple $\mathcal{L} = \langle S, A, \to, \downarrow \rangle$ where:

- S is a set of states,
- A is a set of actions,
- $\to \subseteq S \times A \times S$ is a set of labeled transitions,
- $\downarrow \subseteq V$ is a set of terminating states.

Bisimulation



Bisimulation (forth condition)



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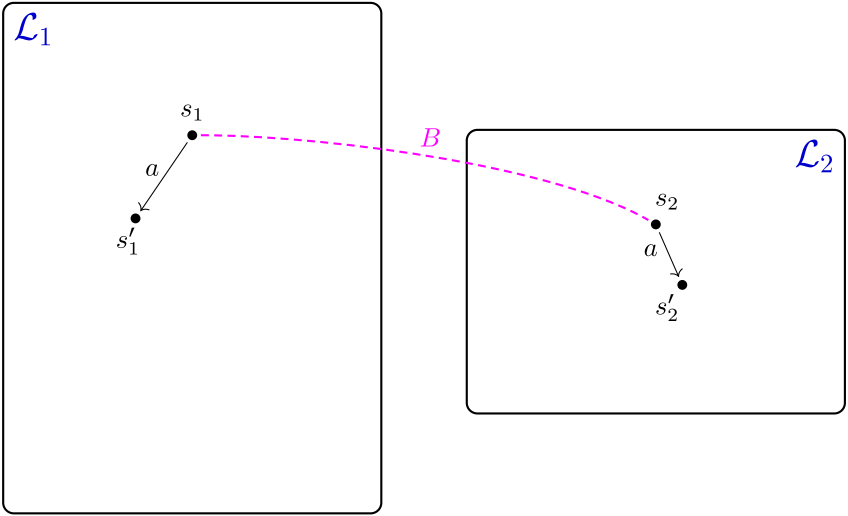

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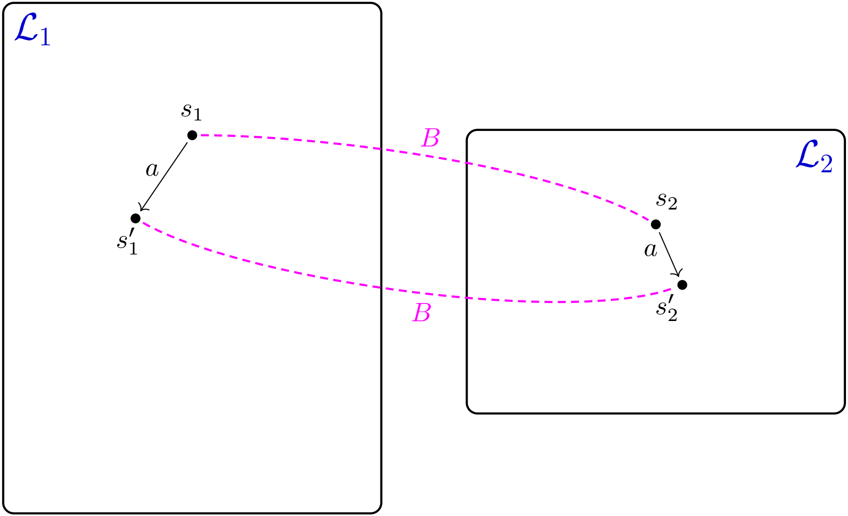

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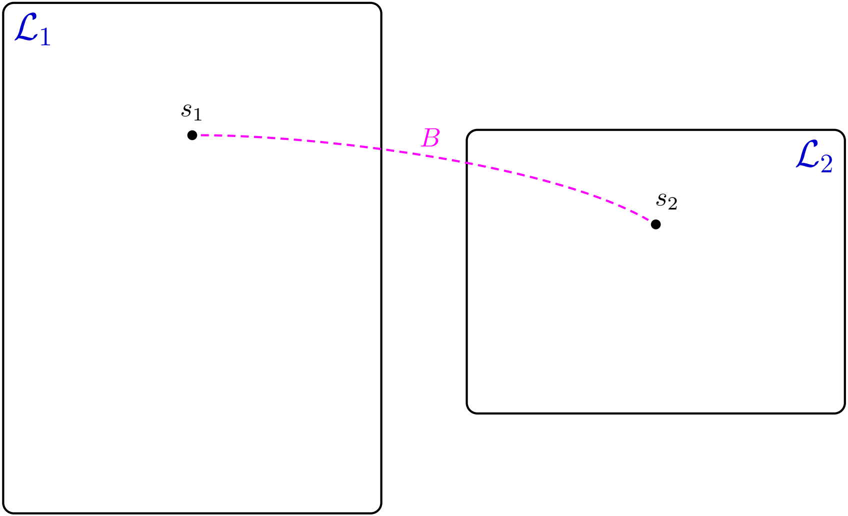

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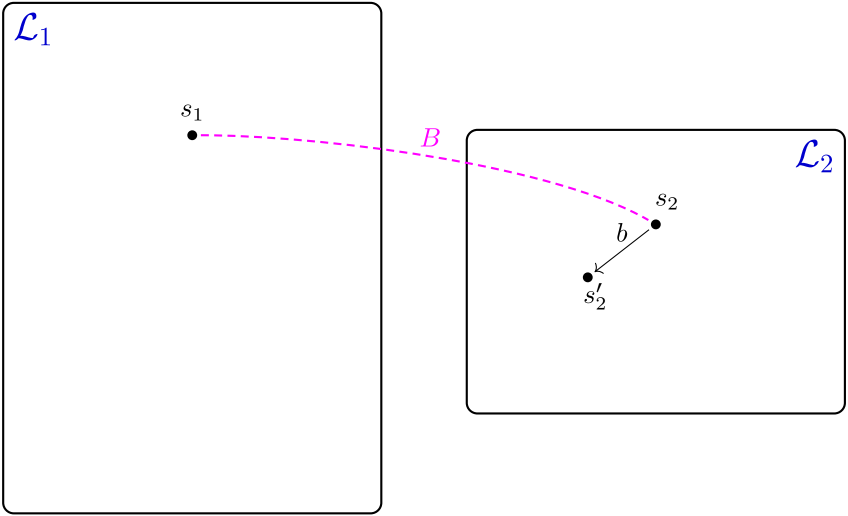

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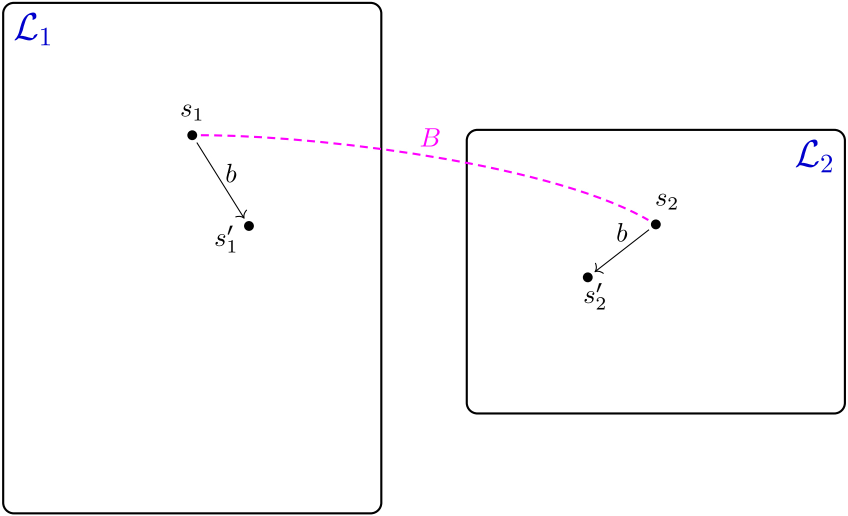

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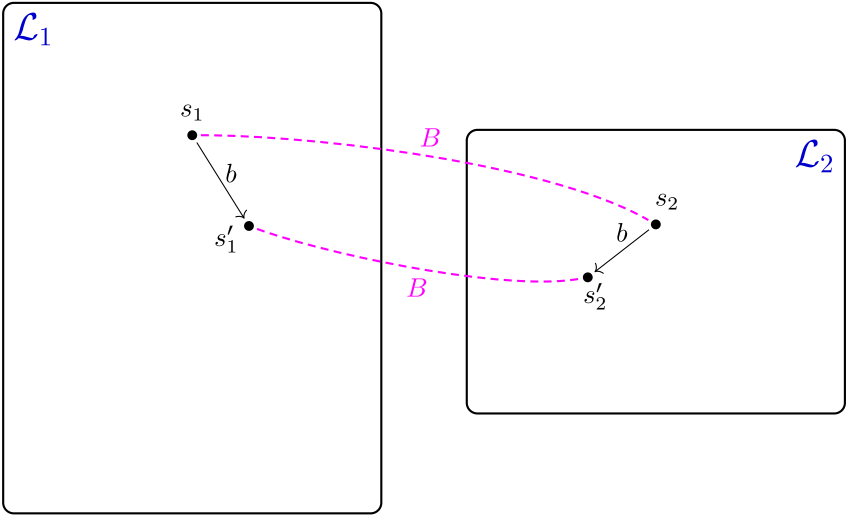

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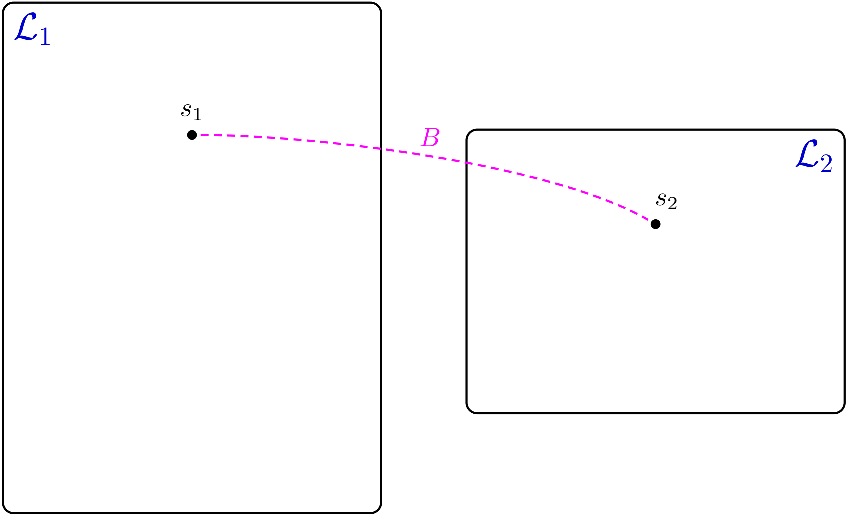

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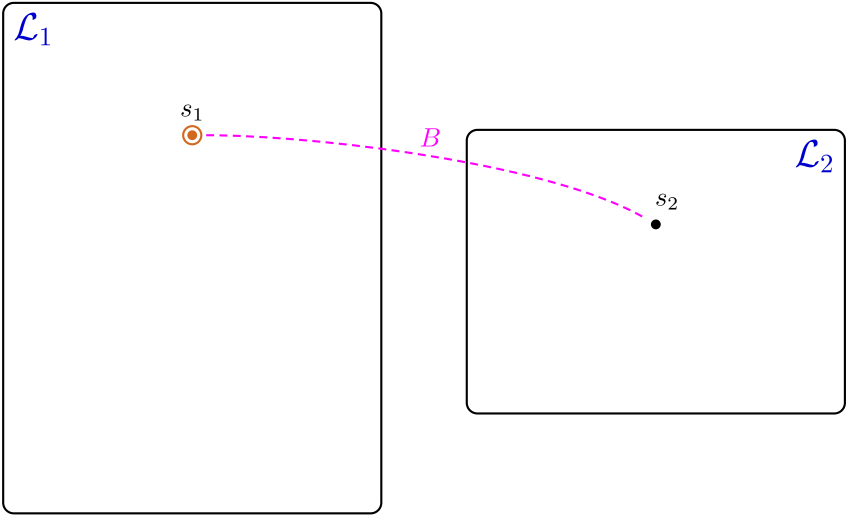

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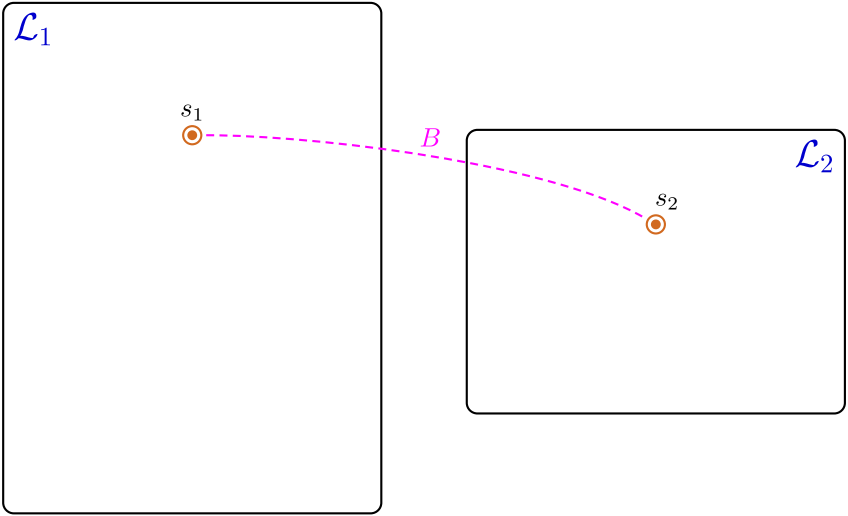

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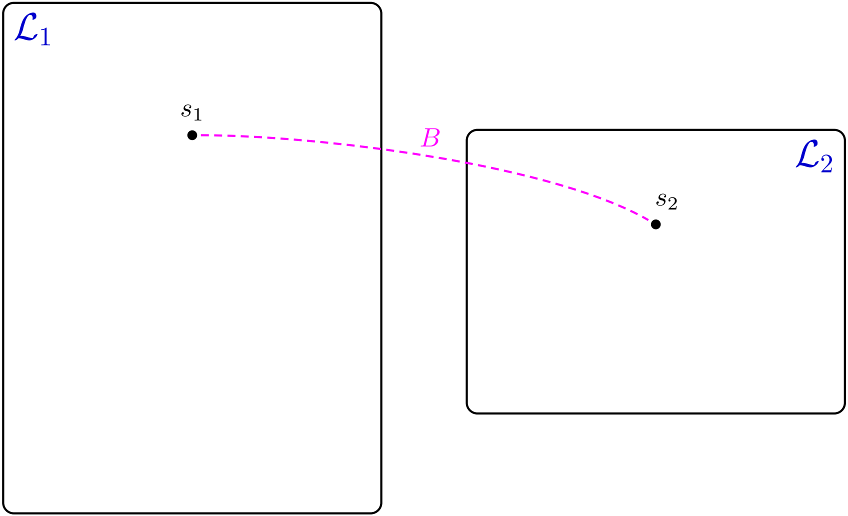

Nearly collapsed LTSs

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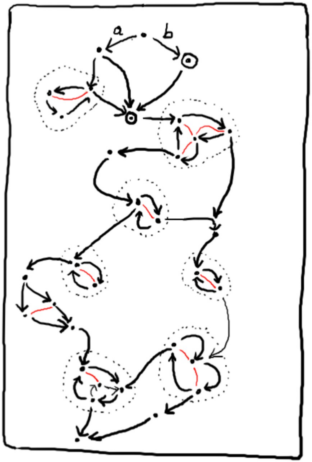

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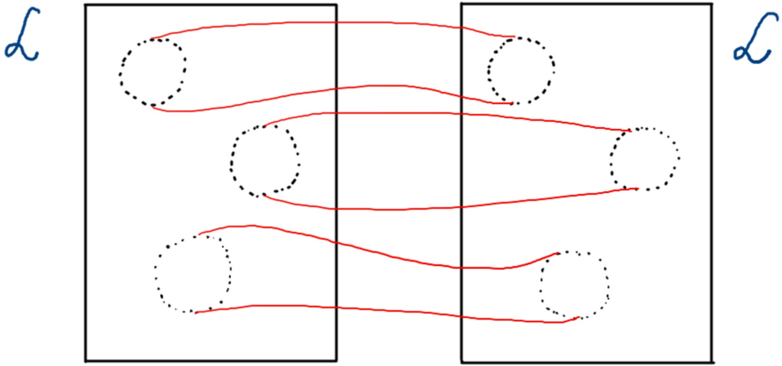
Bisimulation/bisimulating slices



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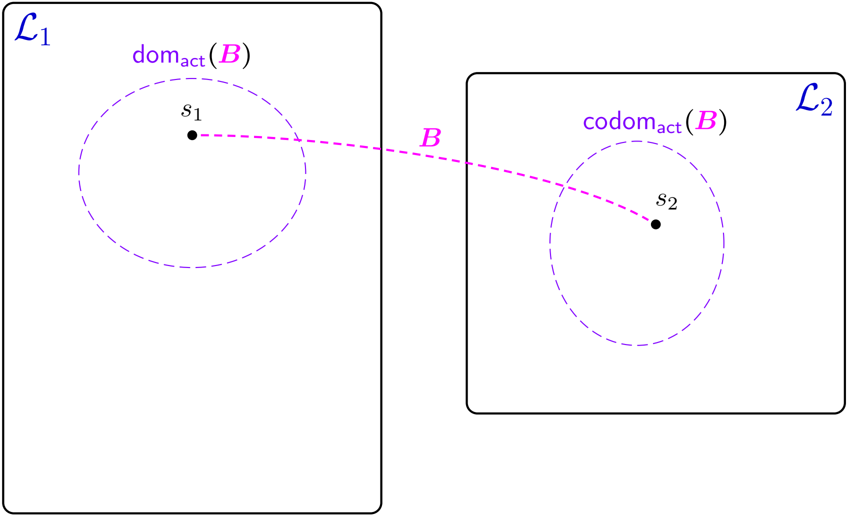
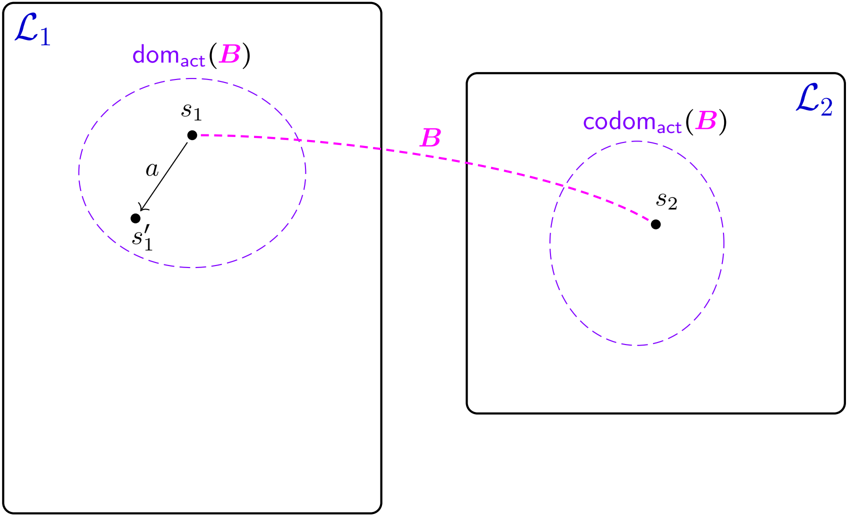


Bisimulating slice (forth condition)



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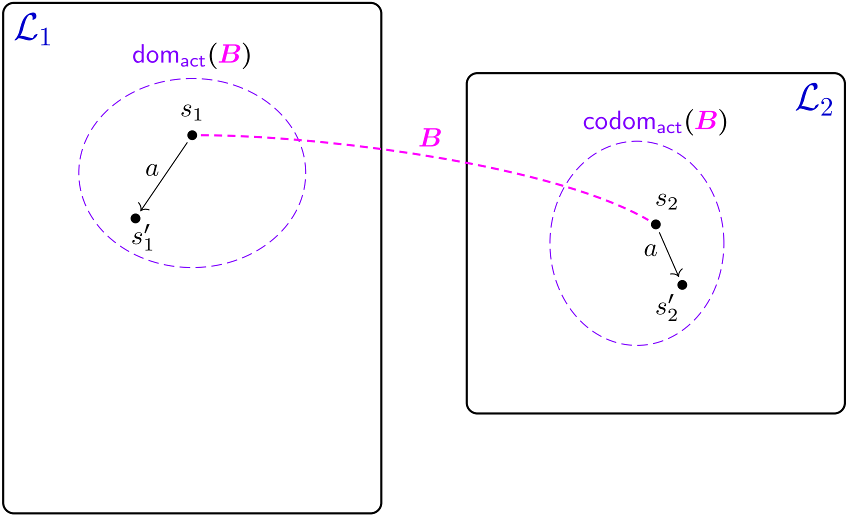

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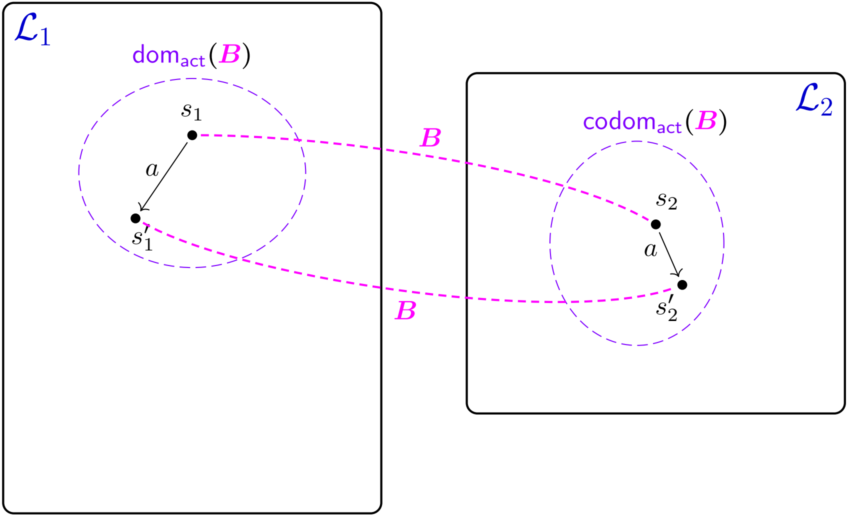

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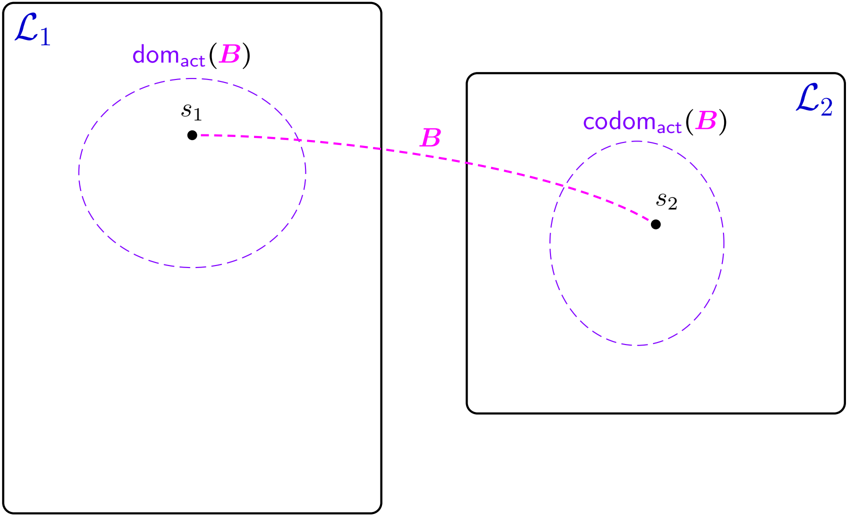

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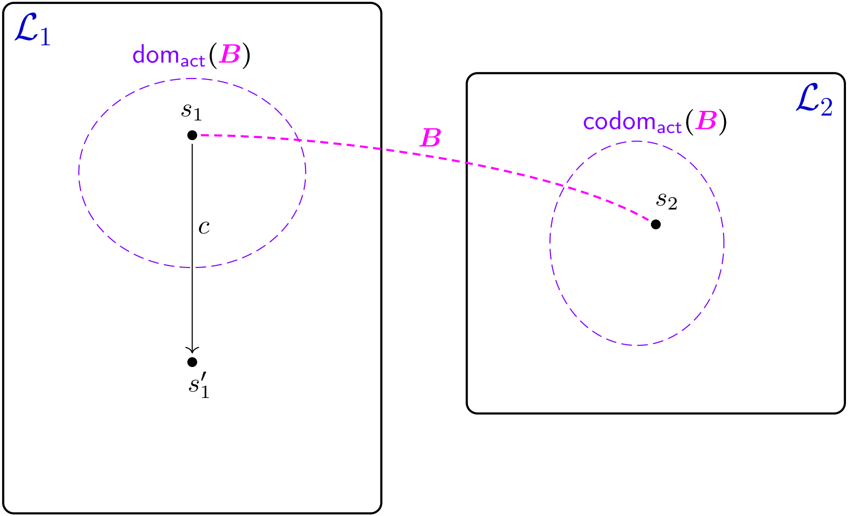

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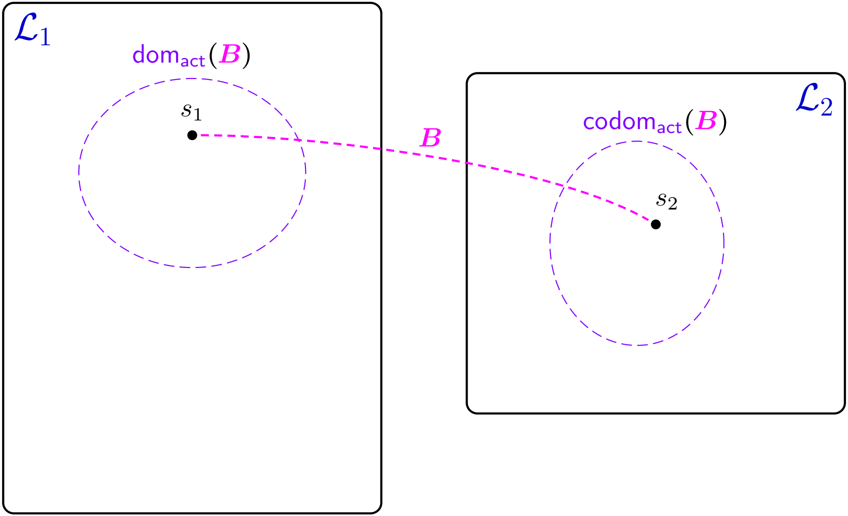

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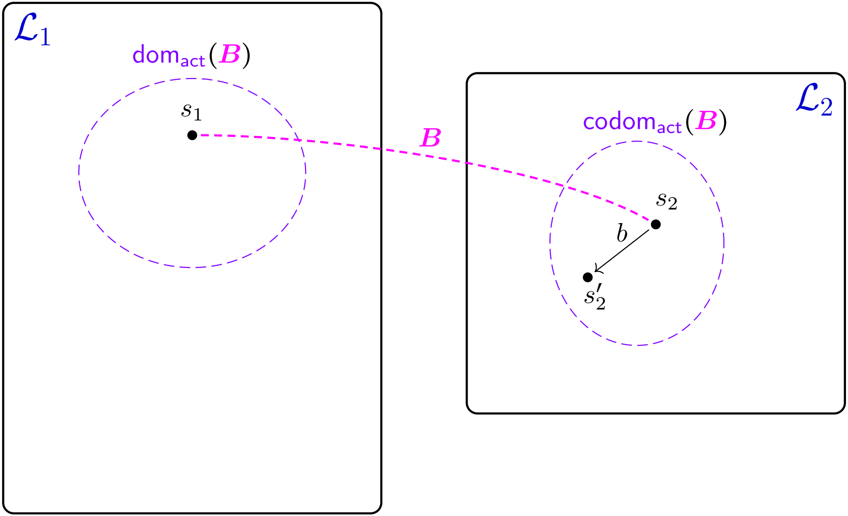

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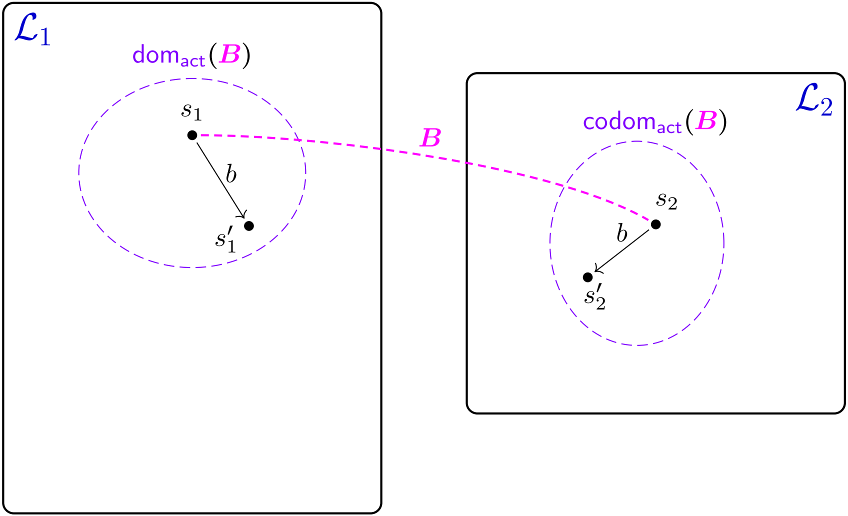

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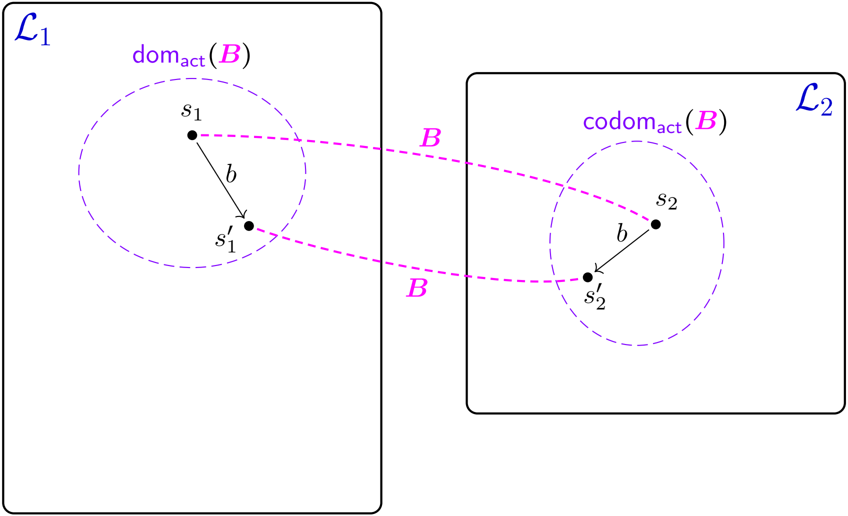

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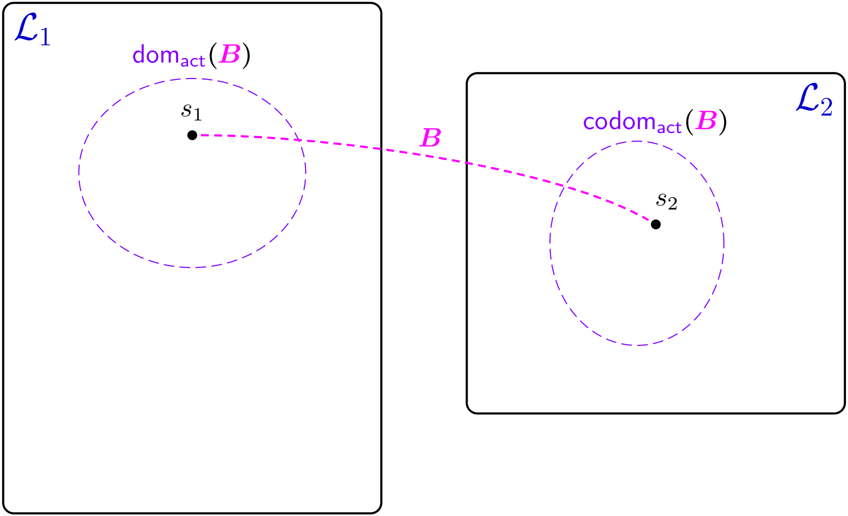

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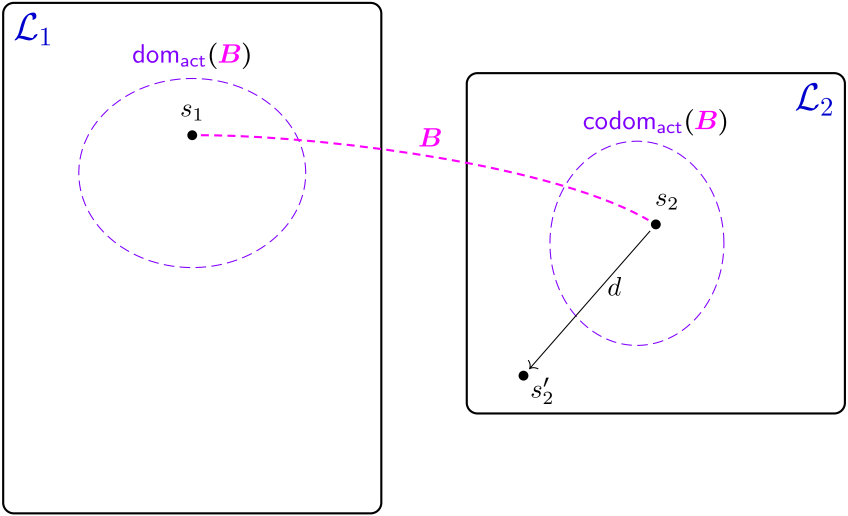

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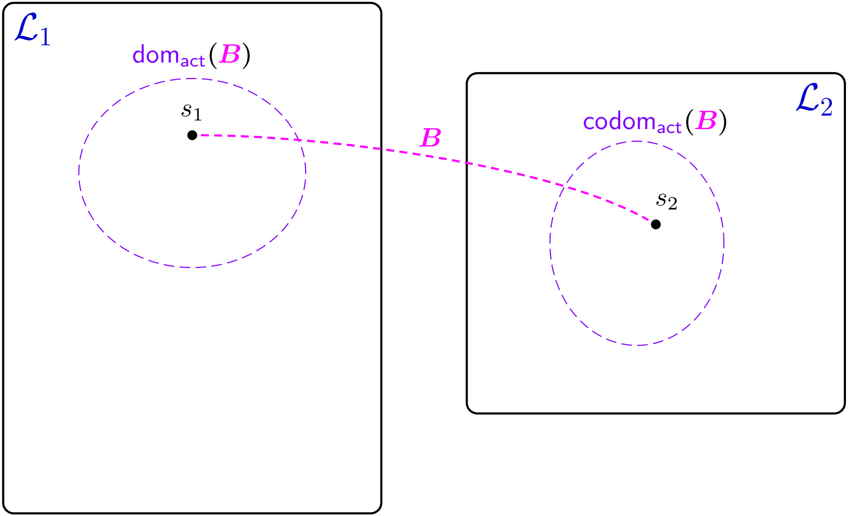

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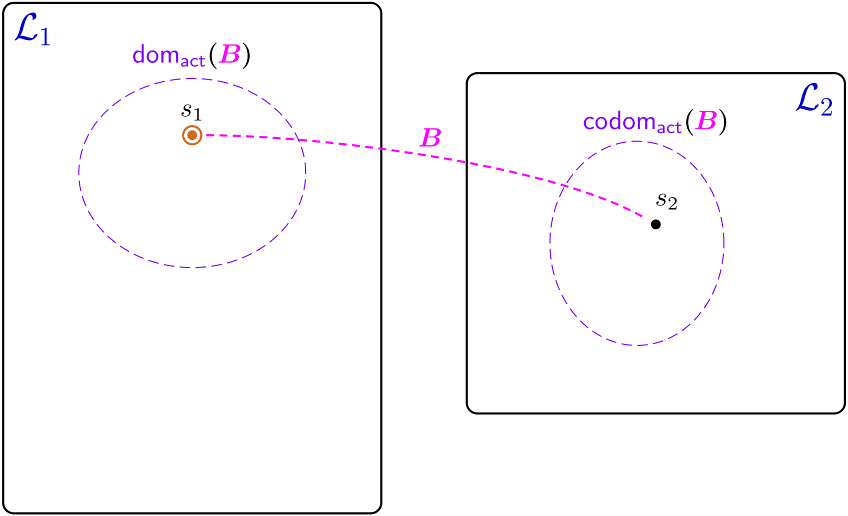

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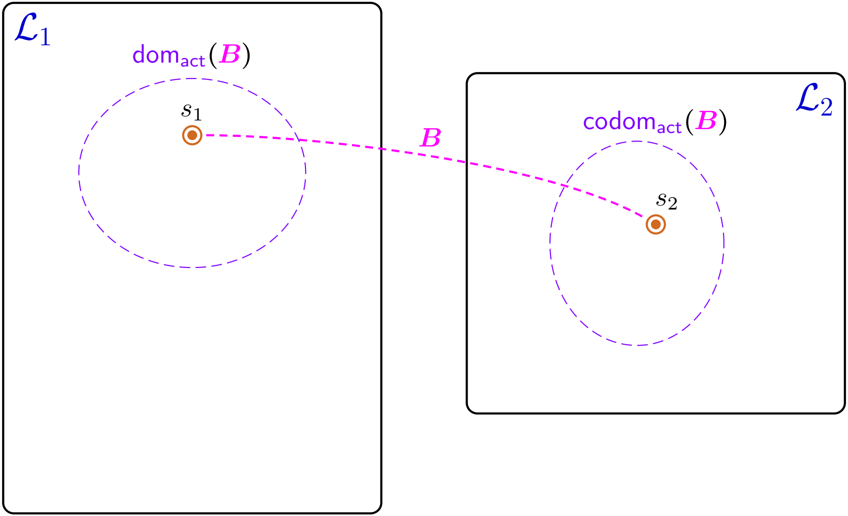

Bisimulating/bisimulation slices: properties

Proposition

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$=$ *bisimulation on full sub-LTSs of active domain/codomain*

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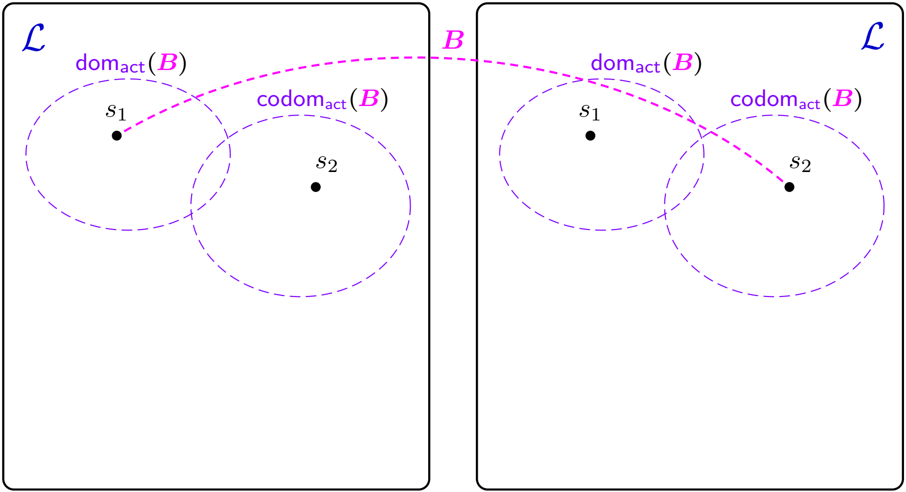
Grounded bisimulation slice



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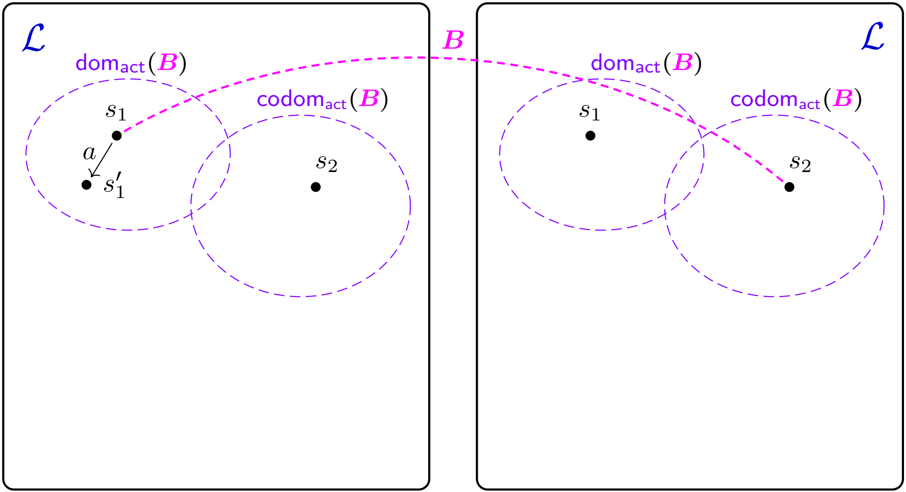

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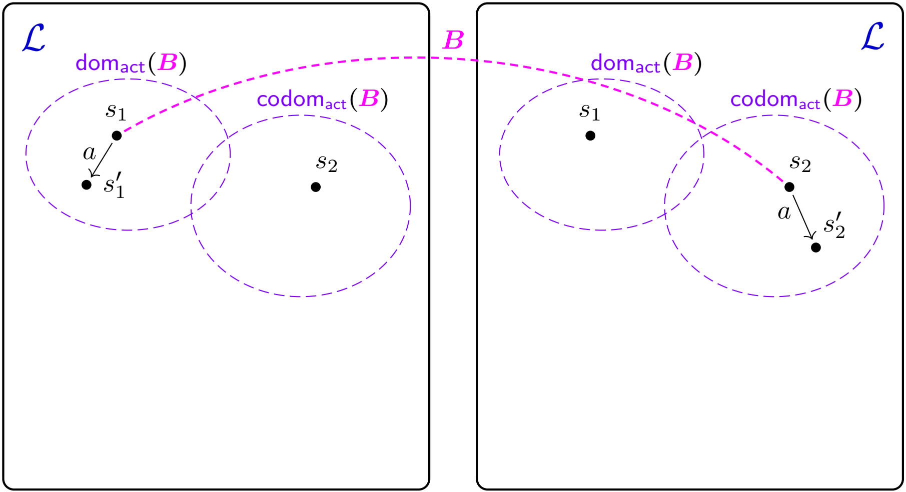

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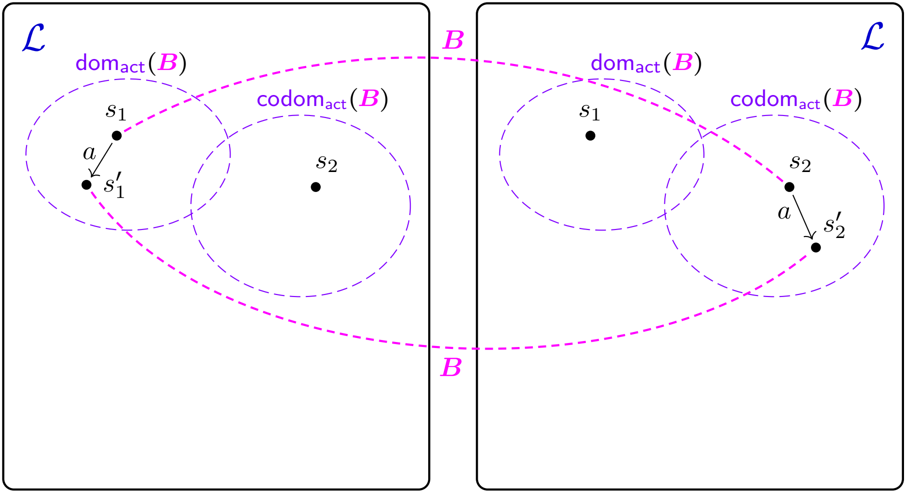

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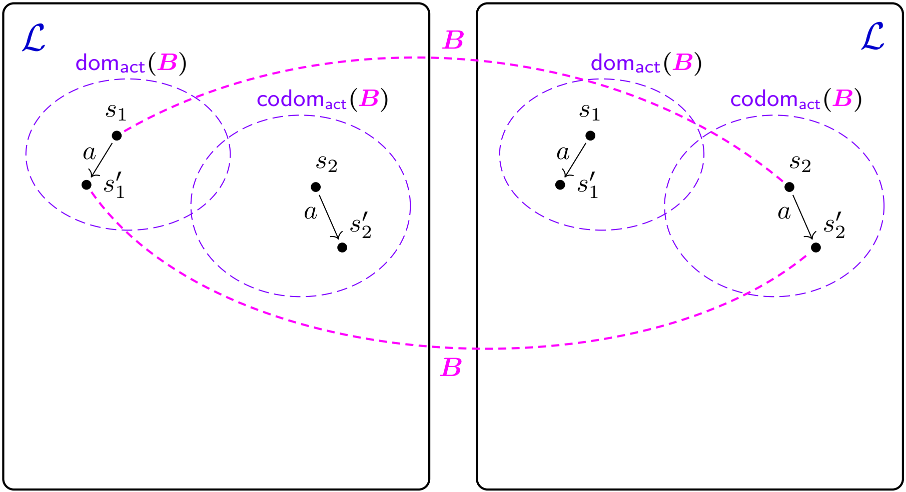

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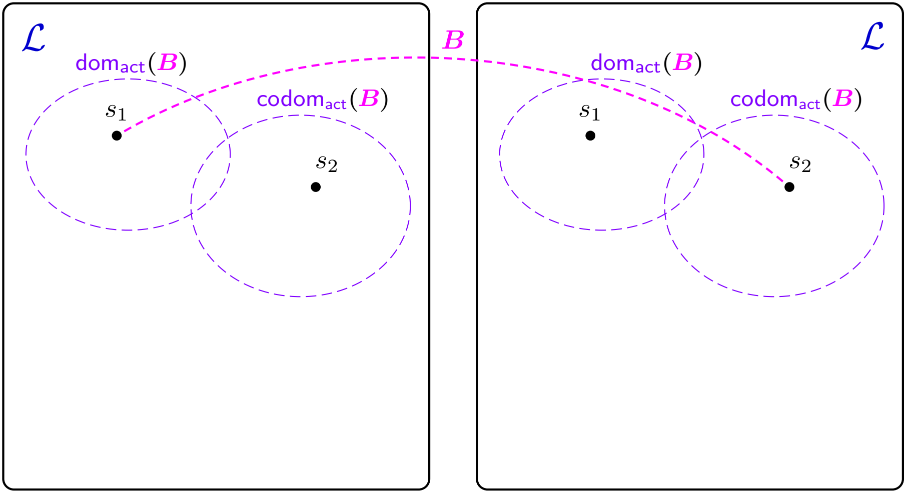

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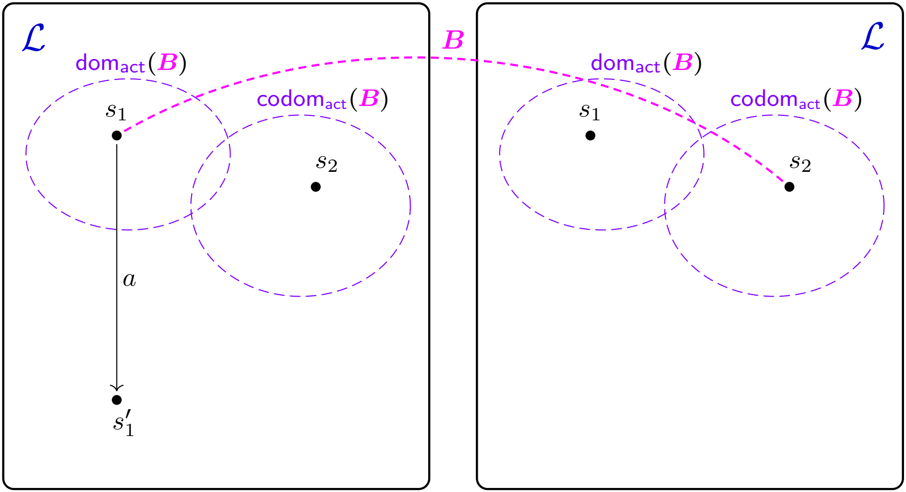

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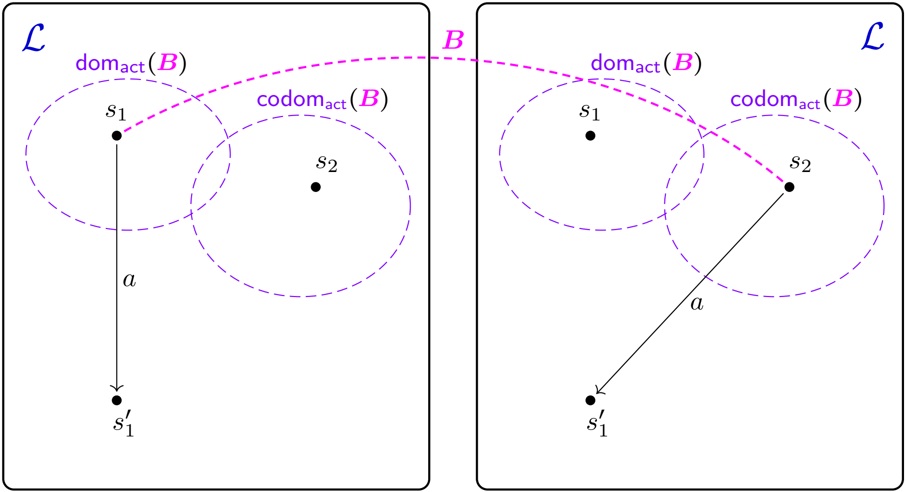

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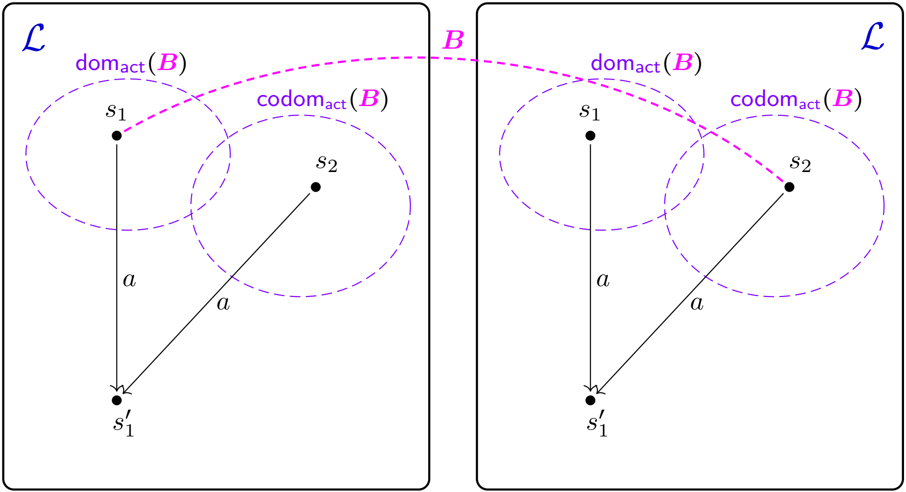

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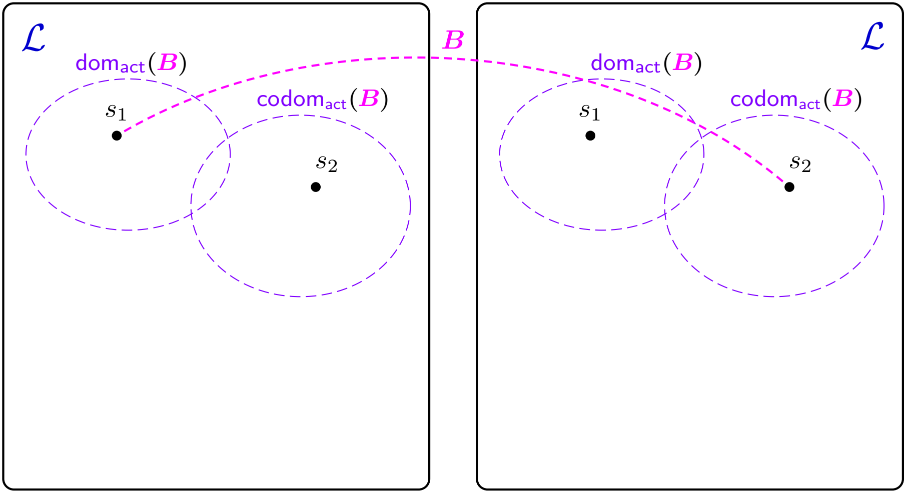

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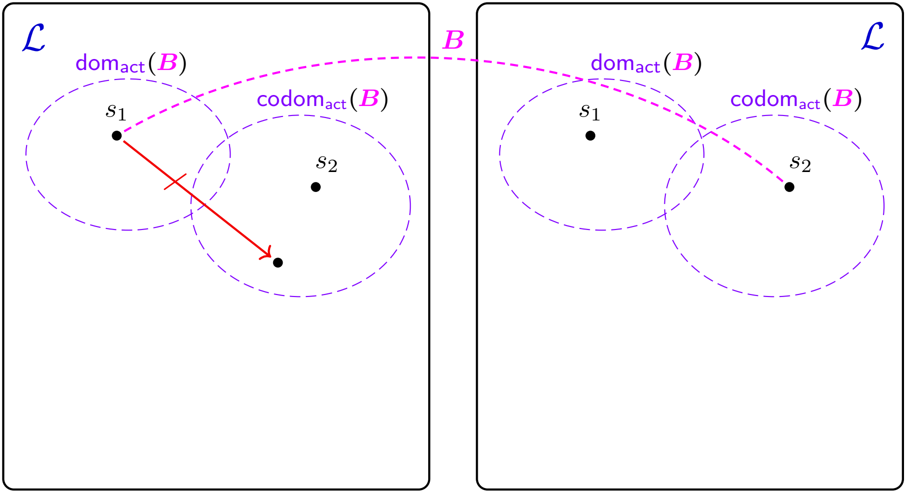

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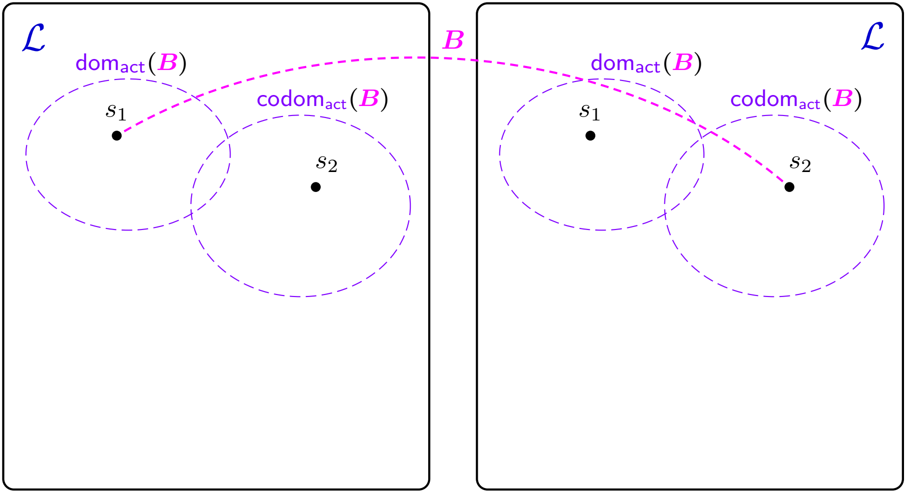

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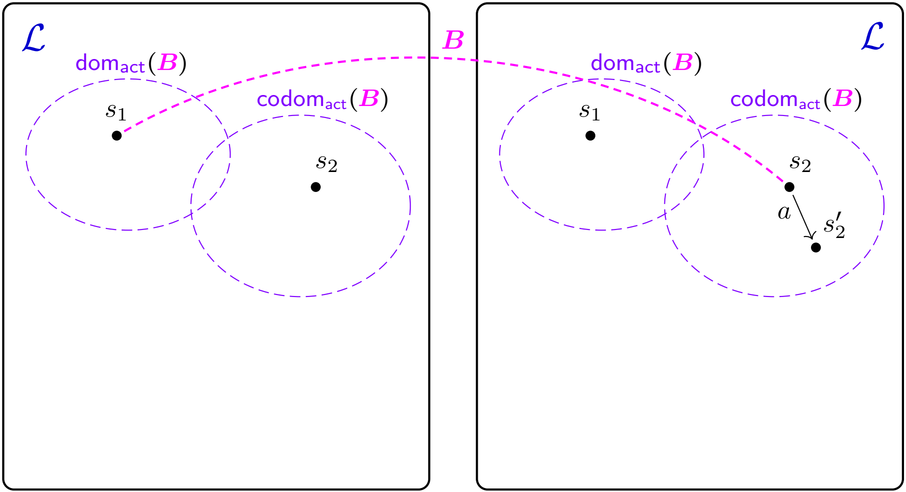

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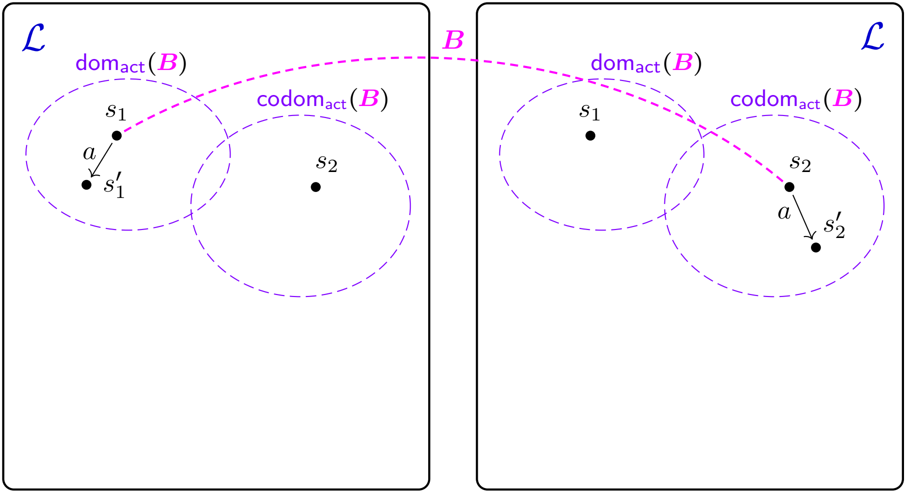

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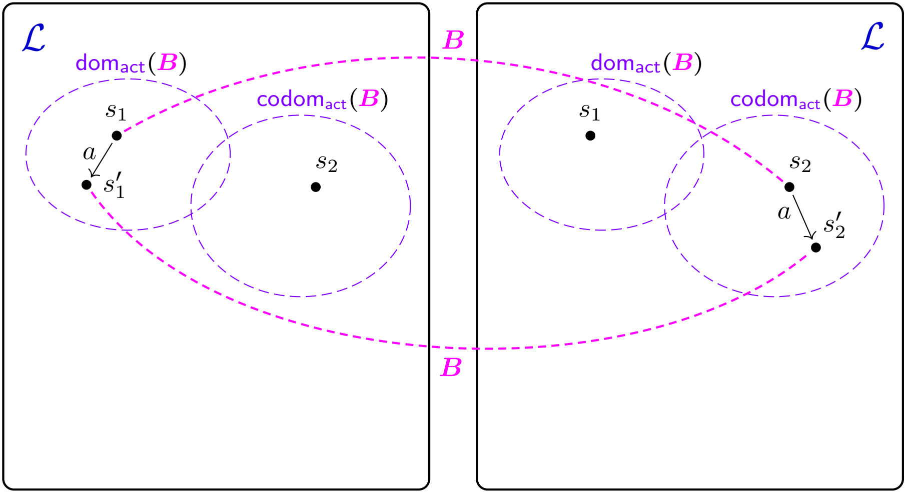

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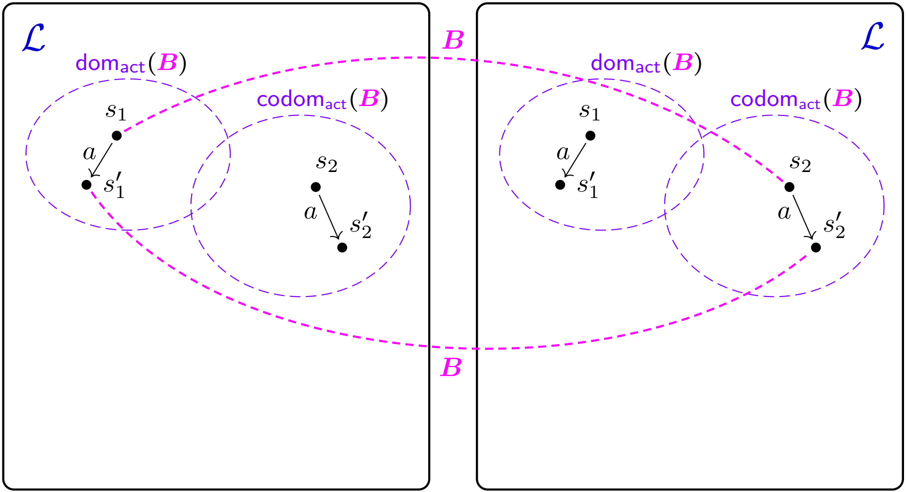

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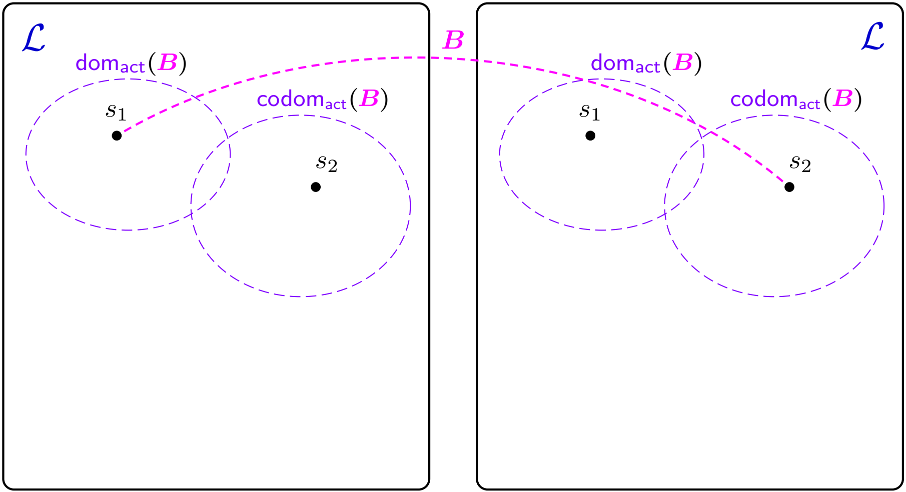

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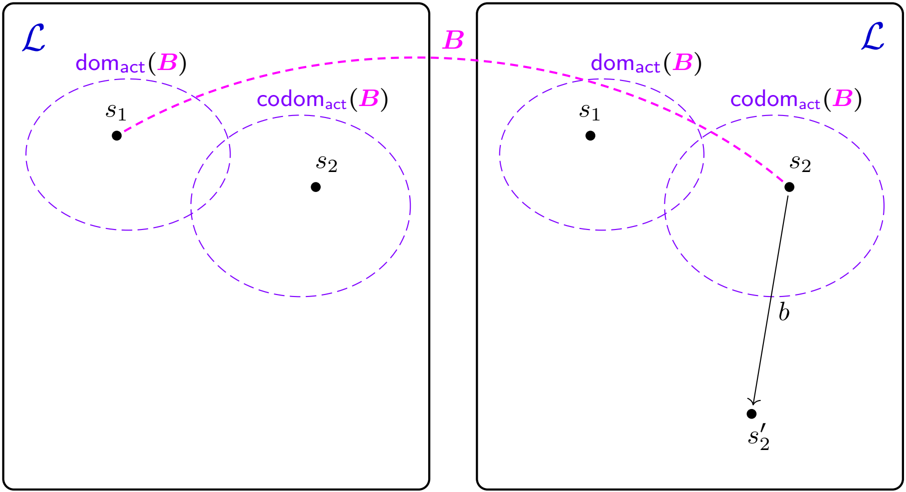

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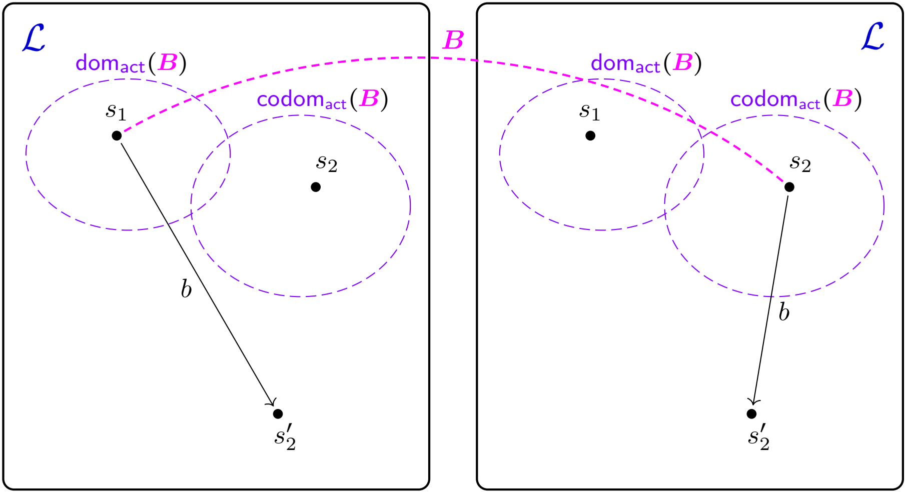

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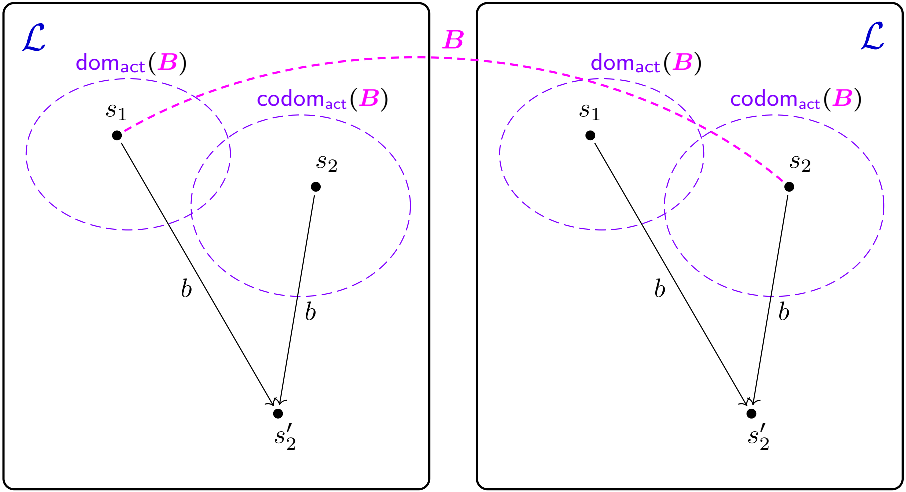

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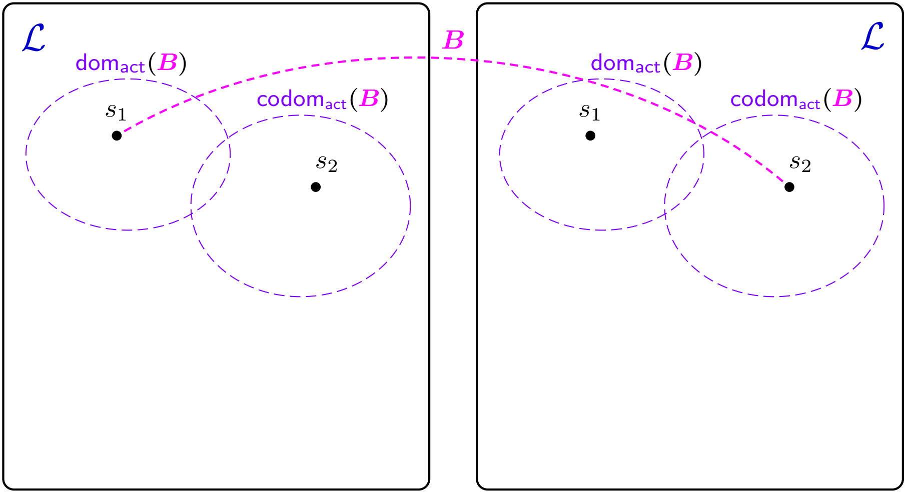

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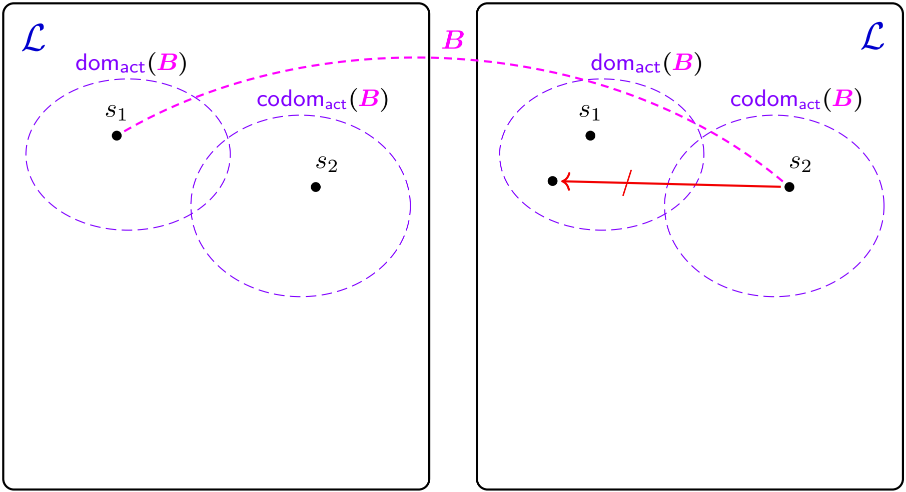

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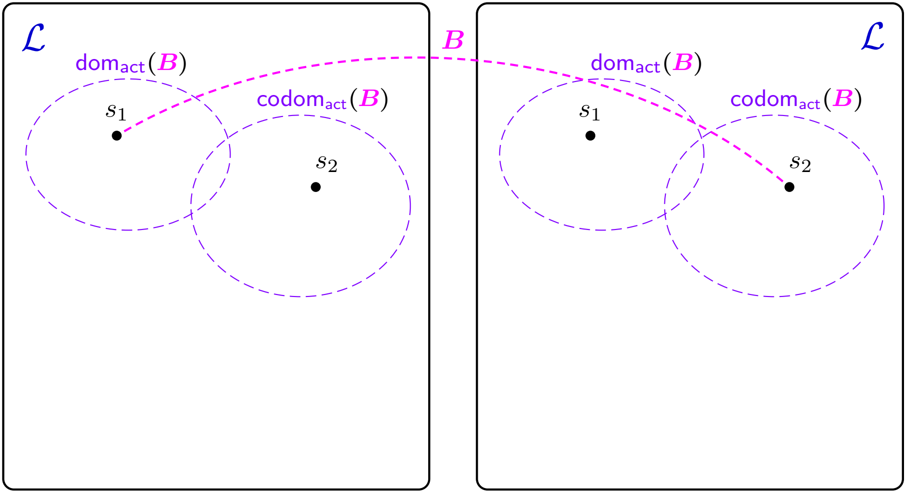
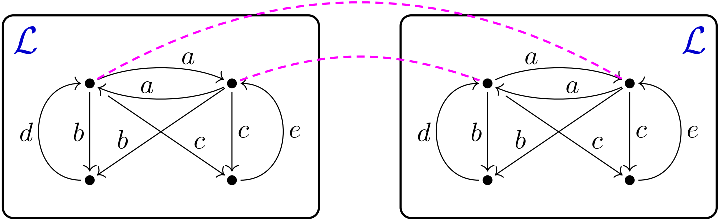


Grounded bisimulation slices: example



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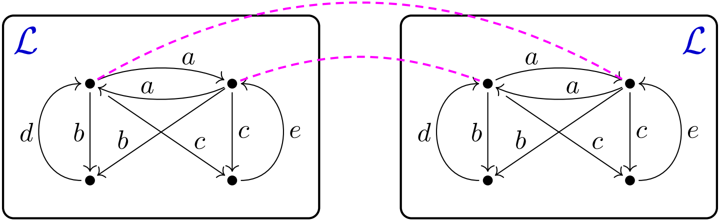

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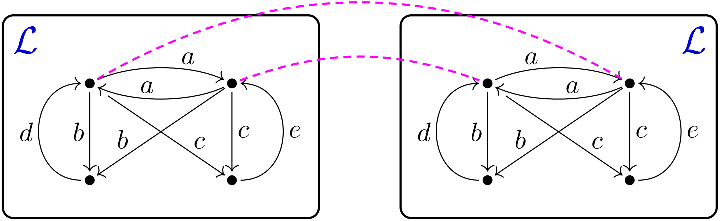
Grounded bisimulation slices: example and property

Proposition (*Grounded bisimulation slices induce bisimulations*)

For every grounded bisimulation slice $\boldsymbol{B} \subseteq S \times S$ on LTS $\mathcal{L} = \langle S, A, \rightarrow, \downarrow \rangle$,

$$\overline{\overline{\boldsymbol{B}}} := \boldsymbol{B} \cup = \qquad \text{is a bisimulation on } \mathcal{L}.$$



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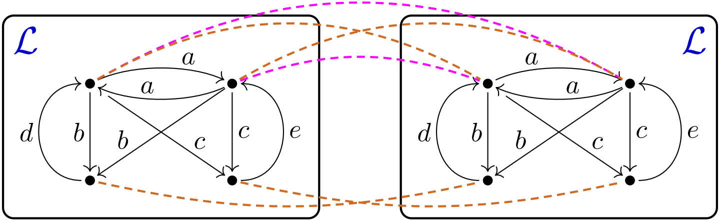

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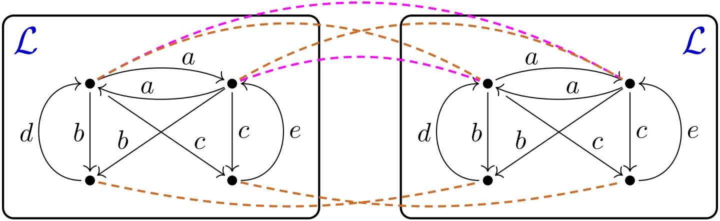

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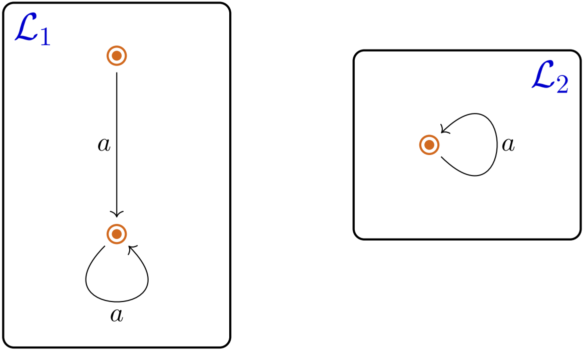
- the bisimulation $\overline{\overline{B}}$ that extends B.

Motivation 2: specification transfer via funct. bisimulations



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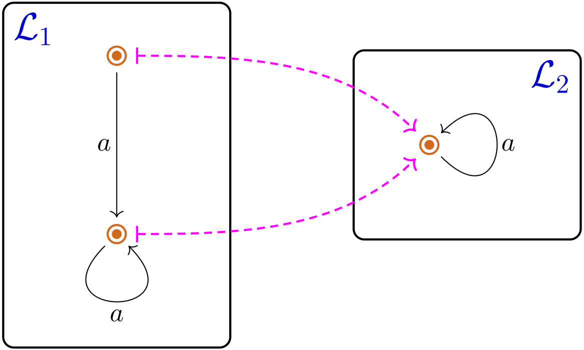
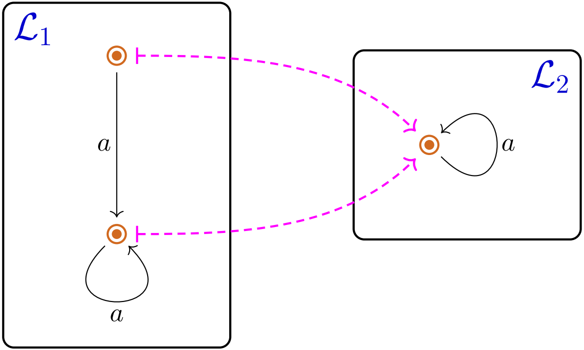
functional bisimulation



Motivation 2: specification transfer via funct. bisimulations

functional bisimulation



specification

a^*

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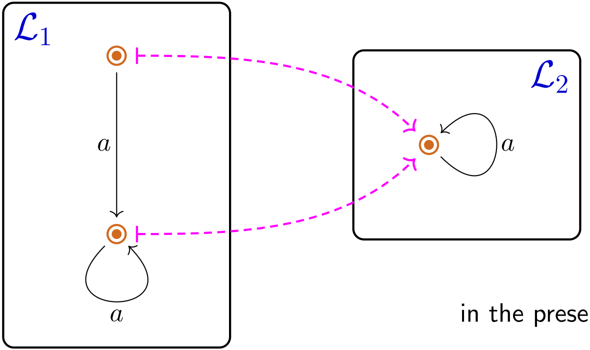
functional bisimulation



specification

a^*

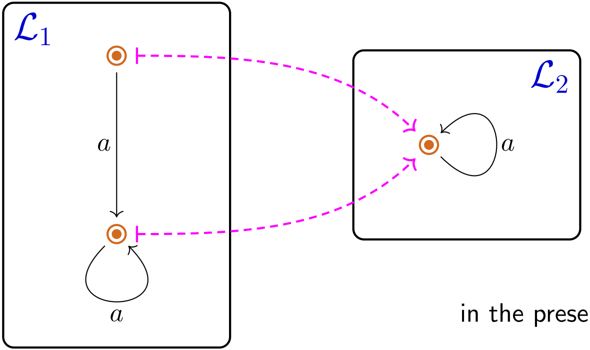
in the presence of axiom:

$$a^* = 1 + a \cdot a^*$$

Motivation 2: specification transfer via funct. bisimulations

pullback via

functional bisimulation



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Motivation 2: specification transfer via funct. bisimulations

specifications

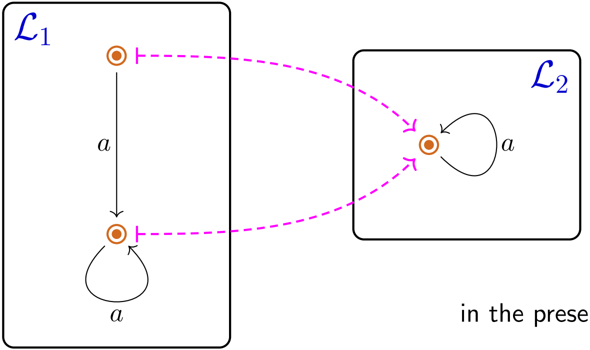
pullback via

functional bisimulation

specification

a^*

a^*



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in the presence of axiom:

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Transfer functions, and local-transfer functions

Transfer function $\hat{=}$ functional bisimulation between LTSs

Definition

A *transfer (partial) function* between LTSs $\mathcal{L}_1$, $\mathcal{L}_2$ is:

- a partial function $\phi : S_1 \rightharpoonup S_2$
  whose graph $\{\langle v, \phi(v)\rangle \mid v \in S_1\}$ is a bisimulation betw. $\mathcal{L}_1$ and $\mathcal{L}_2$.

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Local-transfer function $\hat{=}$ functional grounded bisimulation slice on an LTS

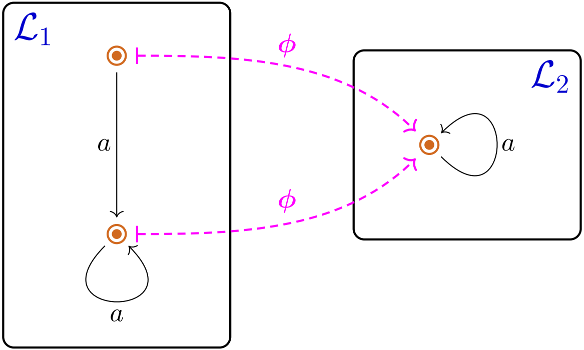
Definition

A *local-transfer function* on an LTS $\mathcal{L}$ is:

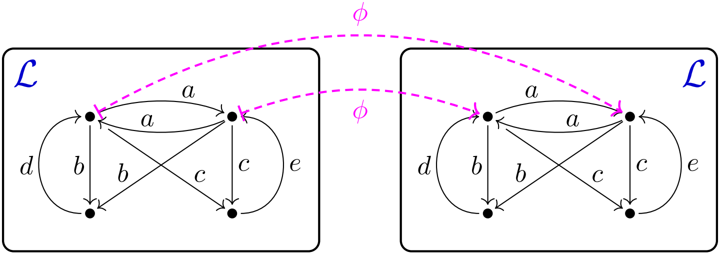
- a partial function $\phi : S_1 \rightharpoonup S_2$
  whose graph $\{\langle v, \phi(v)\rangle \mid v \in S\}$ is a grounded bisimulation slice on $\mathcal{L}$.

Transfer function (example)

ϕ is transfer function

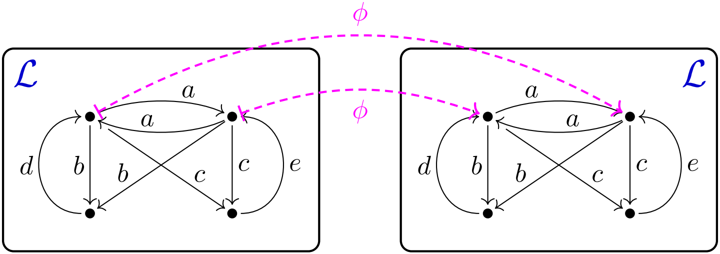


Local transfer function (example)



- ϕ is local transfer function on $\mathcal{L}$,

Local transfer function (example)



- ϕ is local transfer function on $\mathcal{L}$,
- but ϕ cannot be extended into a transfer function on $\mathcal{L}$.

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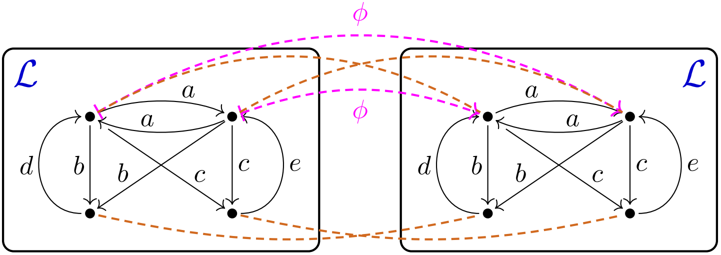

- ϕ is local transfer function on $\mathcal{L}$,
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Goal: Lifting a local transfer function

$$\mathcal{L} \xrightarrow{\quad\phi\quad} \mathcal{L}$$

local transfer function ϕ

Goal: Lifting a local transfer function to a transfer function

$$\mathcal{L} \xrightarrow{\quad\phi\quad} \mathcal{L}$$

local transfer function ϕ

$$\mathsf{E}_W(\mathcal{L}) \xrightarrow{\quad\widehat{\phi}\quad} \mathsf{E}_W(\mathcal{L})$$

transfer function $\widehat{\phi}$

elevation of W above $\mathcal{L}$

for $W = \mathrm{dom}(\phi) \cup \mathrm{ran}(\phi)$

Goal: Lifting a local transfer function to a transfer function

$$\begin{tikzcd}
\mathcal{L} \arrow[r, "\phi"] & \mathcal{L} \\
\mathsf{E}_{W}(\mathcal{L}) \arrow[u, "\pi_1"] \arrow[r, "\widehat{\phi}"] & \mathsf{E}_{W}(\mathcal{L}) \arrow[u, "\pi_1"']
\end{tikzcd}$$

local transfer function ϕ

projections π_1

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elevation of W above $\mathcal{L}$

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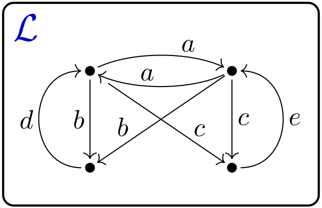
Elevation of a vertex set above LTS (example)



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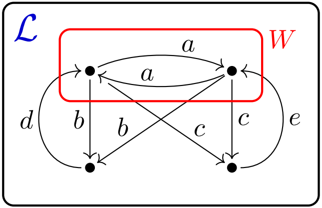

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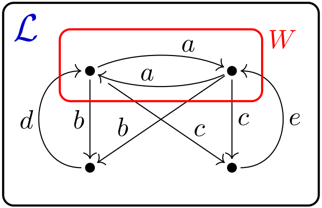

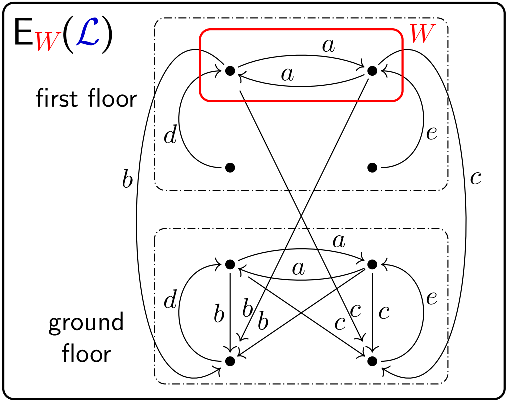

Local-transfer function ⇒ transfer function on elevation

Proposition

$$
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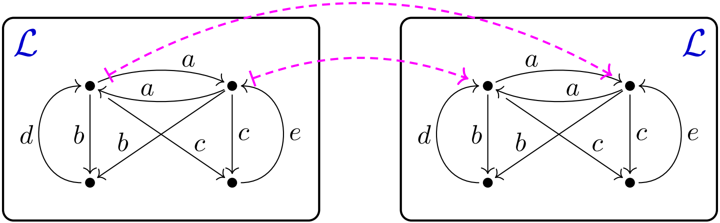

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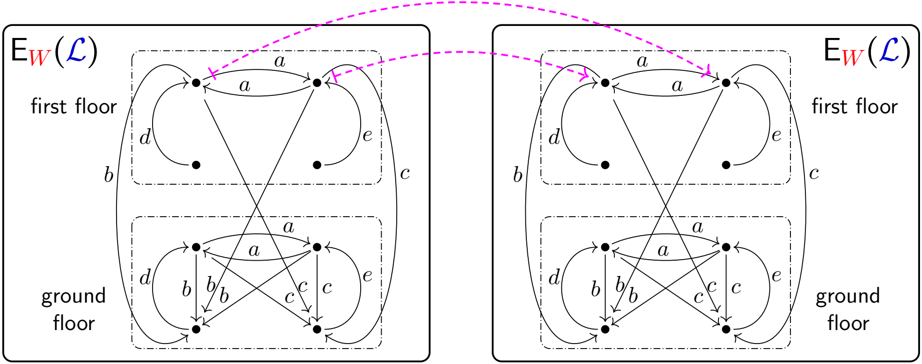

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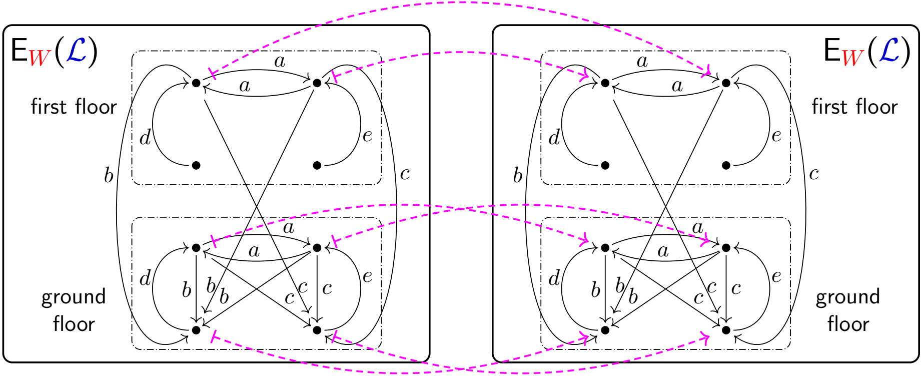

Local-transfer function ⇒ transfer function on elevation

Proposition

Let $\mathcal{L} = \langle S, A, \to, \downarrow \rangle$ be an LTS.
Every local-transfer function $\phi : S \rightharpoonup S$ on $\mathcal{L}$ lifts to a transfer function on the elevation $\mathsf{E}_{\mathsf{field}(\phi)}(\mathcal{L})$ of $W := \mathsf{field}(\phi) = \mathsf{dom}(\phi) \cup \mathsf{ran}(\phi)$ above $\mathcal{L}$

where: $\widehat{\phi} : (S \times \{\mathbf{0}, \mathbf{1}\}) \longrightarrow (S \times \{\mathbf{0}, \mathbf{1}\})$

$$\langle s, \boldsymbol{i} \rangle \longmapsto \widehat{\phi}(\langle s, \boldsymbol{i} \rangle) := \begin{cases} \langle \phi(t), \boldsymbol{i} \rangle & \text{if } \boldsymbol{i} = \mathbf{1} \wedge t \in \mathsf{dom}(\phi) , \\ \text{undefined} & \text{if } \boldsymbol{i} = \mathbf{1} \wedge t \notin \mathsf{dom}(\phi) , \\ \langle t, \boldsymbol{i} \rangle & \text{if } \boldsymbol{i} = \mathbf{0} . \end{cases}$$

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\mathcal{L} \arrow[rr, "\phi"] & & \mathcal{L} \\
\mathsf{E}_W(\mathcal{L})^{\boxed{(\mathbf{1})}} \arrow[u, "\pi_1"] \arrow[rr, "\widehat{\phi}"] & & \mathsf{E}_W(\mathcal{L})^{\boxed{(\mathbf{1})}} \arrow[u, "\pi_1"']
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$$\begin{tikzcd}
\mathcal{L} \arrow[r, "\mathsf{id}_V"] & \mathcal{L} \\
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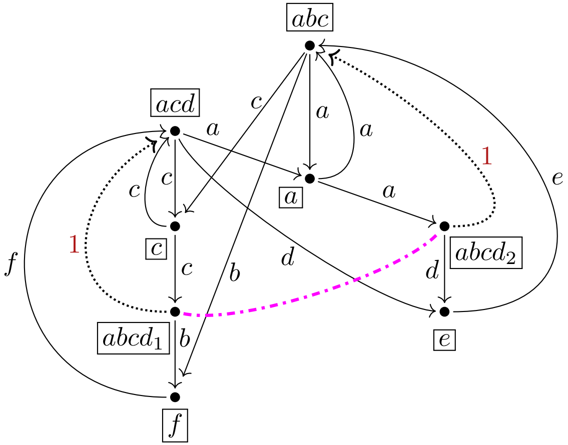
Application aimed at: nearly collapsed 1-LTSs



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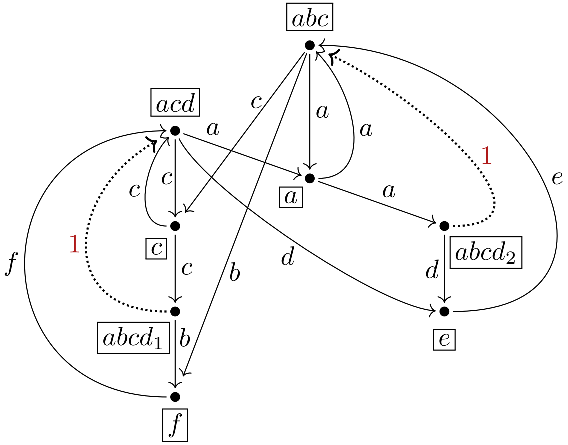

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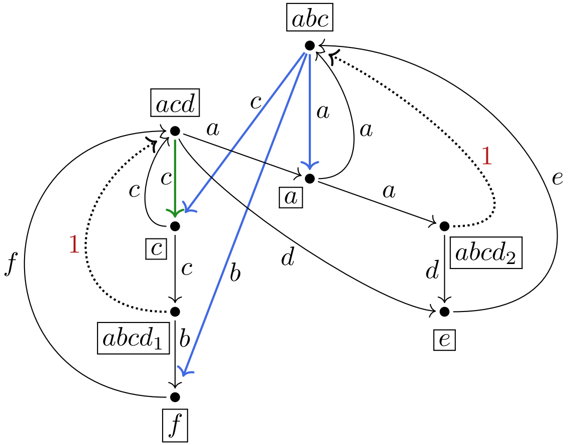

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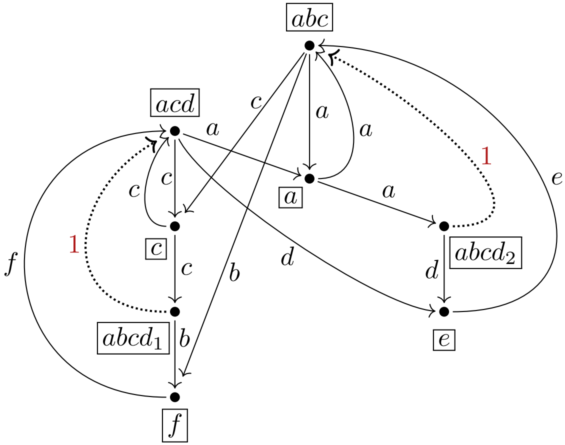

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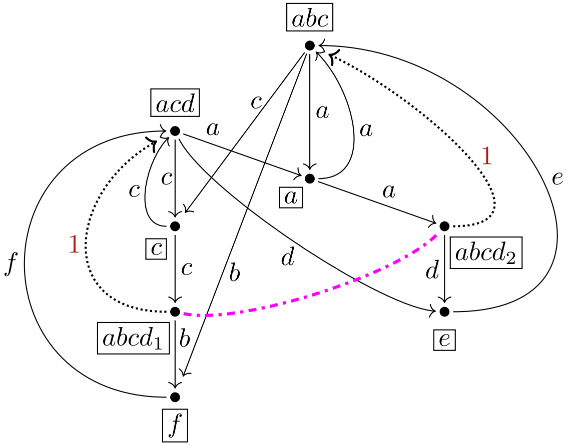

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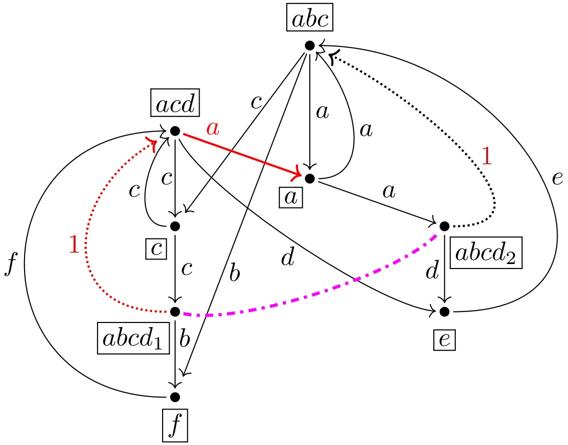

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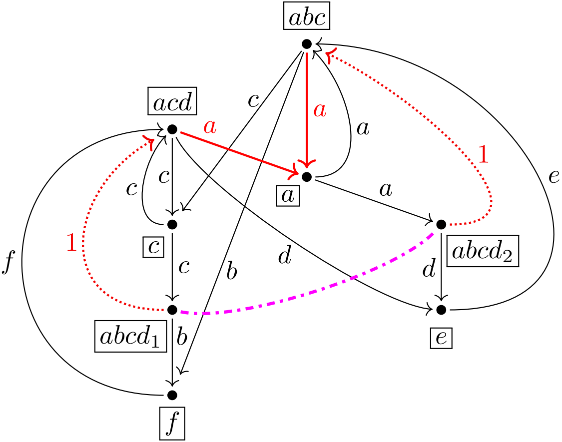

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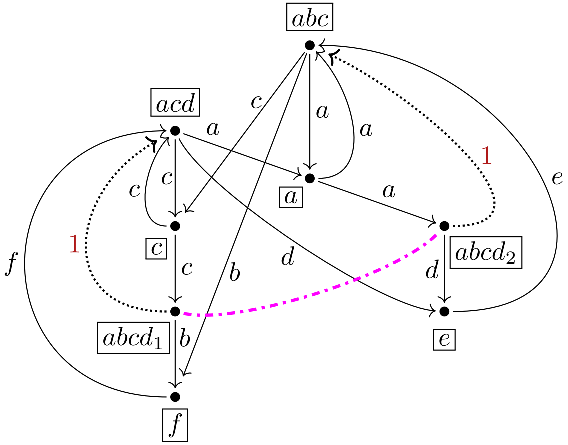

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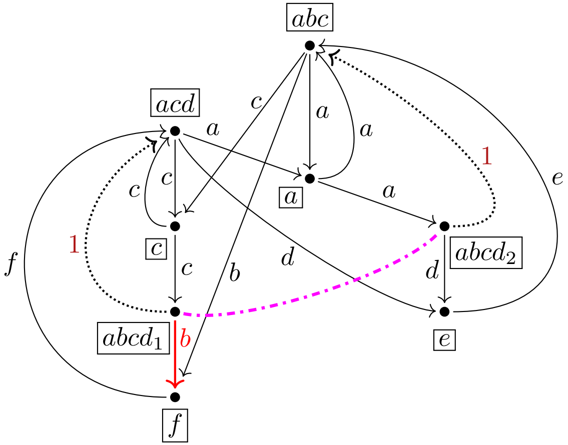

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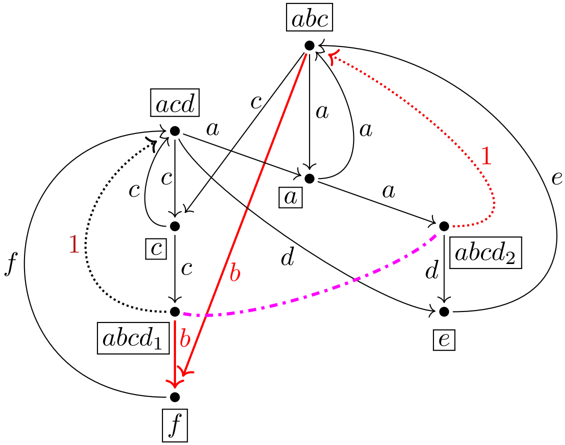

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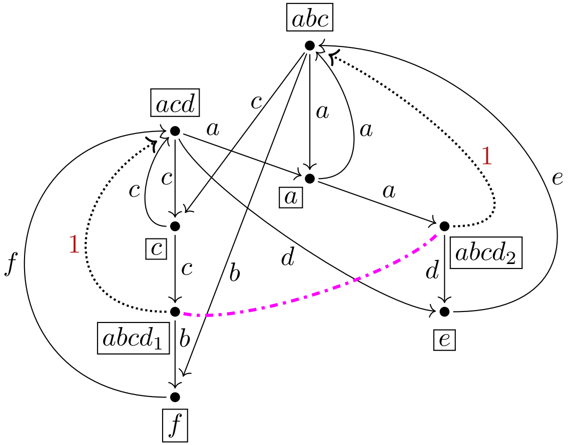

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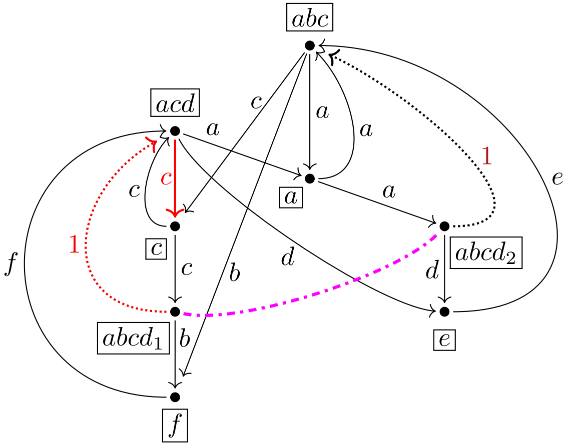

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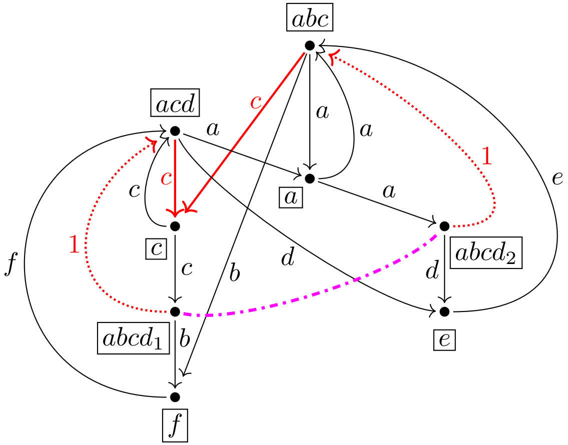

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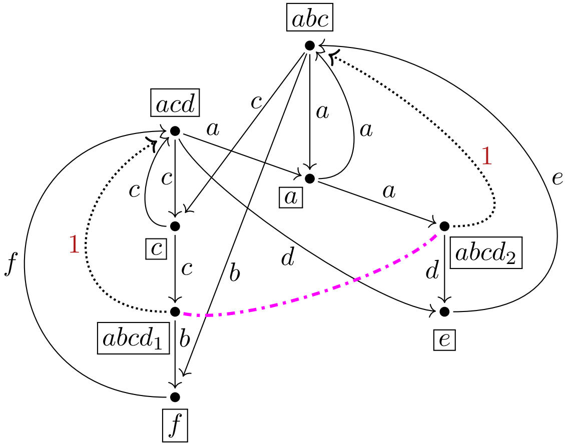

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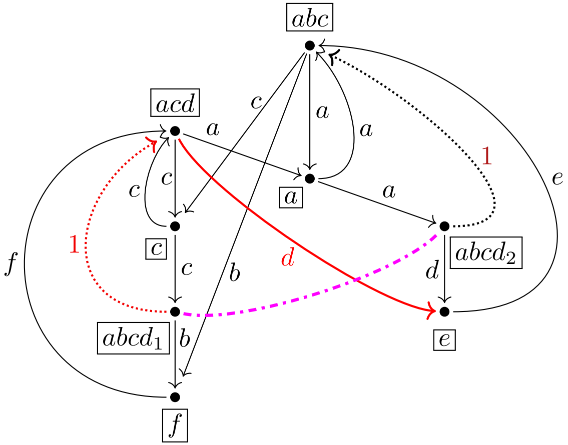

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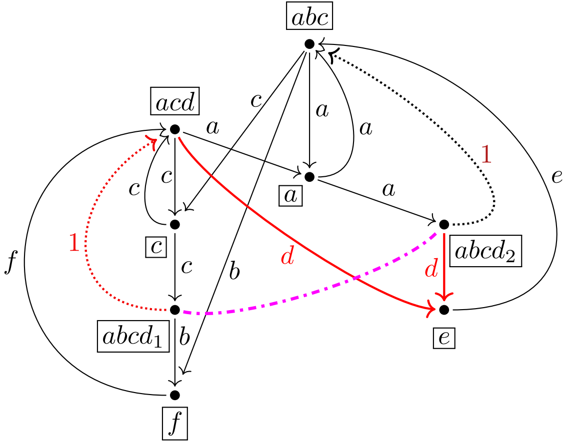

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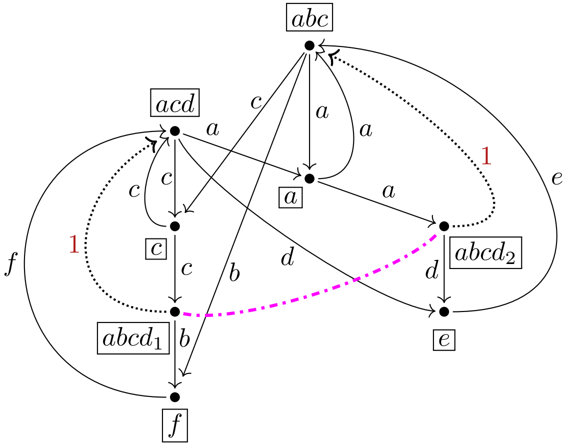
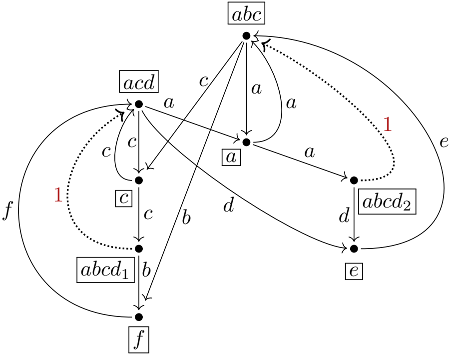
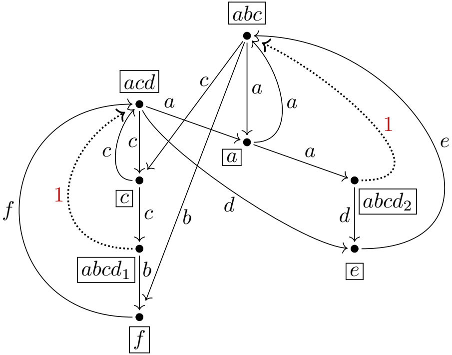


Local transfer function on 1-LTS





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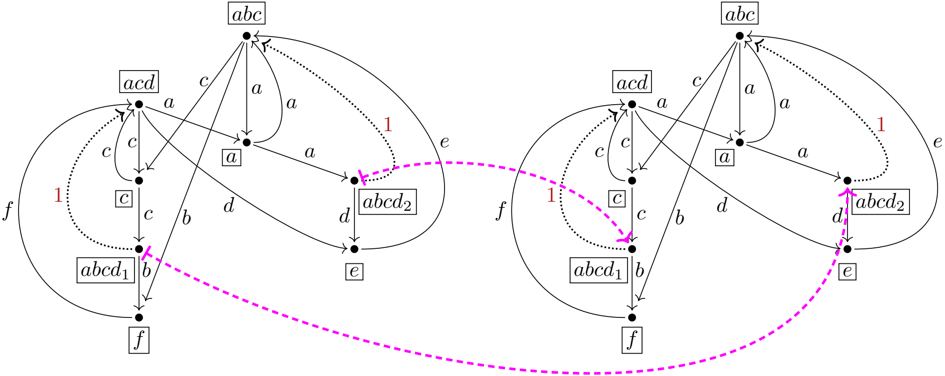

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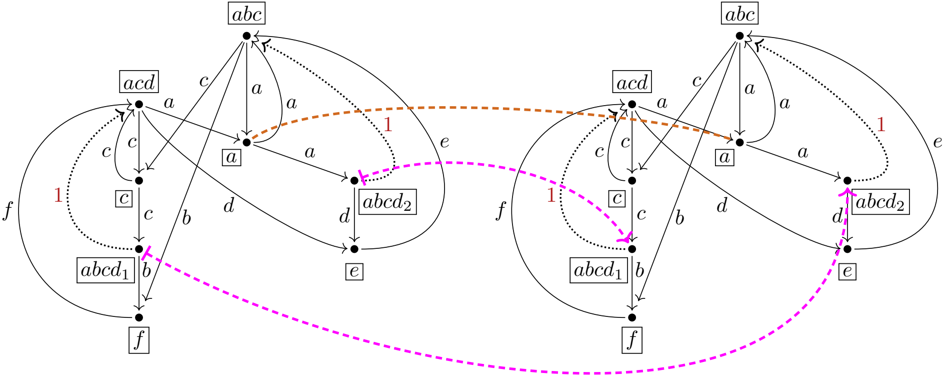

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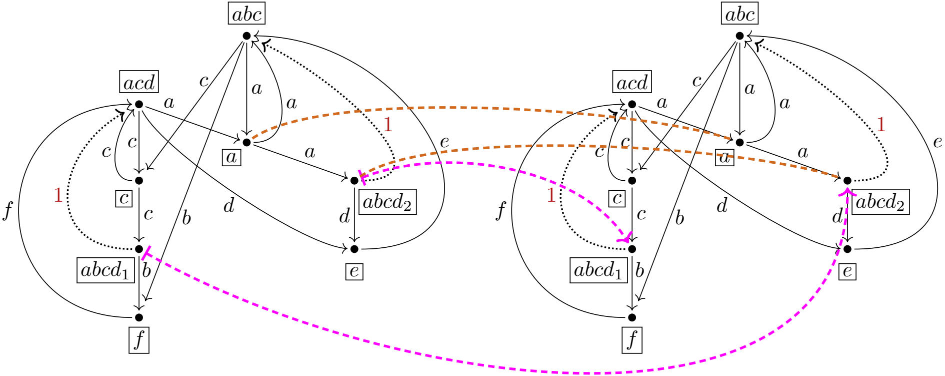

Summary

- fragments of bisimulations
  - bisimulating slices
    - sometimes useful to construct bisimulations
  - grounded bisimulation slices
    - a grounded bisimulation slice B on an LTS $\mathcal{L}$ gives rise to a bisimulation $\overline{\overline{B}} := B \cup =$ on $\mathcal{L}$.
- functional fragments of bisimulations
  - pullback of specifications via functional bisimulations
  - transfer functions $\hat{=}$ functional bisimulations
  - local-transfer functions $\hat{=}$ functional grounded bisimulation slices
    - a local-transfer function ϕ on an LTS $\mathcal{L}$ lifts to a transfer function $\widehat{\phi}$ on the elevation of $\mathrm{E}_{\mathrm{field}(\phi)}(\mathcal{L})$ of field(ϕ) above $\mathcal{L}$.
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  - using local transfer functions on near-collapsed 1-LTSs
  - for transfer of specifications, to prove them equal in a proof system

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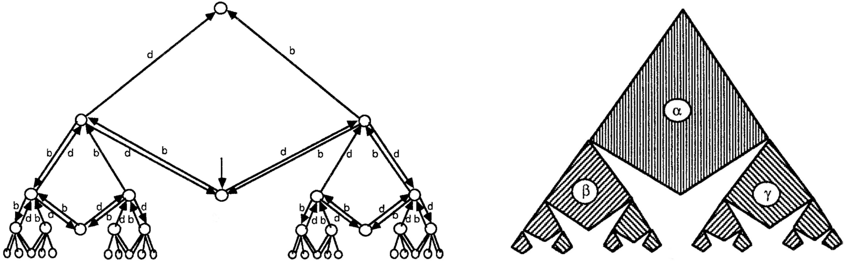
Bisimulating/bisimulation slices: origin and application



$$X = dY + bZ\ , \quad Y = b + bX + dYY\ , \quad Z = d + dX + bZZ$$

Example graph (concrete and abstract form) from:

Baeten, Bergstra, Klop: *Decidability of Bisimulation Equivalence for Processes Generating Context-Free Languages*, 1987

Bisimulating/bisimulation slices: origin and application

If for *each* pair α,β in the k-th slice such a copy α',β' exists, then the partial bisimulation R is called d-*sufficient*.

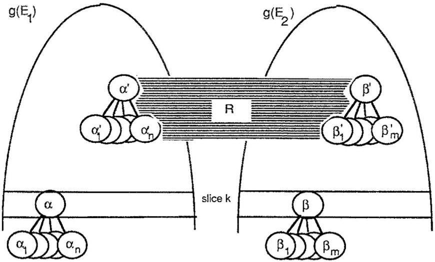


Figure 16

6.4. Let a partial bisimulation R as in 6.3 be given, which is sufficient. Then the *periodical continuation* of R is constructed as follows. Let α,β be as in 6.3. The partial bisimulation R is extended to $(\alpha_1 \cup ... \cup \alpha_n) \times (\beta_1 \cup ... \cup \beta_m)$ by copying the restriction of R to $(\alpha_1' \cup ... \cup \alpha_n') \times (\beta_1' \cup ... \cup \beta_m')$. This is done for all pairs α,β in slice k of $g(E_1)$, $g(E_2)$. It is

Baeten, Bergstra, Klop: *Decidability of Bisimulation Equivalence for Processes Generating Context-Free Languages*, 1987

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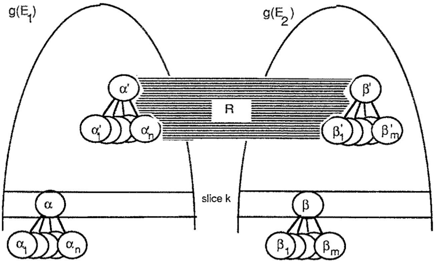

Figure 16

6.4. Let a partial bisimulation R as in 6.3 be given, which is sufficient. Then the *periodical continuation* of R is constructed as follows. Let α,β be as in 6.3. The partial bisimulation R is extended to $(\alpha_1 \cup ... \cup \alpha_n) \times (\beta_1 \cup ... \cup \beta_m)$ by copying the restriction of R to $(\alpha_1' \cup ... \cup \alpha_n') \times (\beta_1' \cup ... \cup \beta_m')$. This is done for all pairs α,β in slice k of $g(E_1)$, $g(E_2)$. It is

Baeten, Bergstra, Klop: *Decidability of Bisimulation Equivalence for Processes Generating Context-Free Languages*, 1987

Bisimulation, functional bisimulation

Definition

Let $\mathcal{L}_i = \langle S_i, A, \to_i, \downarrow_i \rangle$, for $i \in \{1,2\}$ be LTSs.

A *bisimulation between $\mathcal{L}_1$ and $\mathcal{L}_2$* is a non-empty relation $\boldsymbol{B} \subseteq S_1 \times S_2$ such that for all $\langle v_1, v_2 \rangle \in \boldsymbol{B}$:

$$\text{(forth)} \quad \forall v_1' \in S_1 \; \forall a \in A \; \big(v_1 \xrightarrow{a}_1 v_1' \implies \exists v_2' \in S_2 \big(v_2 \xrightarrow{a}_2 v_2' \wedge \langle v_1', v_2' \rangle \in \boldsymbol{B} \big) \big) \,,$$

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$$\text{(forth)} \quad \forall v_1' \in S_1 \, \forall a \in A \quad \left(v_1 \xrightarrow{a}_1 v_1' \implies \exists v_2' \in S_2 \left(v_2 \xrightarrow{a}_2 v_2' \land \langle v_1', v_2' \rangle \in \boldsymbol{B} \right) \right),$$

$$\text{(back)} \quad \forall v_2' \in S_2 \forall a \in A \quad \left(\exists v_1' \in S_1 \left(v_1 \xrightarrow{a}_1 v_1' \land \langle v_1', v_2' \rangle \in \boldsymbol{B} \right) \Longleftarrow v_2 \xrightarrow{a}_2 v_2' \right),$$

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$$\text{(forth)} \quad \forall v_1' \in S_1 \, \forall a \in A \; \Big(v_1 \xrightarrow{a}_1 v_1' \implies \exists v_2' \in S_2 \big(v_2 \xrightarrow{a}_2 v_2' \wedge \langle v_1', v_2' \rangle \in \boldsymbol{B} \big) \Big),$$

$$\text{(back)} \quad \forall v_2' \in S_2 \forall a \in A \; \Big(\exists v_1' \in S_1 \big(v_1 \xrightarrow{a}_1 v_1' \wedge \langle v_1', v_2' \rangle \in \boldsymbol{B} \big) \impliedby v_2 \xrightarrow{a}_2 v_2' \Big),$$

$$\text{(termination)} \quad v_1 \downarrow_1 \iff v_2 \downarrow_2 .$$

Bisimulation, functional bisimulation

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Let $\mathcal{L}_i = \langle S_i, A, \to_i, \downarrow_i \rangle$, for $i \in \{1,2\}$ be LTSs.

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$$\begin{array}{rl} \text{(forth)} & \forall v_1' \in S_1 \; \forall a \in A \\ & \quad \Big(v_1 \xrightarrow{a}_1 v_1' \implies \exists v_2' \in S_2 \big(v_2 \xrightarrow{a}_2 v_2' \wedge \langle v_1', v_2' \rangle \in \boldsymbol{B} \big) \Big) \,, \\ \text{(back)} & \forall v_2' \in S_2 \forall a \in A \\ & \quad \Big(\exists v_1' \in S_1 \big(v_1 \xrightarrow{a}_1 v_1' \wedge \langle v_1', v_2' \rangle \in \boldsymbol{B} \big) \impliedby v_2 \xrightarrow{a}_2 v_2' \Big) \,, \\ \text{(termination)} & v_1 {\downarrow_1} \iff v_2 {\downarrow_2} \,. \end{array}$$

We denote it by $\mathcal{L}_1 \leftharpoondown \boldsymbol{B} \rightharpoonup \mathcal{L}_2$.

Bisimulation, functional bisimulation

Definition

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We denote it by $\mathcal{L}_1 \leftharpoondown \boldsymbol{B} \rightharpoondown \mathcal{L}_2$.

A *functional bisimulation between $\mathcal{L}_1$ and $\mathcal{L}_2$* is a bisimulation that is the graph $\mathrm{gr}(\phi)$ of a partial function $\phi : S_1 \rightharpoonup S_2$. We write $\mathcal{L}_1 = \phi \rightharpoondown \mathcal{L}_2$.

Bisimulating/bisimulation slices

Definition

Let $\mathcal{L}_i = \langle S_i, A, \to_i, \downarrow_i \rangle$ for $i \in \{1, 2\}$ be LTSs.
A *bisimulating slice between $\mathcal{L}_1$, $\mathcal{L}_2$* is a non-empty relation $\boldsymbol{B} \subseteq S_1 \times S_2$ with active domain $W_1 := \mathrm{dom}_{\mathsf{act}}(\boldsymbol{B}) := \{x \mid \langle x, y \rangle \in \boldsymbol{B}\}$, active codomain $W_2 := \mathrm{cod}_{\mathsf{act}}(\boldsymbol{B}) := \{y \mid \langle x, y \rangle \in \boldsymbol{B}\}$ s.th. for all $\langle v_1, v_2 \rangle \in \boldsymbol{B}$:

$$\begin{aligned}
(\text{forth})_{\mathsf{s}} \quad & \forall a \in A\ \forall v_1' \in S_1 \\
& \quad \big(v_1 \xrightarrow{a}_1 v_1' \wedge \boxed{v_1' \in W_1} \\
& \qquad \Longrightarrow\ \exists v_2' \in S_2 \big(v_2 \xrightarrow{a}_2 v_2' \wedge \langle v_1', v_2' \rangle \in \boldsymbol{B} \big) \big) ,
\end{aligned}$$

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(\text{forth})_{\mathsf{s}} \quad & \forall a \in A \; \forall v_1' \in S_1 \\
& \Big(v_1 \xrightarrow{a}_1 v_1' \wedge \boxed{v_1' \in W_1} \\
& \qquad \Longrightarrow \; \exists v_2' \in S_2 \big(v_2 \xrightarrow{a}_2 v_2' \wedge \langle v_1', v_2' \rangle \in \boldsymbol{B} \big) \Big) \,, \\
(\text{back})_{\mathsf{s}} \quad & \forall a \in A \; \forall v_2' \in S_2 \\
& \Big(\exists v_1' \in S_1 \big(v_1 \xrightarrow{a}_1 v_1' \wedge \langle v_1', v_2' \rangle \in \boldsymbol{B} \\
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(\text{forth})_{\mathsf{s}} \quad & \forall a \in A \; \forall v_1' \in S_1 \\
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& \qquad \Longrightarrow \; \exists v_2' \in S_2 \big(v_2 \xrightarrow{a}_2 v_2' \wedge \langle v_1', v_2' \rangle \in \boldsymbol{B} \big) \Big) , \\
(\text{back})_{\mathsf{s}} \quad & \forall a \in A \; \forall v_2' \in S_2 \\
& \Big(\exists v_1' \in S_1 \big(v_1 \xrightarrow{a}_1 v_1' \wedge \langle v_1', v_2' \rangle \in \boldsymbol{B} \\
& \qquad \Longleftarrow \; v_2 \xrightarrow{a}_2 v_2' \wedge \boxed{v_2' \in W_2} \Big) , \\
(\text{termination})_{\mathsf{s}} \quad & v_1 \downarrow_1 \;\Longleftrightarrow\; v_2 \downarrow_2 .
\end{aligned}$$

Bisimulating/bisimulation slices

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Let $\mathcal{L}_i = \langle S_i, A, \to_i, \downarrow_i \rangle$ for $i \in \{1,2\}$ be LTSs.
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$$
\begin{aligned}
&(\text{forth})_{\mathsf{s}} \quad \forall a \in A\, \forall v_1' \in S_1 \\
&\qquad \Big(v_1 \xrightarrow{a}_1 v_1' \wedge \boxed{v_1' \in W_1} \\
&\qquad\qquad \Longrightarrow \exists v_2' \in S_2 \big(v_2 \xrightarrow{a}_2 v_2' \wedge \langle v_1', v_2' \rangle \in \boldsymbol{B} \big) \Big)\,,
\end{aligned}
$$

$$
\begin{aligned}
&(\text{back})_{\mathsf{s}} \quad \forall a \in A\, \forall v_2' \in S_2 \\
&\qquad \Big(\exists v_1' \in S_1 \big(v_1 \xrightarrow{a}_1 v_1' \wedge \langle v_1', v_2' \rangle \in \boldsymbol{B} \\
&\qquad\qquad \Longleftarrow v_2 \xrightarrow{a}_2 v_2' \wedge \boxed{v_2' \in W_2} \Big)\,,
\end{aligned}
$$

$$
(\text{termination})_{\mathsf{s}} \quad v_1{\downarrow_1} \iff v_2{\downarrow_2}\,.
$$

A *bisimulation slice between* $\mathcal{L}_1$ *and* $\mathcal{L}_2$ is a bisimulating slice between $\mathcal{L}_1$ and $\mathcal{L}_2$ that is contained in a bisimulation between $\mathcal{L}_1$ and $\mathcal{L}_2$.

Transfer functions: conditions

Proposition

Let $\mathcal{L}_i = \langle S_i, A, \to_i, \downarrow_i \rangle$ for $i \in \{1, 2\}$ be LTSs.

A partial function $\phi : S_1 \rightharpoonup S_2$ with domain $W_1 := \mathrm{dom}(\phi)$ is a transfer function between $\mathcal{L}_1$ and $\mathcal{L}_2$ if and only if $W_1 \neq \varnothing$, and for all $s_1, s_1', s_2' \in S$ and $a \in A$:

$$s_1 \in W_1 \wedge s_1 \xrightarrow{a}_1 s_1' \implies \phi(s_1) \xrightarrow{a}_2 \phi(s_1') \ ,$$

Transfer functions: conditions

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$$\exists s_1' \in S_1 \big(\, s_1 \xrightarrow{a}_1 s_1' \wedge \phi(s_1') = s_2' \, \big) \impliedby s_1 \in W_1 \wedge \phi(s_1) \xrightarrow{a}_2 s_2' \ ,$$

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$$s_1 \in W_1 \wedge s_1 \xrightarrow{a}_1 s_1' \implies \phi(s_1) \xrightarrow{a}_2 \phi(s_1') \ ,$$

$$\exists s_1' \in S_1 \big(\, s_1 \xrightarrow{a}_1 s_1' \wedge \phi(s_1') = s_2' \, \big) \impliedby s_1 \in W_1 \wedge \phi(s_1) \xrightarrow{a}_2 s_2' \ ,$$

$$s_1 \in W_1 \implies \big(\, s_1{\downarrow_1} \iff \phi(s_1){\downarrow_2} \, \big) \ .$$

Grounded bisimulation slices

Definition

Let $\mathcal{L} = \langle S, A, \rightarrow, \downarrow \rangle$ be an LTS.

A *grounded* bisimulation slice *on* $\mathcal{L}$ is a bisimulating slice $\boldsymbol{B} \subseteq S \times S$ with $\mathrm{dom}_{\mathsf{act}}(\boldsymbol{B}) := W_1$, $\mathrm{cod}_{\mathsf{act}}(\boldsymbol{B}) := W_2$ such that for every $\langle v_1, v_2 \rangle \in \boldsymbol{B}$ holds:

(forth)$_{\mathsf{g}}$ $\quad \forall a \in A \, \forall v_1' \in S_1$

$$\left(v_1 \xrightarrow{a} v_1' \wedge \boxed{v_1' \notin W_1} \implies v_2 \xrightarrow{a} v_1' \wedge \boxed{v_1' \notin W_2} \right),$$

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$$(\text{back})_{\mathsf{g}} \quad \forall a \in A \, \forall v_2' \in S_1 \; \Big(v_1 \xrightarrow{a} v_2' \wedge \boxed{v_2' \notin W_1} \impliedby v_2 \xrightarrow{a} v_2' \wedge \boxed{v_2' \notin W_2} \Big) .$$

Elevation of a vertex set above LTS

Definition

Let $\mathcal{L} = \langle S, A, \to, \downarrow\rangle$ be an LTS, and $W \subseteq S$ a subset of the vertices of $\mathcal{L}$.

The *elevation of W above $\mathcal{L}$* is the LTS $\mathsf{E}_W(\mathcal{L}) = \langle S_{\mathsf{E}_W}, A, \to_{\mathsf{E}_W}, \downarrow_{\mathsf{E}_W}\rangle$:

$$S_{\mathsf{E}_W} := S \times \{\mathbf{0}, \mathbf{1}\} \;,$$

$$\begin{aligned}\to_{\mathsf{E}_W} := \;& \big\{\langle\langle v_1, \mathbf{1}\rangle, a, \langle v_2, \mathbf{1}\rangle\rangle \;\big|\; \langle v_1, a, v_2\rangle \in \to \wedge\, a \in A \wedge v_2 \in W\big\} \\ \cup\; & \big\{\langle\langle v_1, \mathbf{1}\rangle, a, \langle v_2, \mathbf{0}\rangle\rangle \;\big|\; \langle v_1, a, v_2\rangle \in \to \wedge\, a \in A \wedge v_2 \notin W\big\} \\ \cup\; & \big\{\langle\langle v_1, \mathbf{0}\rangle, a, \langle v_2, \mathbf{0}\rangle\rangle \;\big|\; \langle v_1, a, v_2\rangle \in \to\big\} \;,\end{aligned}$$

$$\downarrow_{\mathsf{E}_W} := \big\{\langle v, i\rangle \;\big|\; v \in S,\, i \in \{\mathbf{0}, \mathbf{1}\}\,,\, v\downarrow\big\} \;.$$

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$$\downarrow_{\mathsf{E}_W} := \big\{ \langle v, i \rangle \;\big|\; v \in S,\, i \in \{\mathbf{0}, \mathbf{1}\},\, v\downarrow \big\} \;.$$

Proposition (elevation above LTS functionally bisimilar with LTS)

Let $\mathcal{L} = \langle S, A, \to, \downarrow \rangle$ be an LTS, and $W \subseteq S$.

Then $\mathsf{E}_W(\mathcal{L}) =\!\!\pi_1\!\!\Rightarrow \mathcal{L}$ holds, where $\pi_1 : S \times \{\mathbf{0}, \mathbf{1}\} \to S,\ \langle v, \boldsymbol{i} \rangle \mapsto v$ projection function.