Loop Elimination in Process Graphs is Confluent when Pruning Steps are Added

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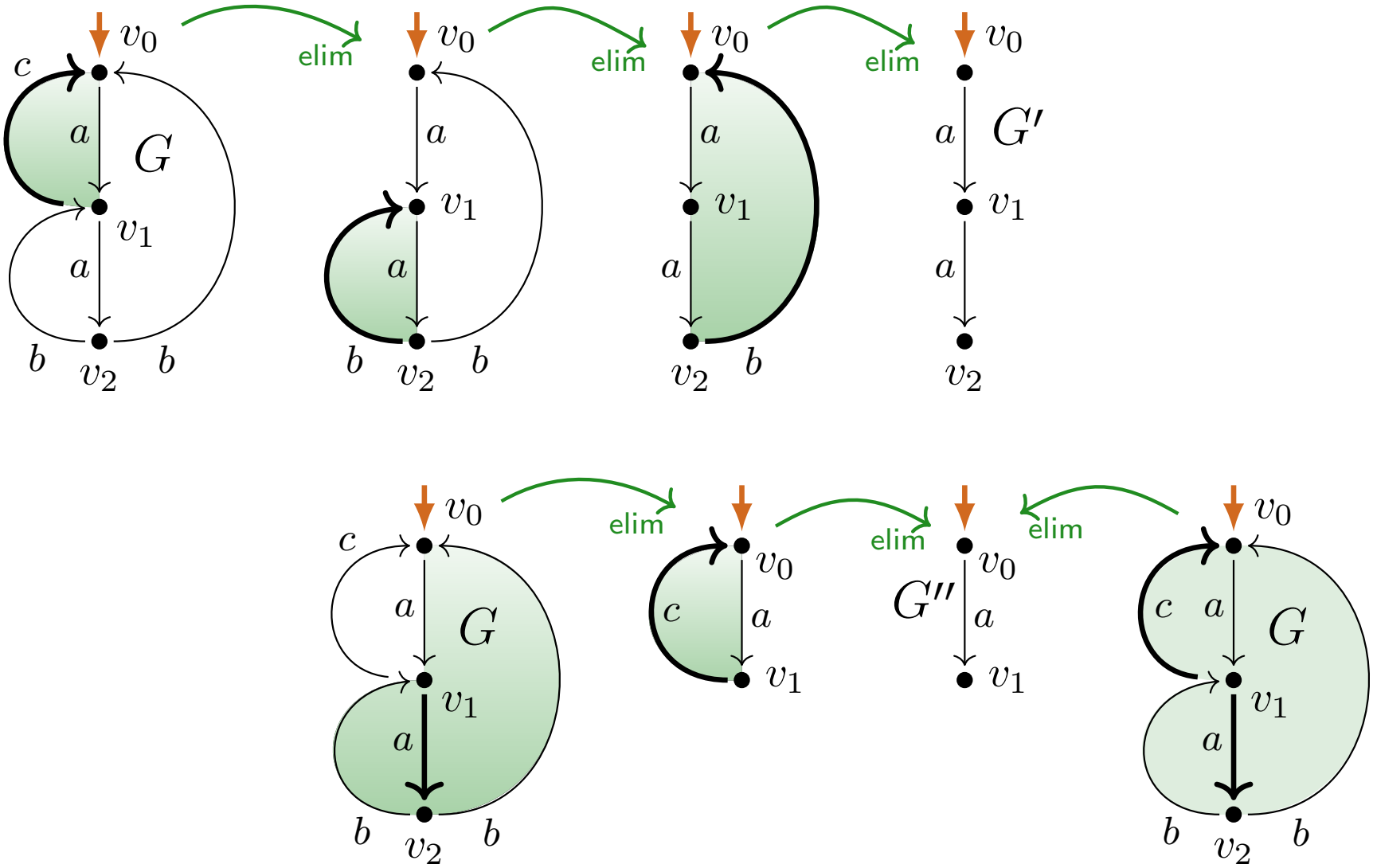
Overview

- Loop elimination $\to_{\mathrm{elim}}$ in process graphs
  - Milner's process interpretation P of regular expressions
  - Loop existence and elimination property LEE
    LEE $\hat{=}$ interpretations of $(*/\pm)$ reg. expr's
  - $\to_{\mathrm{elim}}$ is not confluent
- Completion of loop elimination $\to_{\mathrm{elim}}$ with pruning steps $\to_{\mathrm{prune}}$
  - 'critical pair' lemma for 'bi-loop' elimination steps
  - use of decreasing diagrams
- Application 1: LEE is decidable in polynomial time
- Application 2: Layered LEE = LEE
- Further questions

Loop elimination $\to_{\text{elim}}$



Process interpretation P of regular expressions *(Milner, 1984)*

$\mathbf{0} \xmapsto{P}$ deadlock δ, no observable behaviour

$a \xmapsto{P}$ atomic action a, then terminate

$e_1 + e_2 \xmapsto{P}$ *(choice)* execute $P(e_1)$ or $P(e_2)$

$e_1 \cdot e_2 \xmapsto{P}$ *(sequentialization)* execute $P(e_1)$, then $P(e_2)$

$e^* \xmapsto{P}$ *(iteration)* repeat (terminate or execute $P(e)$)

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Process semantics $[\![\cdot]\!]_P$ of regular expressions *(Milner, 1984)*

$\mathbf{0} \xrightarrow{P}$ deadlock $\boldsymbol{\delta}$, no observable behaviour

$\mathbf{1} \xrightarrow{P}$ empty-step process $\boldsymbol{\epsilon}$, then terminate

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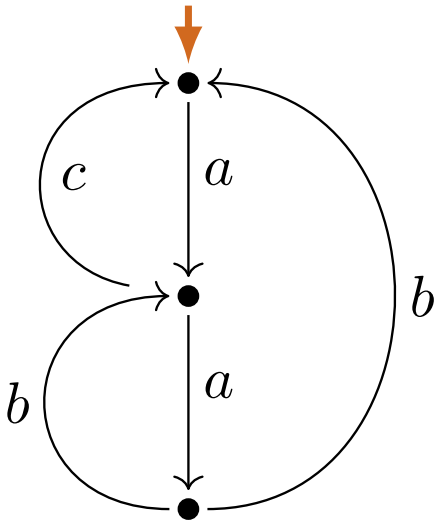
$[\![e]\!]_P := [P(e)]_{\leftrightarrow}$ (bisimilarity equivalence class of process $P(e)$)

Process interpretation P (example)

G



$P(uf) \simeq G$

$$\overbrace{((1 \cdot a) \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0}^{uf}$$

$$\downarrow a$$

$$(1 \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0$$

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Process interpretation P (example) has property LEE

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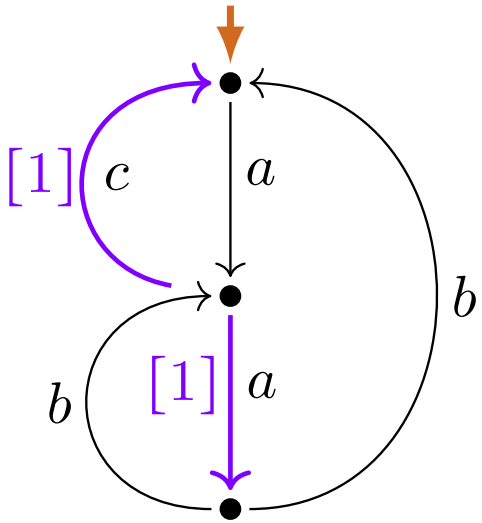

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Loop graphs (interpretations of innermost iterations without 1)

Definition

A process graph is a loop graph if:

(L1) There is an infinite path from the start vertex.

(L2) Every infinite path from the start vertex returns to it.

(L3) Immediate termination is only possible at the start vertex.

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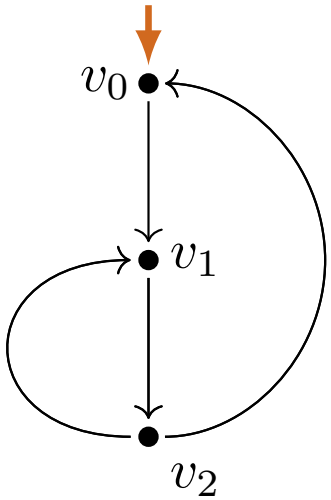

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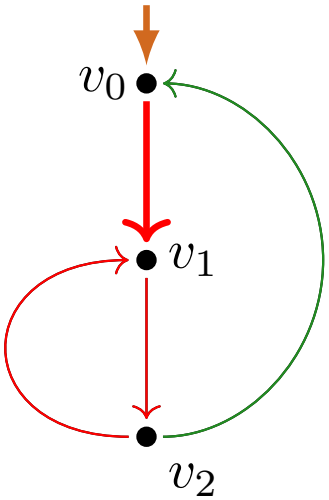

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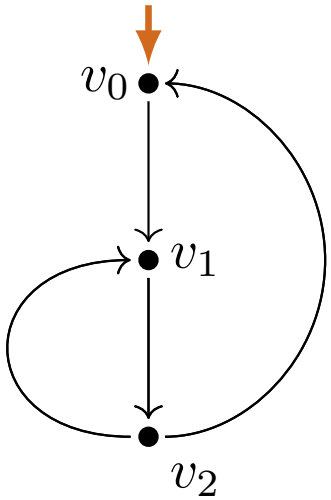

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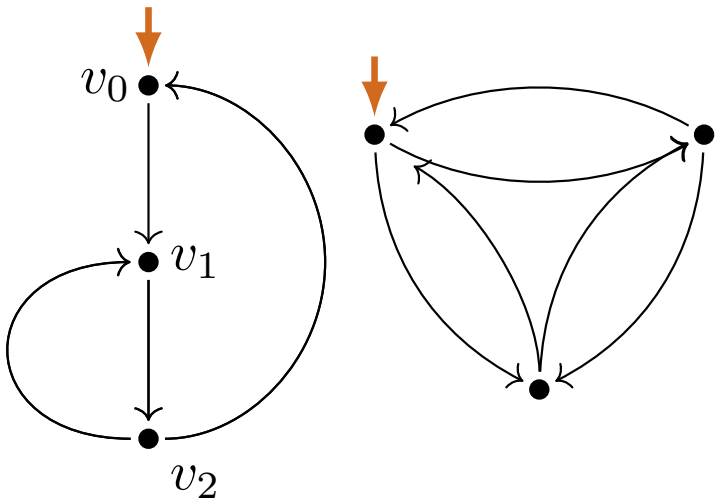

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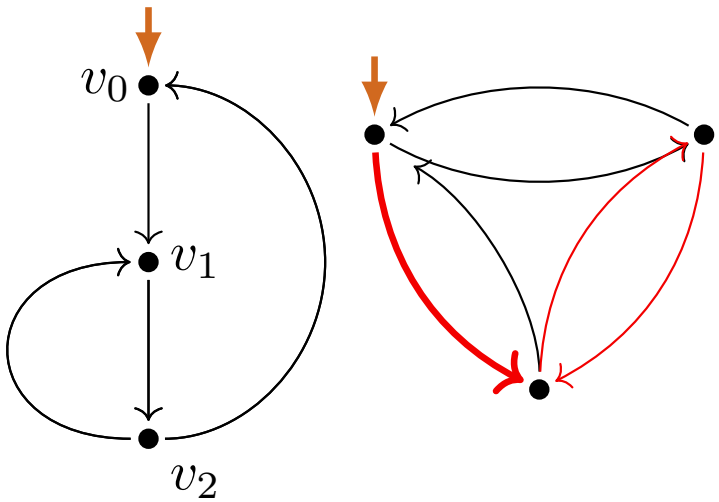

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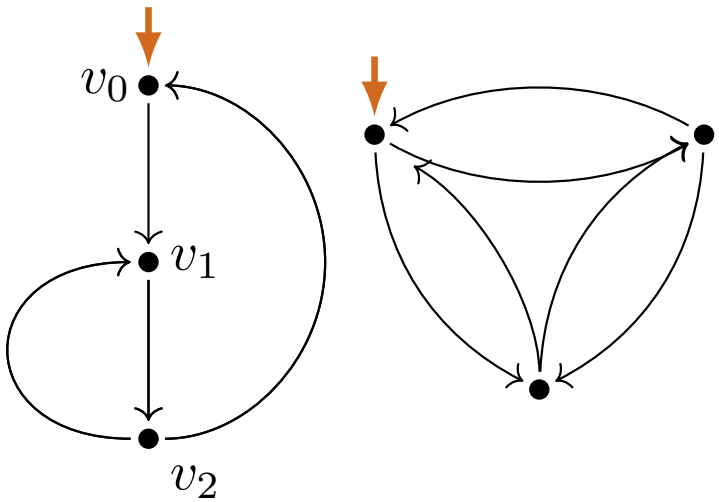

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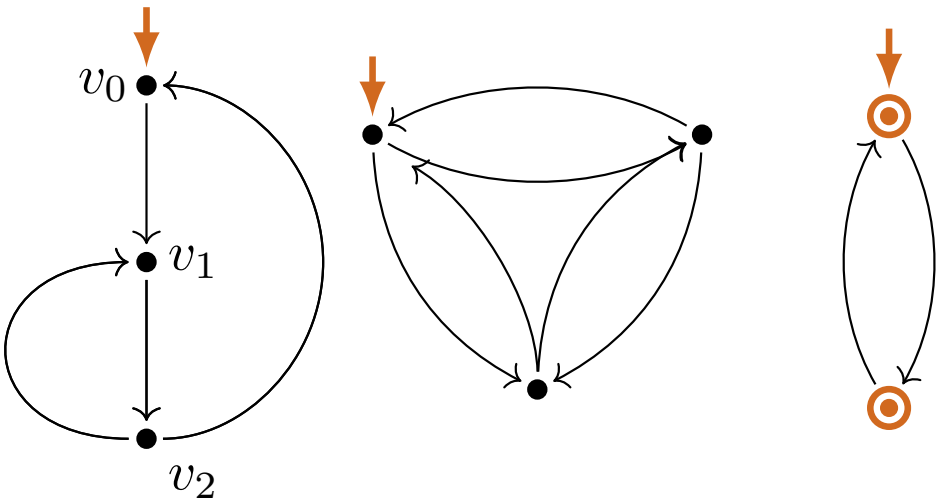

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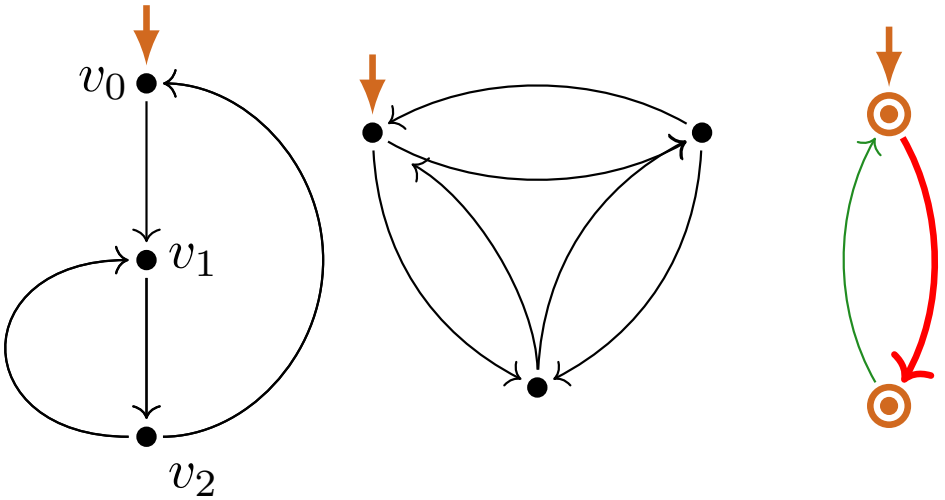

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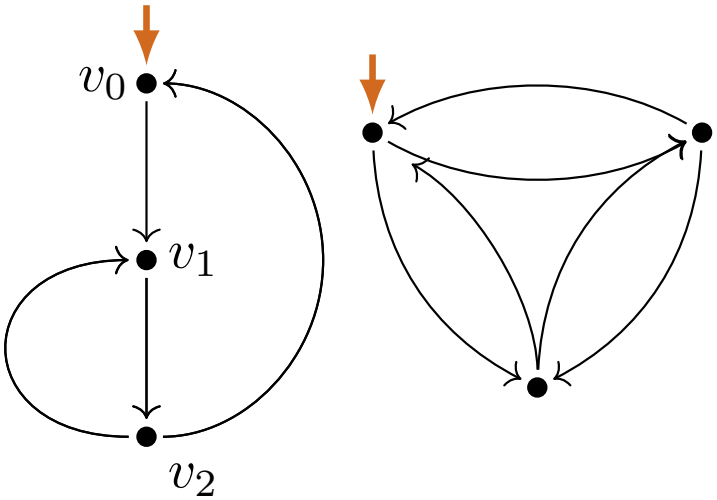

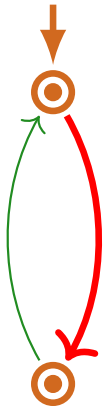

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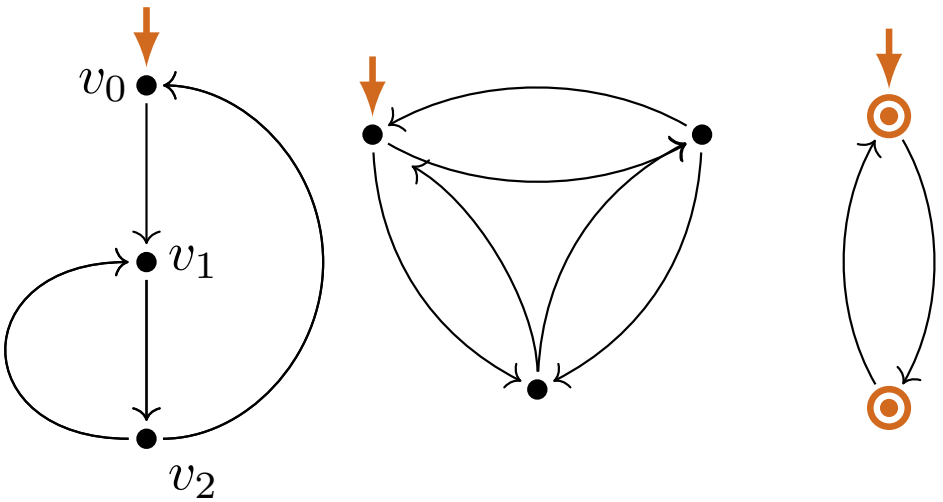

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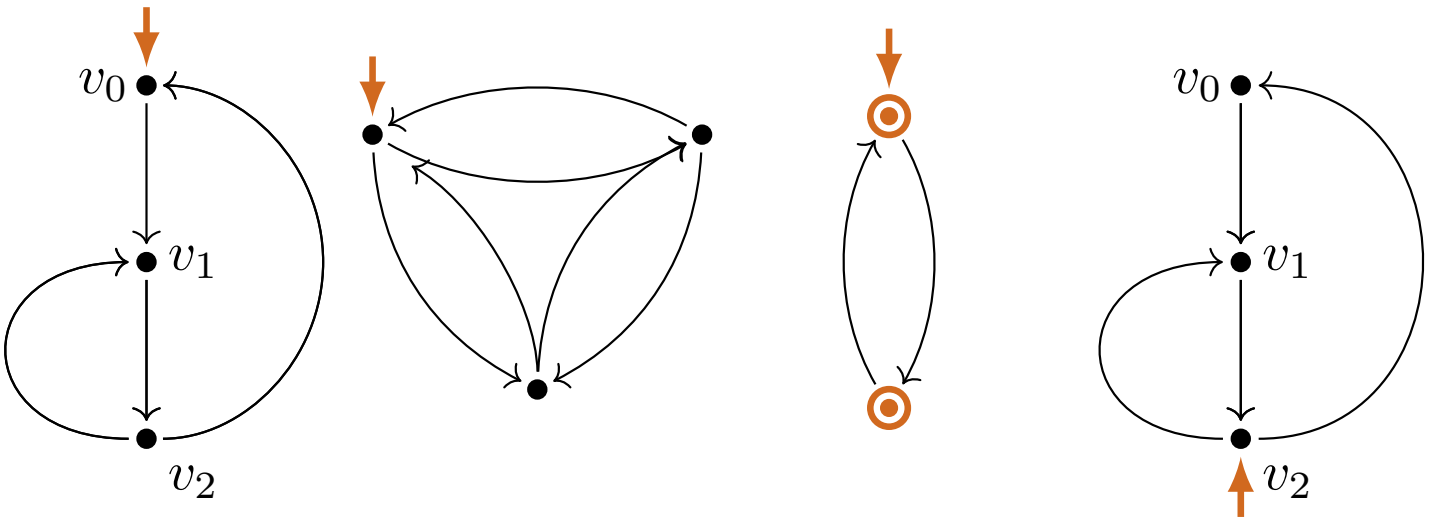

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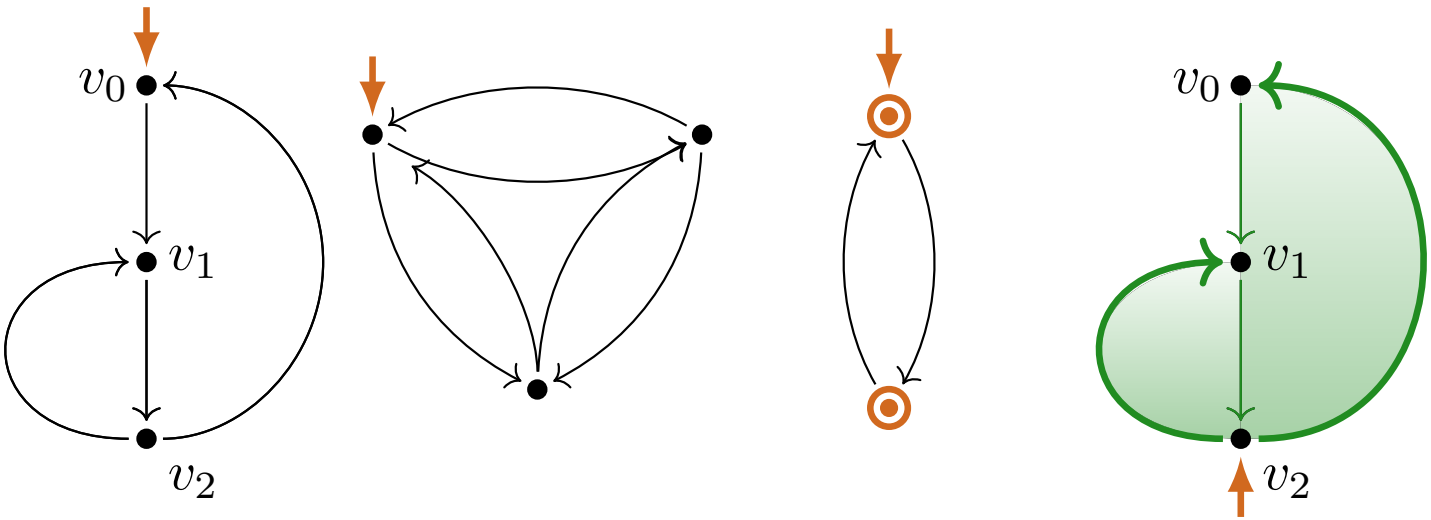

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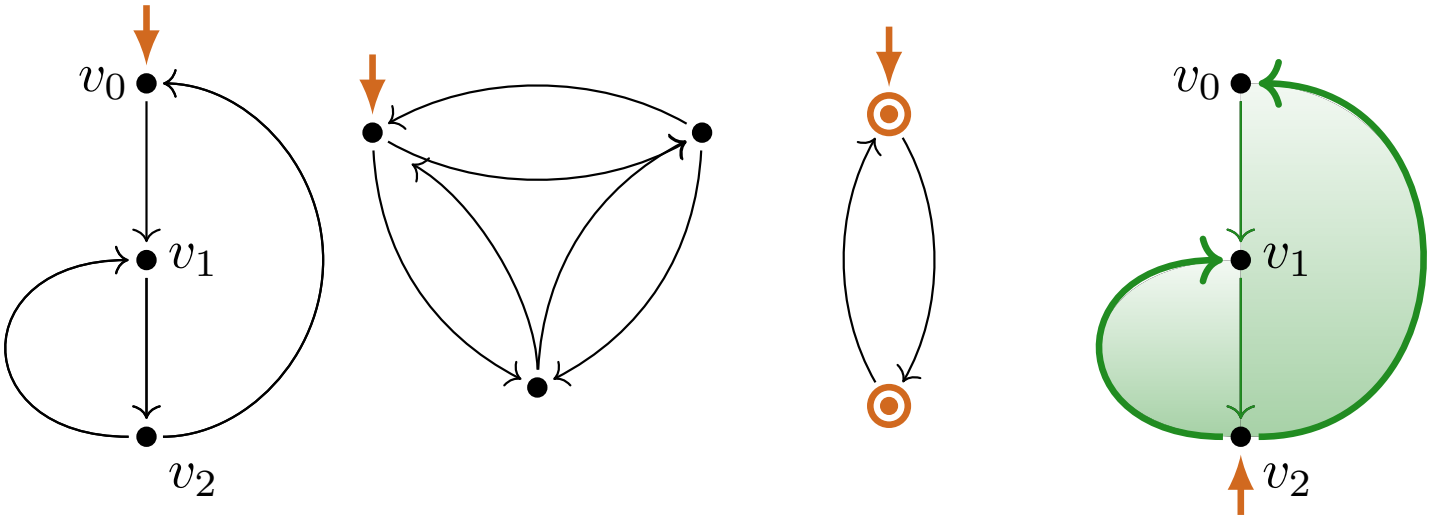

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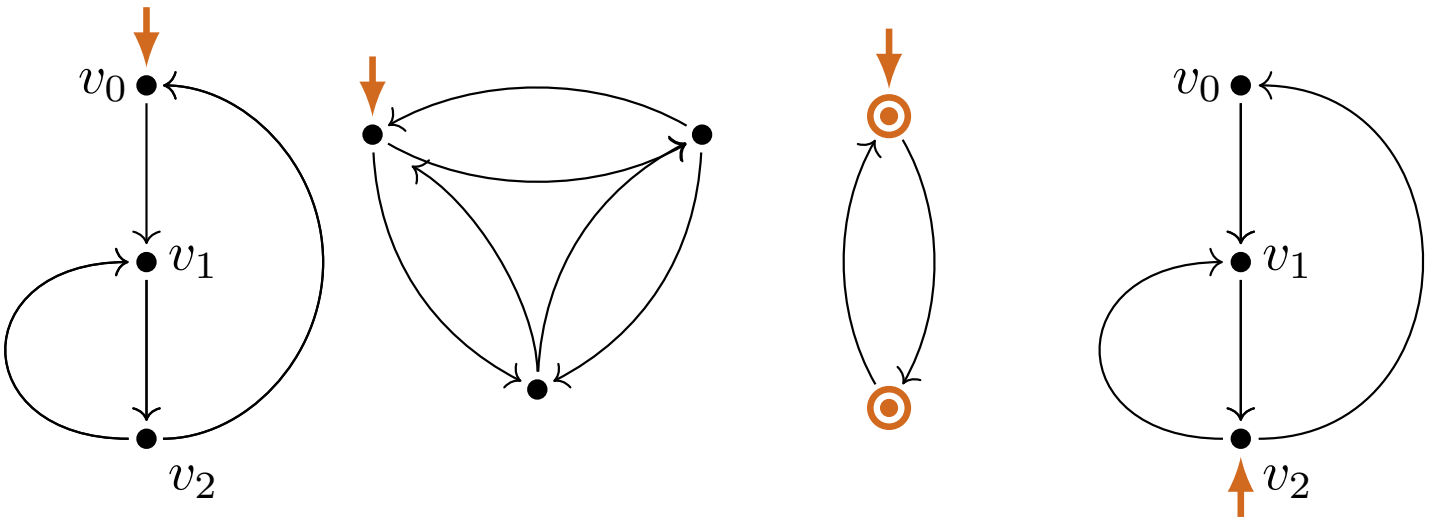

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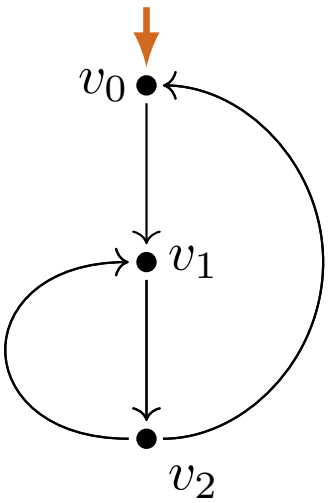

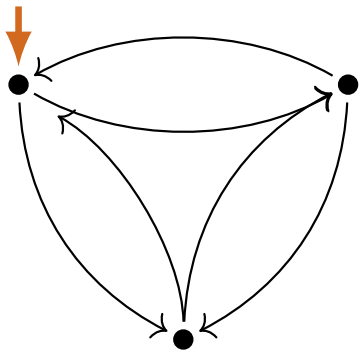

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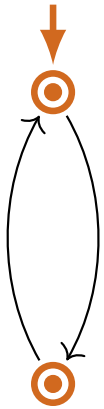

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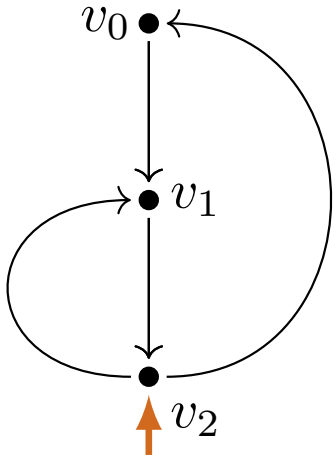

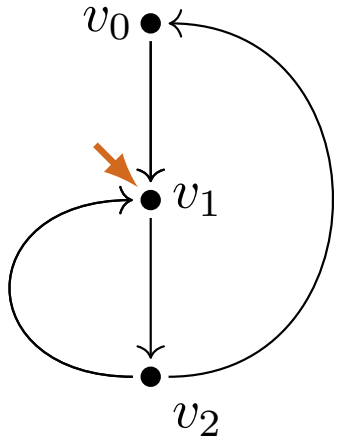

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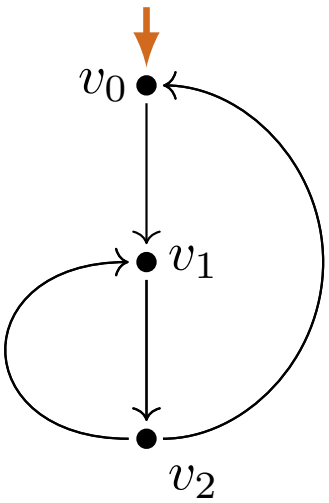

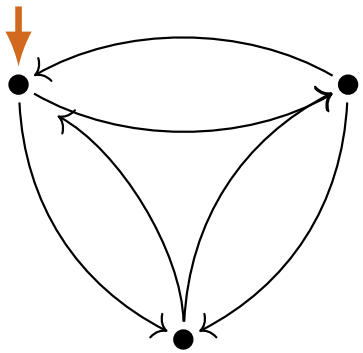

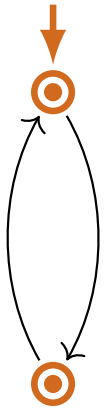

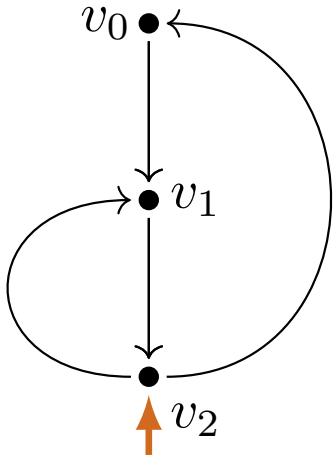

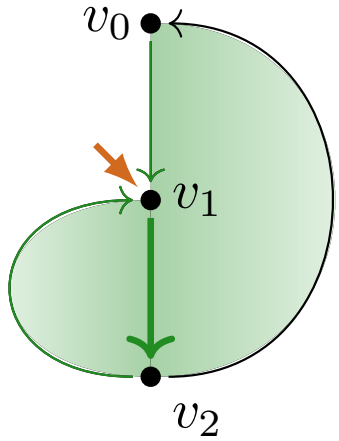

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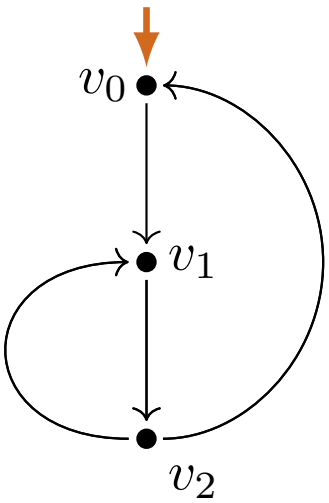

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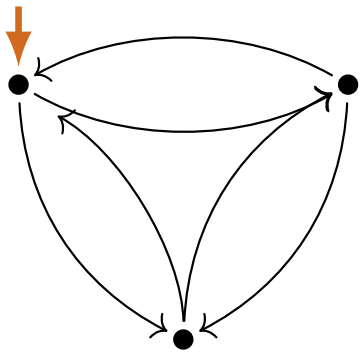

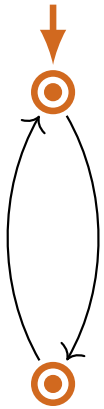

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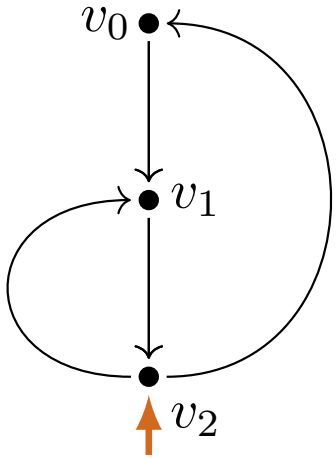

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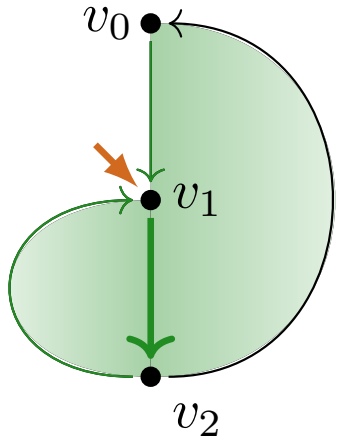

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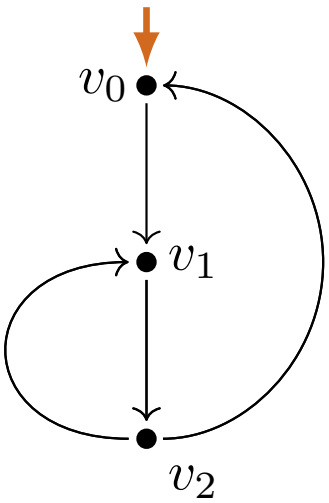

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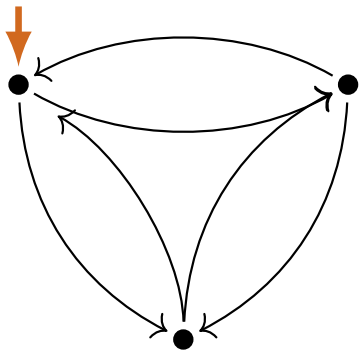

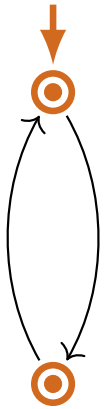

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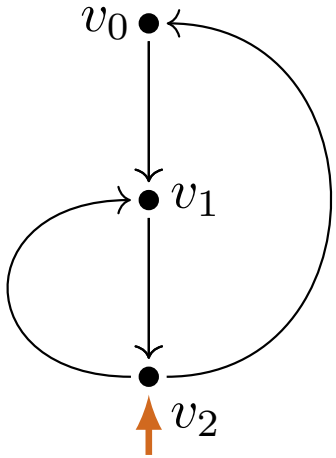

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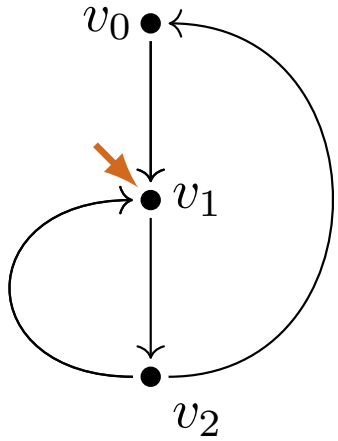

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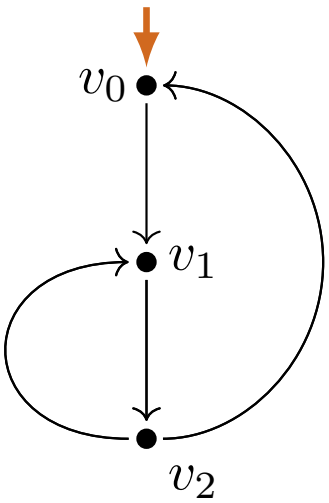
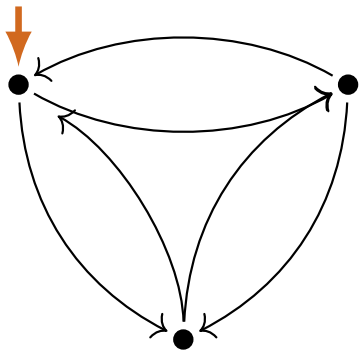
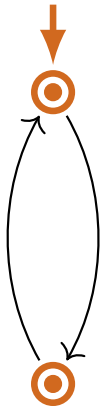




(L1),~~(L2)~~



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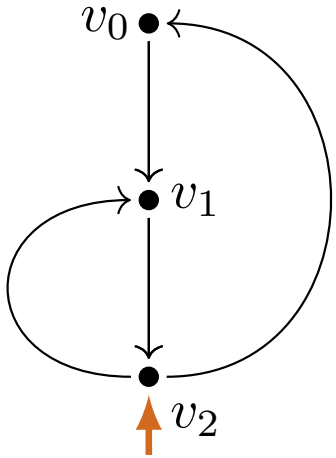

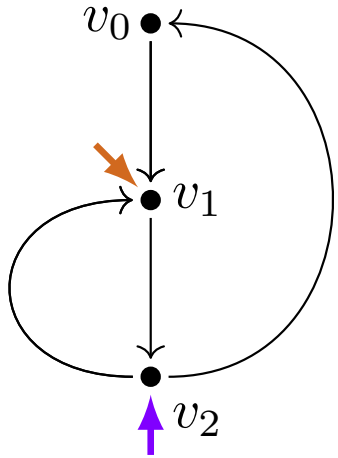

loop chart

Loop graphs (interpretations of innermost iterations without 1)

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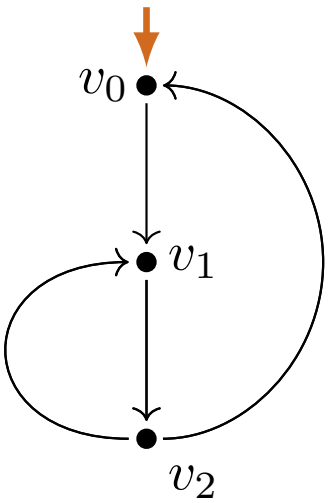

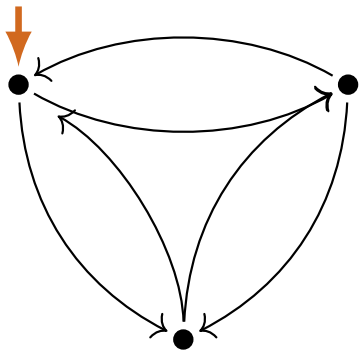

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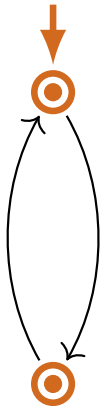

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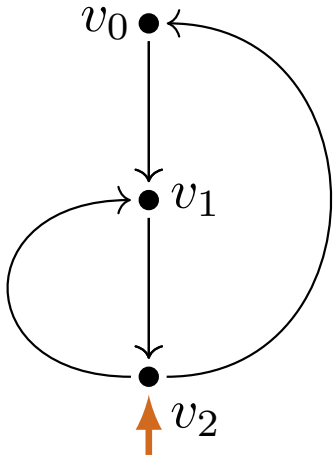
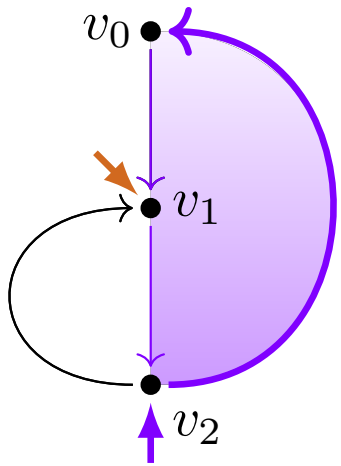


loop chart



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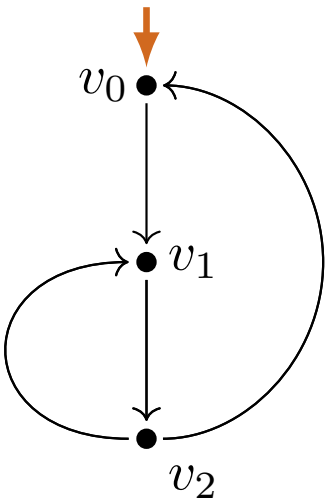

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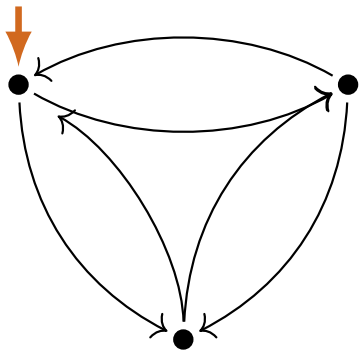

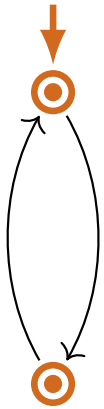

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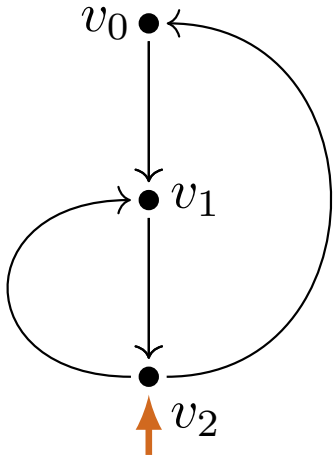
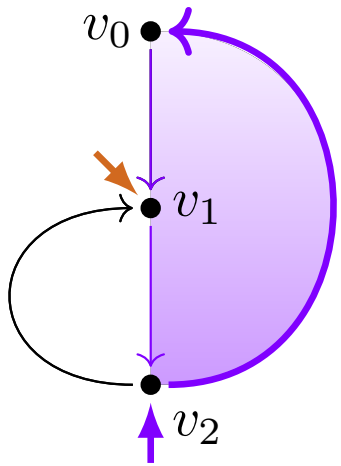


loop chart



loop subchart

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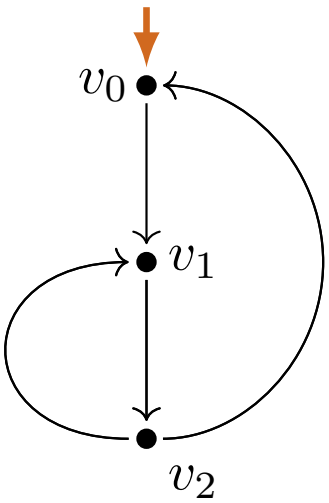

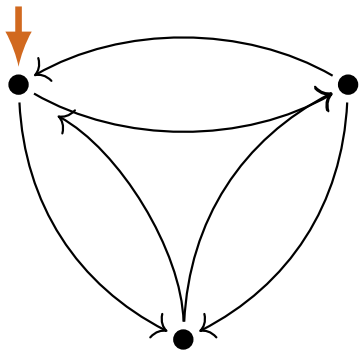

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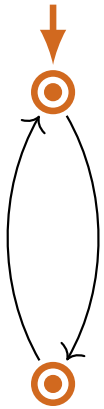

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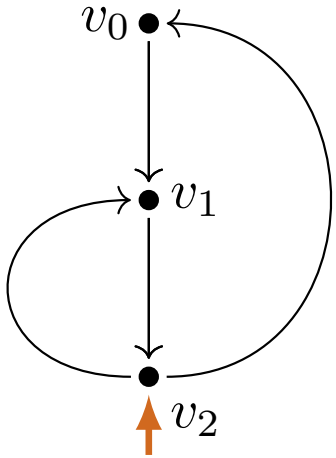
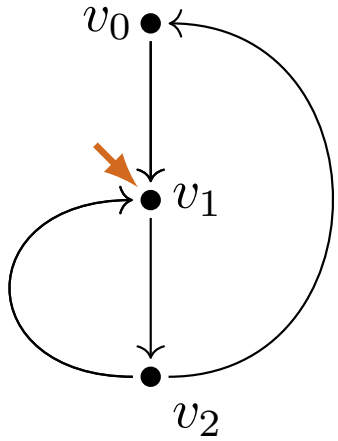
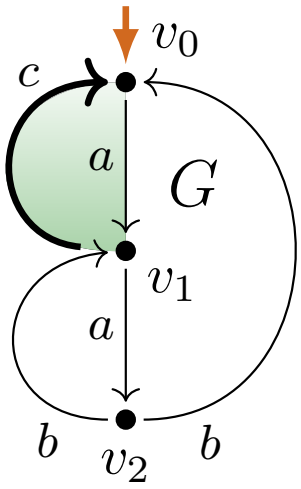


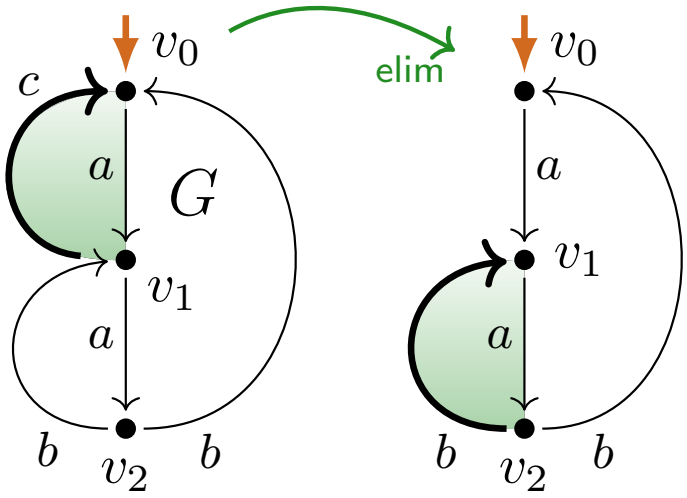


loop chart

Loop elimination $\to_{\mathrm{elim}}$



Loop elimination $\rightarrow_{\text{elim}}$



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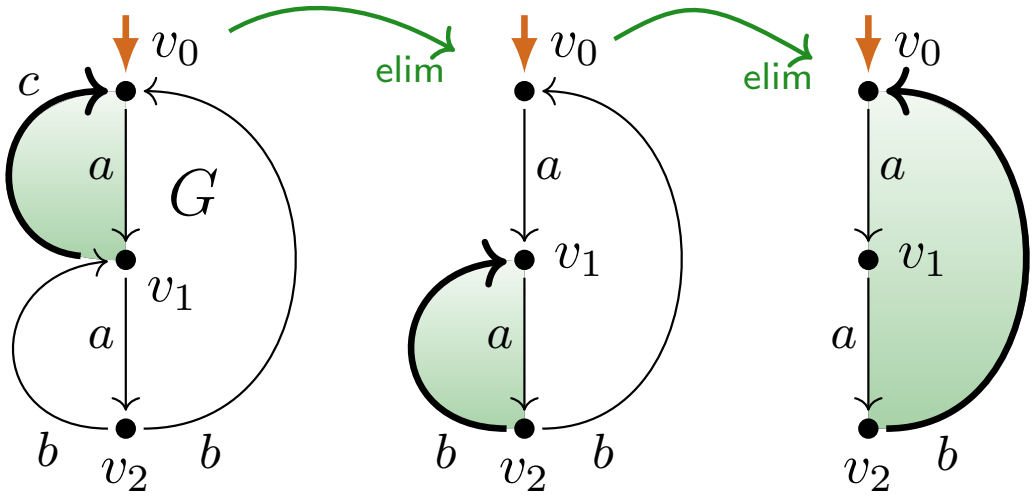
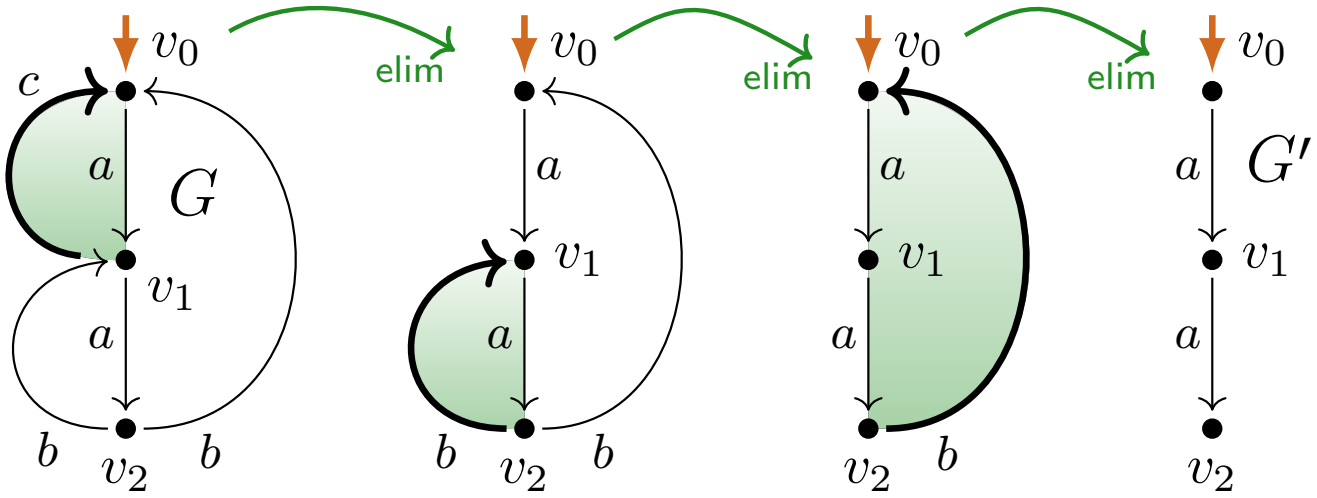
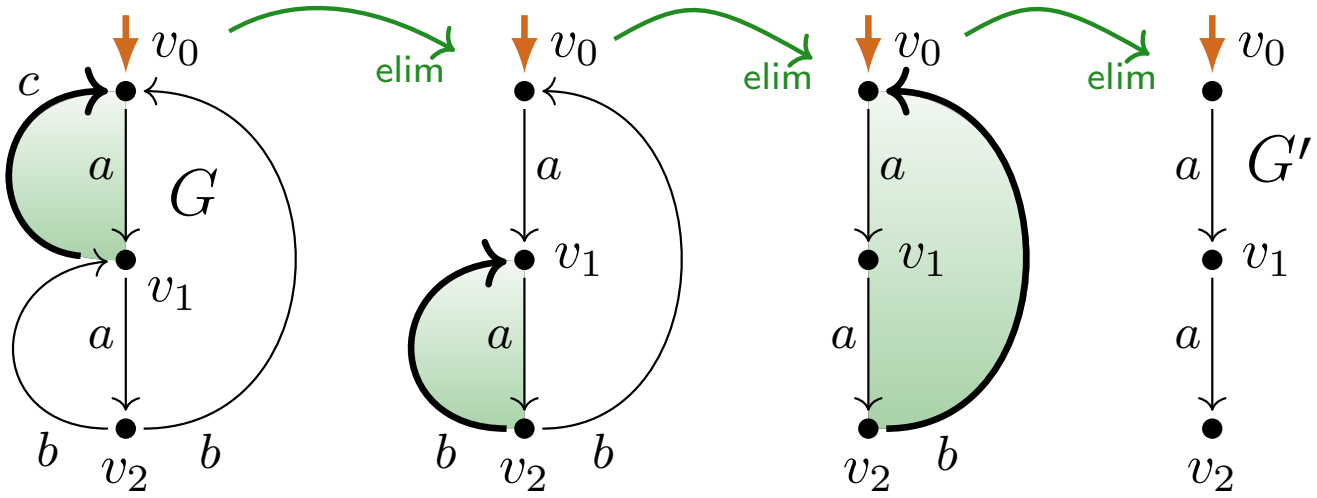
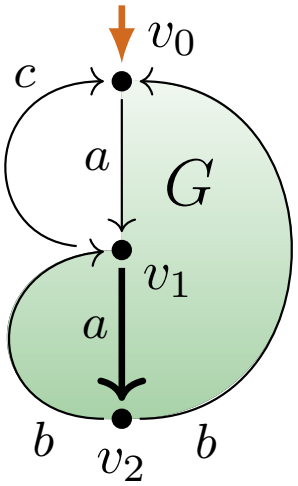


Loop elimination $\rightarrow_{\mathrm{elim}}$

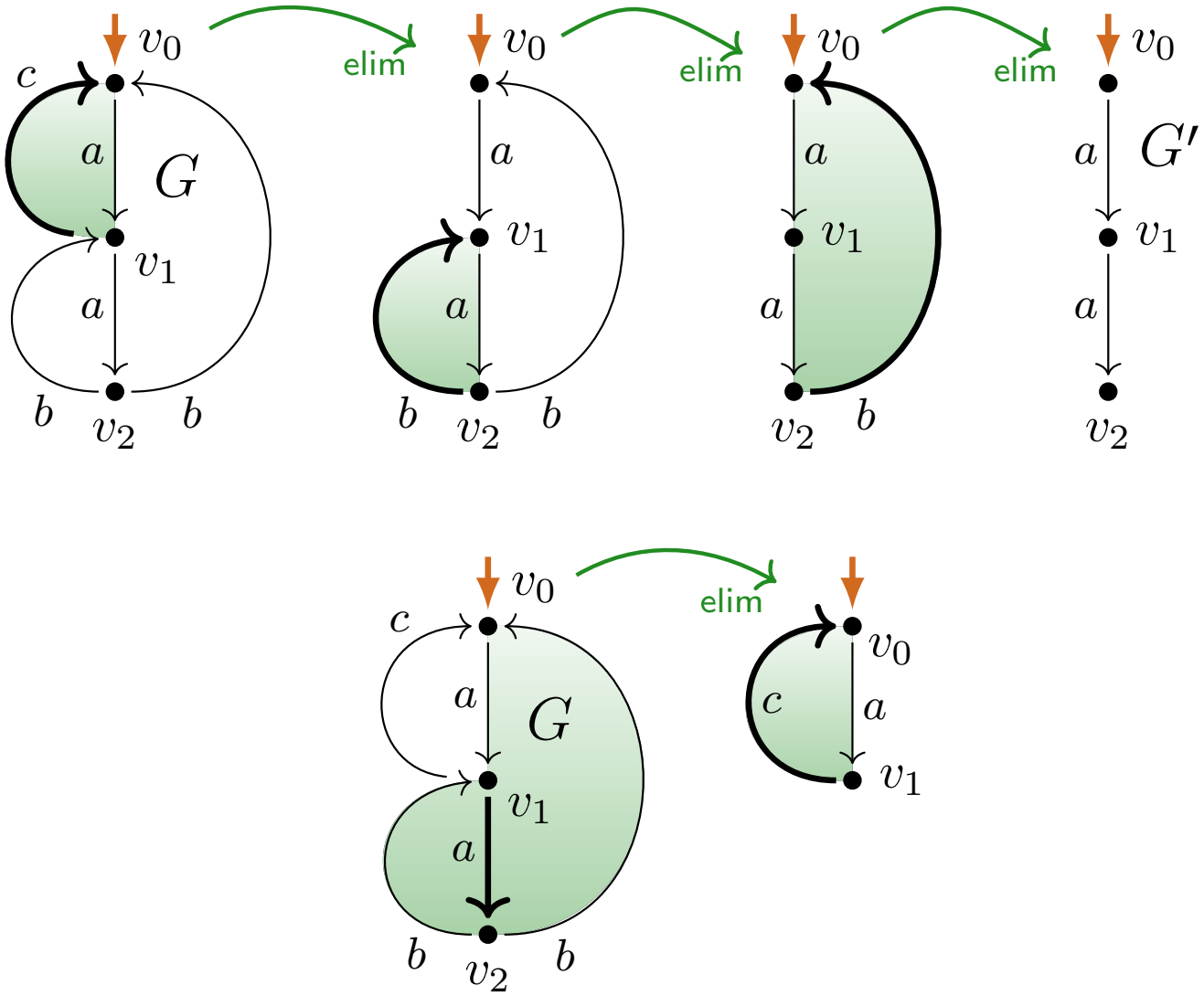


Loop elimination $\rightarrow_{\text{elim}}$

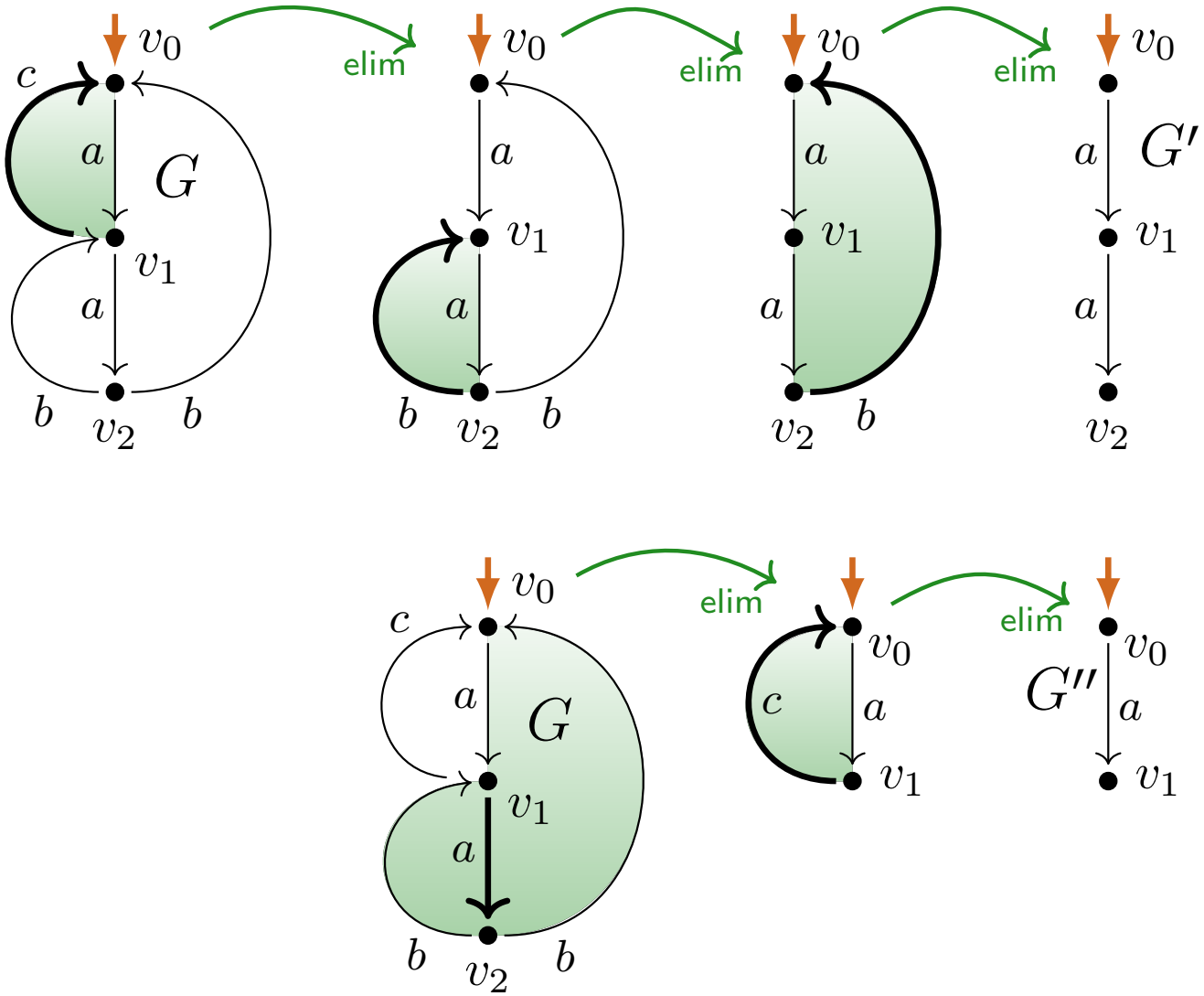




Loop elimination $\rightarrow_{\text{elim}}$



Loop elimination $\rightarrow_{\text{elim}}$



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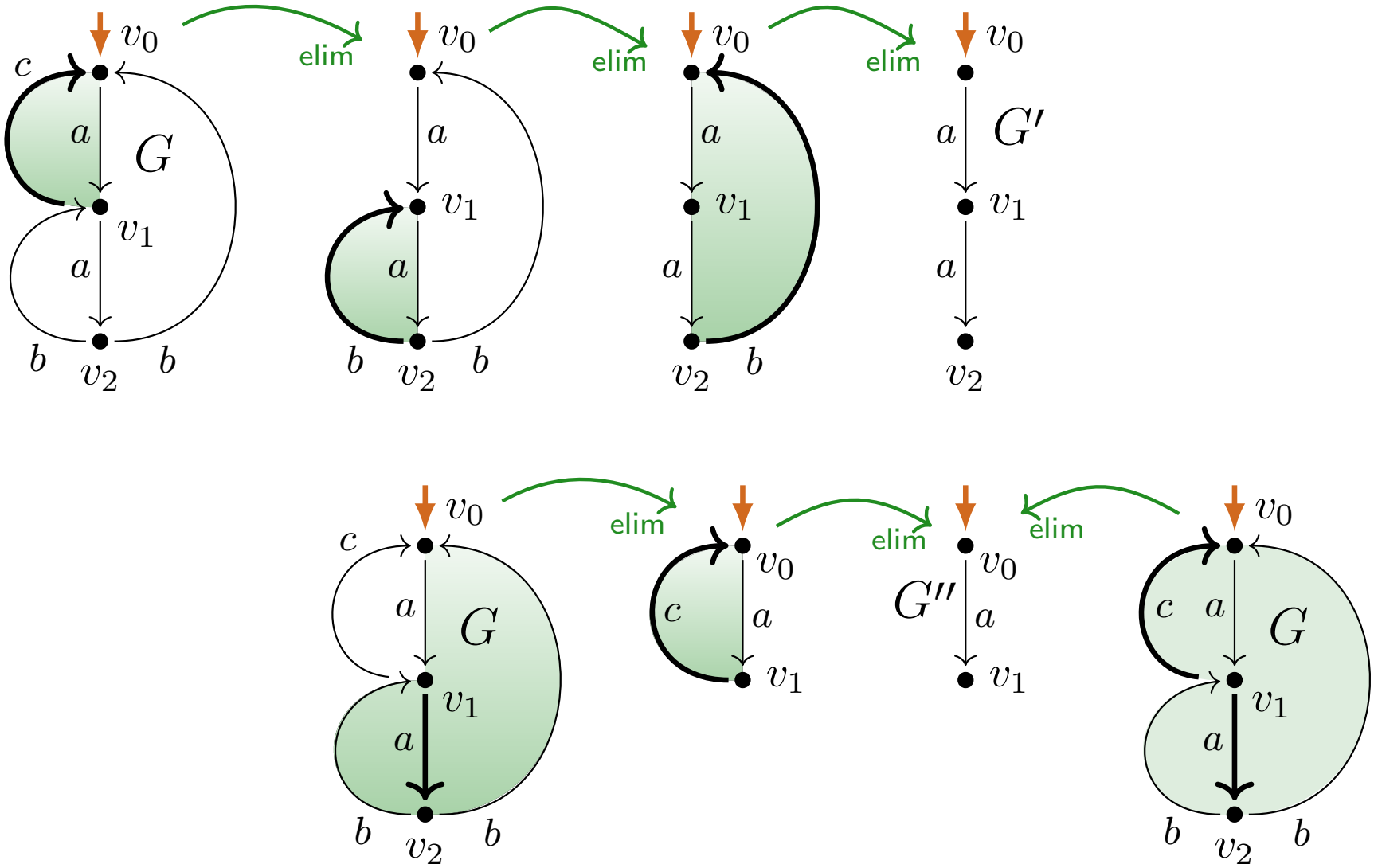

LEE

Definition

A graph G satisfies LEE (*loop existence and elimination*) if:

$$\exists G_0 \left(G \to^*_{\text{elim}} G_0 \not\to_{\text{elim}} \quad \wedge \quad \underbrace{\neg\infty(G_0)}_{G_0 \text{ permits no infinite trace}} \right).$$

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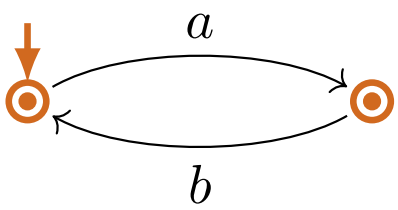
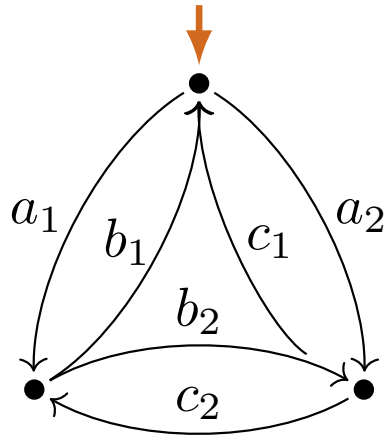
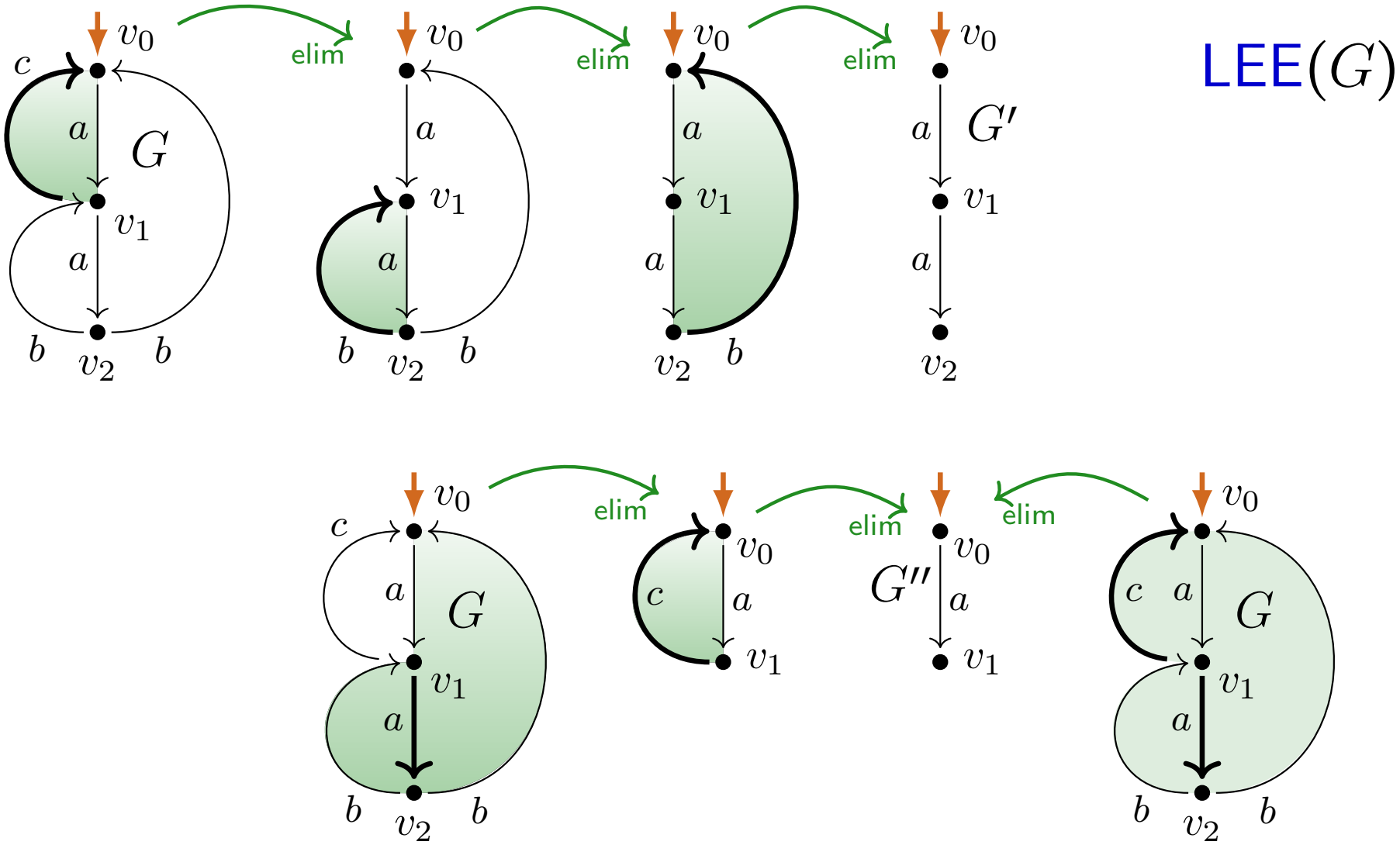


not LEE



not LEE

Loop elimination $\to_{\text{elim}}$



LEE $\triangleq$ P-/$P^\bullet$-expressible graphs for $(*/\pm)$ reg. expr's

Lemma (*G–Fokkink, LICS'20*)

For all process graphs G:

$$\mathsf{LEE}(G) \implies \exists\, uf \in \mathit{RExp}^{(*/\pm)} \big(\underbrace{G \leftrightarrows P(uf)}_{G \in [\![uf]\!]_P} \big),$$

$$\mathsf{LEE}(G) \impliedby \exists\, uf \in \mathit{RExp}^{(*/\pm)} \big(G \simeq P(uf) \big).$$

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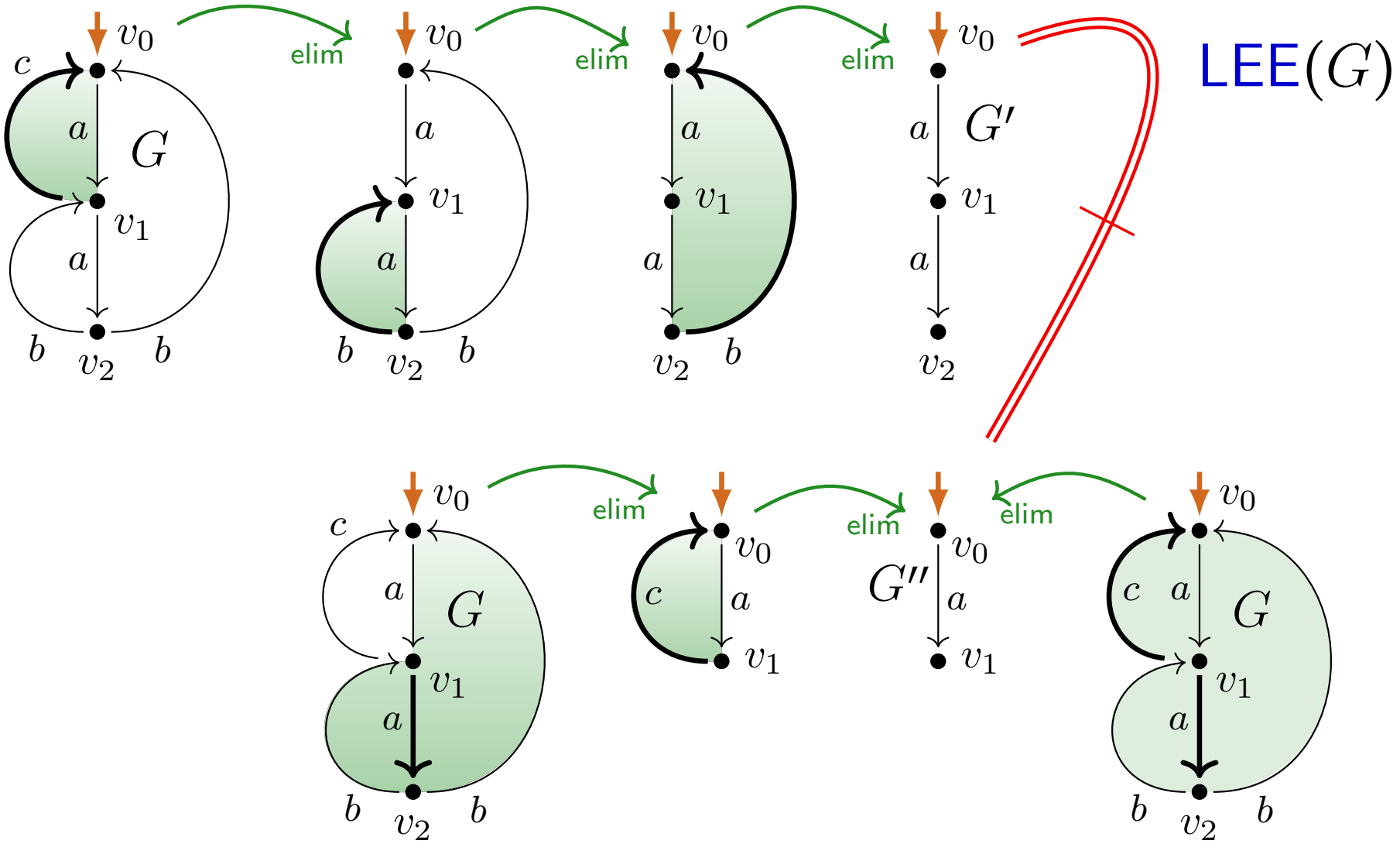
Theorem (*TERMGRAPH '26*, LEE = LLEE *from IWC'26*)

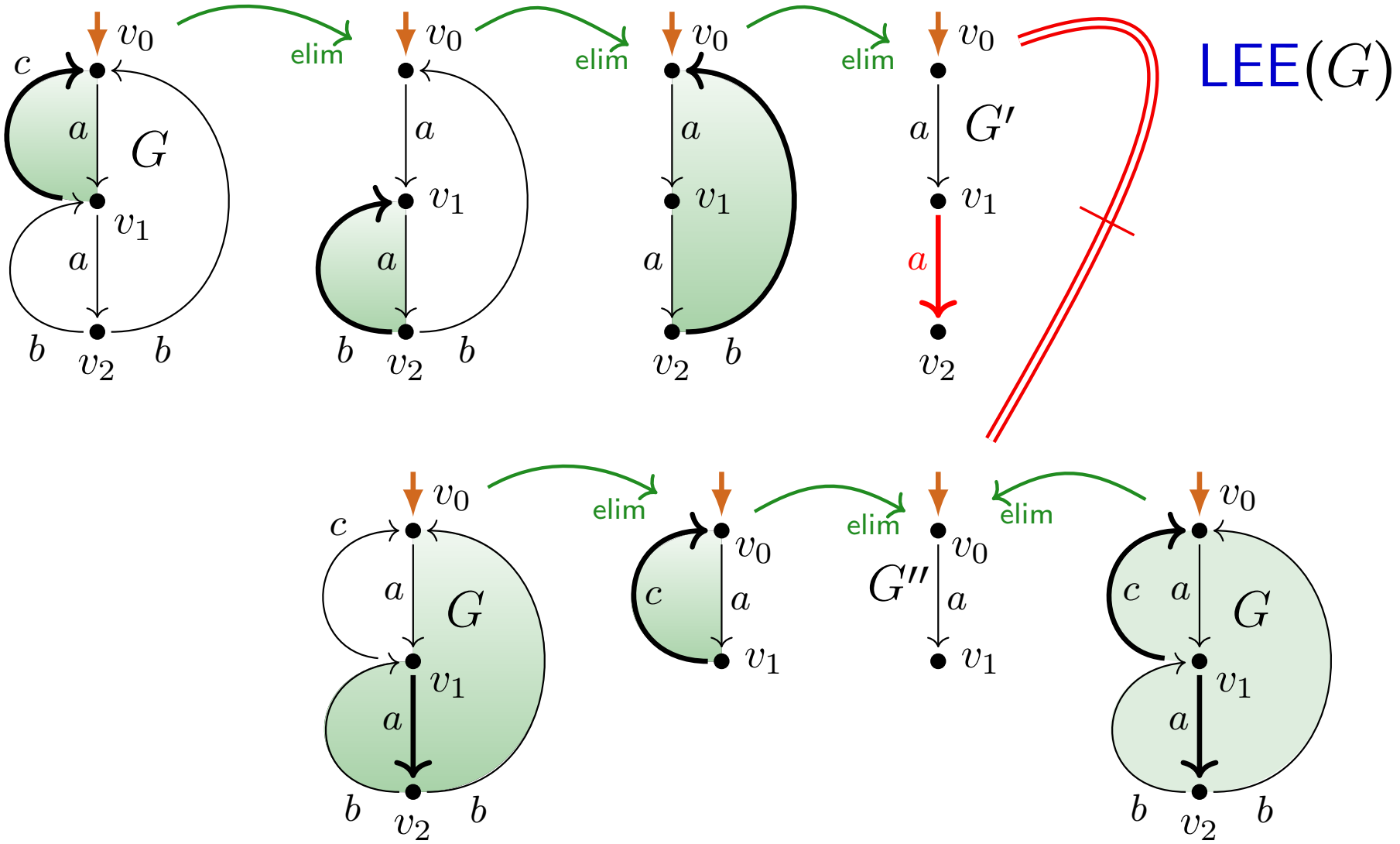
LEE $\triangleq$ compact process interpretations $P^\bullet(uf)$ of $(*/\pm)$ reg. expr's uf:

$$\mathsf{LEE}(G) \iff \exists\, uf \in \mathit{RExp}^{(*/\pm)} \big(G \simeq P^\bullet(uf) \big).$$

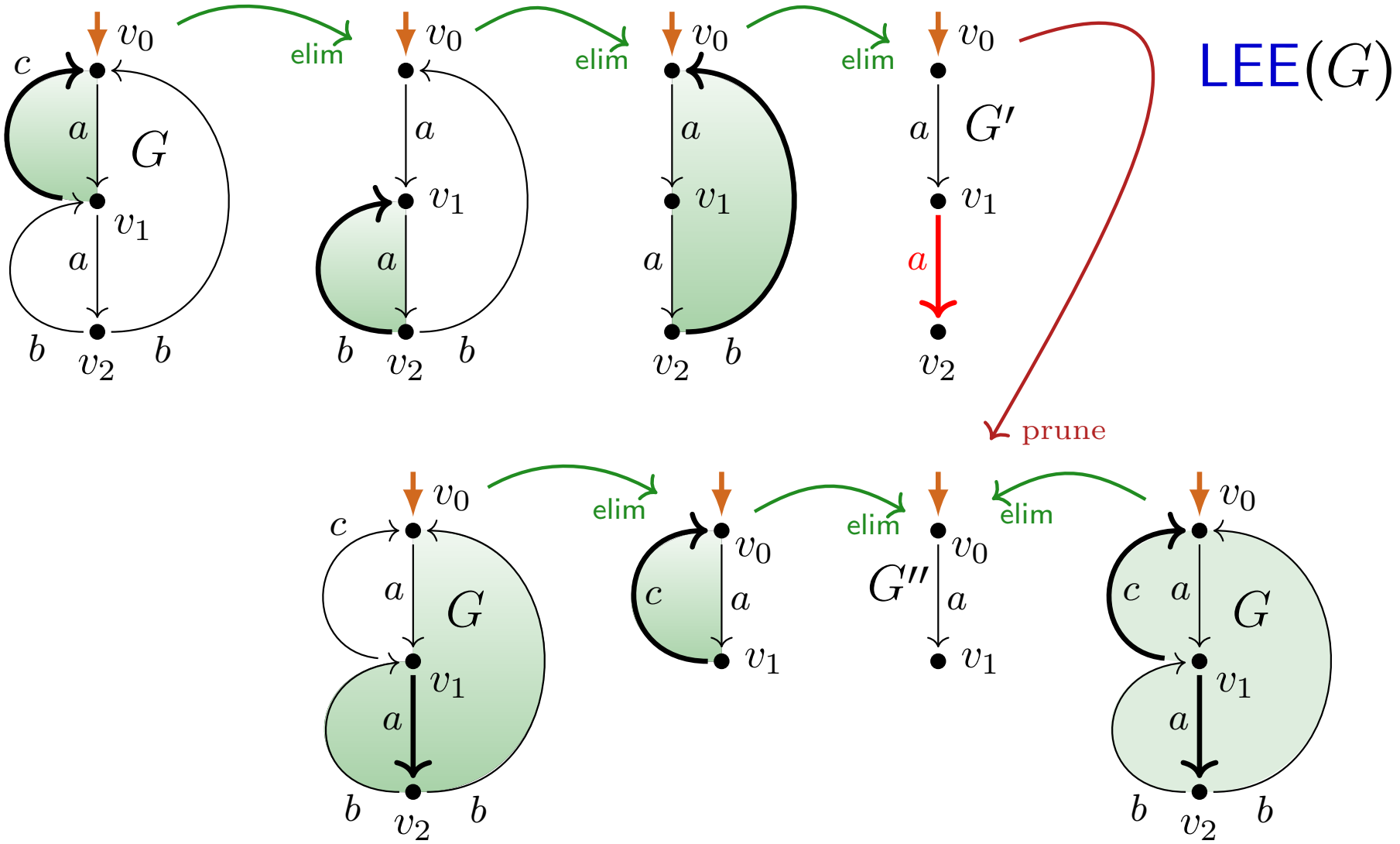
Loop elimination $\rightarrow_{\text{elim}}$ is **not** confluent



Loop elimination $\rightarrow_{\text{elim}}$ is **not** confluent



Joining an $\rightarrow_{\text{elim}}$ fork with a pruning $\rightarrow_{\text{prune}}$ step



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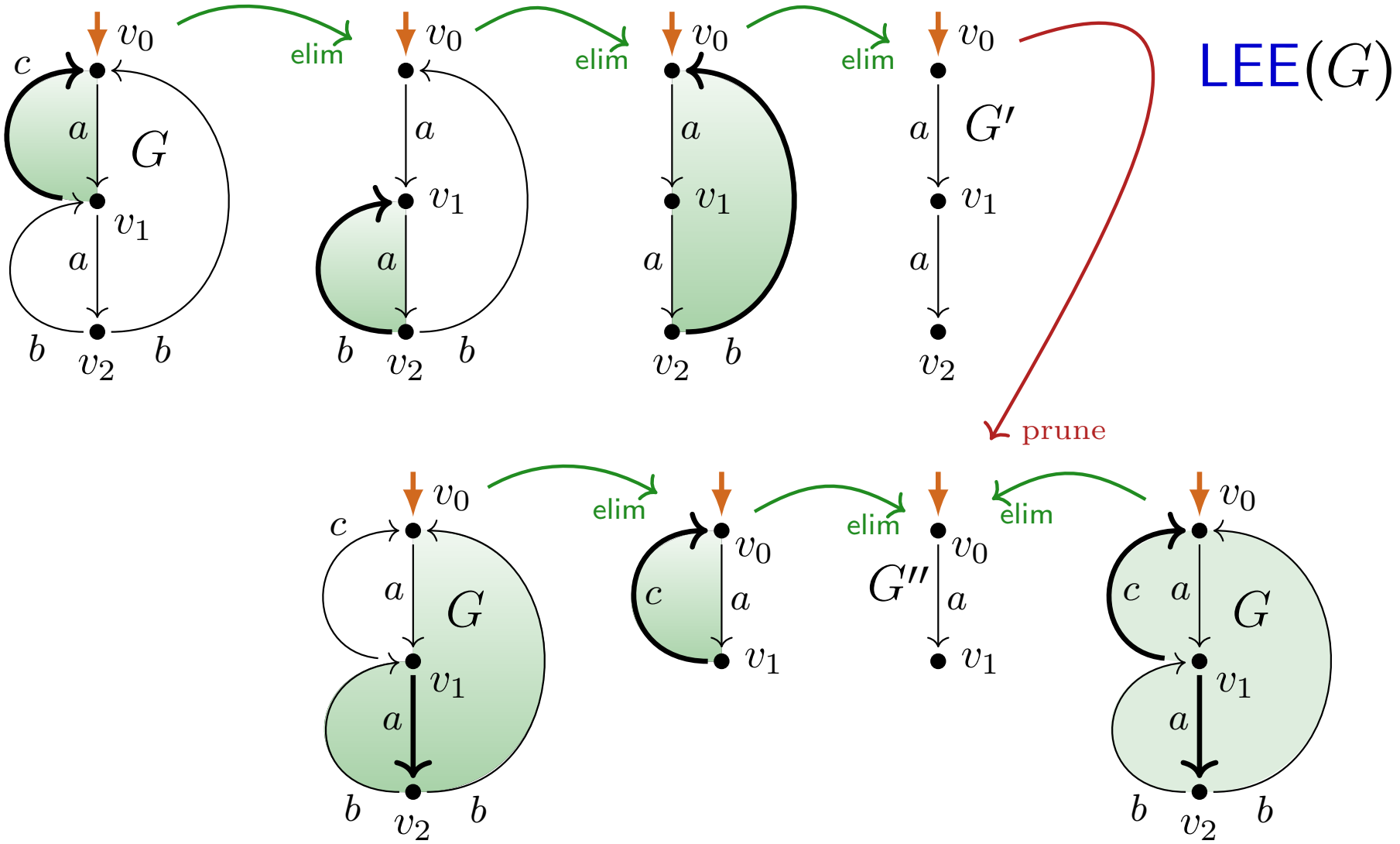

Overview

- Loop elimination $\to_{\mathrm{elim}}$ in process graphs
  - Milner's process interpretation P of regular expressions
  - Loop existence and elimination property LEE
    LEE $\triangleq$ interpretations of $(*/1)$ reg. expr's
  - $\to_{\mathrm{elim}}$ is not confluent
- Completion of loop elimination $\to_{\mathrm{elim}}$ with pruning steps $\to_{\mathrm{prune}}$
  - 'critical pair' lemma for 'bi-loop' elimination steps
  - use of decreasing diagrams
- Application 1: LEE is decidable in polynomial time
- Application 2: Layered LEE = LEE
- Further questions

‘Critical Pair Lemma’

Lemma (joining $\rightarrow_{\mathrm{elim}}$ forks)

Forks $G_2 \leftarrow_{\mathrm{elim}} G \rightarrow_{\mathrm{elim}} G_1$ in which loop subgraphs $G{\downarrow_*^{v_2,T_2}}$, $G{\downarrow_*^{v_1,T_1}}$ of G are eliminated, can be joined with $\rightarrow_{\mathrm{prune}}$ steps in the three situations:

‘Critical Pair Lemma’

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(i) Root-overlap: $v_1 = v_2$, and the loops form bigger loop $G{\downarrow_*^{v_1,T_1 \cup T_2}}$.

(ii) Parallel/nested: $v_1 \neq v_2$, the cyclic parts $\circlearrowleft(G{\downarrow_*^{v_1,T_1}})$, $\circlearrowleft(G{\downarrow_*^{v_2,T_2}})$ of the loops are vertex-disjoint.

(iii) Non-root overlap: $v_1 \neq v_2$, the cyclic parts $\circlearrowleft(G{\downarrow_*^{v_1,T_1}})$, and $\circlearrowleft(G{\downarrow_*^{v_2,T_2}})$ have common vertices.

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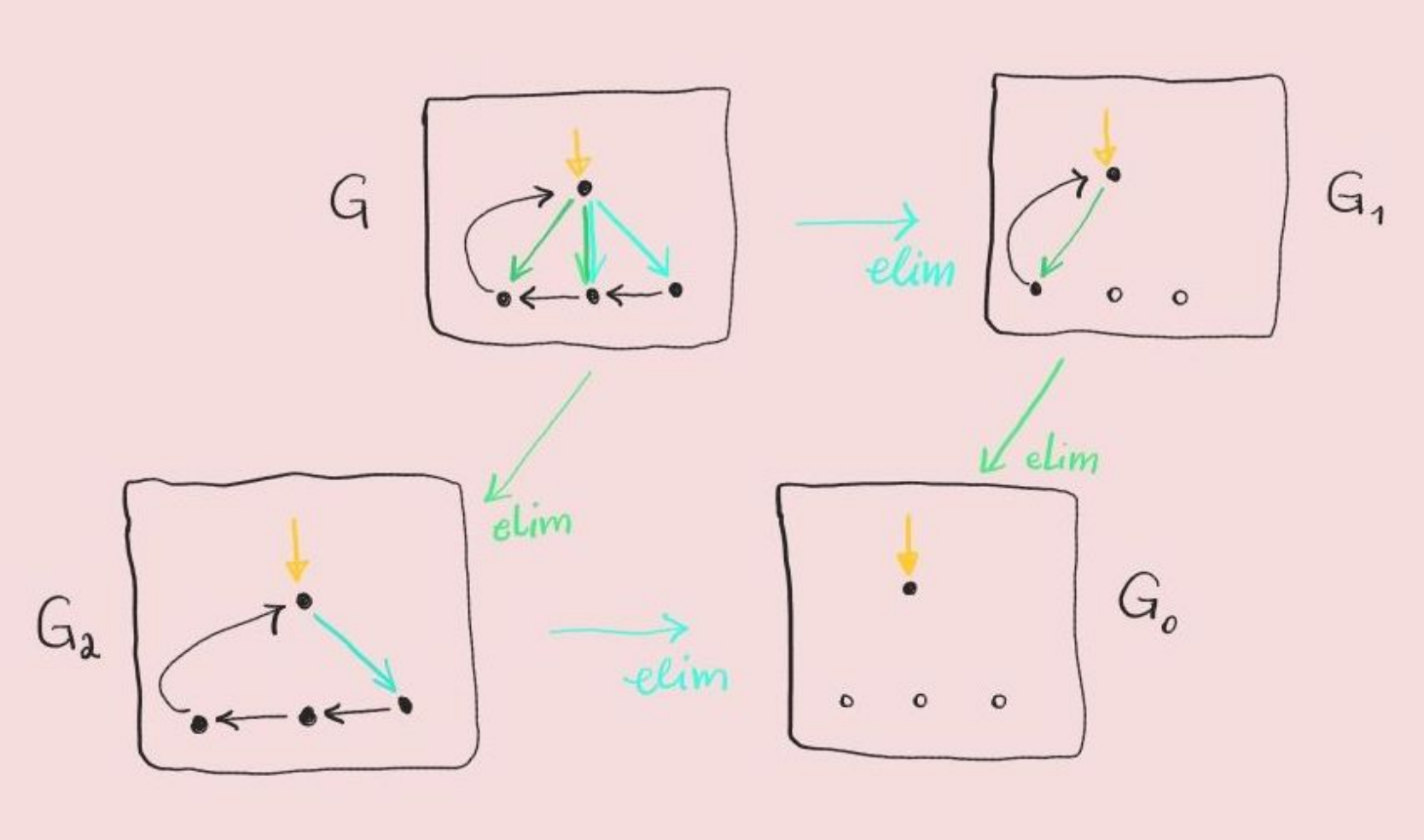
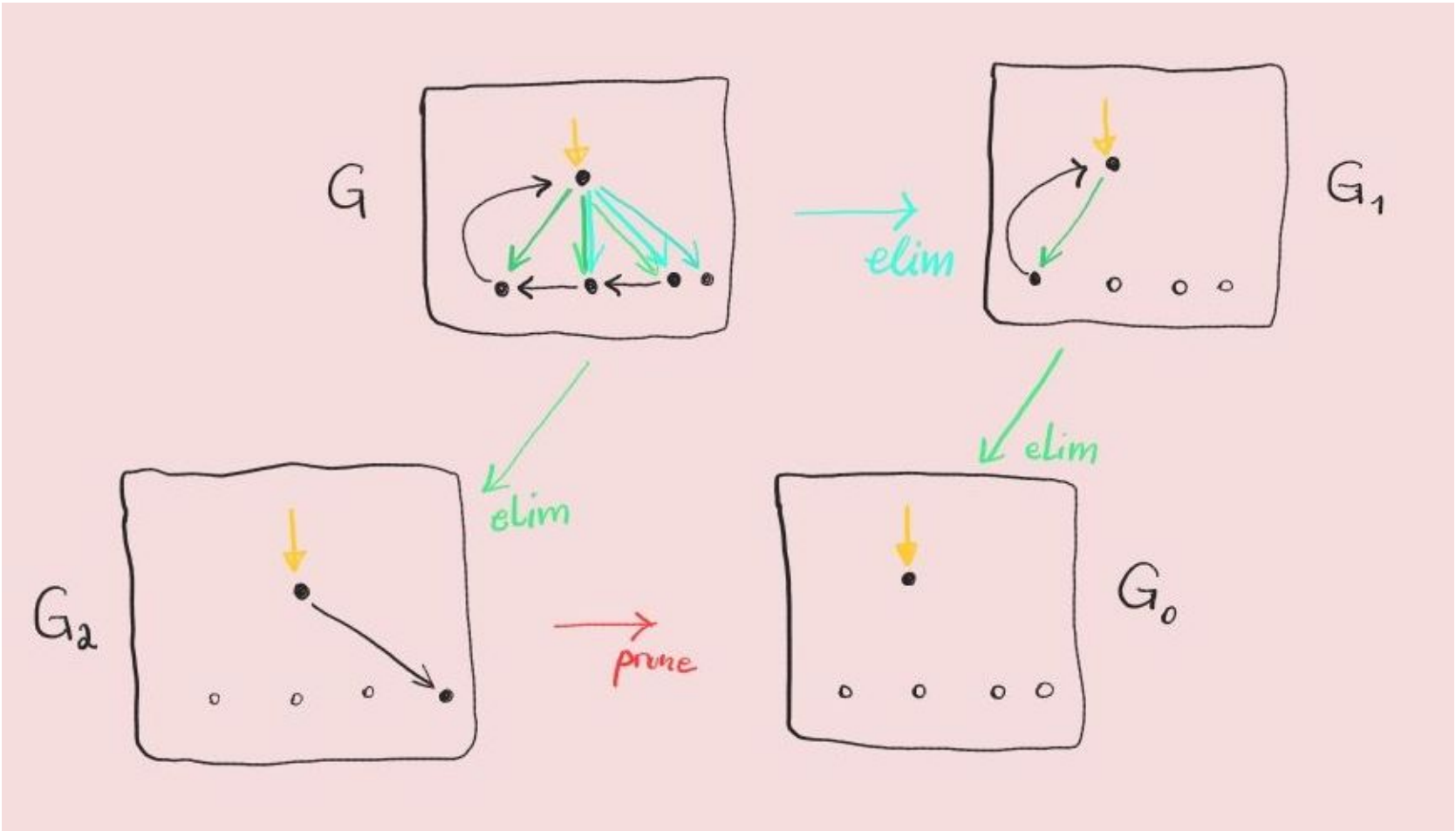
Root-overlapping loop subgraphs (example a)



Root-overlap of loop subgraphs (example b)



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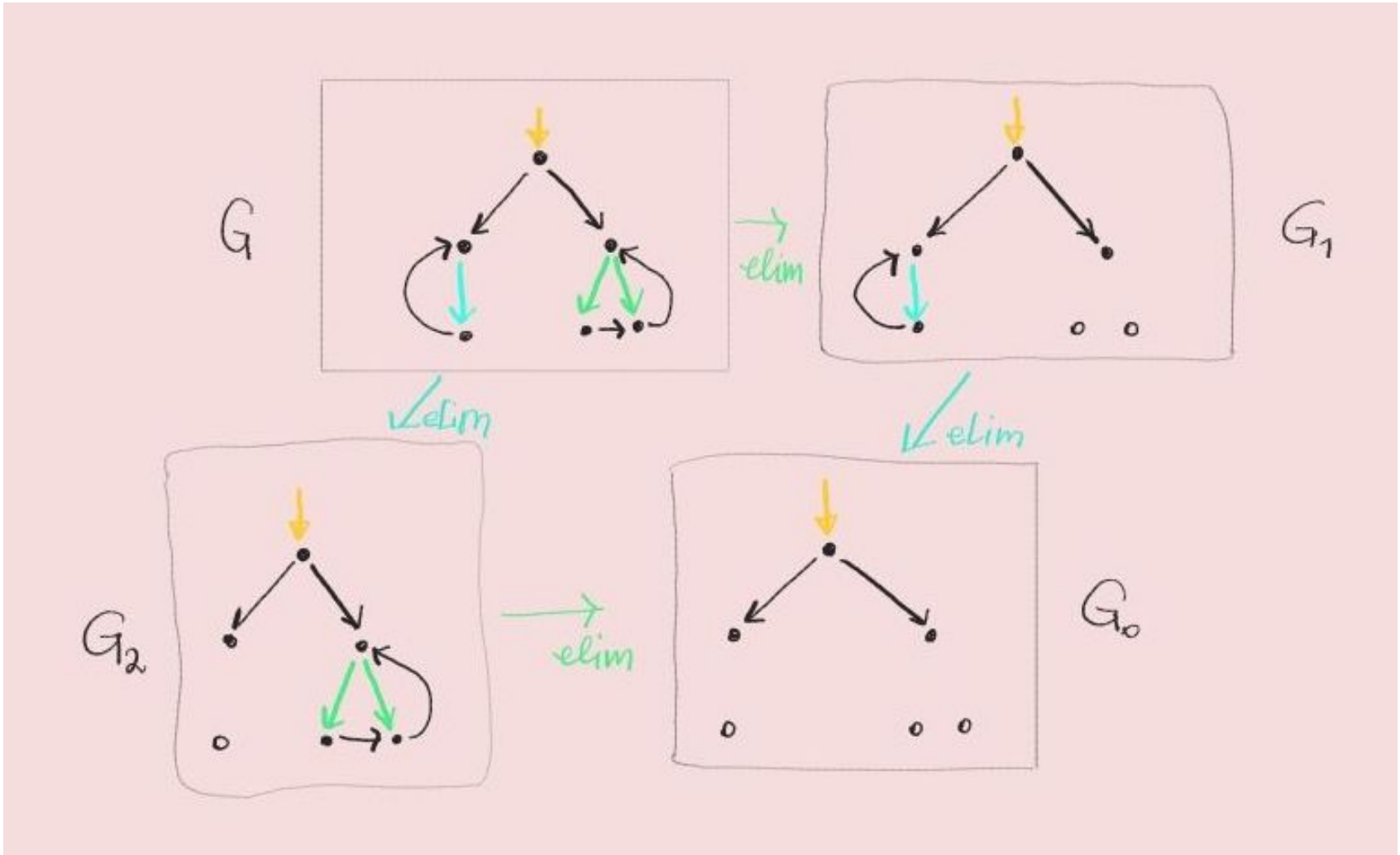
(ii) Parallel/nested: $v_1 \neq v_2$, & the cyclic parts $\circlearrowleft(G{\downarrow_*^{v_1,T_1}})$, $\circlearrowleft(G{\downarrow_*^{v_2,T_2}})$ of the loops are vertex-disjoint. Then the $\to_{\text{elim}}$ steps are independent of each other, except that the elimination of one loop may eliminate the other in the garbage collection operation. The fork can always be joined as follows: $G_2 \to_{\text{elim}}^{=} G_0 \leftarrow_{\text{elim}}^{=} G_1$.

Parallel loop subgraphs (example)



'Critical Pair Lemma'

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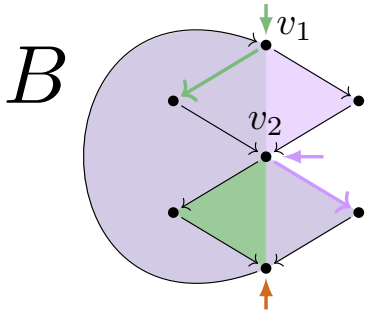
Non-Root overlap /'critical pair' (example)



Non-Root overlap /'critical pair' (example)

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w.r.t v_1, v_2
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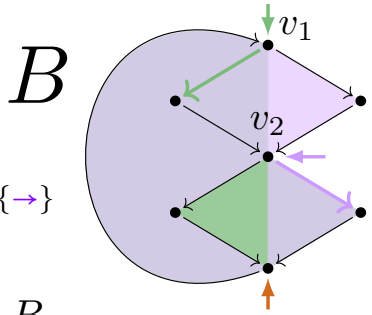

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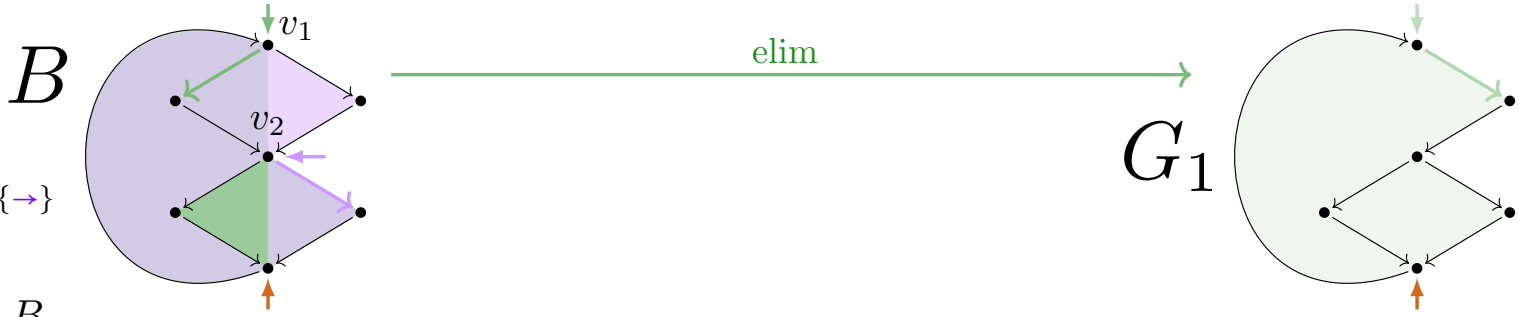

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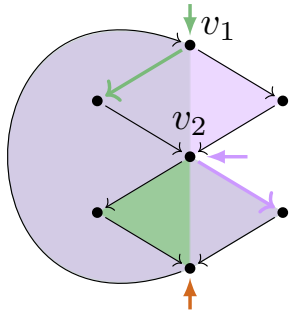
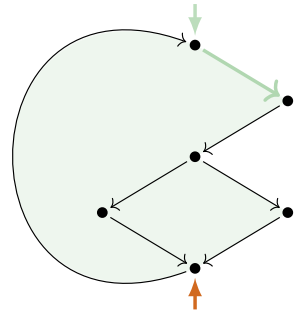


elim



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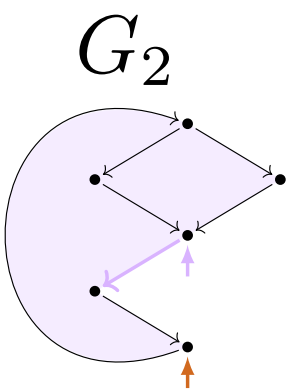

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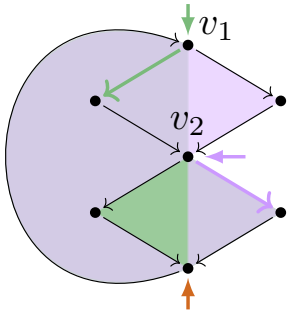

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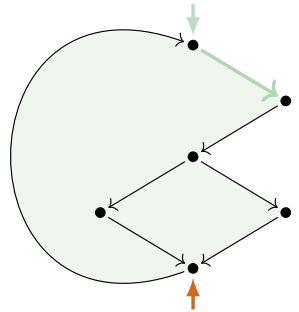

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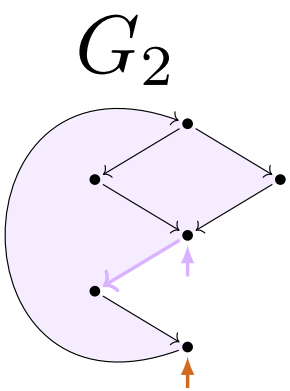

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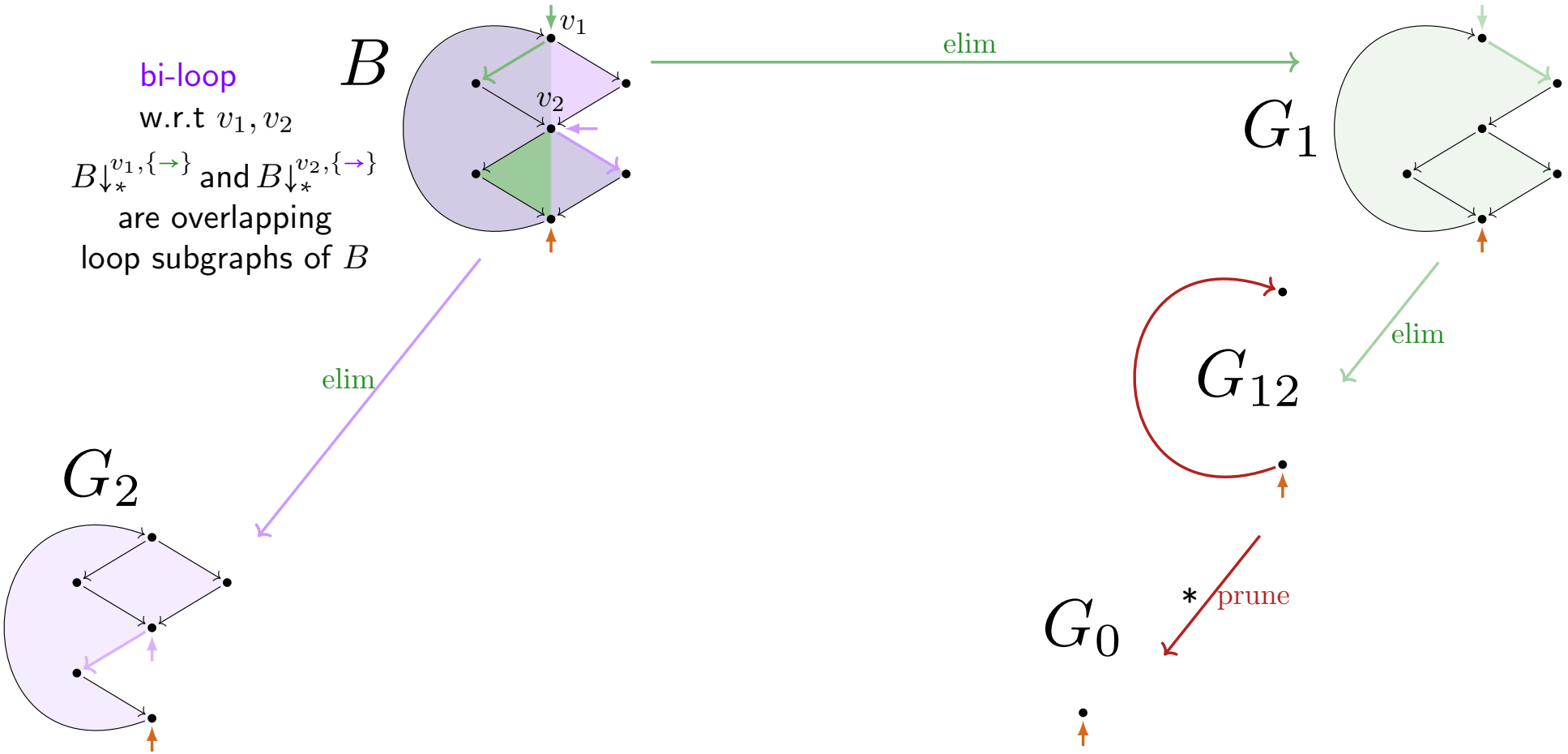

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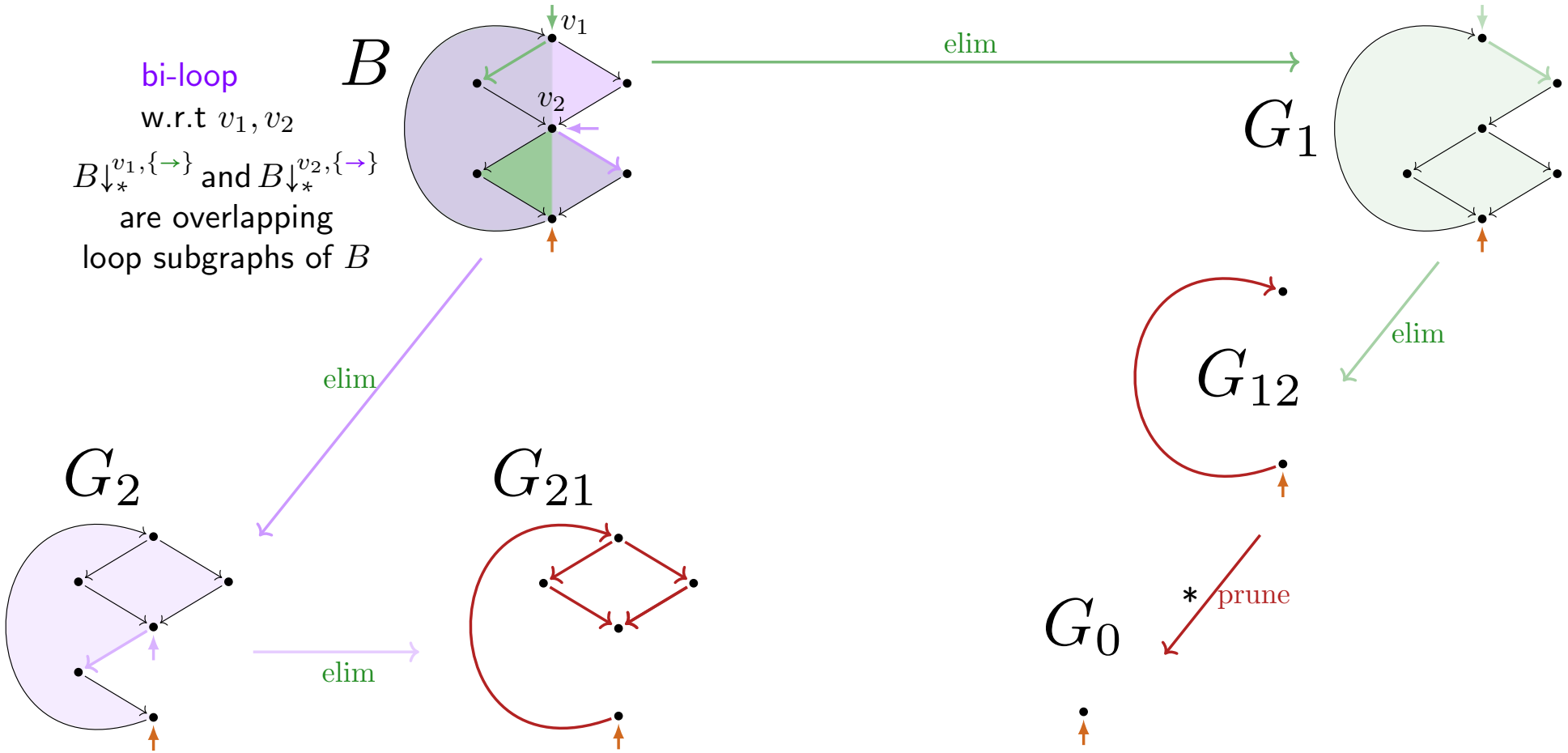

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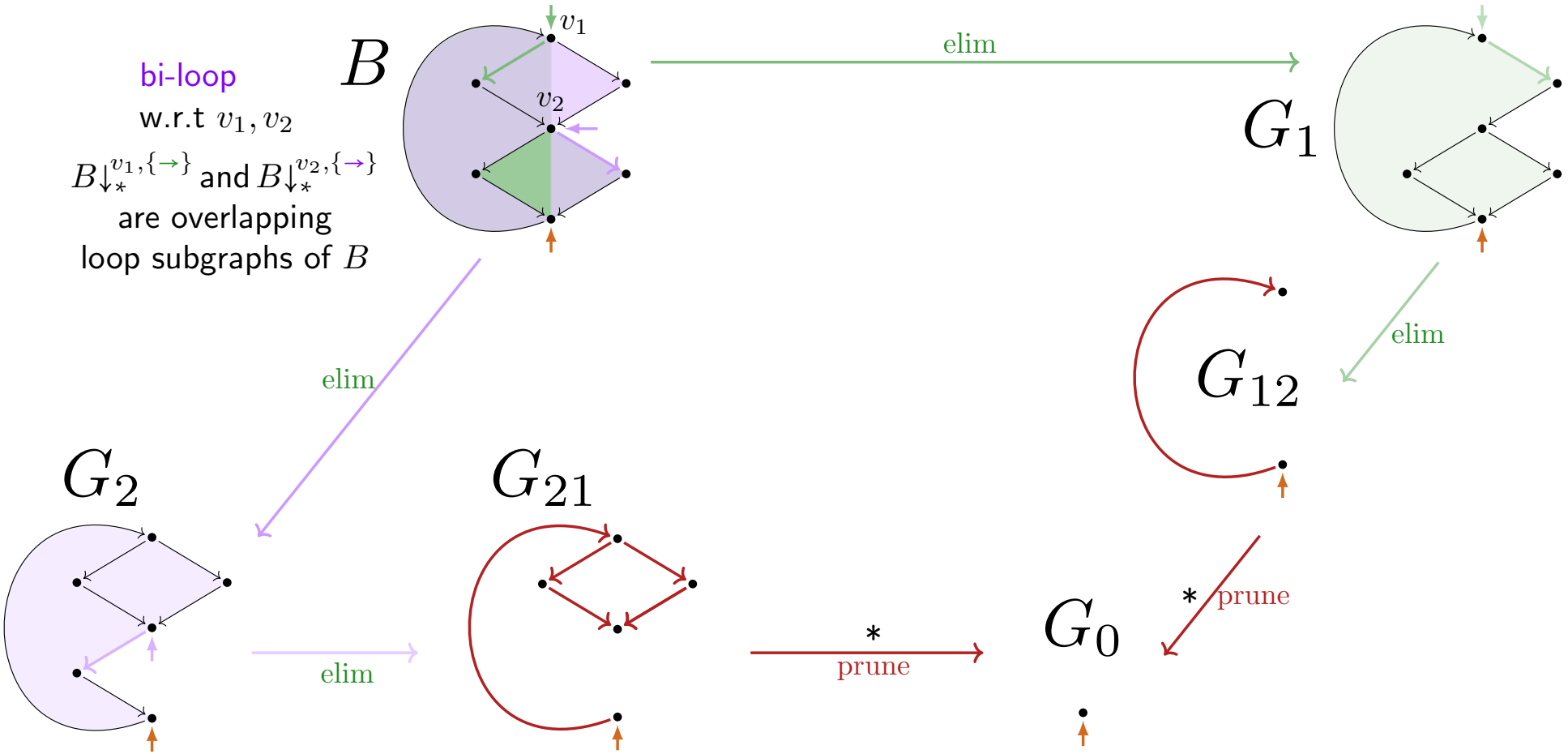

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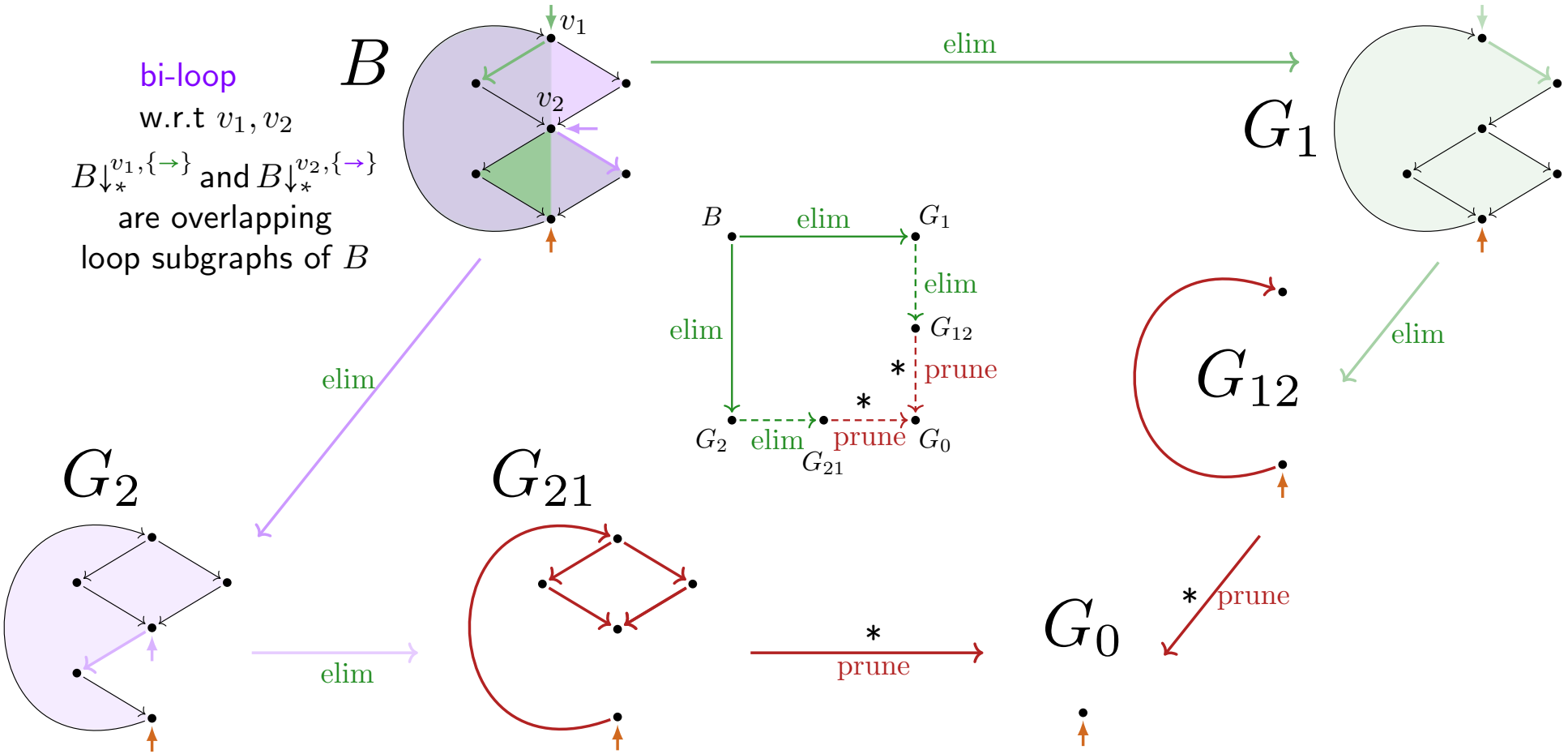


'Critical Pair Lemma'

Lemma (joining $\to_{\text{elim}}$ forks)

Forks $G_2 \leftarrow_{\text{elim}} G \to_{\text{elim}} G_1$ in which loop subgraphs $G{\downarrow_*^{v_2,T_2}}$, $G{\downarrow_*^{v_1,T_1}}$ of G are eliminated, can be joined with $\to_{\text{prune}}$ steps in the three situations:

(i) Root-overlap: $v_1 = v_2$, and the loops form bigger loop $G{\downarrow_*^{v_1,T_1 \cup T_2}}$. The $\to_{\text{elim}}$ steps may overlap in their loop-entry transitions, but the fork can always be joined by eliminating residual loop-entries as follows: $G_2 \to_{\text{elim}}^{=} \cdot \to_{\text{prune}}^{*} G_0 \leftarrow_{\text{prune}}^{*} \cdot \leftarrow_{\text{elim}}^{=} G_1$.

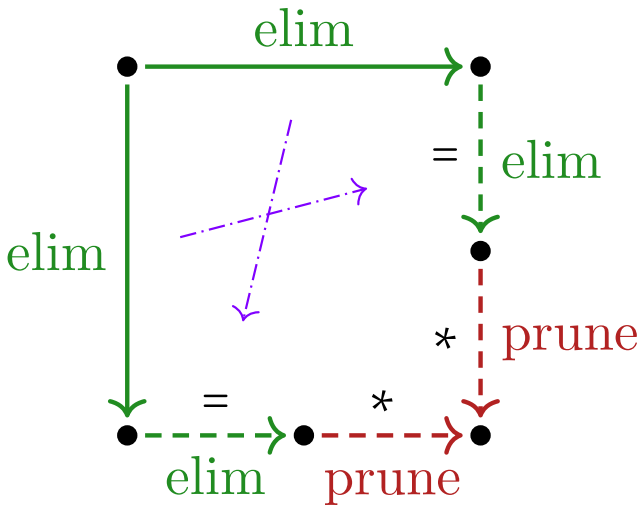
(ii) Parallel/nested: $v_1 \neq v_2$, and the cyclic parts $\circlearrowleft(G{\downarrow_*^{v_1,T_1}})$, $\circlearrowleft(G{\downarrow_*^{v_2,T_2}})$ of the loops are vertex-disjoint. Then the $\to_{\text{elim}}$ steps are independent of each other, except that the elimination of one loop may eliminate the other in the garbage collection operation. The fork can always be joined as follows: $G_2 \to_{\text{elim}}^{=} G_0 \leftarrow_{\text{elim}}^{=} G_1$.

(iii) Non-root overlap: $v_1 \neq v_2$, the start-vertex connected cyclic parts $\circlearrowleft(G{\downarrow_*^{v_1,T_1}})$, and $\circlearrowleft(G{\downarrow_*^{v_2,T_2}})$ have common vertices. The two loops together form a bi-loop w.r.t. v_1, v_2. The fork can always be joined as follows: $G_2 \to_{\text{elim}}^{=} \cdot \to_{\text{prune}}^{*} G_0 \leftarrow_{\text{prune}}^{*} \cdot \leftarrow_{\text{elim}}^{=} G_1$.

Elementary diagrams

Lemma

$\to_{\mathrm{ep}}$ is (locally) decreasing with respect to $\{\to_\ell\}_{\ell \in \{\mathrm{elim},\mathrm{prune}\}}$ for precedence $\mathrm{prune} < \mathrm{elim}$.



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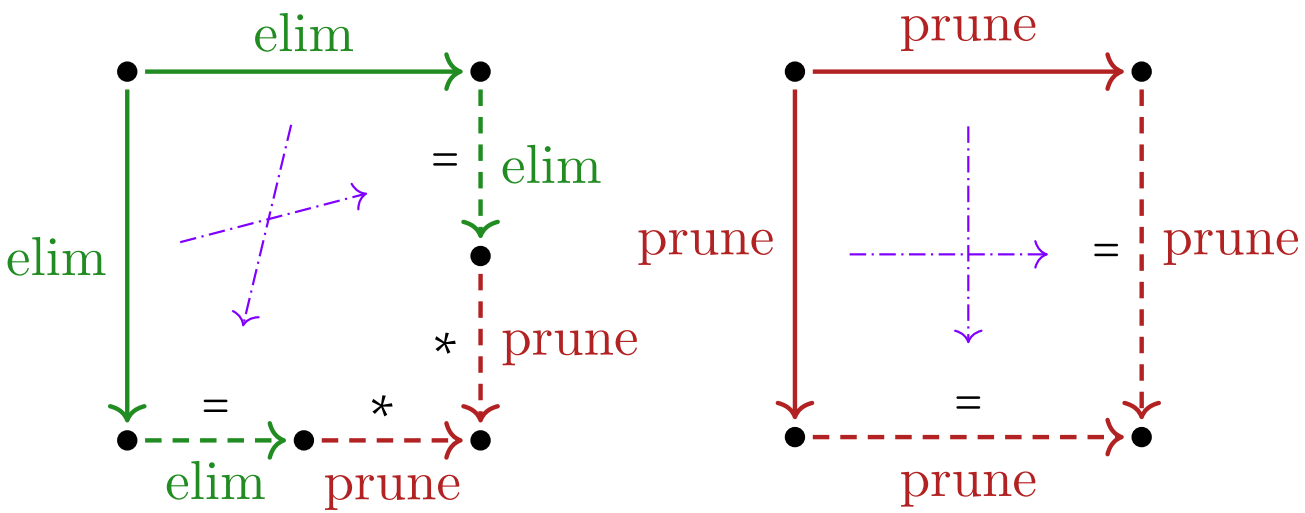

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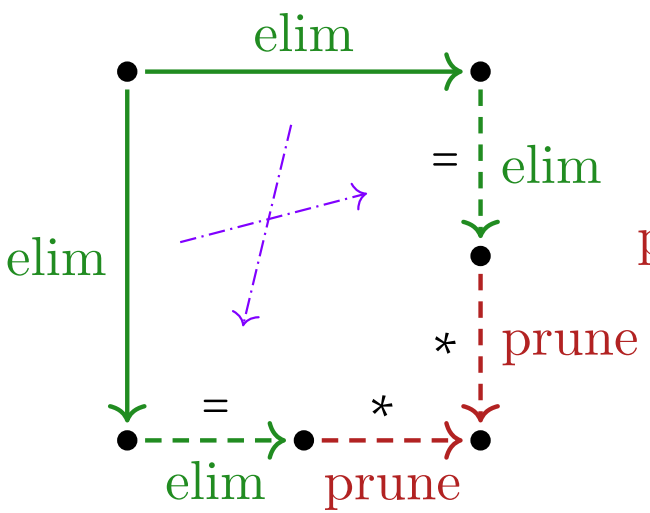

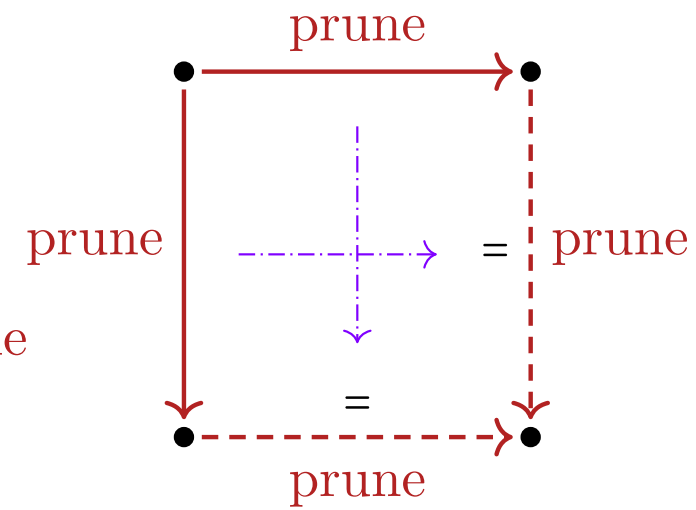

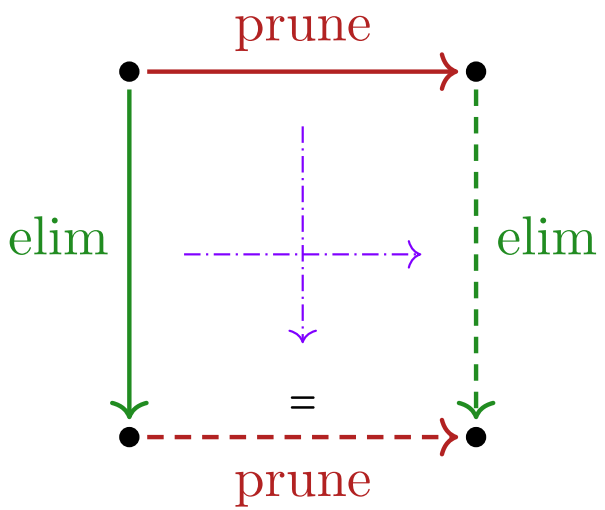

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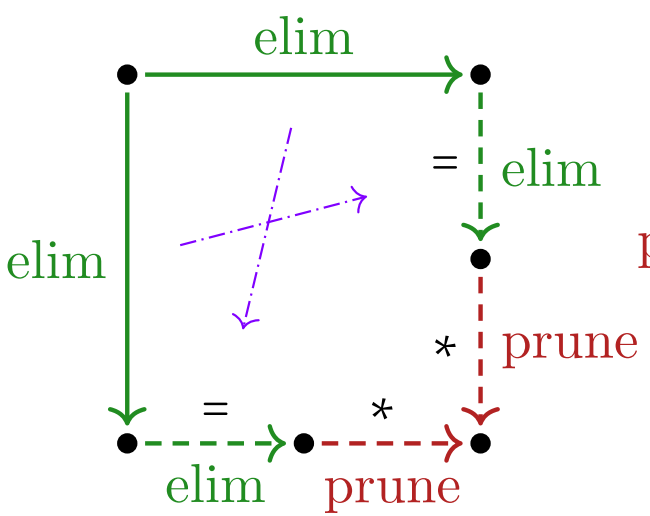

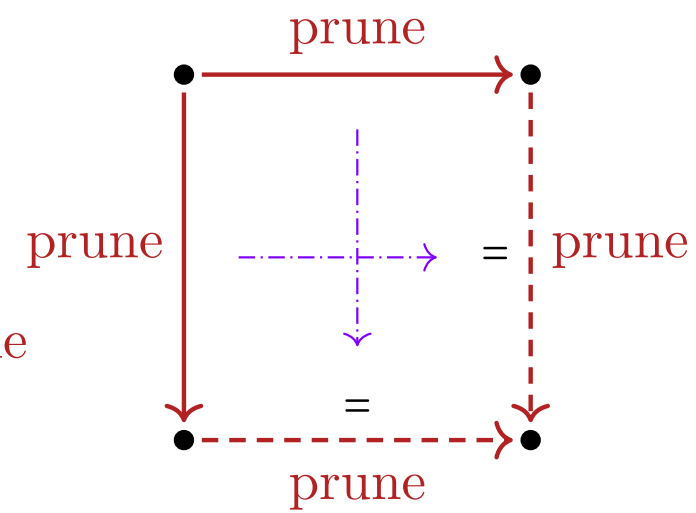

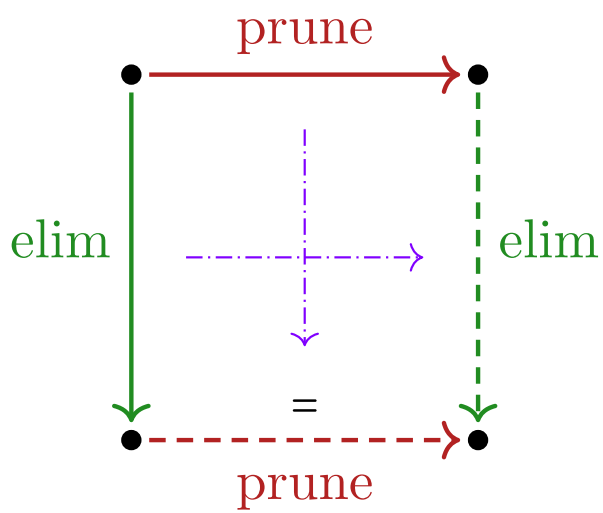

Theorem

$\to_{\mathrm{ep}} = \to_{\mathrm{elim}} \cup \to_{\mathrm{prune}}$ is confluent.

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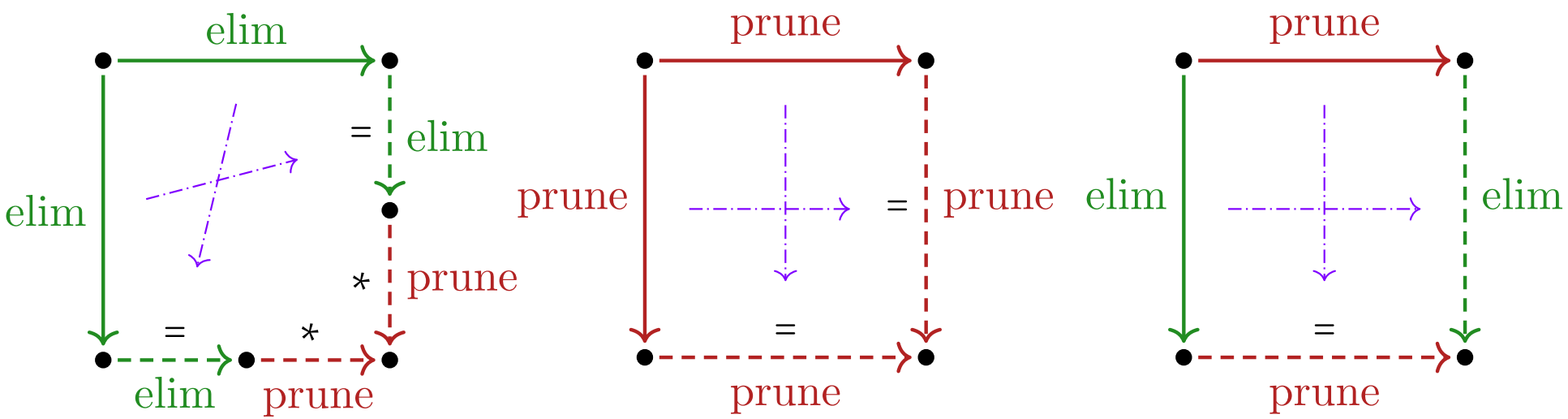

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Proof.

By the Decreasing Diagrams Theorem (van Oostrom ~'94/de Bruijn). □

Overview

- Loop elimination $\to_{\mathrm{elim}}$ in process graphs
  - Milner's process interpretation P of regular expressions
  - Loop existence and elimination property LEE
    LEE $\hat{=}$ interpretations of $(*/\not{1})$ reg. expr's
  - $\to_{\mathrm{elim}}$ is not confluent
- Completion of loop elimination $\to_{\mathrm{elim}}$ with pruning steps $\to_{\mathrm{prune}}$
  - 'critical pair' lemma for 'bi-loop' elimination steps
  - use of decreasing diagrams
- Application 1: LEE is decidable in polynomial time
- Application 2: Layered LEE = LEE
- Further questions

Efficient decidability of LEE

Corollary (of confluence of $\to_{\mathrm{ep}}$)

For every finite process graph G, TFAE:

(i) LEE(G): there is an $\to_{\mathrm{elim}}$ normal form of G without infinite trace.

(ii) The unique $\to_{\mathrm{ep}}$ normal form of G does not enable an infinite trace.

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Theorem

For finite graphs G, LEE is decidable in $\leq \mathrm{O}(|G|^3) \subseteq \mathrm{PTIME}$.

Proof

Due to (i) $\Rightarrow$ (iii) in the Corollary, to decide LEE(G) it suffices to construct any $\to_{\mathrm{elim}}$ rewrite sequence $G \to^*_{\mathrm{elim}} G_0 \not\to_{\mathrm{elim}}$ to normal form G_0, and check if $\neg\infty(G_0)$ holds. That is possible in $\leq \mathrm{O}(|G|^3)$ time. $\square$

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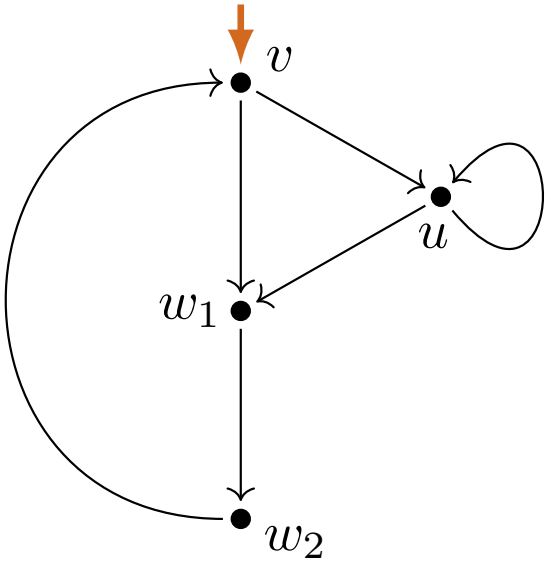

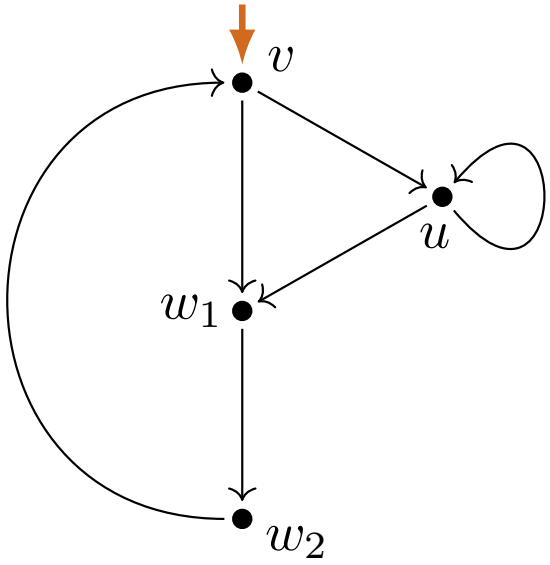

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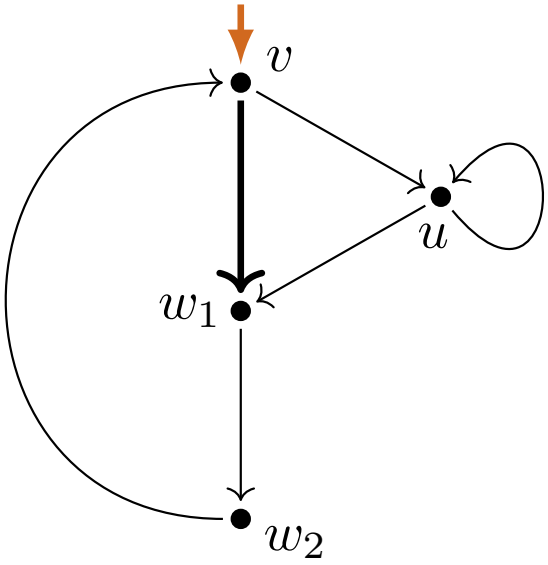

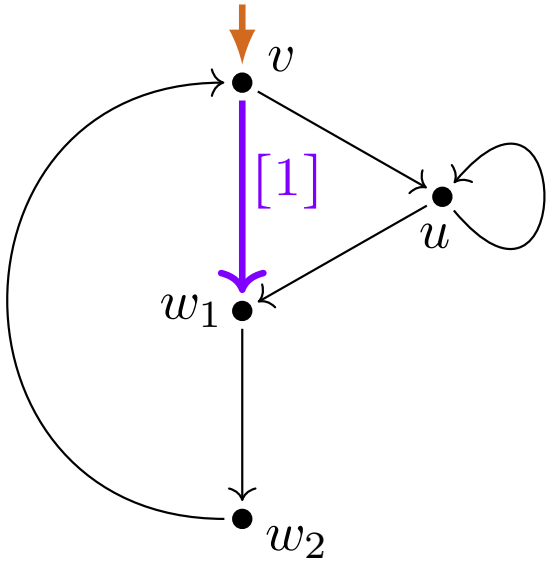

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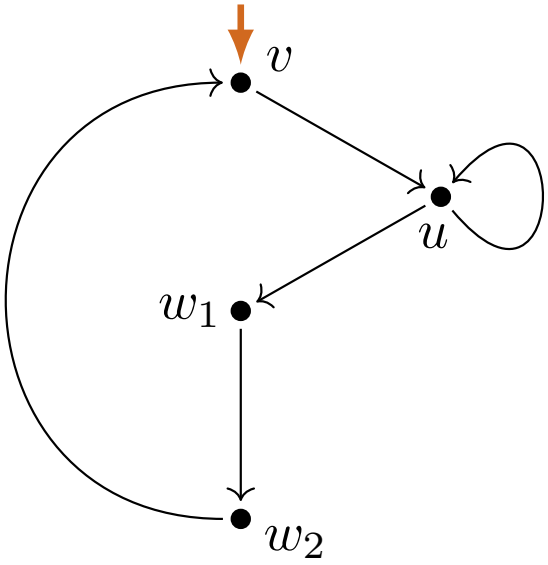

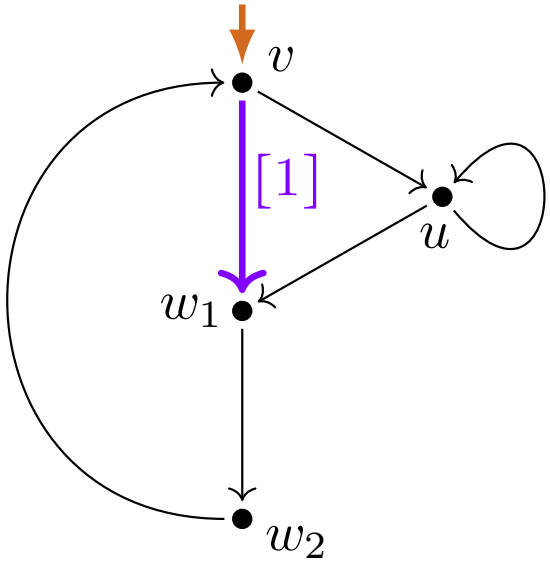

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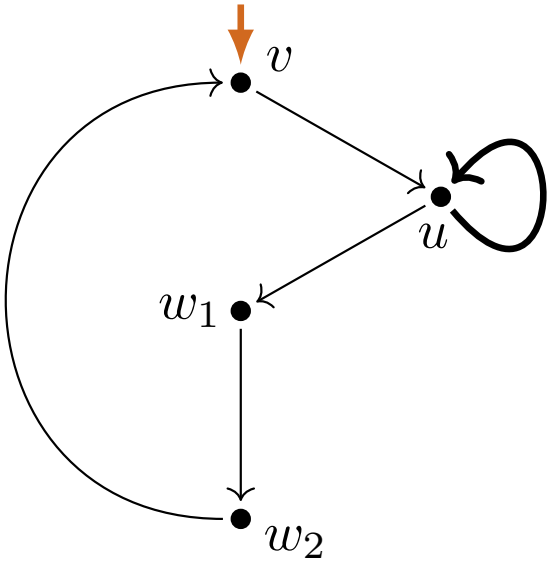

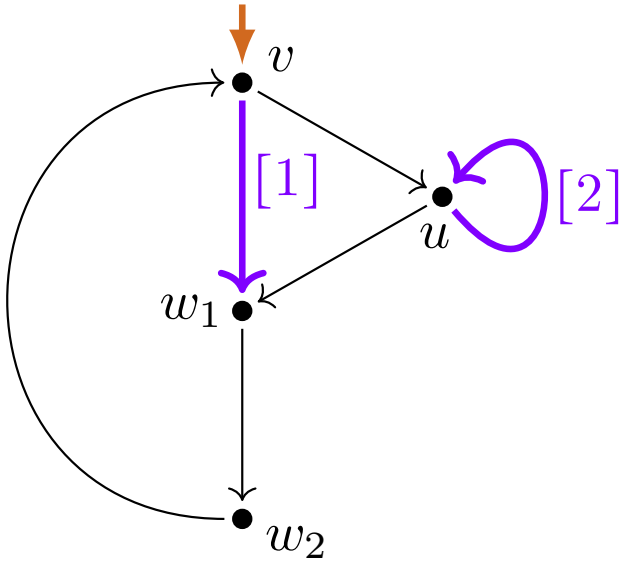

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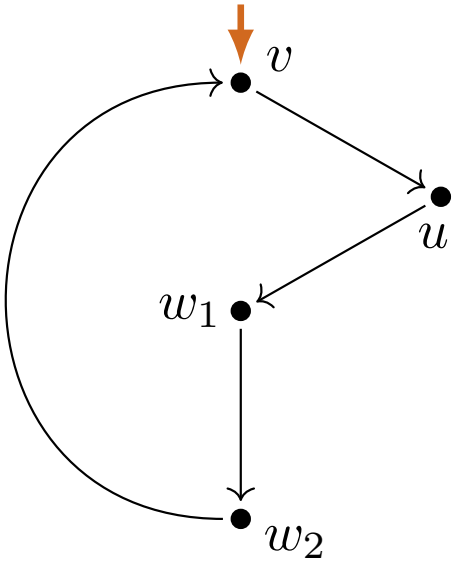

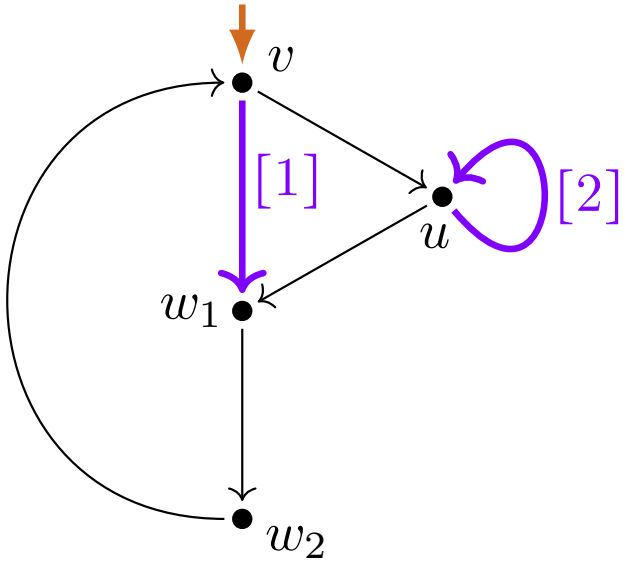

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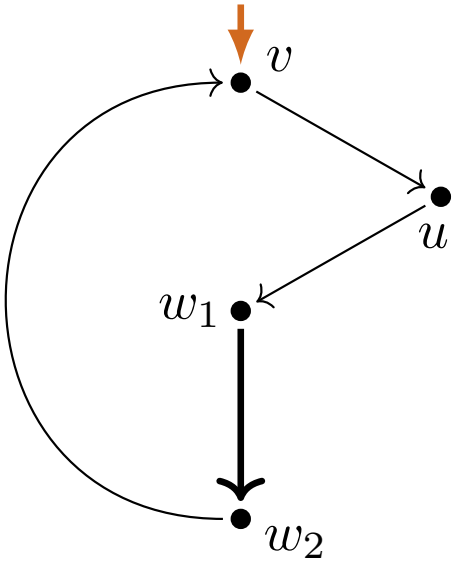

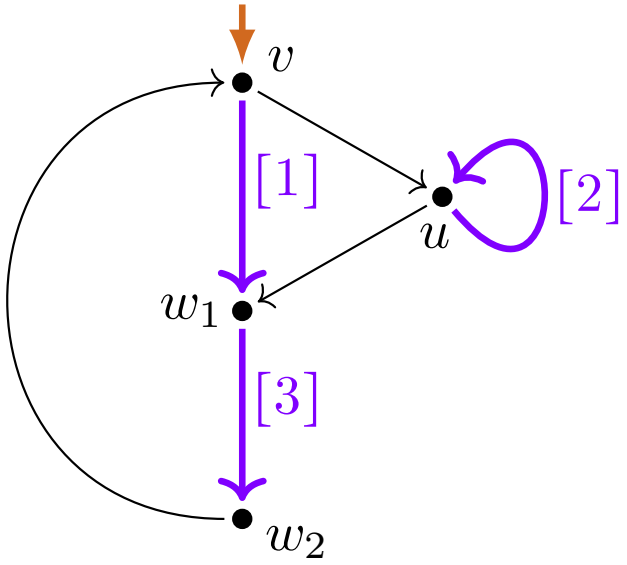

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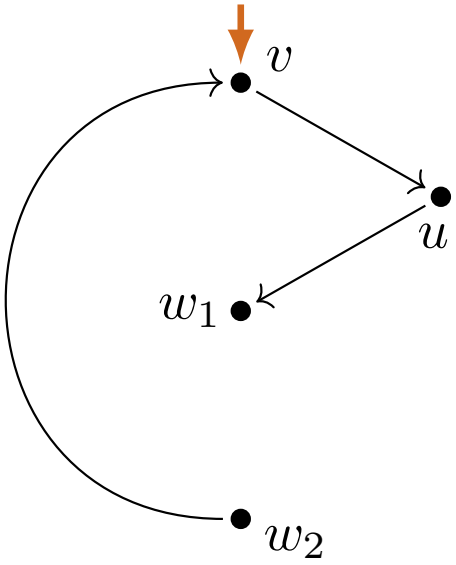

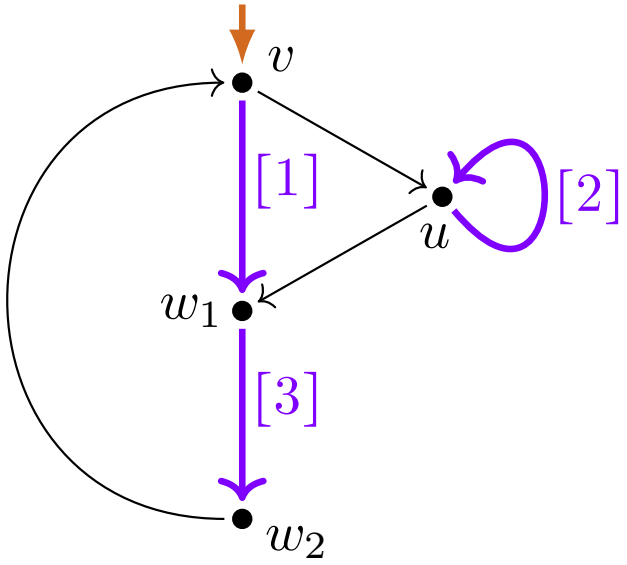

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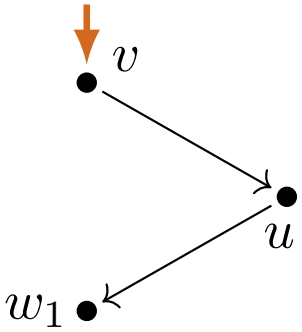

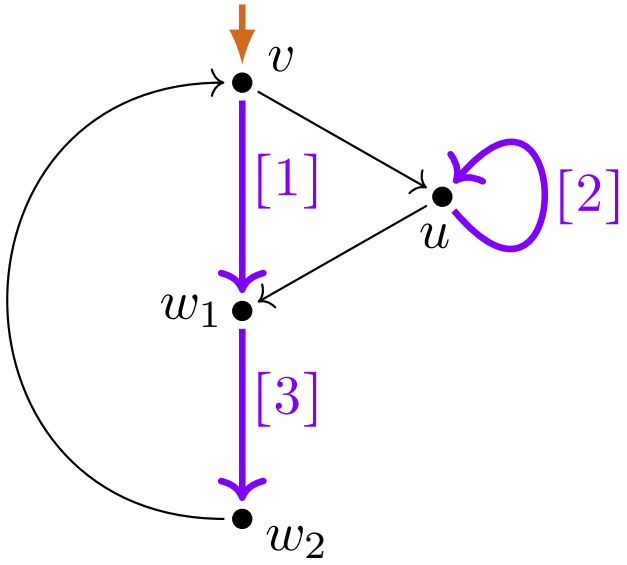

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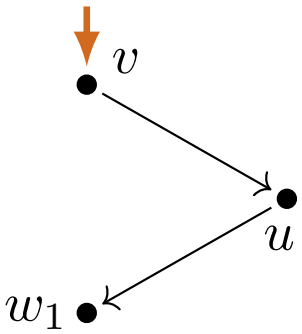

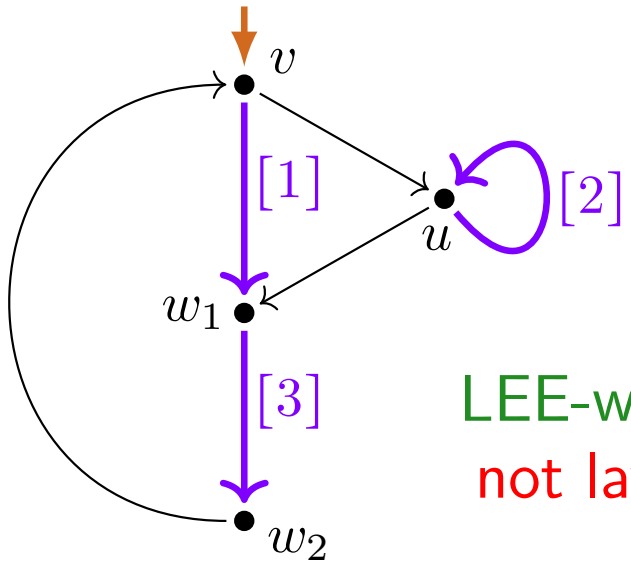

LEE-witness
not layered

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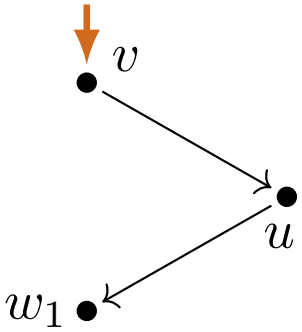

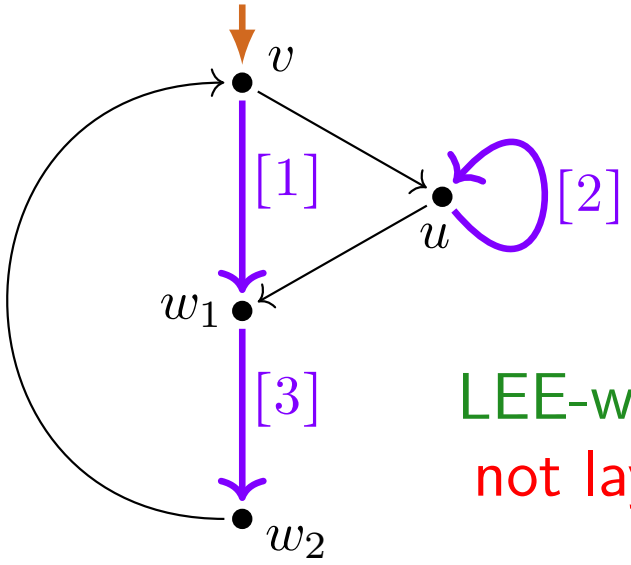

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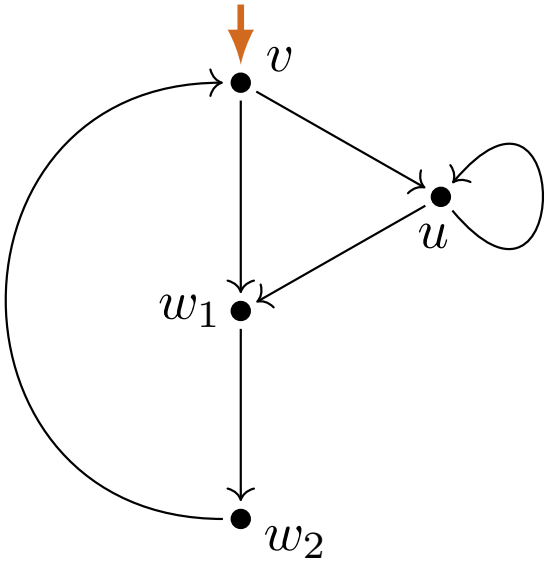

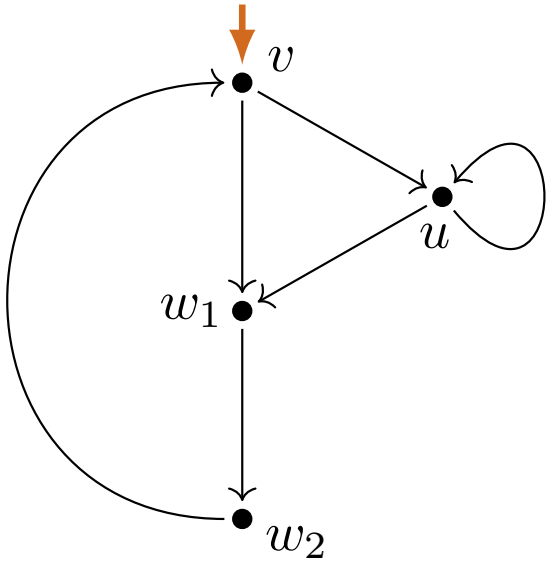

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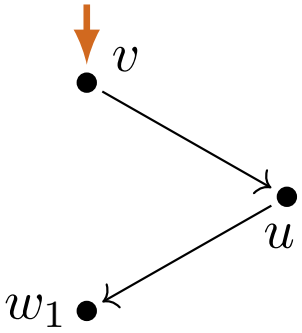

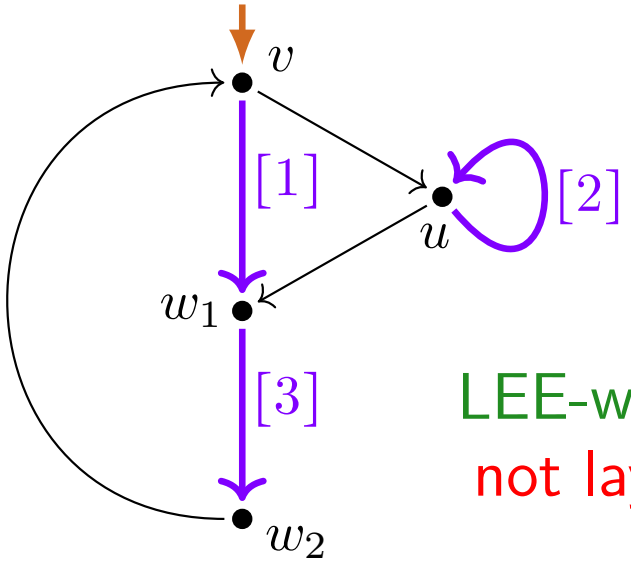

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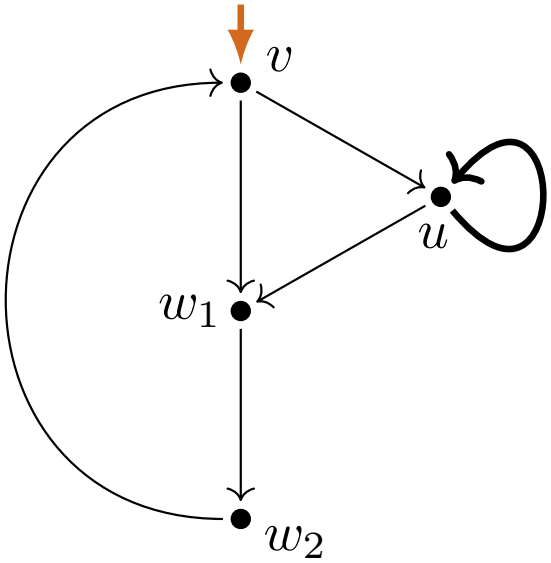
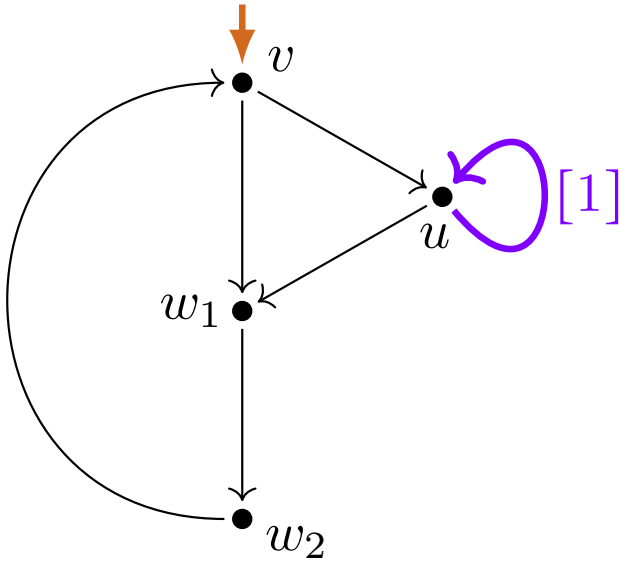
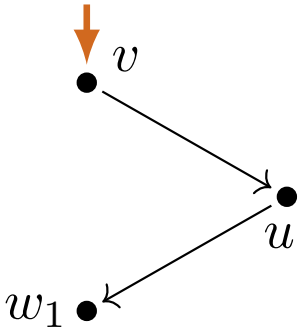
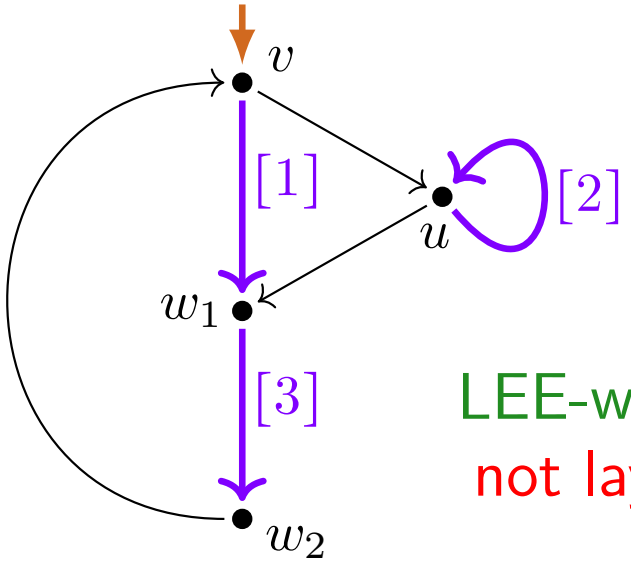




Not Layered / Layered Loop Elimination





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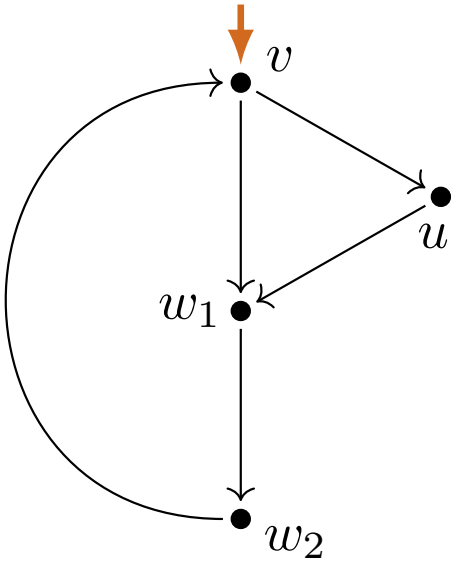

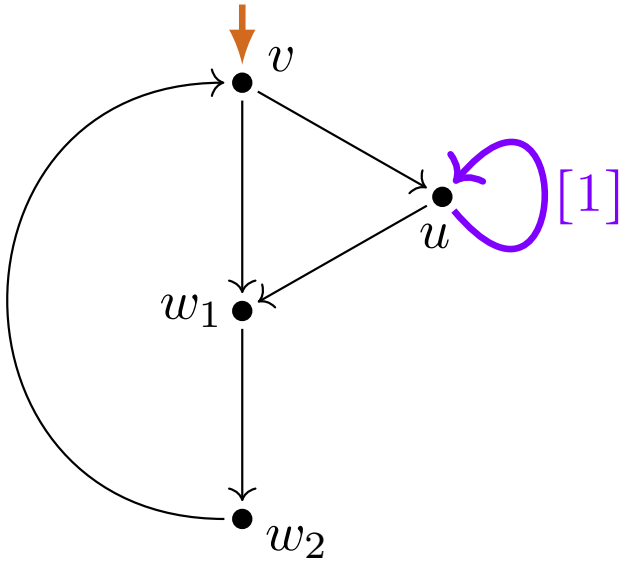

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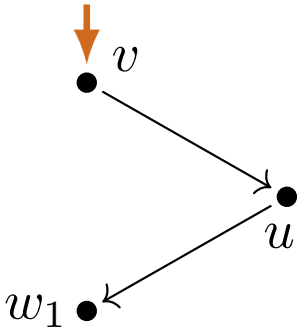

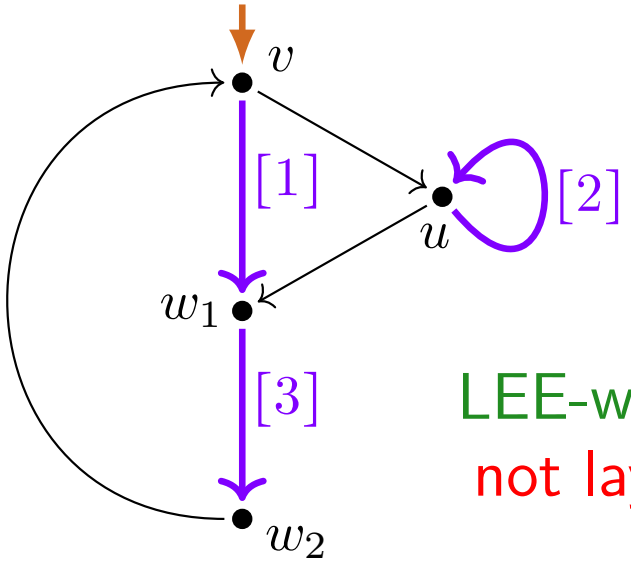

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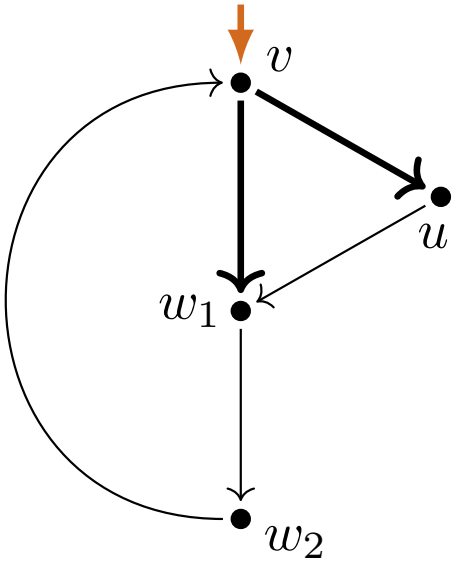

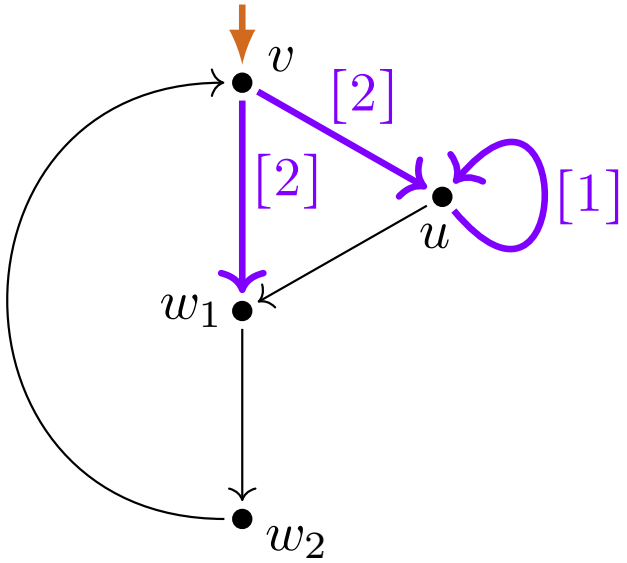

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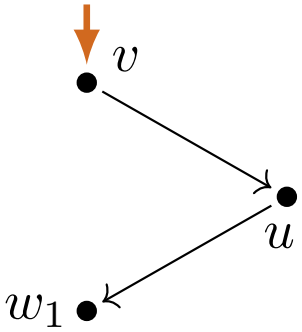

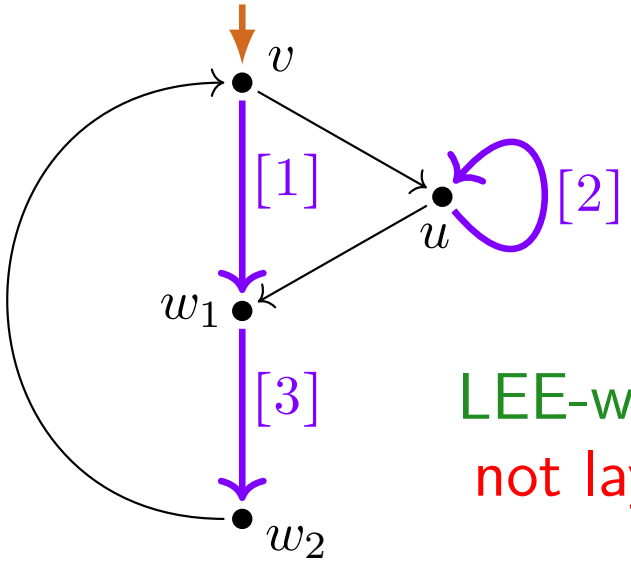

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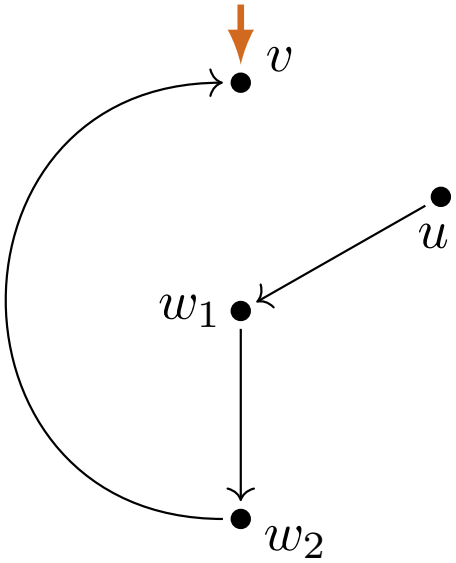

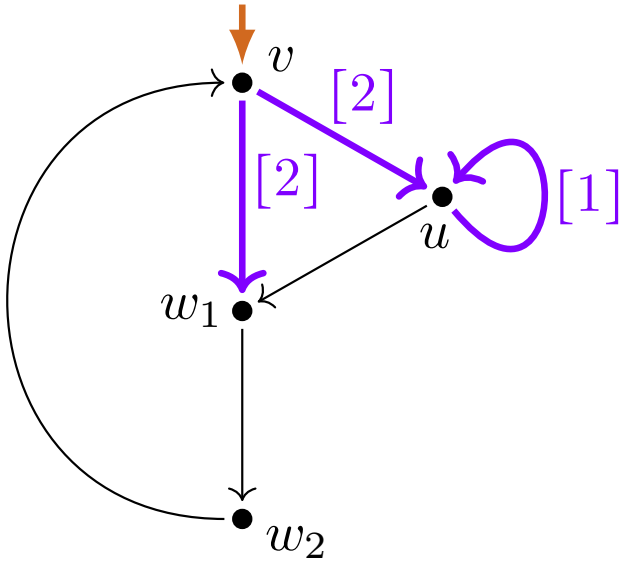

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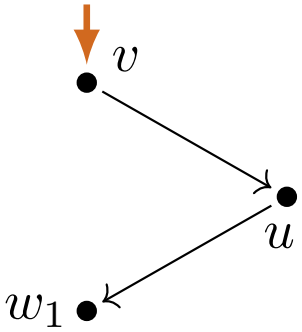

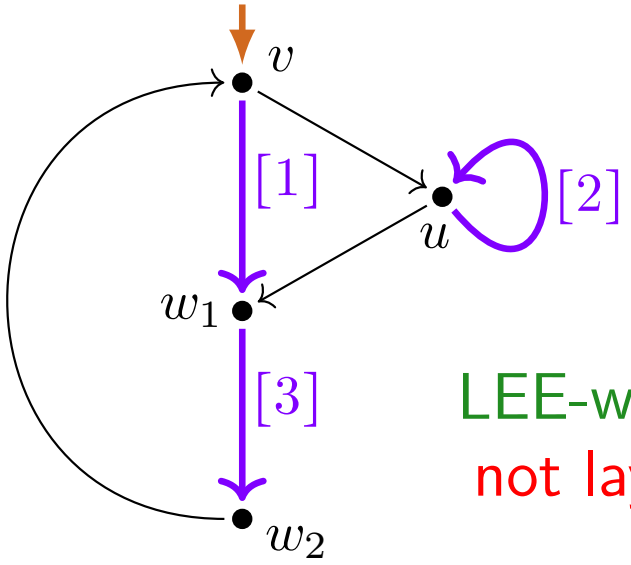

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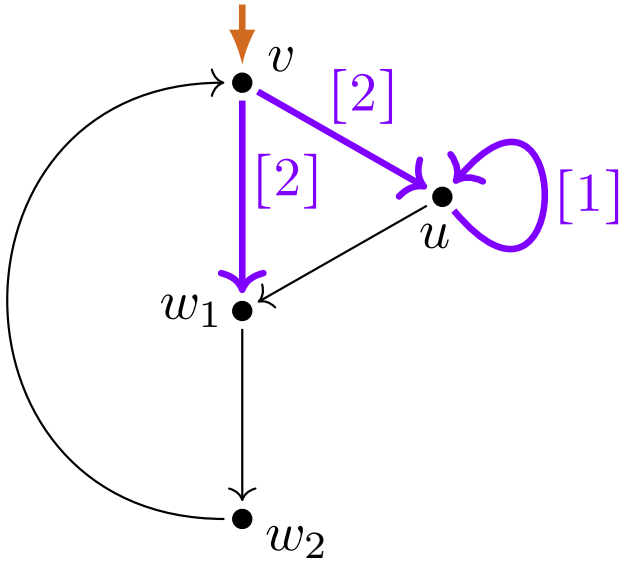

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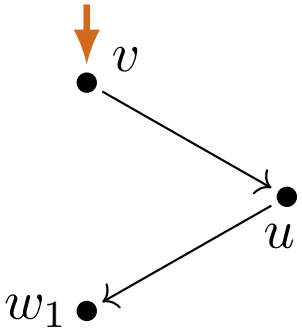

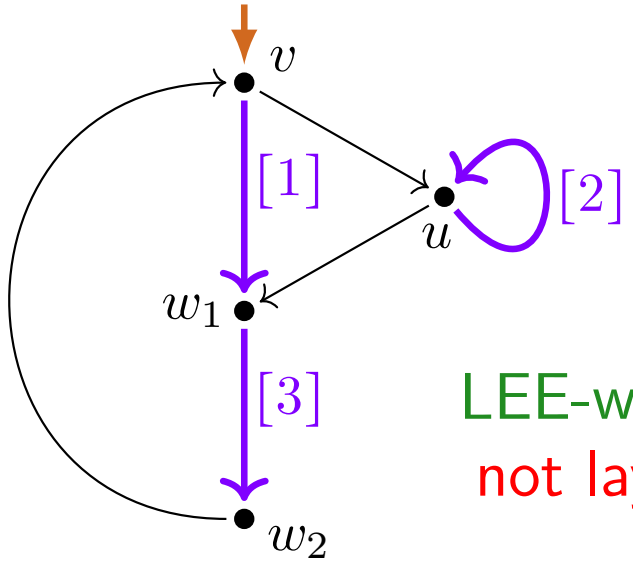

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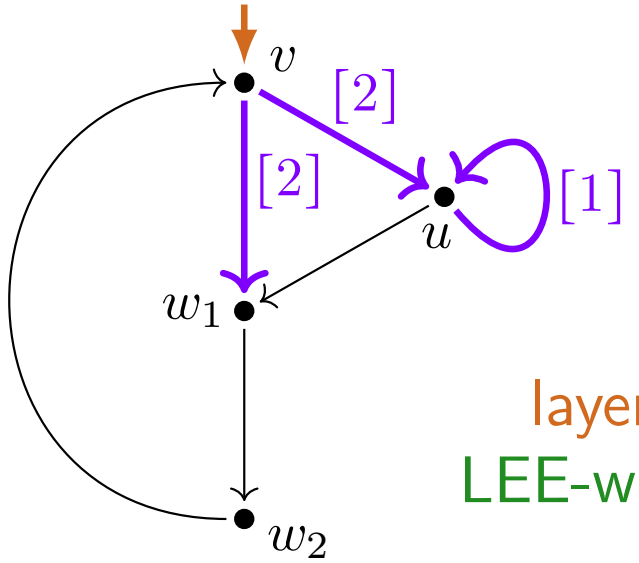

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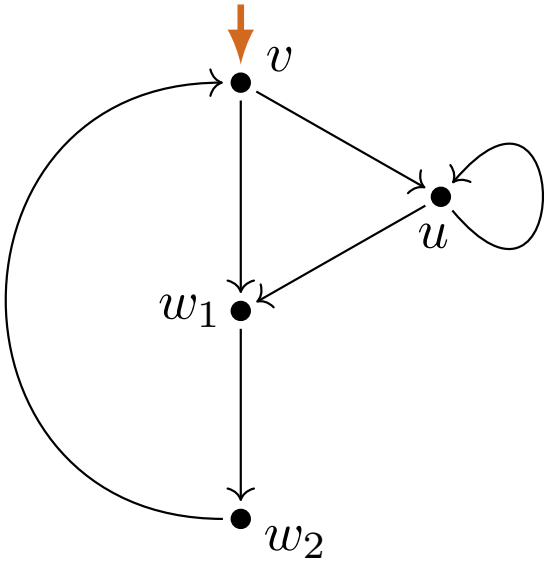

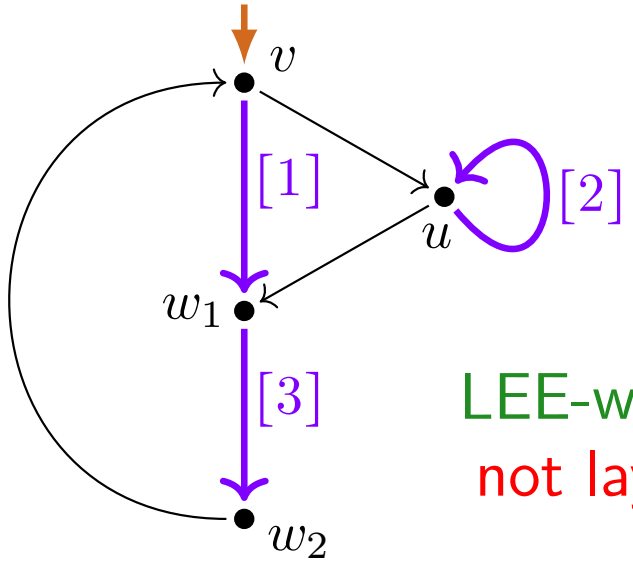

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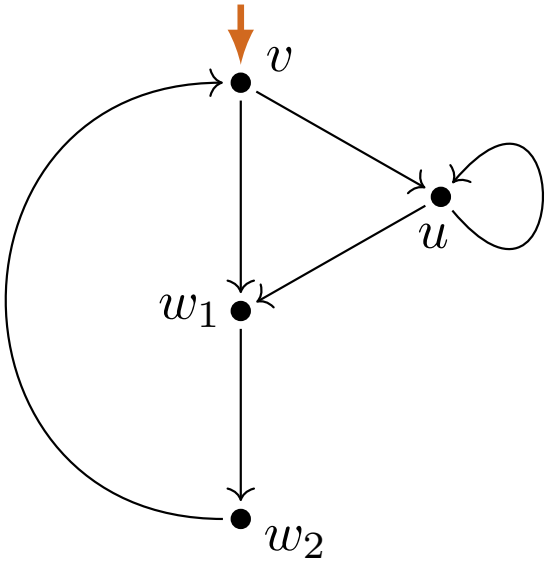

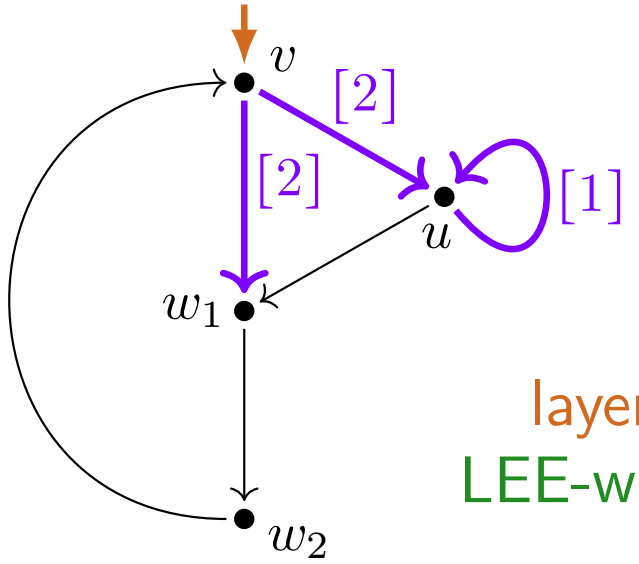

layered
LEE-witness

Layered Loop Elimination Condition LLEE

Layered loop elimination is defined on vertex-marked graphs $^{\circ}G$ in order to:

- mark vertices in the loop bodies of eliminated loops,
- permit the elimination of loop subcharts only at unmarked vertices.

Definition

The Layered Loop Existence & Elimination Condition LLEE for graphs G:

$$\mathrm{LLEE}(G) \;:\Longleftrightarrow\; \exists\, {}^{\bullet}G_0 \Big(\big({}^{\circ}G \overset{\circ}{\rightarrow}{}^{*}_{\mathrm{elim}} {}^{\bullet}G_0 \overset{\circ}{\not\rightarrow}_{\mathrm{elim}} \big) \;\wedge\; \neg\infty\big({}^{\bullet}G_0\big) \Big) .$$

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$$ {}^{\bullet}G_1 \xrightarrow{\circ}_{\mathrm{elim}} {}^{\bullet}G_1' \implies |{}^{\bullet}G_1|_{\circ} \to_{\mathrm{elim}} |{}^{\bullet}G_1'|_{\circ} $$

$$ {}^{\circ}G \xrightarrow{\circ}{}^{*}_{\mathrm{elim}} {}^{\bullet}G_1 \implies \Big[\exists\, {}^{\bullet}G_1'' \Big[\begin{matrix} {}^{\bullet}G_1 \xrightarrow{\circ}_{\mathrm{elim}} {}^{\bullet}G_1'' \\ \wedge\; |{}^{\bullet}G_1''|_{\circ} \to^{*}_{\mathrm{prune}} \cdot \leftarrow^{*}_{\mathrm{prune}} \cdot \leftarrow^{=}_{\mathrm{elim}} G_1' \end{matrix} \Big] \impliedby |{}^{\bullet}G_1|_{\circ} \to_{\mathrm{elim}} G_1' \Big] $$

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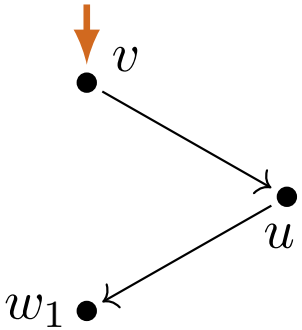
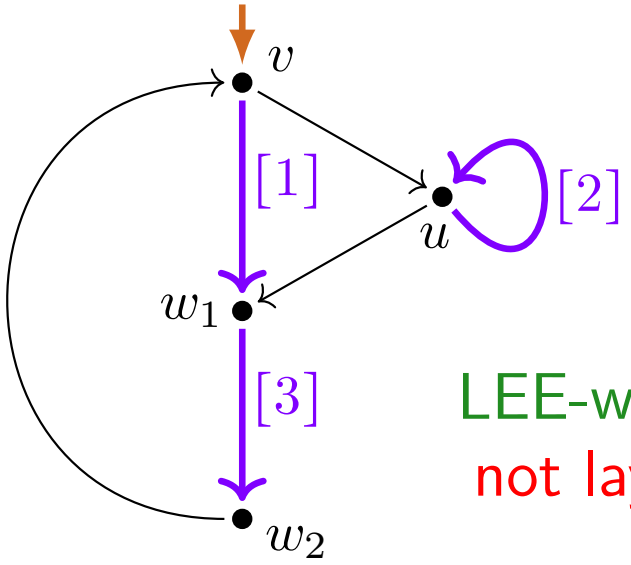
Theorem

$\mathrm{LLEE}(G) \Leftrightarrow \mathrm{LEE}(G)$ for all graphs G.

Making Layered





LEE-witness
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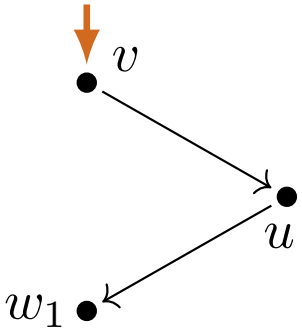
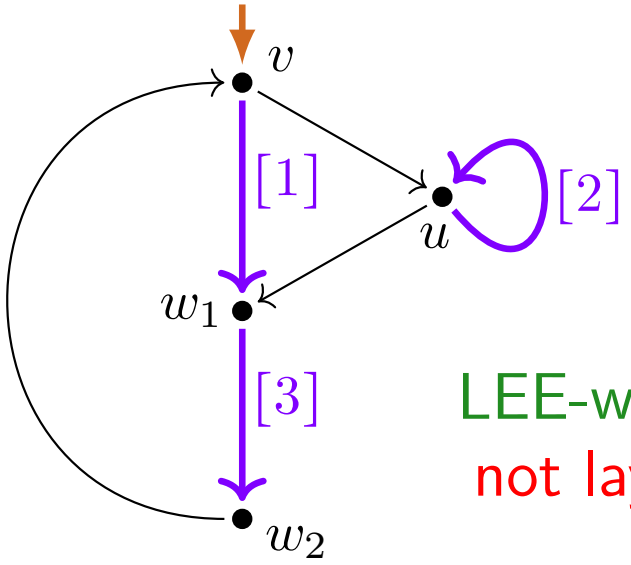
Making Layered





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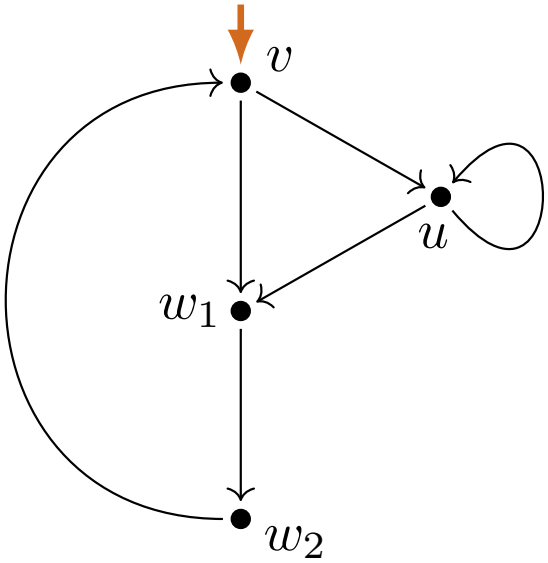

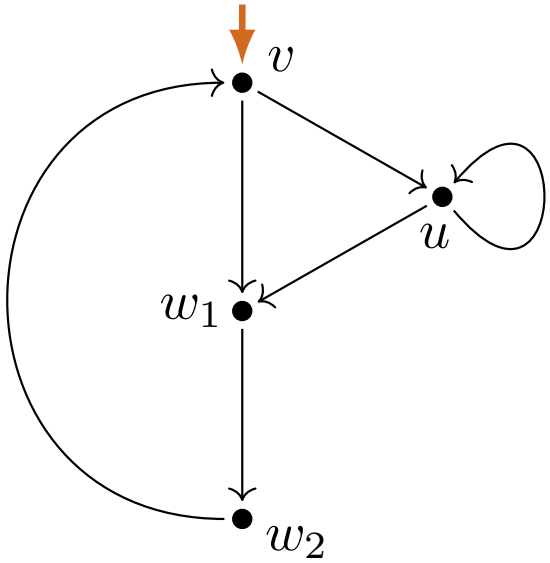

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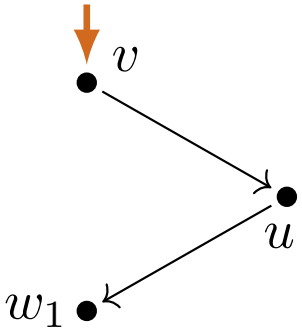

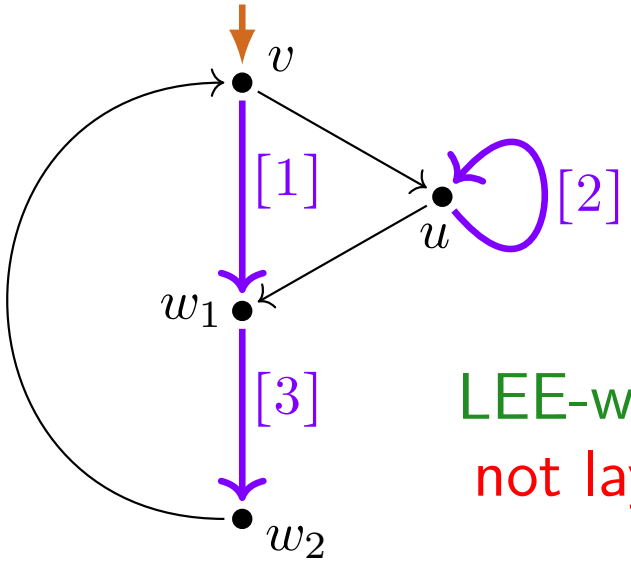

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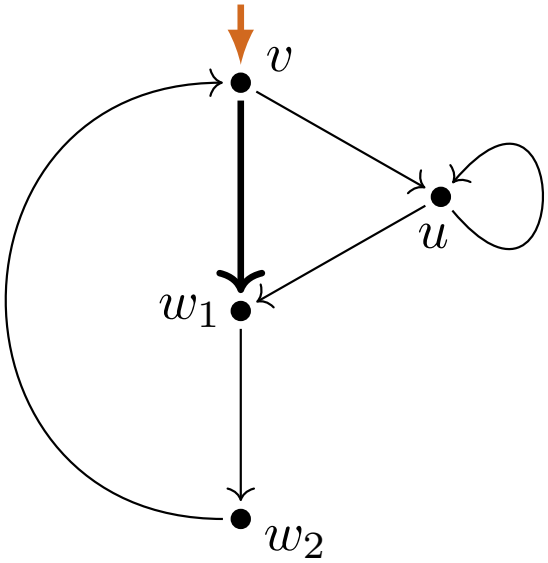

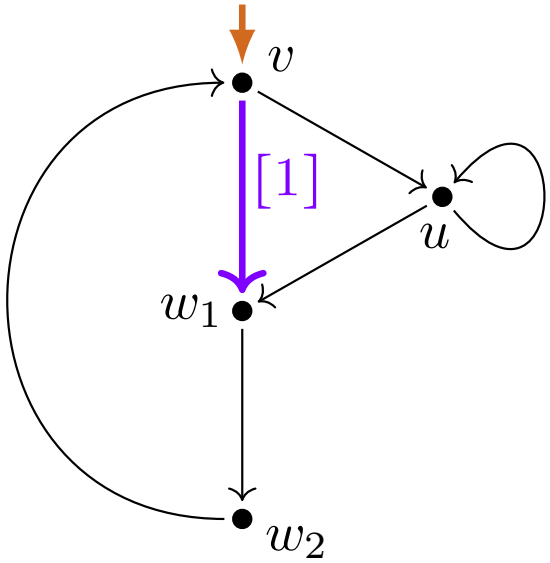

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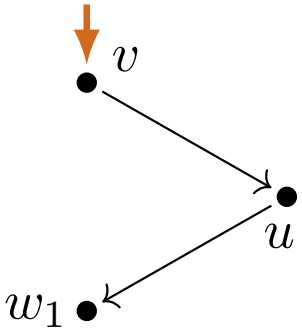

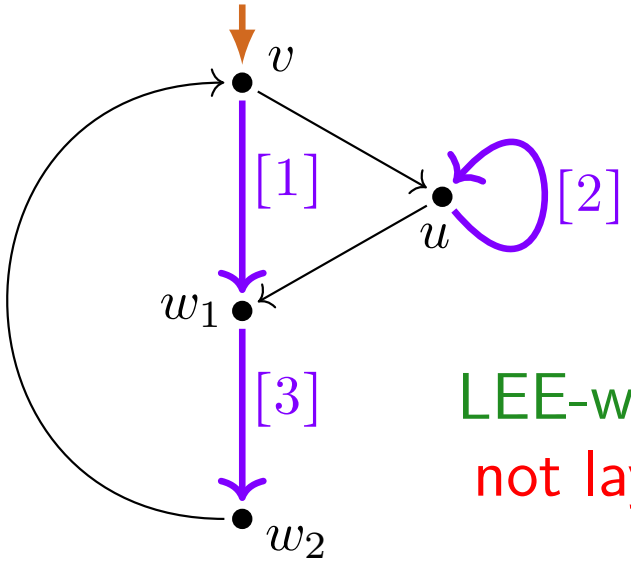

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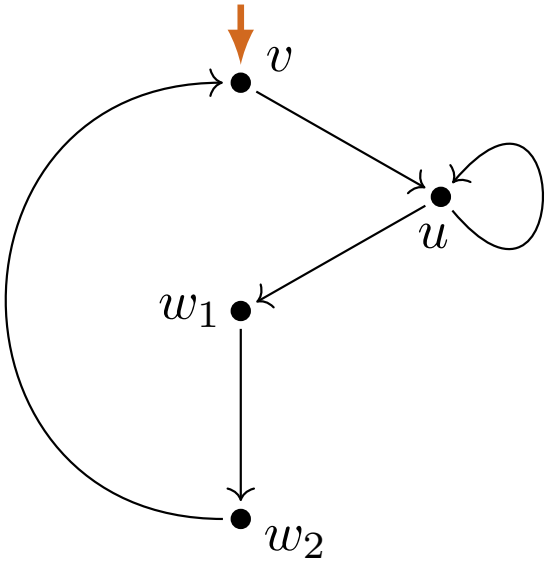

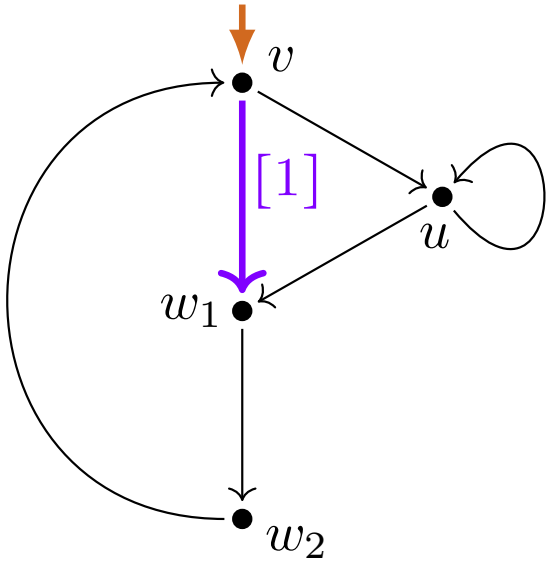

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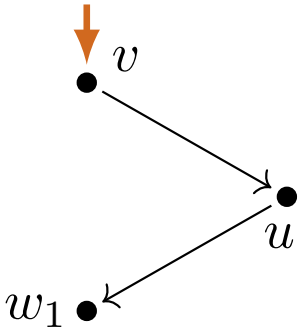

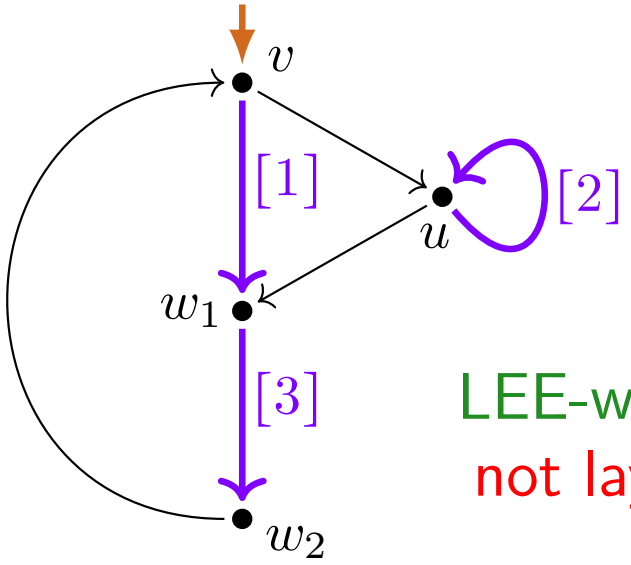

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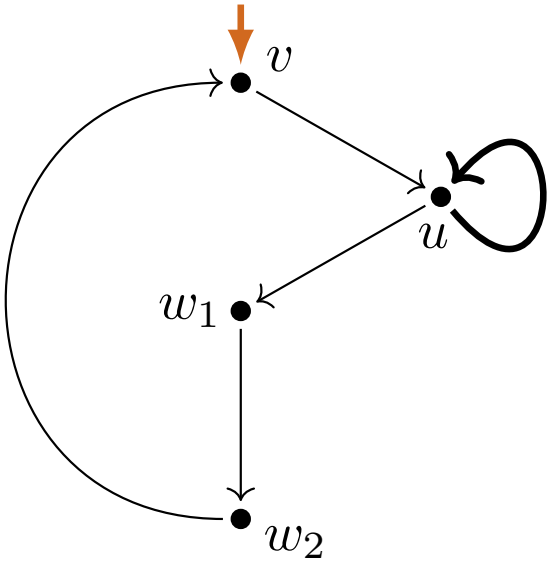

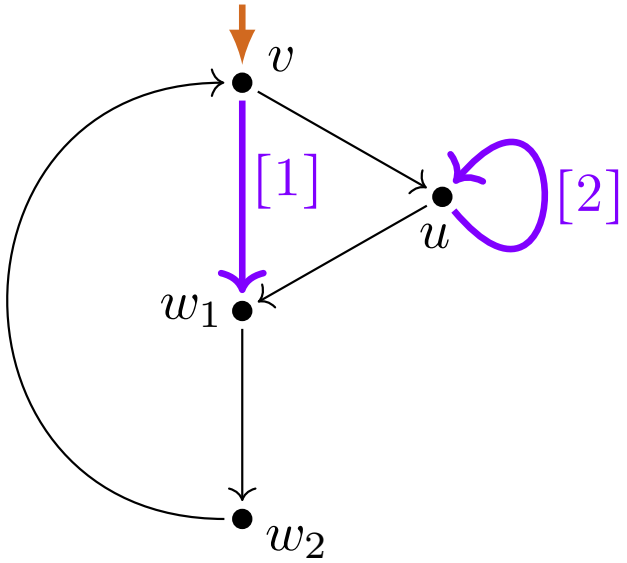

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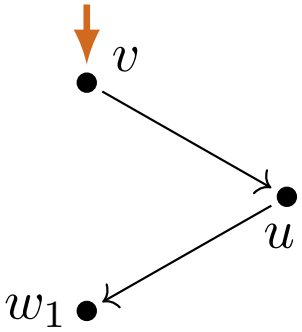

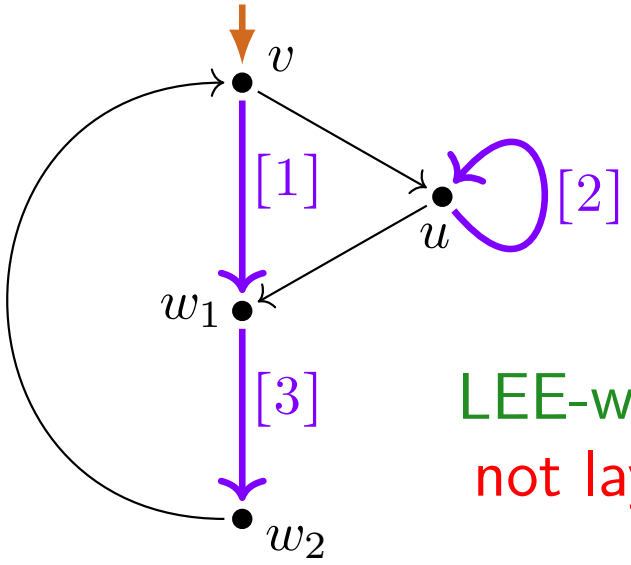

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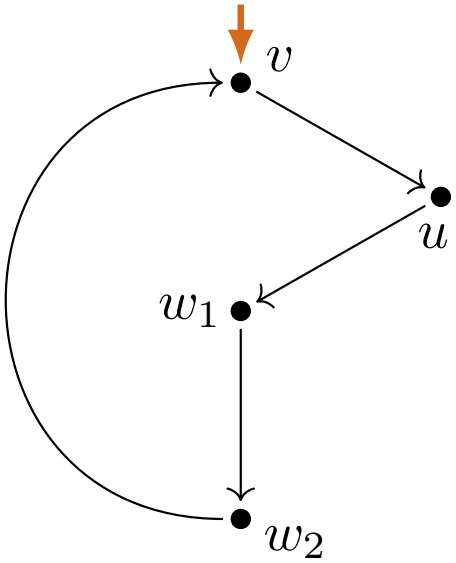
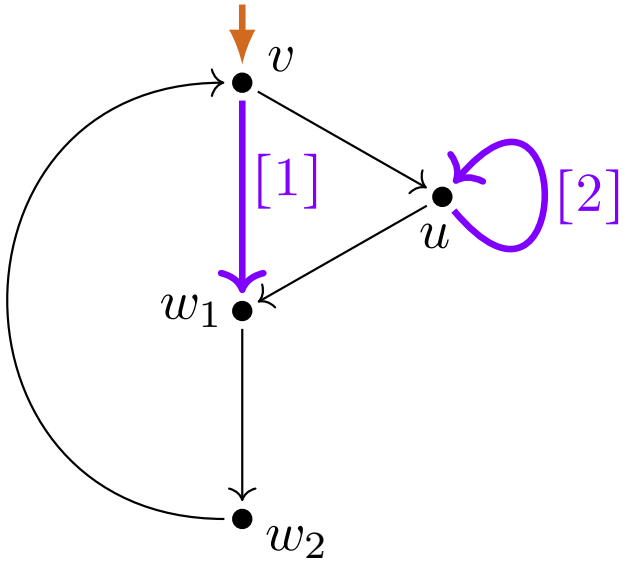
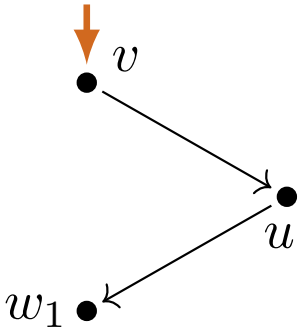
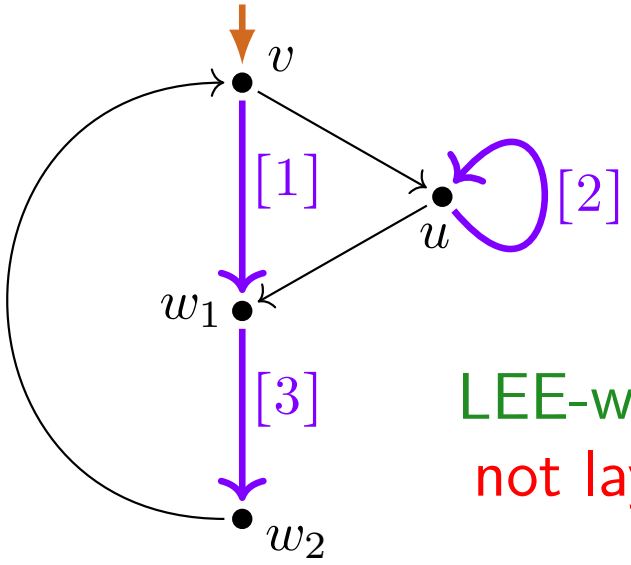


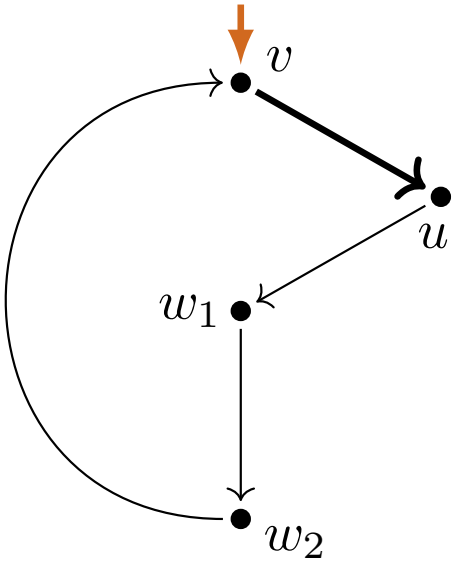


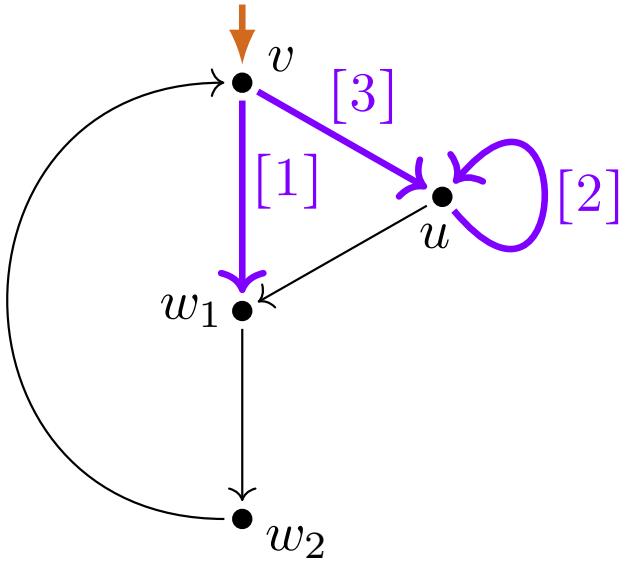
Making Layered





LEE-witness
not layered





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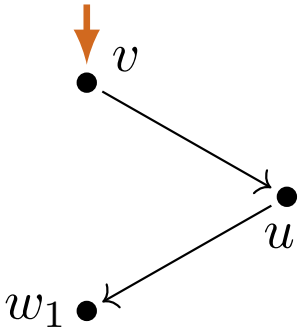

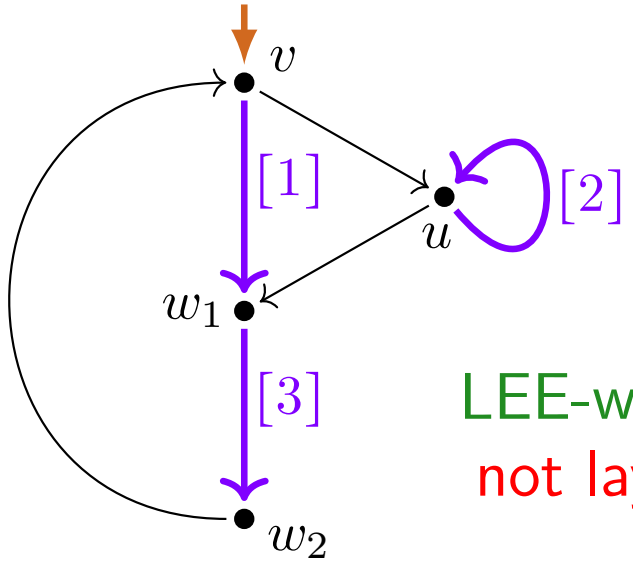

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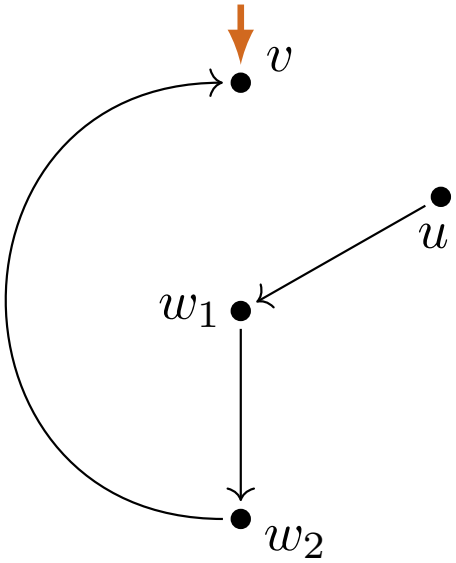
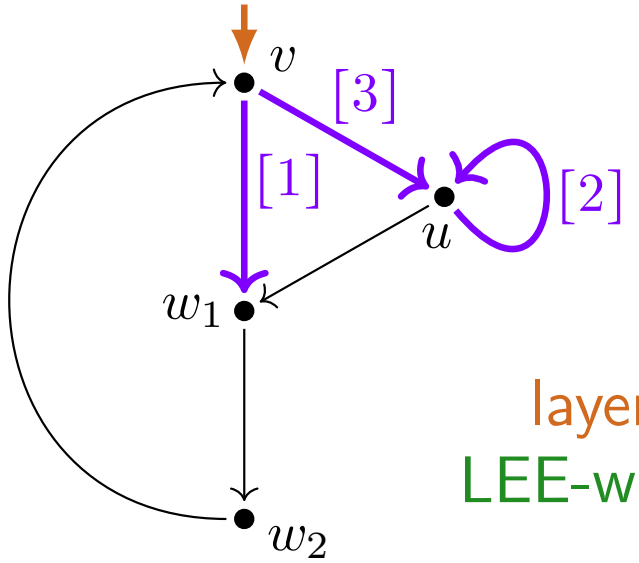
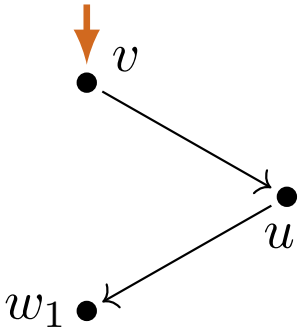
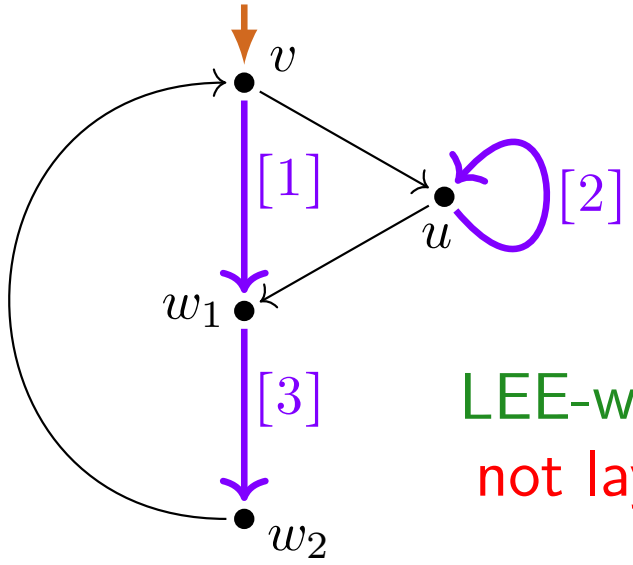




layered
LEE-witness

Making Layered





LEE-witness
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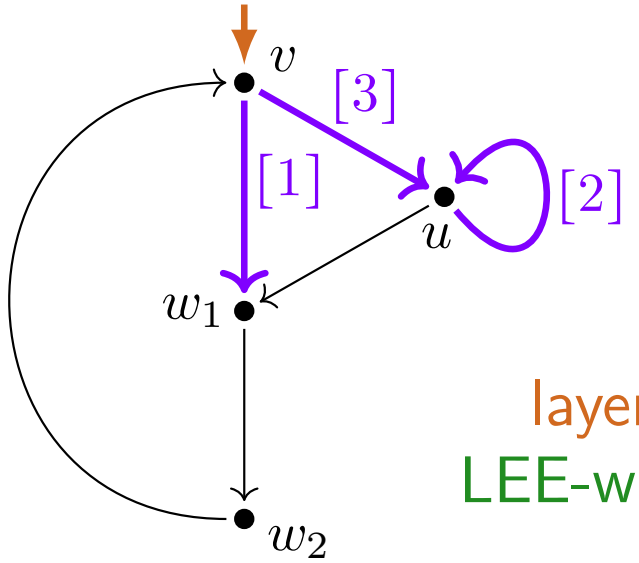

layered
LEE-witness

Further questions

(Q1) How efficient ($\leq O((n+k)^3)$) can LEE be decided?

- To try: characterisation of LEE as flow-graph co-reducibility (Allen / Hecht–Ullman, 1970/72, Laarakker–Kappé, TERMGRAPH 2026).

(Q2) Is 1-LEE decidable in polynomial time?
(1-LEE $\hat{=}$ generalised loop elimination, which is not confluent.)

(Q3) How can, for a graph G with LEE, any two regular expressions e_1, e_2 with $P(e_1) = P(e_2) \simeq G$ that are extracted from maximal $\to_{\mathrm{elim}}$ rewrite sequences from G be proved equivalent with respect to $[\![\cdot]\!]_P$?

(Q4) Can the Decreasing Diagrams Method be adapted to show pre-/post-ponement? (Here postponement of $\overset{\circ}{\to}_{\mathrm{elim}}$ over $\to_{\mathrm{elim}}$ is closely connected to confluence of $\to_{\mathrm{ep}}$.)

- Vincent van Oostrom referred me to his IWC 2020 article.

Summary

- Loop elimination $\to_{\mathrm{elim}}$ in process graphs
  - Milner's process interpretation P of regular expressions
  - Loop existence and elimination property LEE
    LEE = compact process interpretations of $(*/\not{1})$ reg. expr's
  - $\to_{\mathrm{elim}}$ is not confluent
- Completion of loop elimination $\to_{\mathrm{elim}}$ with pruning steps $\to_{\mathrm{prune}}$
  - 'critical pair' lemma for joining elimination forks $\leftarrow_{\mathrm{elim}} \cdot \to_{\mathrm{elim}}$
  - use of decreasing diagrams for $\to_{\mathrm{ep}} = \to_{\mathrm{elim}} \cup \to_{\mathrm{prune}}$
- Application 1: LEE is decidable in polynomial time
- Application 2: Layered LEE = LEE

IFIP-1.6 and TERMGRAPH talks

TERMGRAPH (last Sunday)

CG: *Towards a Characterisation of the Graph Structure of Process Interpretations of Regular Expressions*

Ext. Abstract https://clegra.github.io/lf/pi-struc-TG-26.pdf

Slides https://clegra.github.io/lf/TG-26.pdf

- LEE $\hat{=}$ image of $P^\bullet|_{RExp^{(*/\not{1})}}$
- 1-LEE $\hat{=}$ image of $P^\bullet$

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IFIP-1.6 meeting (last Saturday)

CG: *Finding Small Steps for Hard Problems, in Work on the Process Interpretation of Regular Expressions*

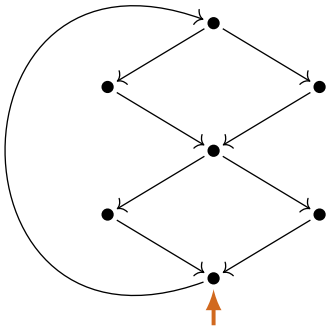
Slides https://clegra.github.io/lf/IFIP-1_6-2026.pdf

- Reducing for Graphs Representing Regular Expressions under Bisimilarity

Resources

- Slides/extended abstract on clegra.github.io
  - slides: .../lf/IWC-26.pdf
  - extended abstract: .../lf/le-conf-IWC-26.pdf
- CG: Milner's Proof System for Regular Expressions Modulo Bisimilarity is Complete
  - LICS 2022, arXiv:2209.12188, poster.
- CG: The Image of the Process Interpretation of Regular Expressions is Not Closed under Bisimulation Collapse
  - arXiv:2303.08553, 2021/2023.
- CG, Wan Fokkink: A Complete Proof System for 1-Free Regular Expressions Modulo Bisimilarity,
  - LICS 2020, arXiv:2004.12740, video on youtube.
- CG: Structure-Constrained Process Graphs for the Process Semantics of Regular Expressions
  - TERMGRAPH 2020, EPTCS 334, arXiv:2012.10869.
- CG: Modeling Terms by Graphs with Structure Constraints
  - TERMGRAPH 2018, EPTCS 288, arXiv:1902.02010.

‘Critical pair’: bi-loop elimination



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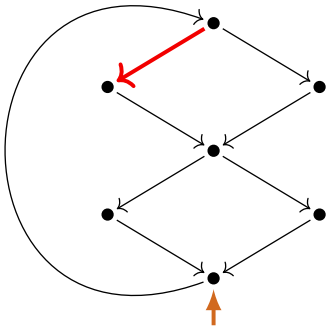

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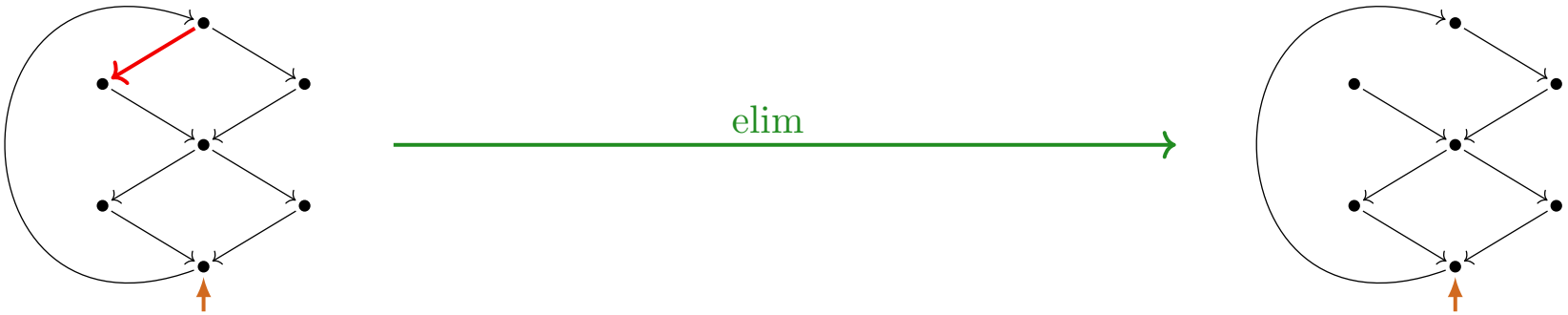

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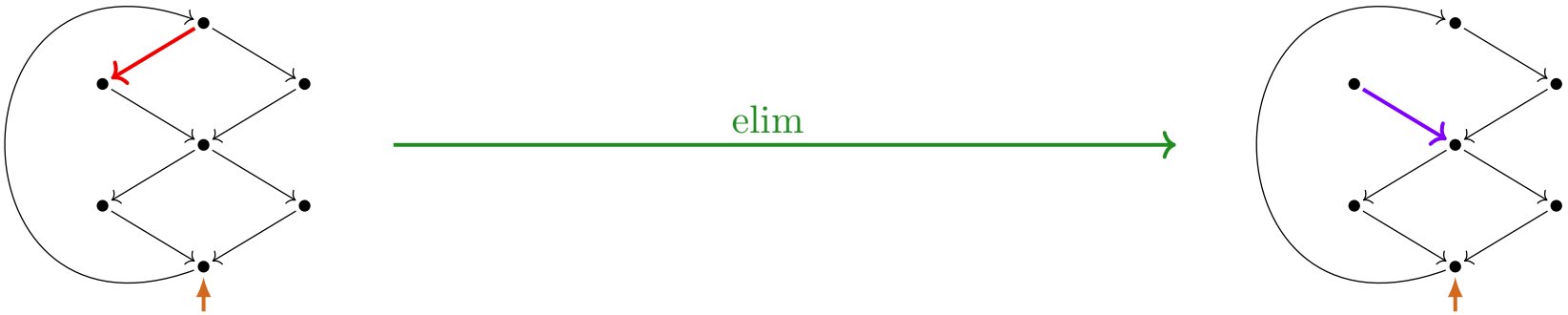

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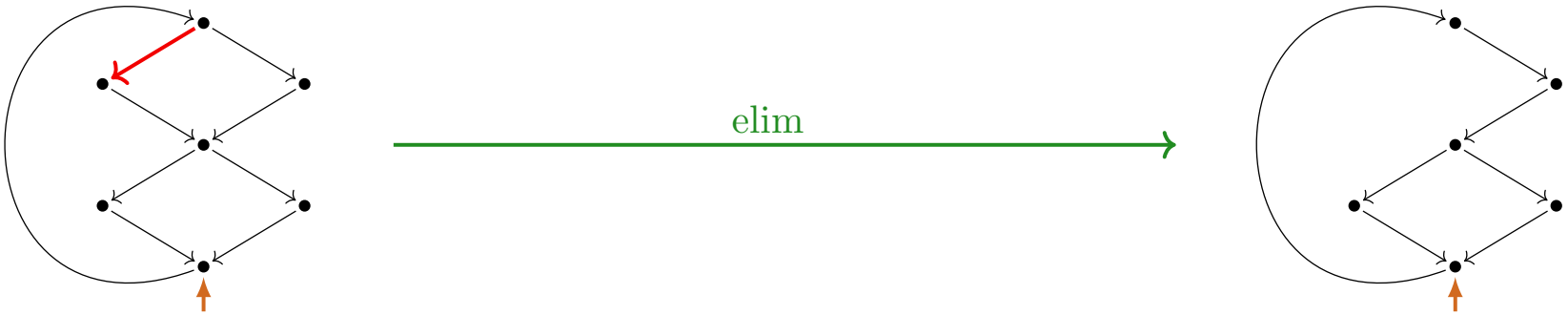

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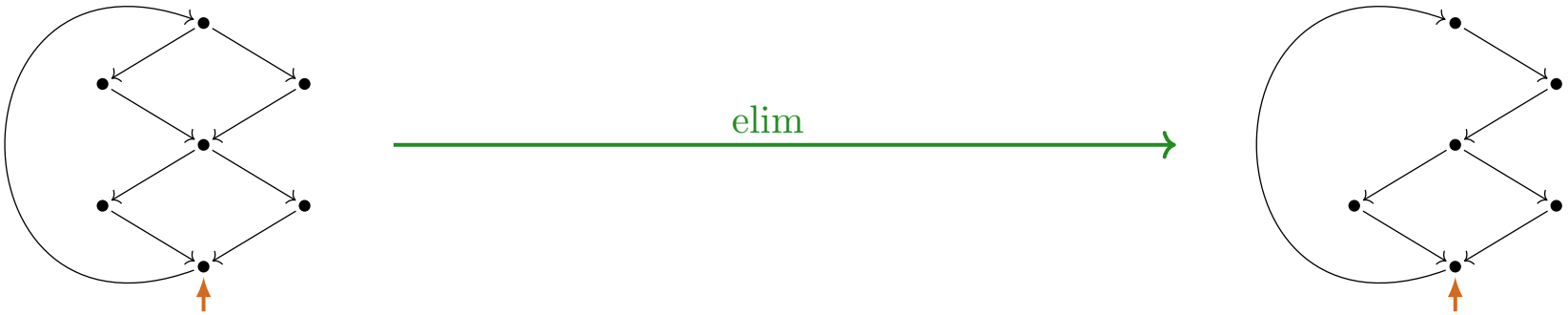

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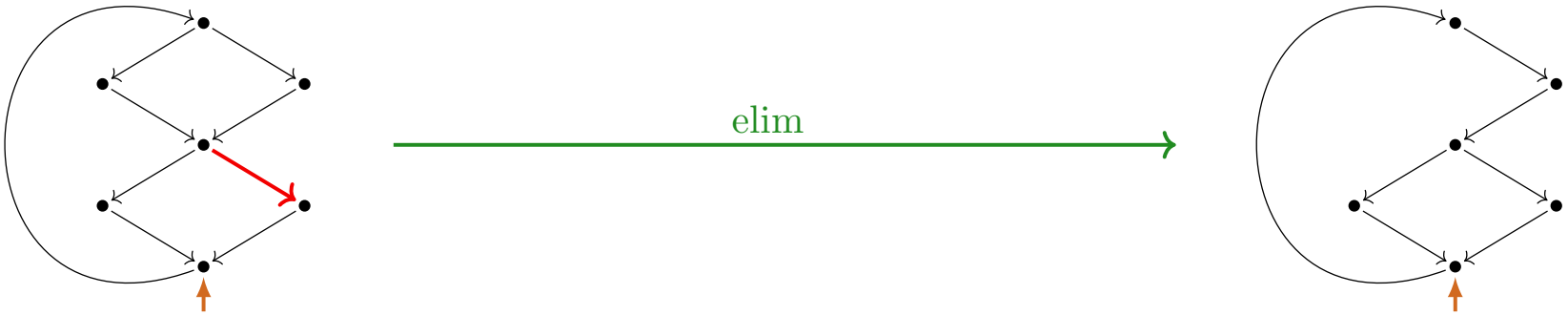

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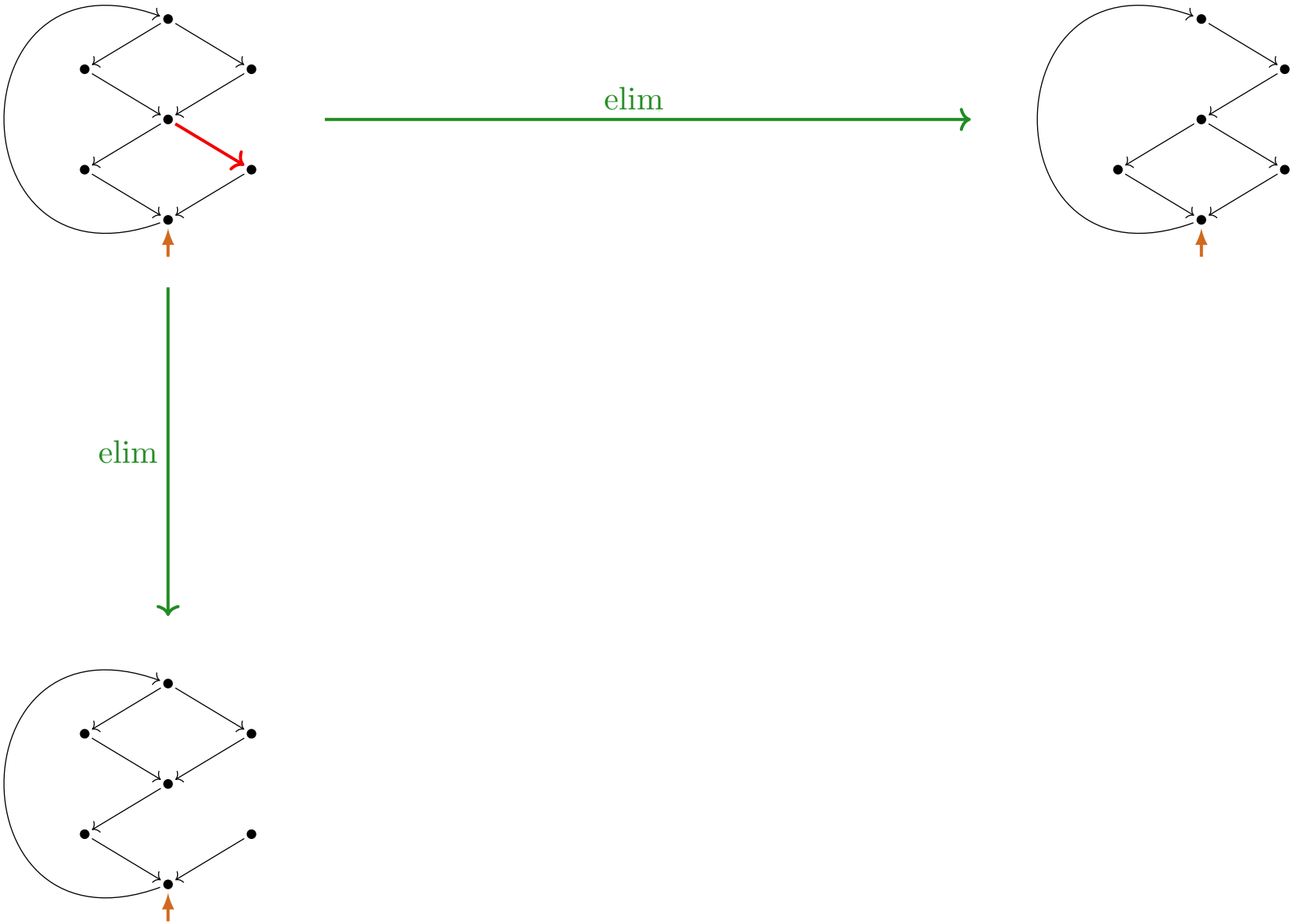

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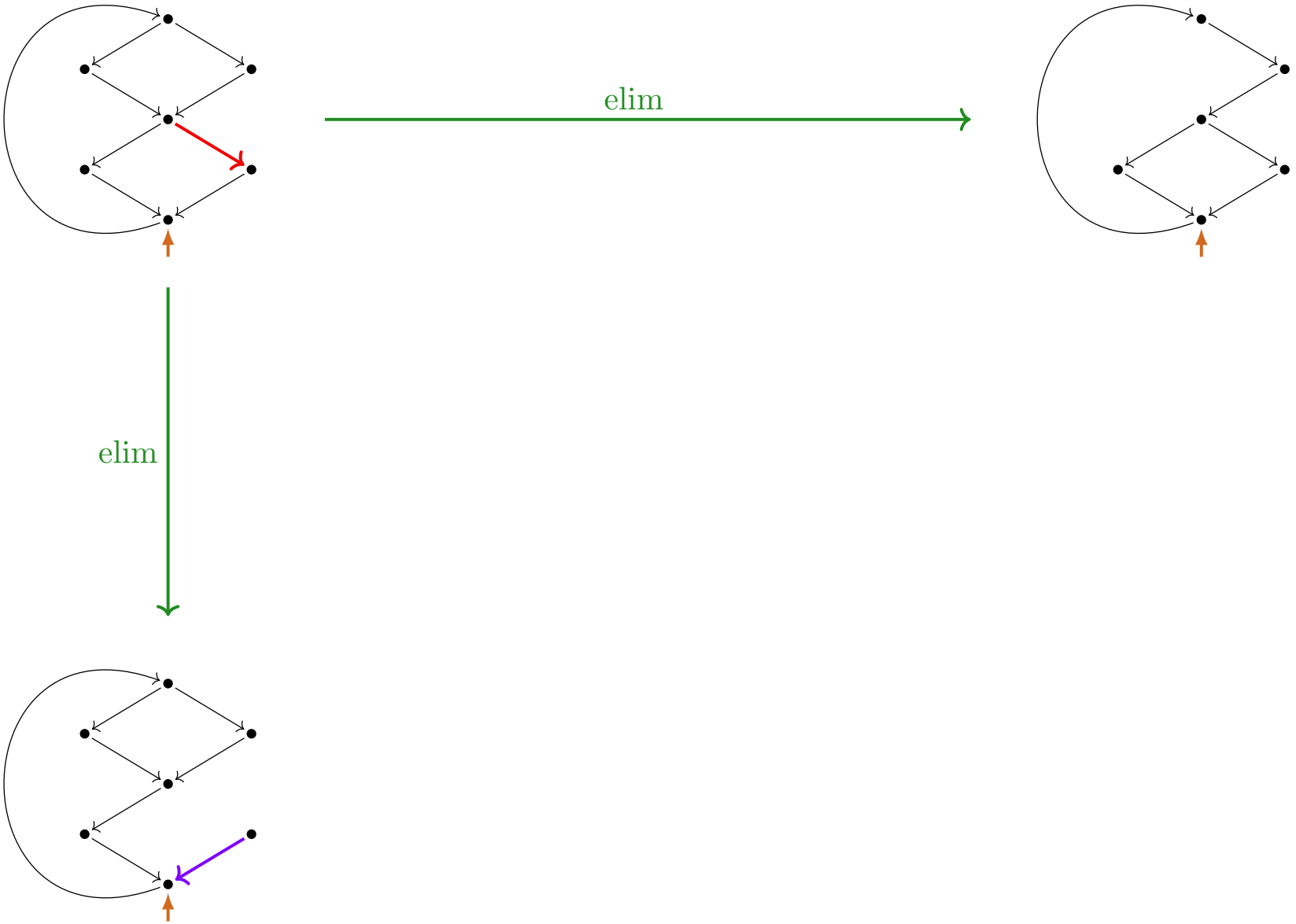

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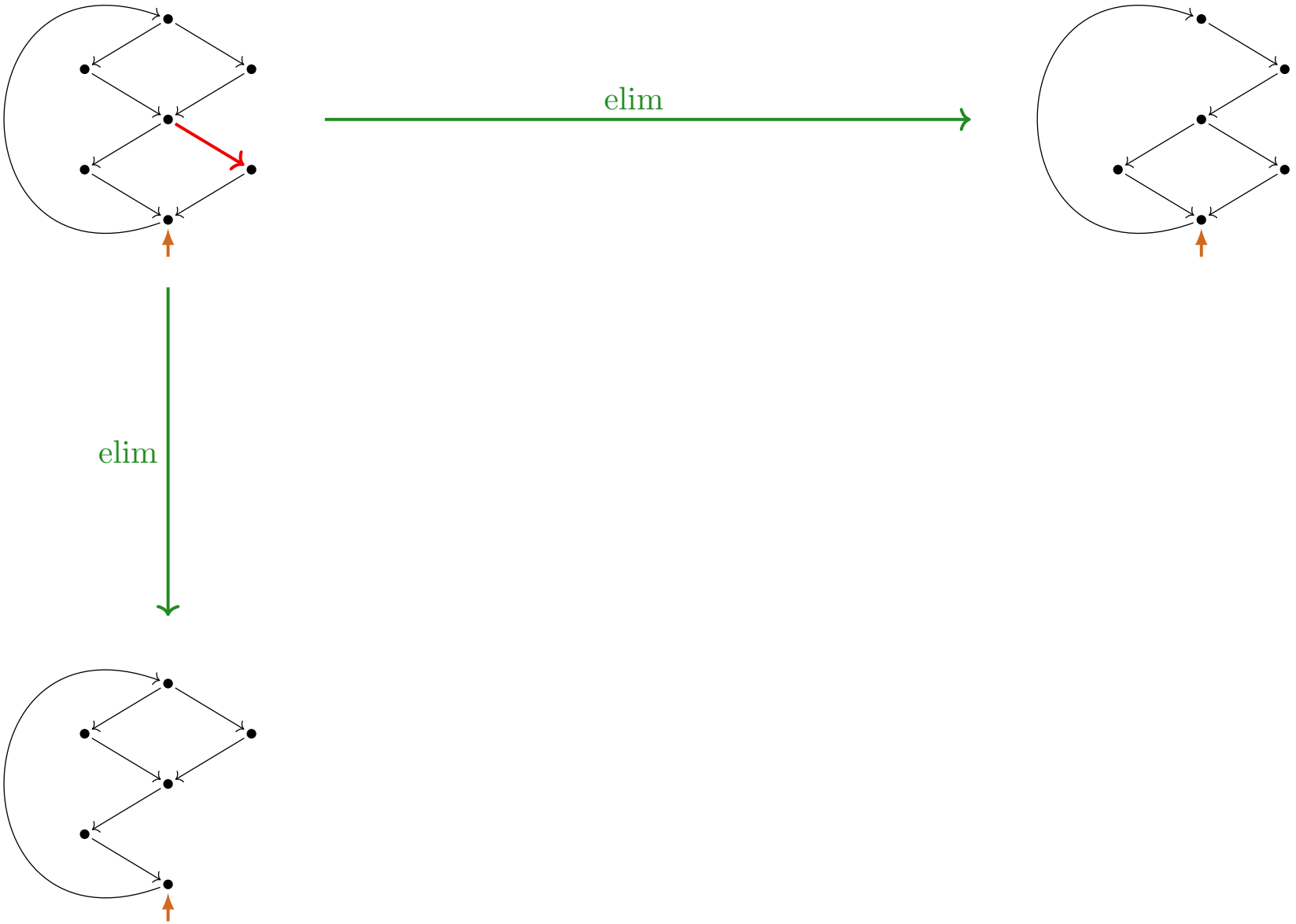

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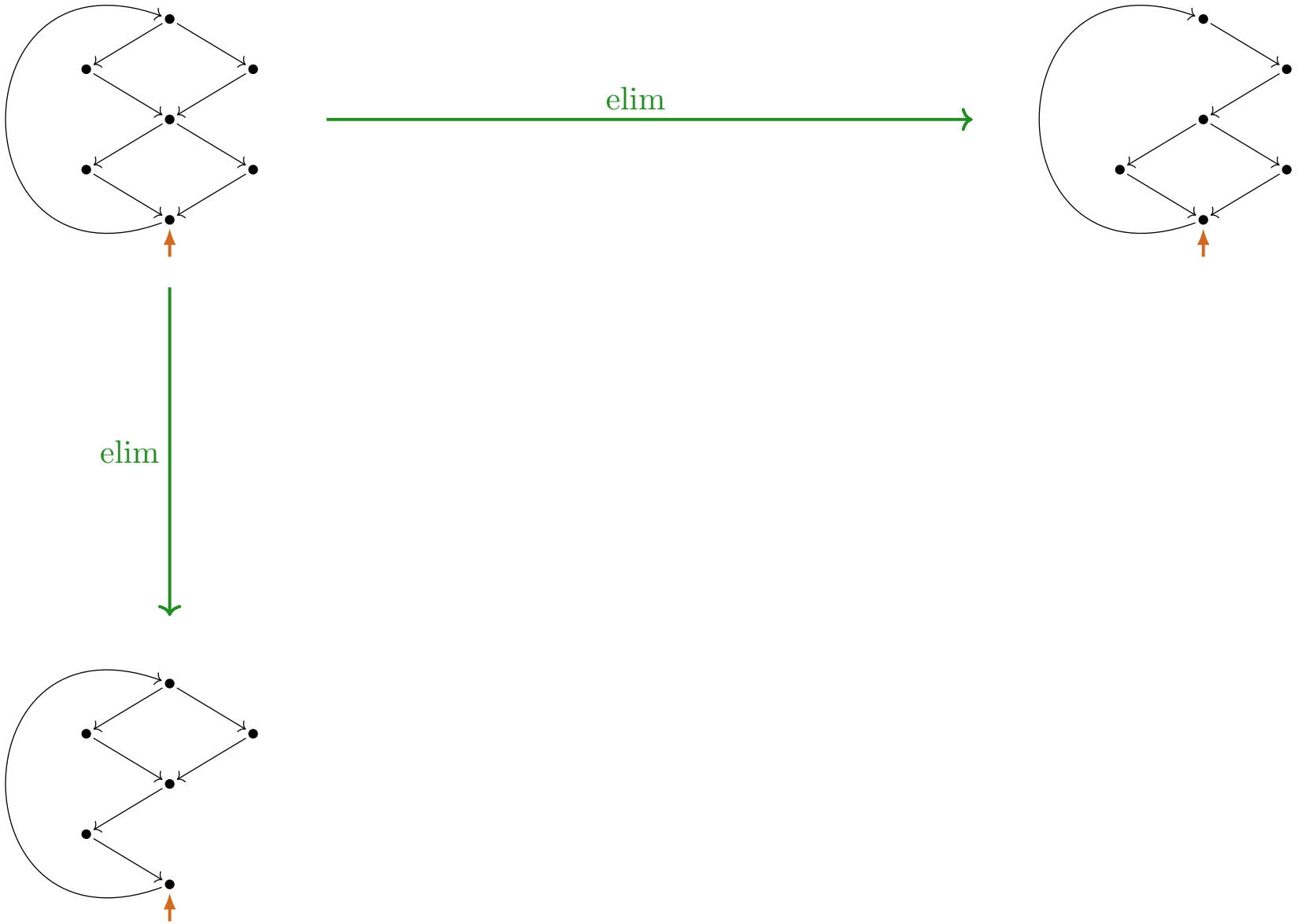

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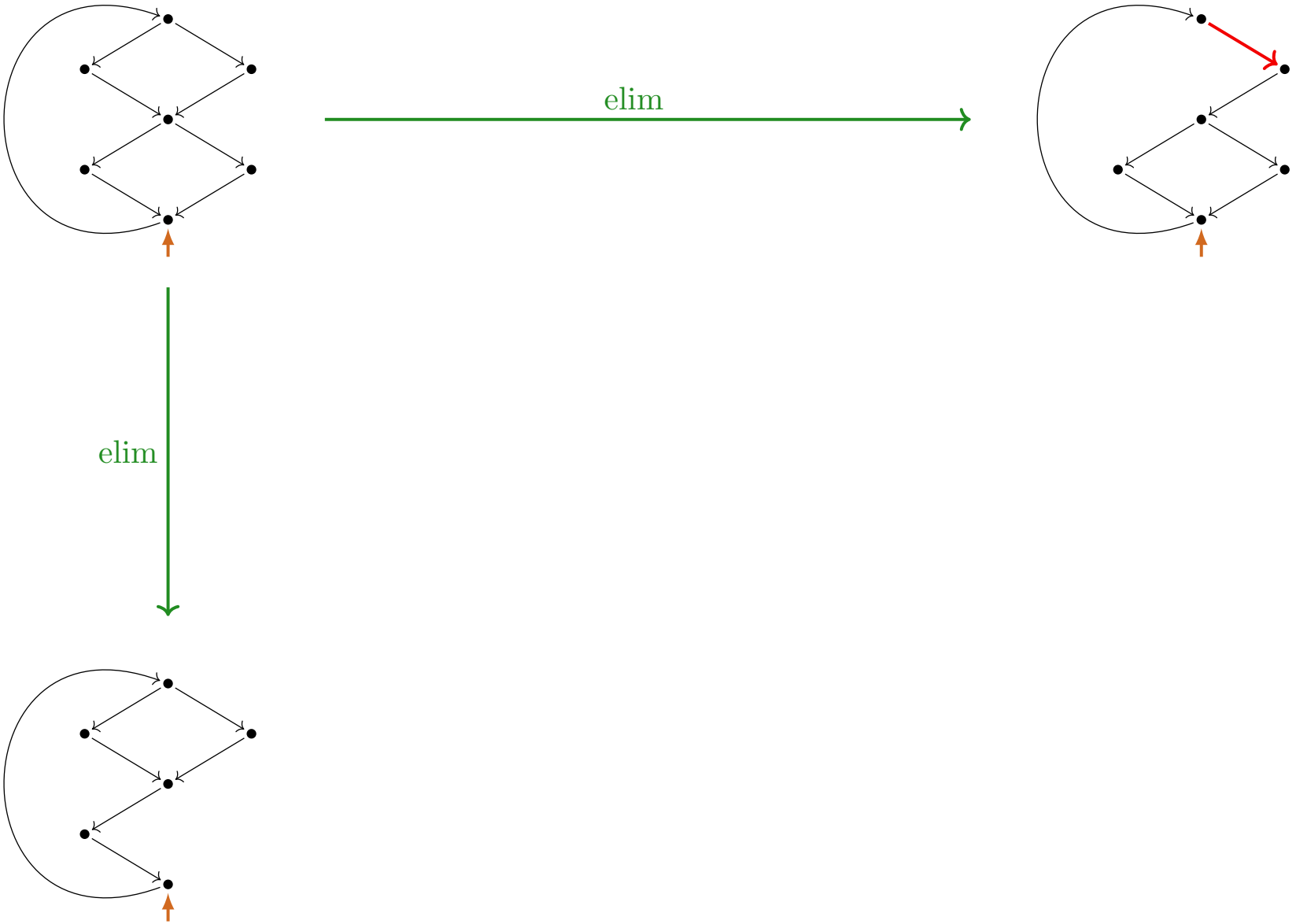

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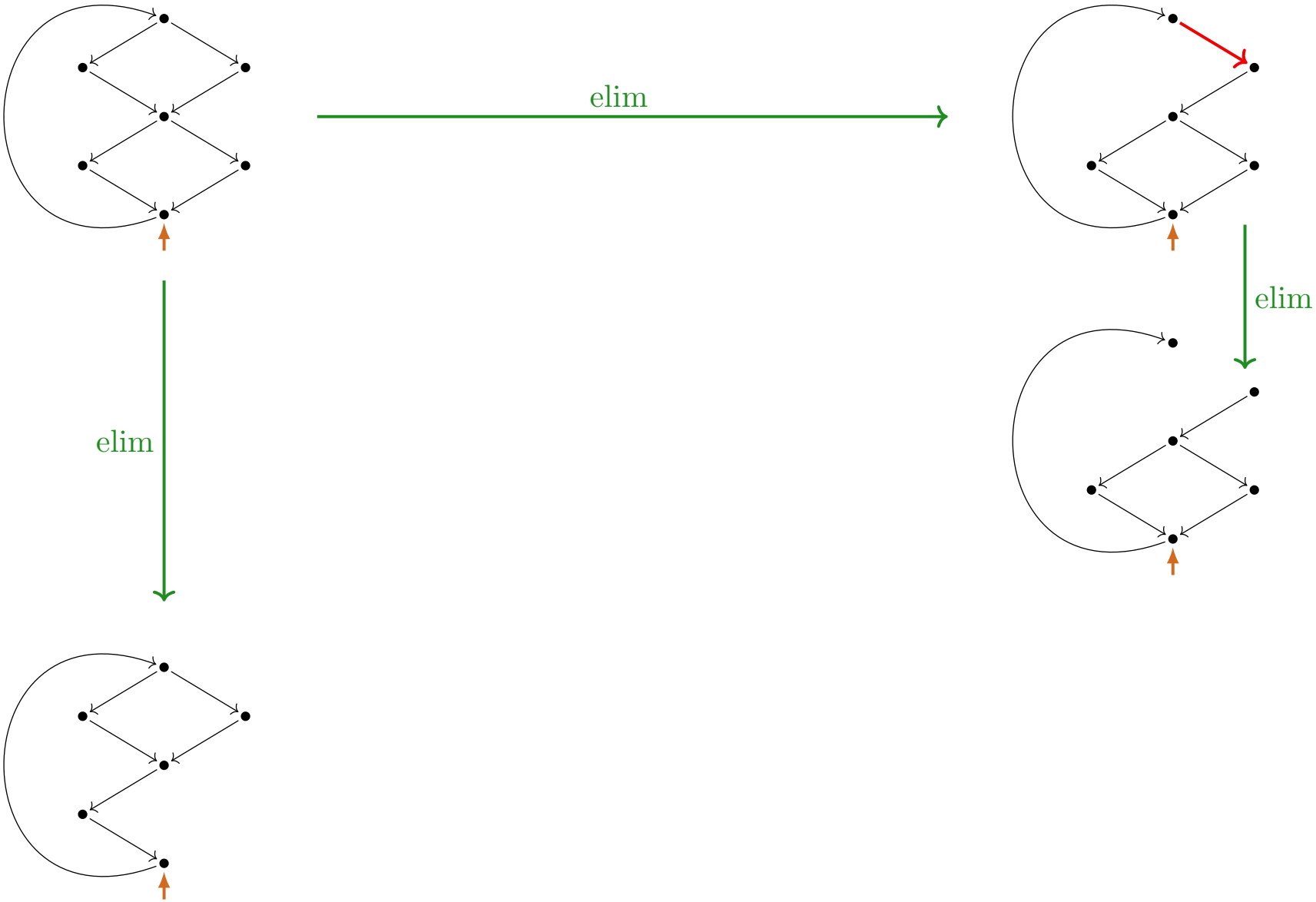

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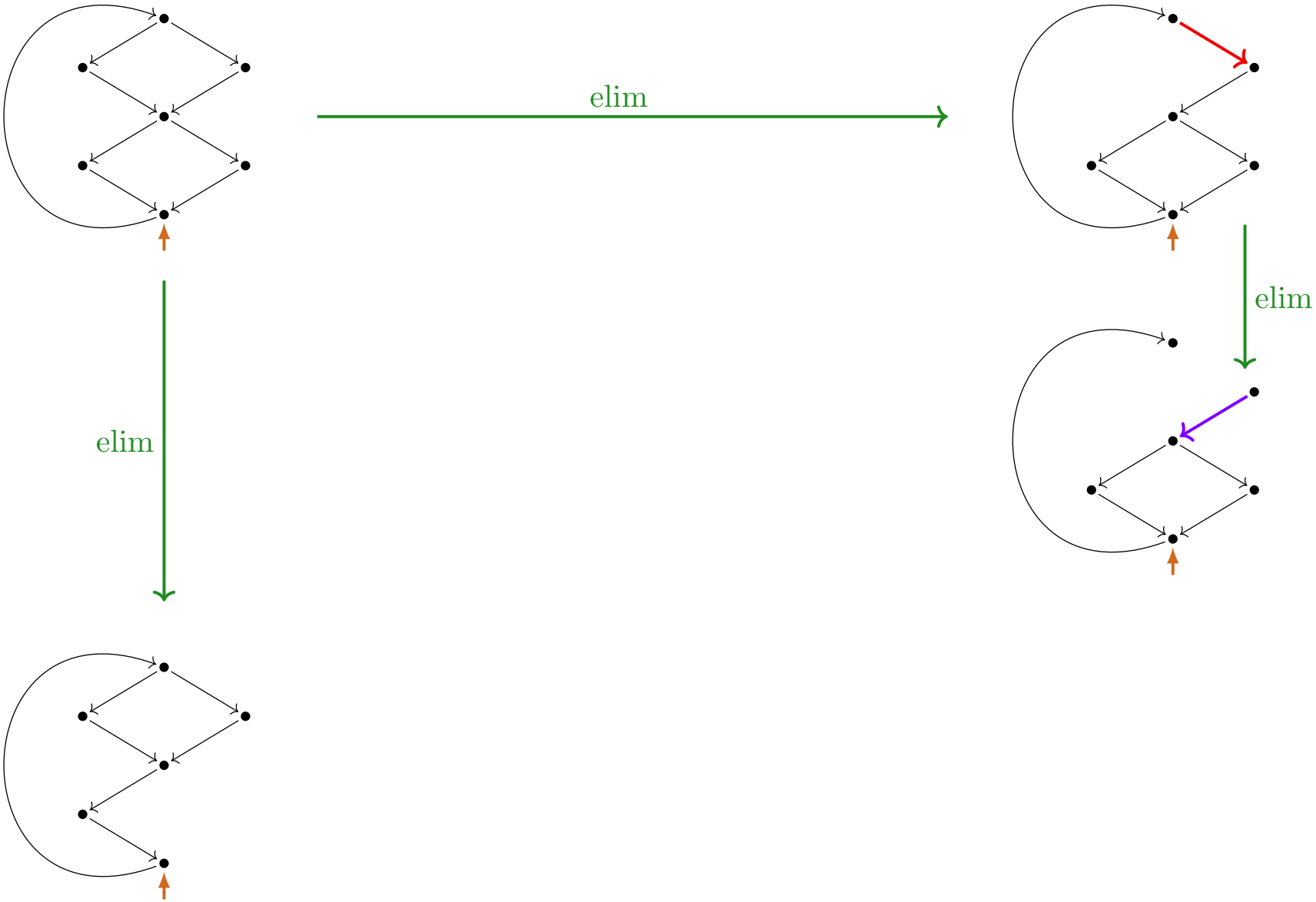

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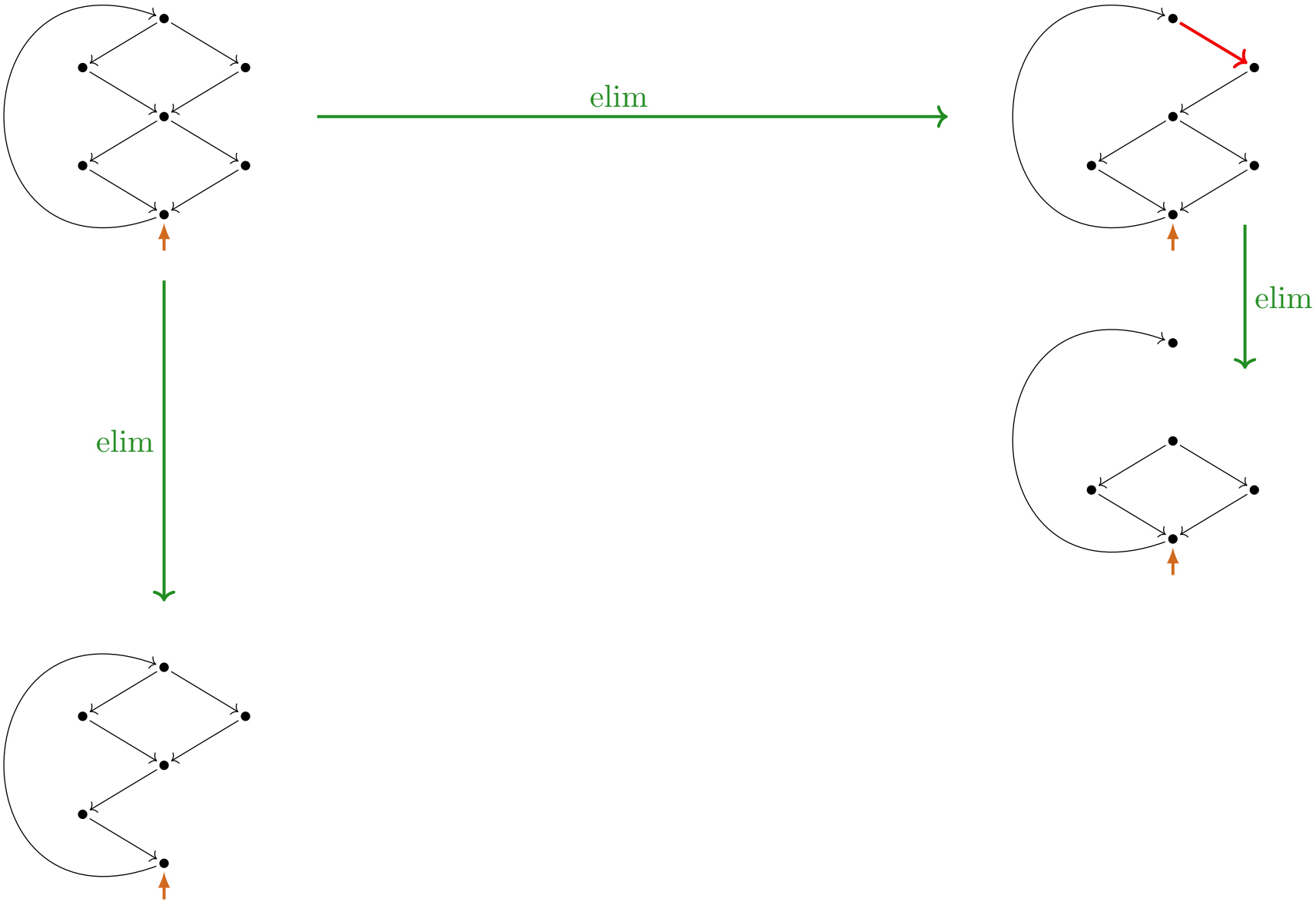

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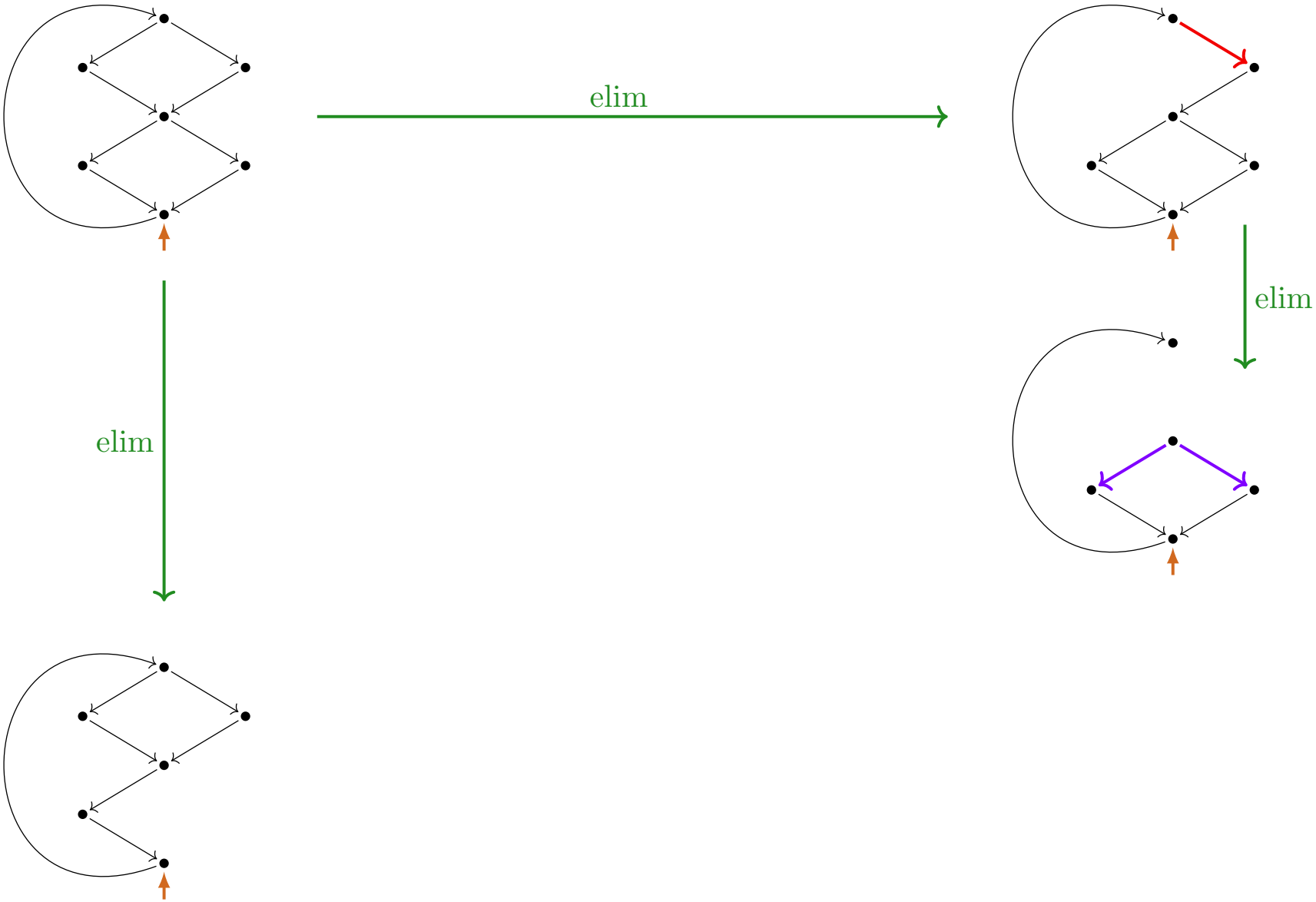

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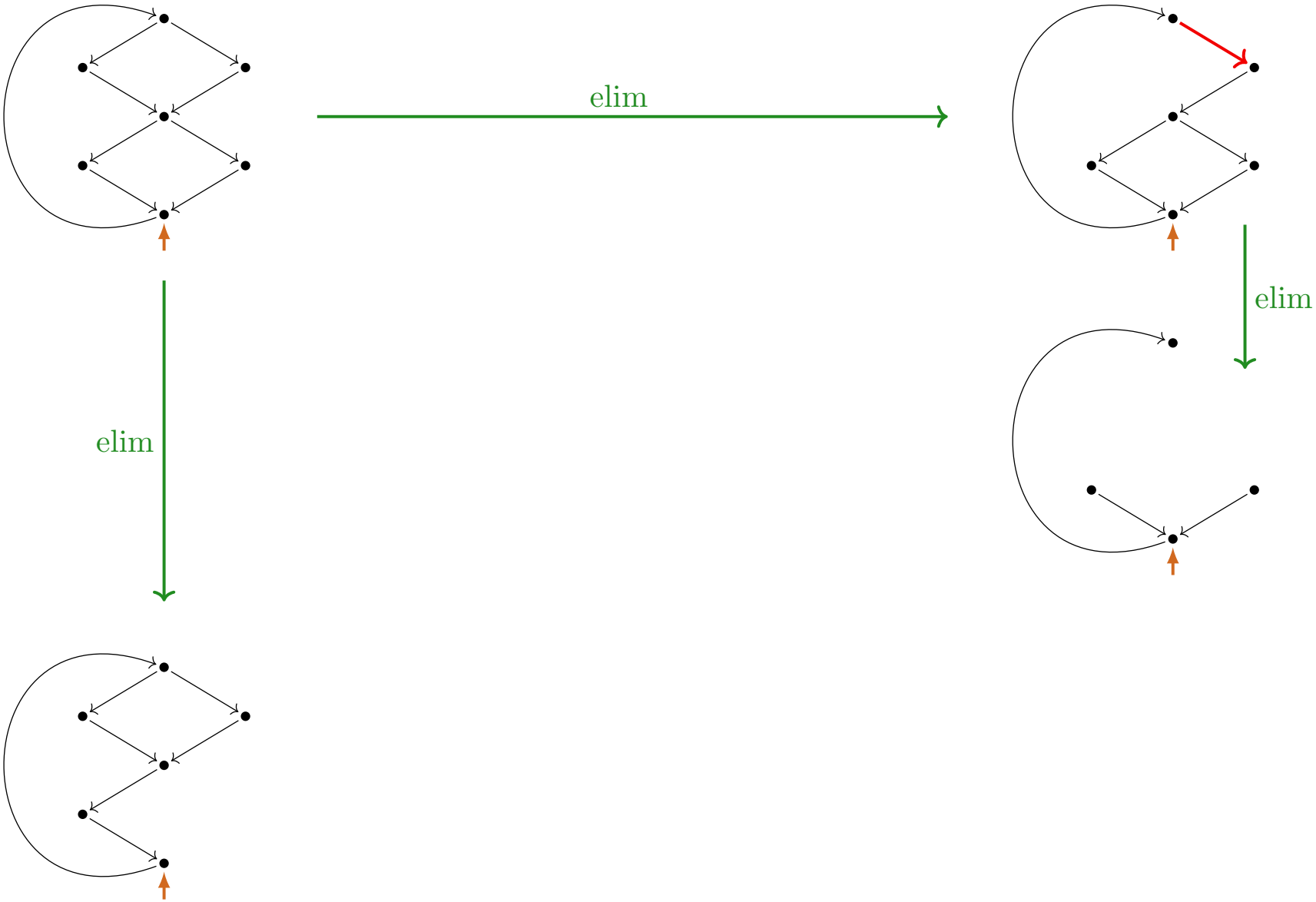

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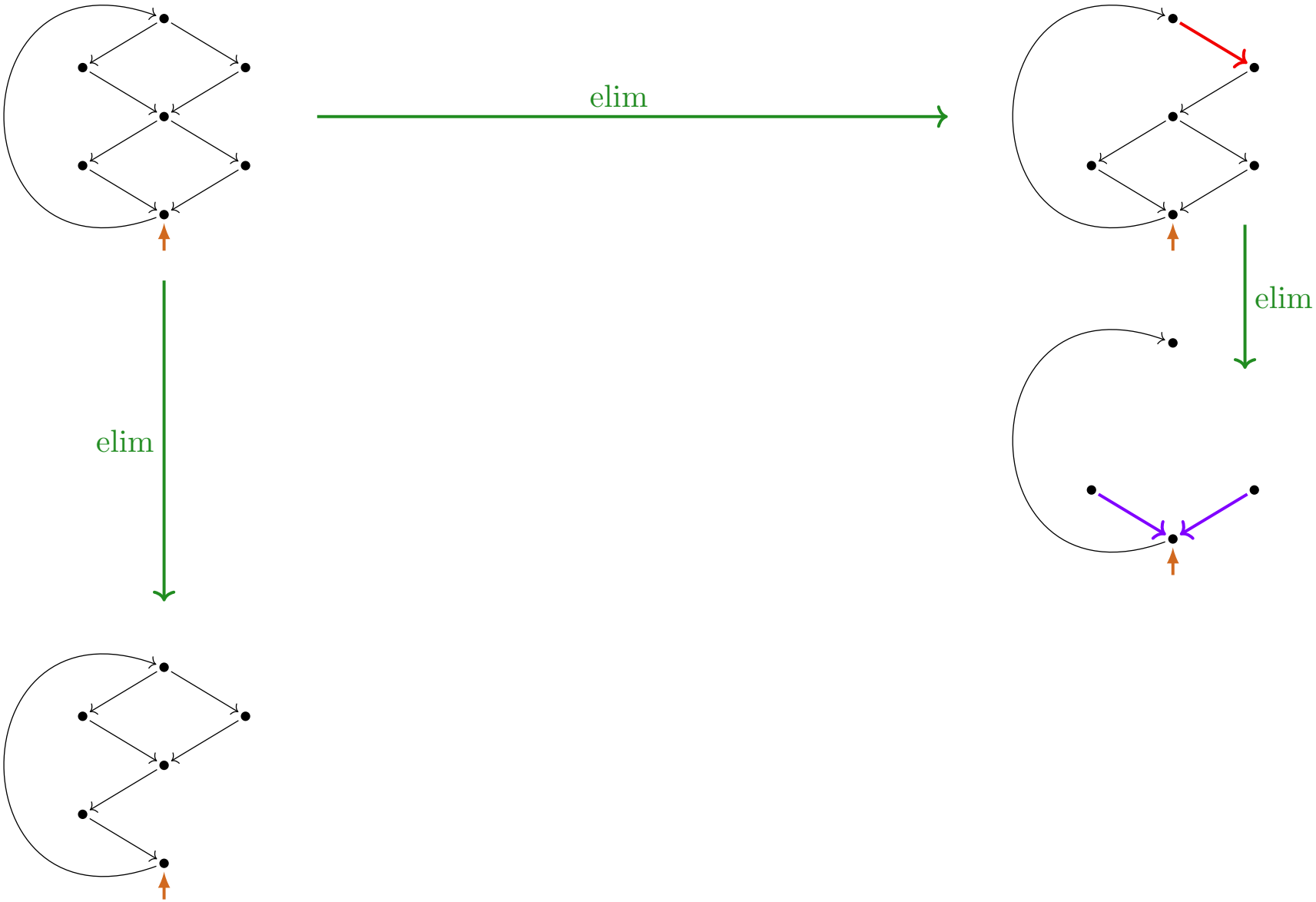

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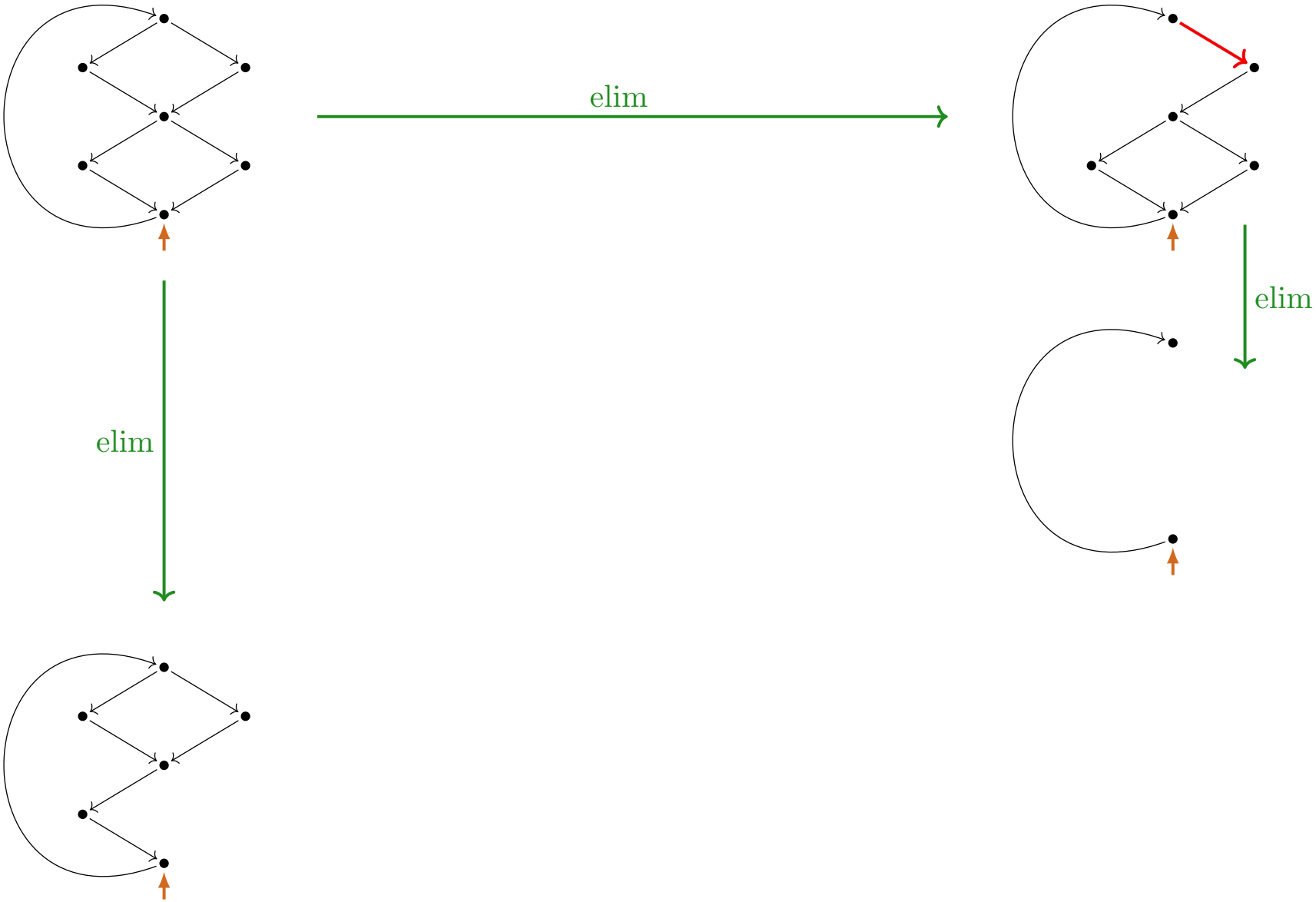

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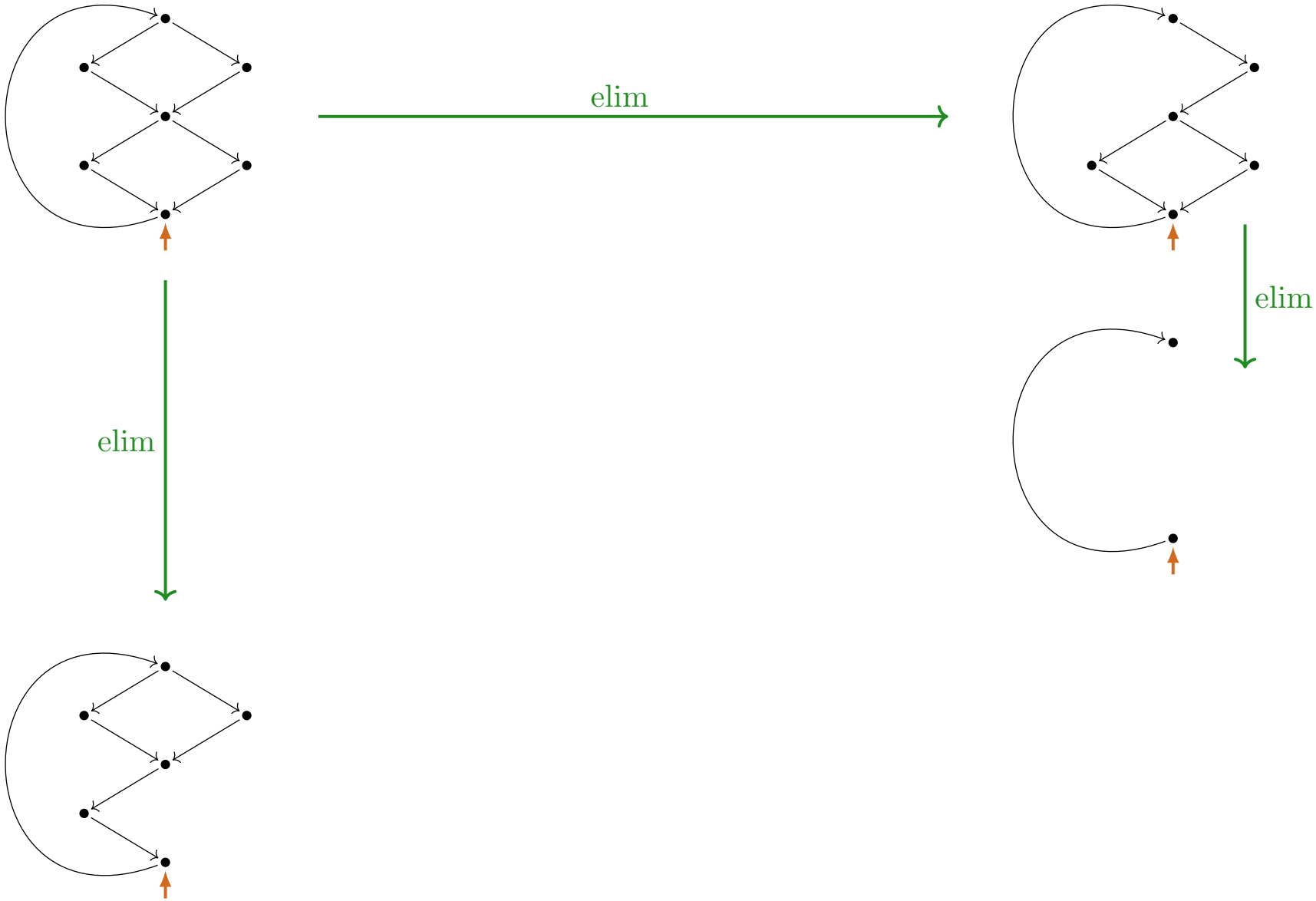

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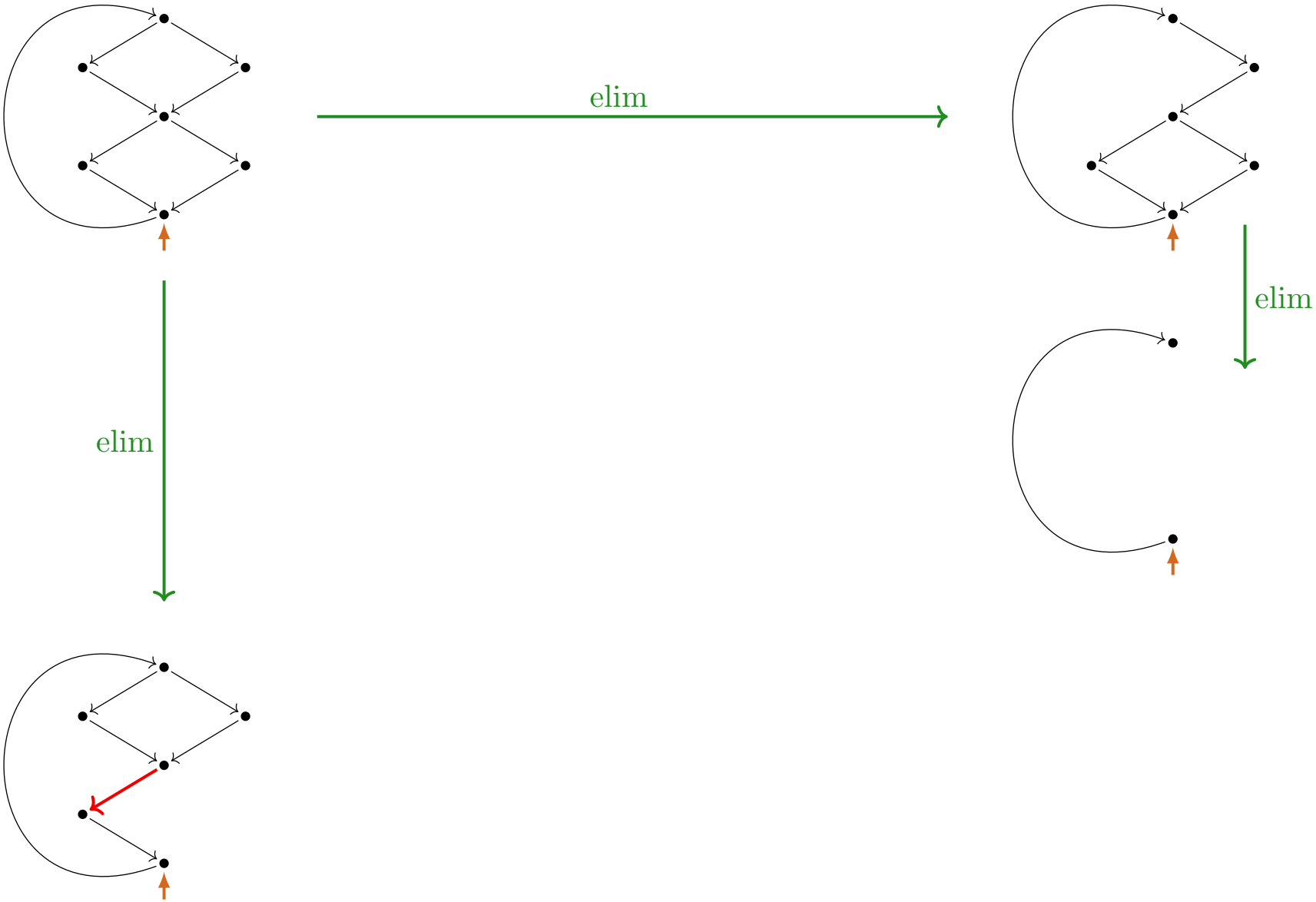

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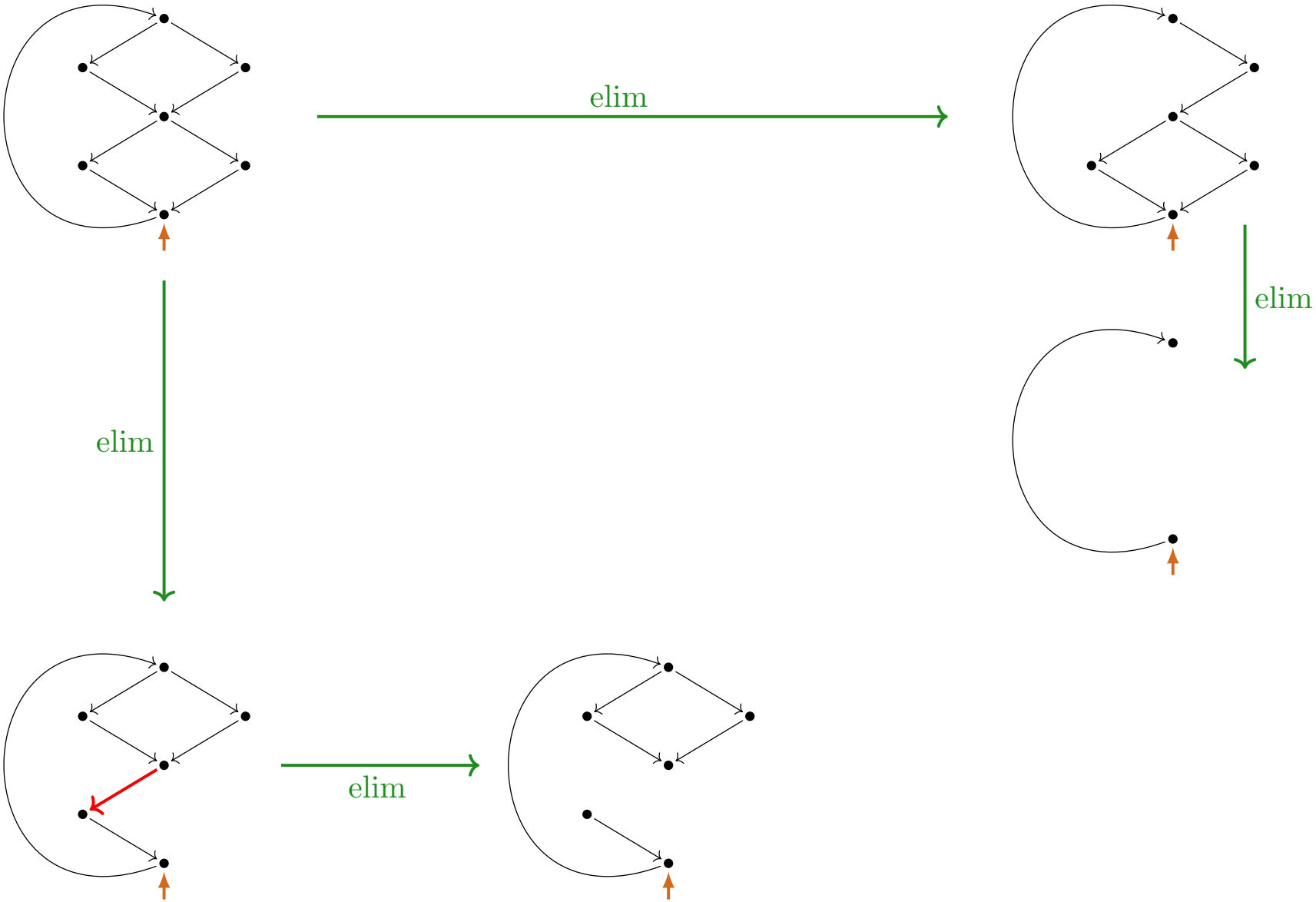

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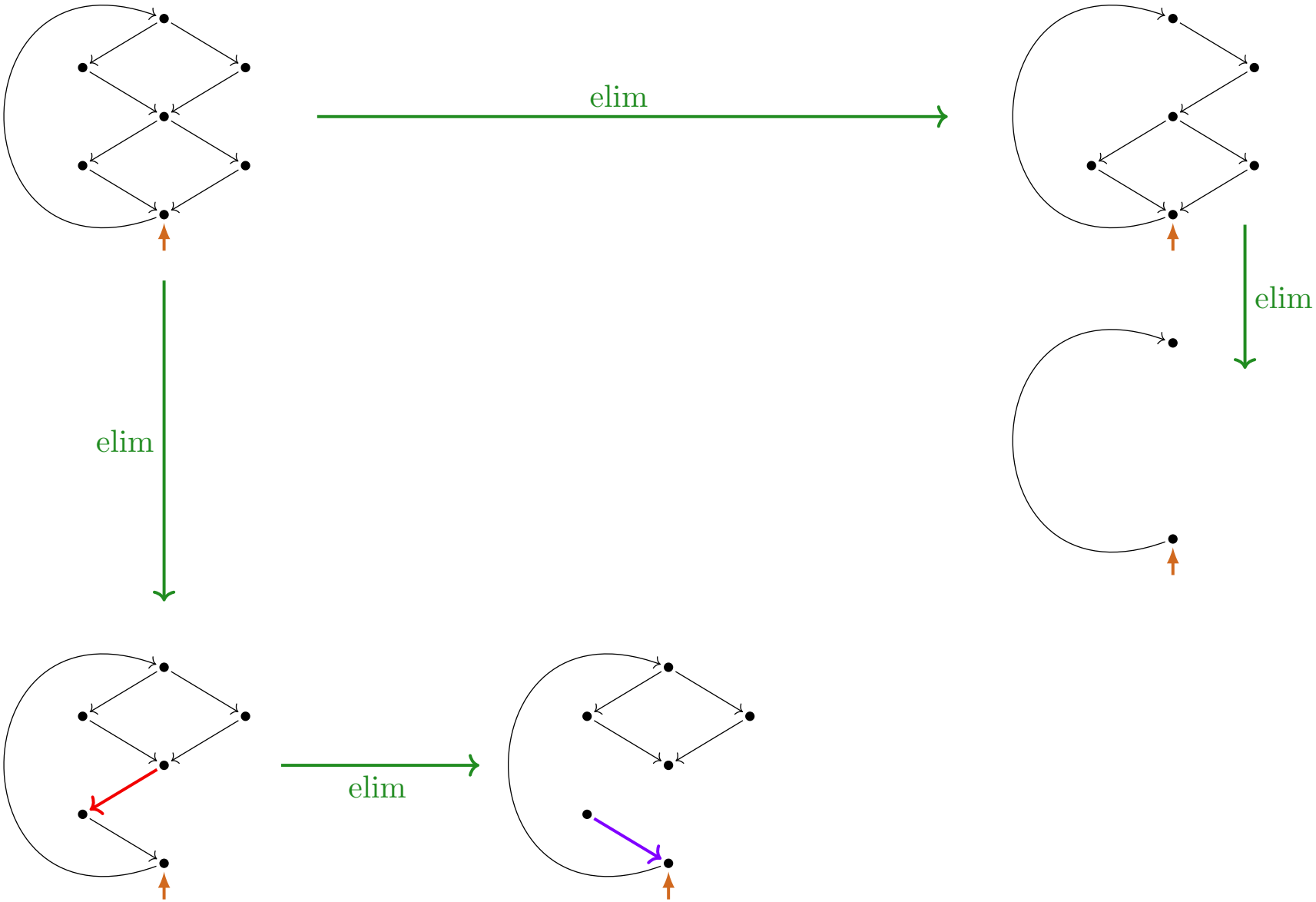

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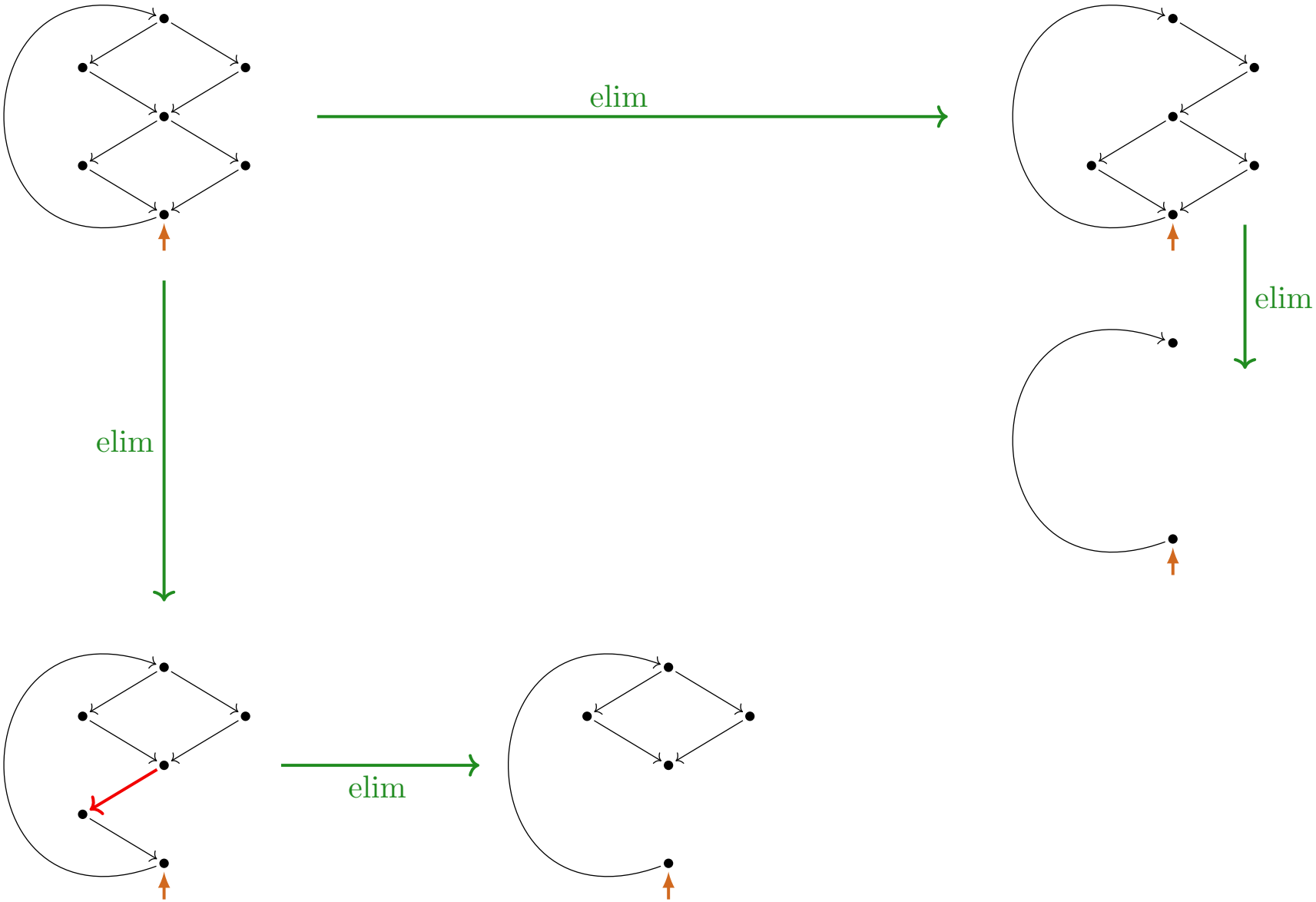

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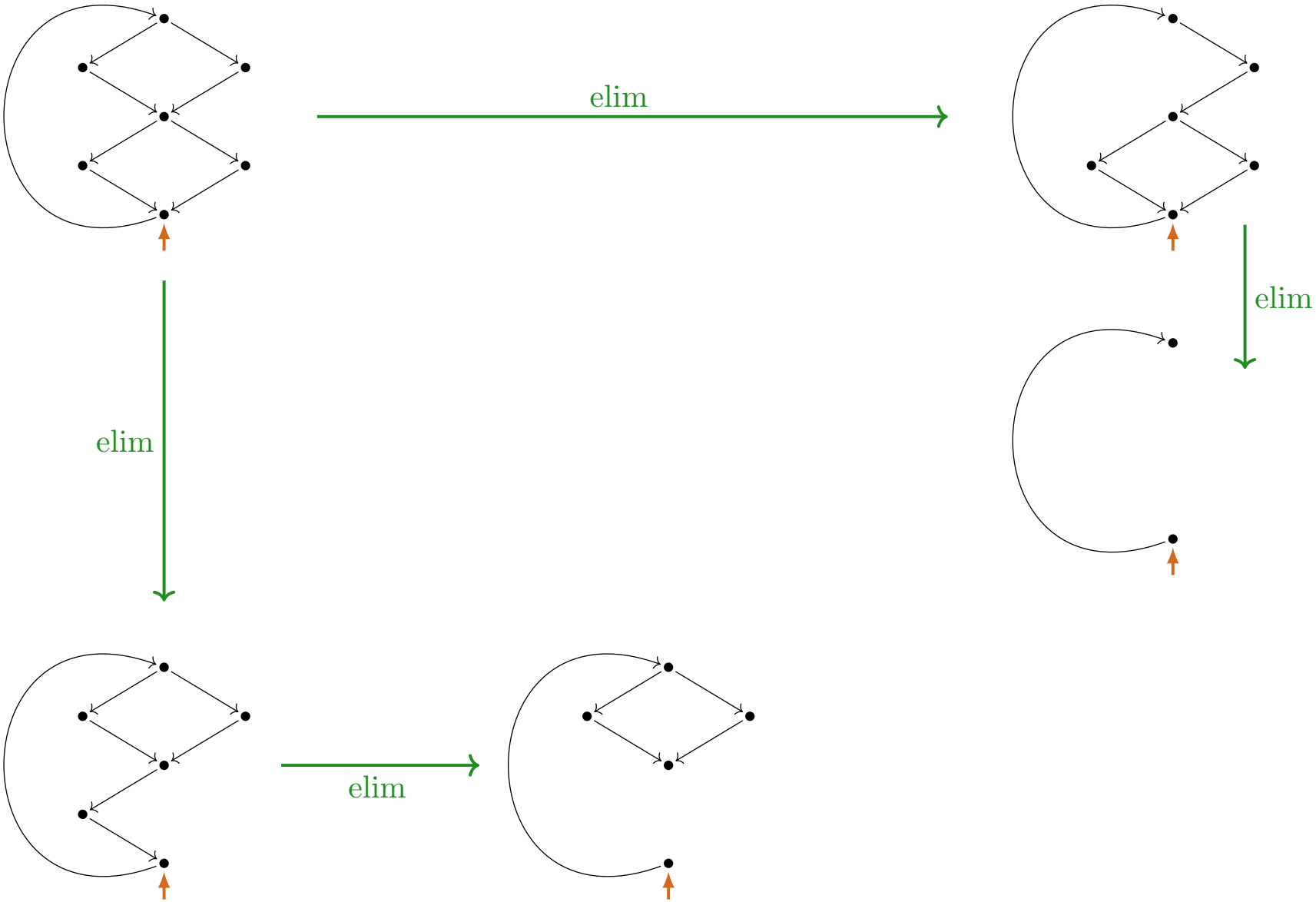

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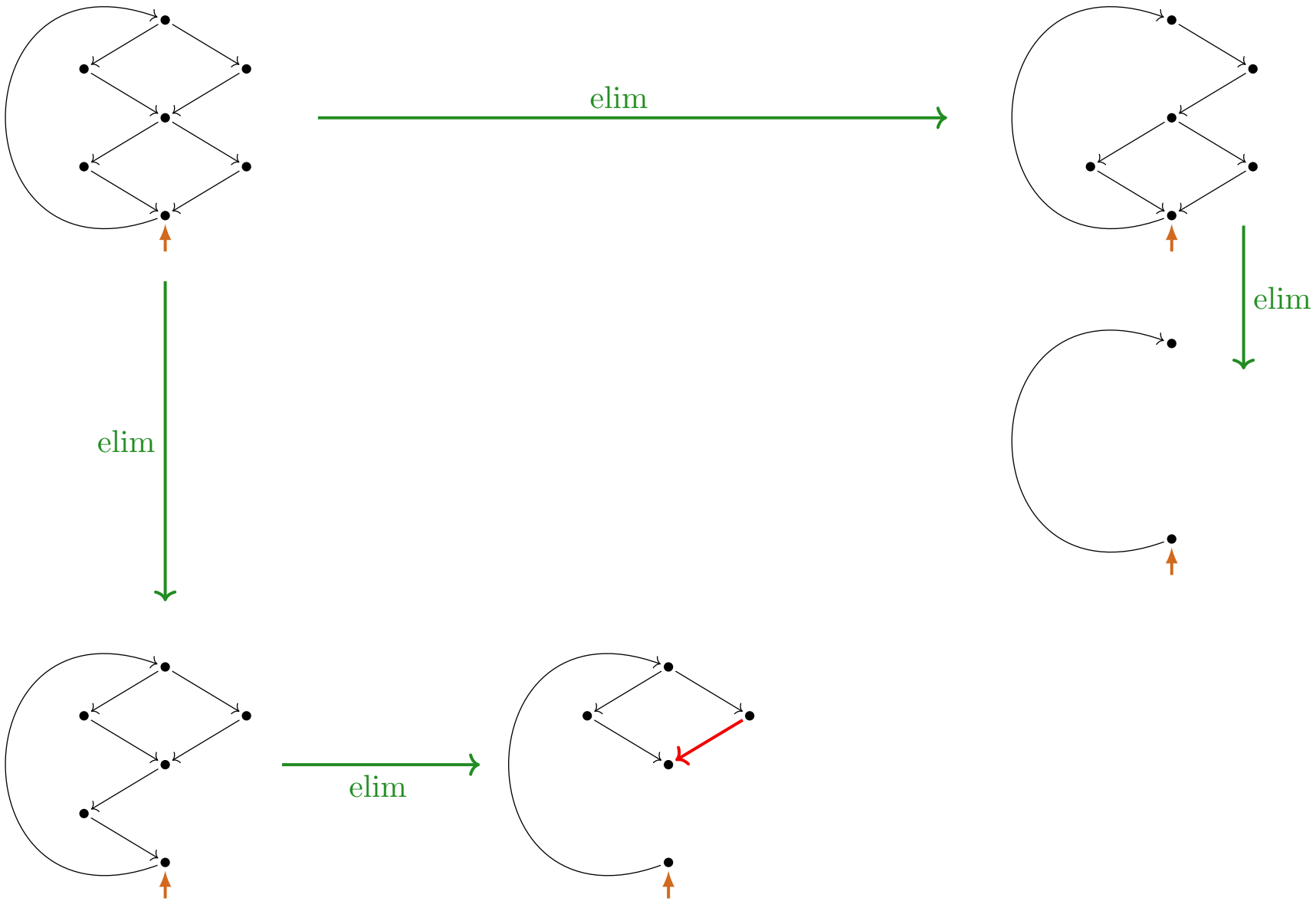

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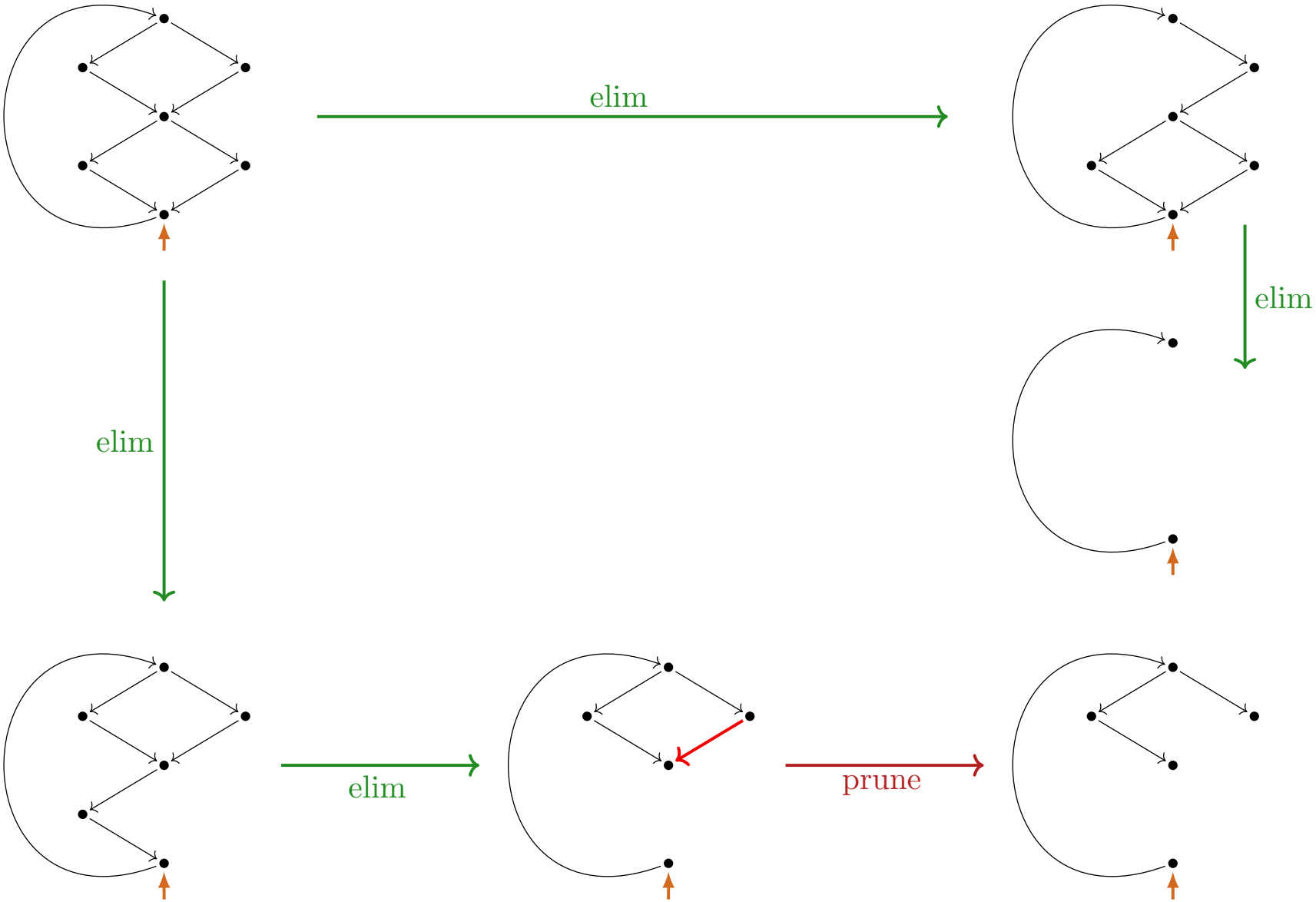

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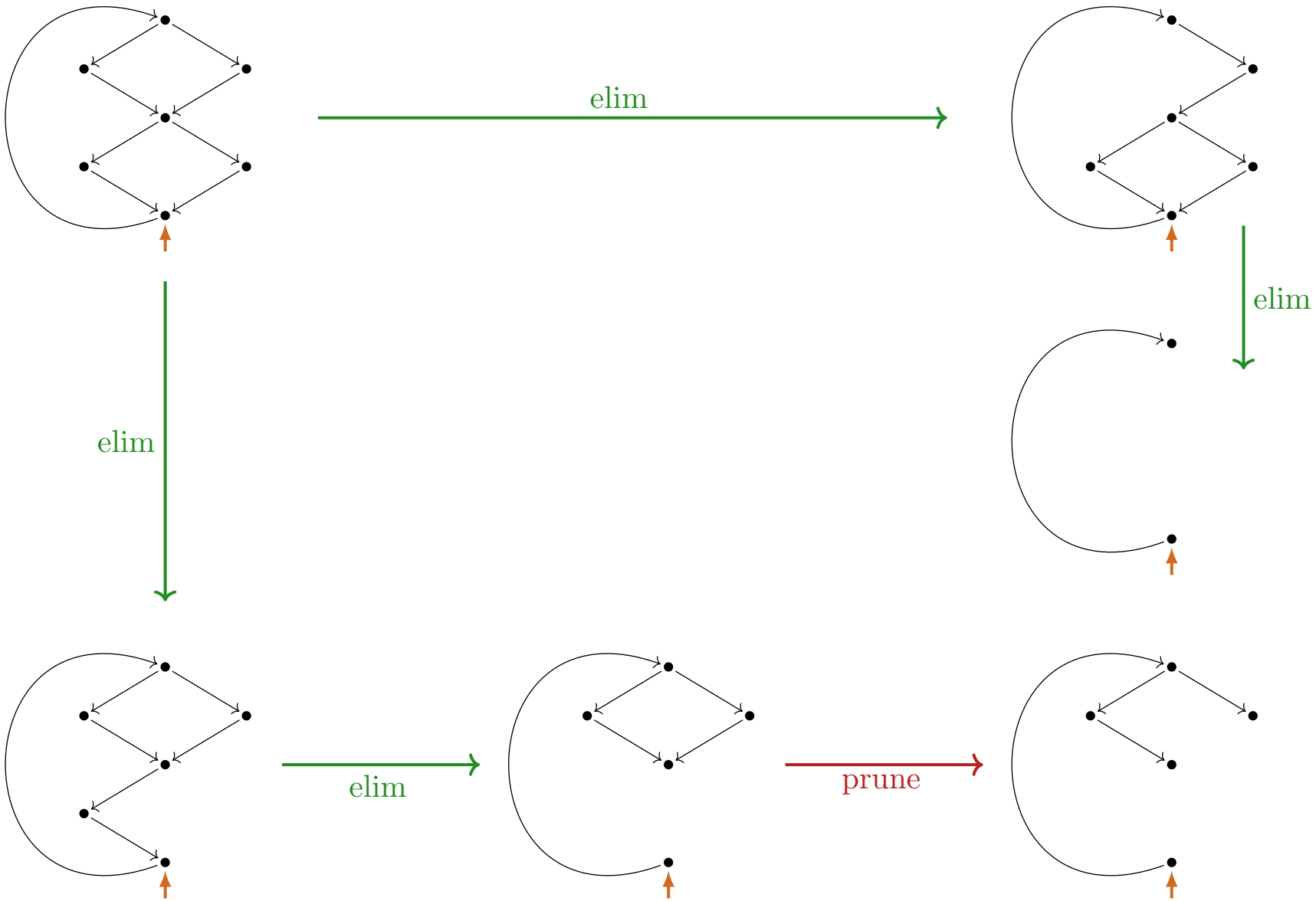

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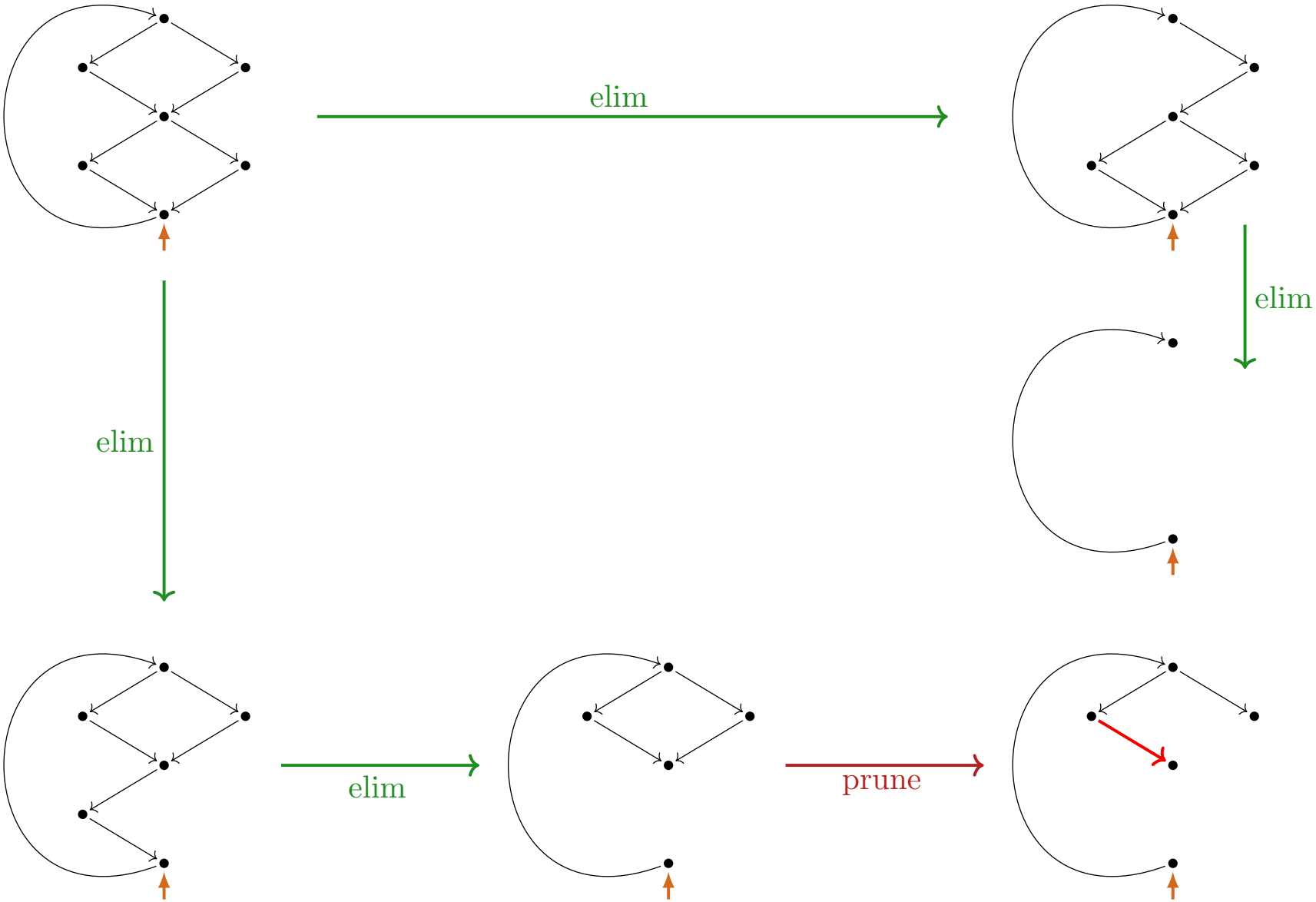

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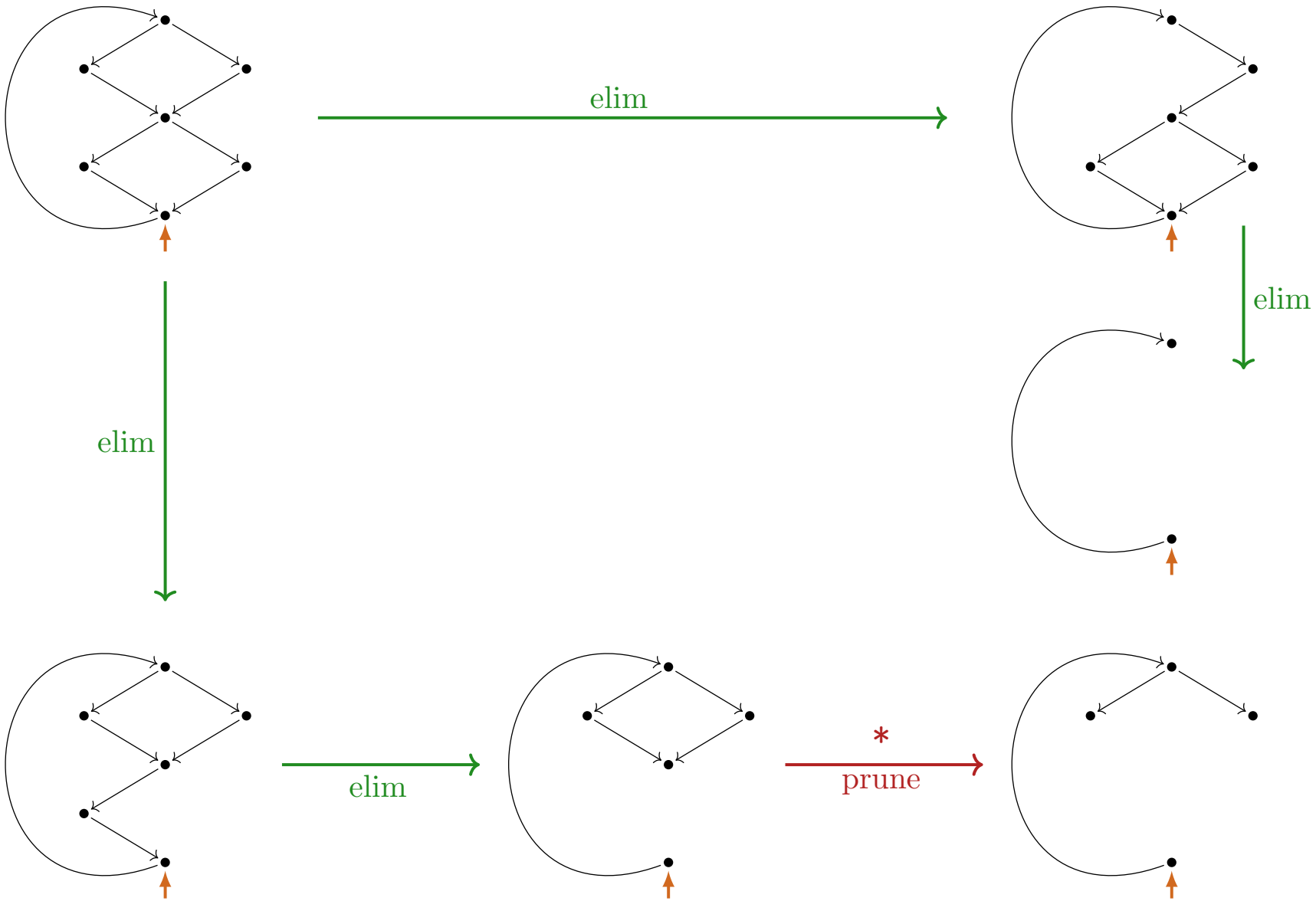

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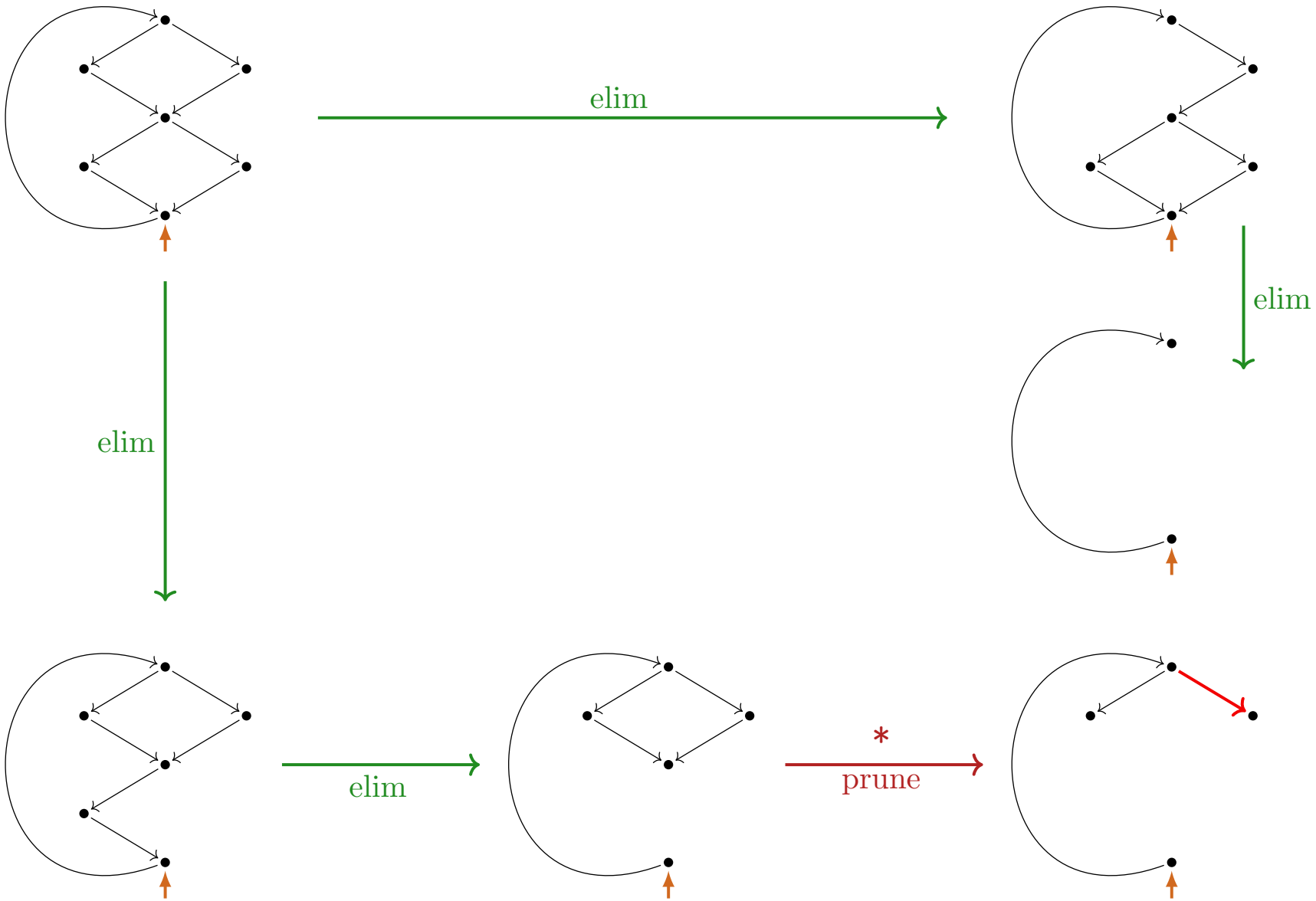

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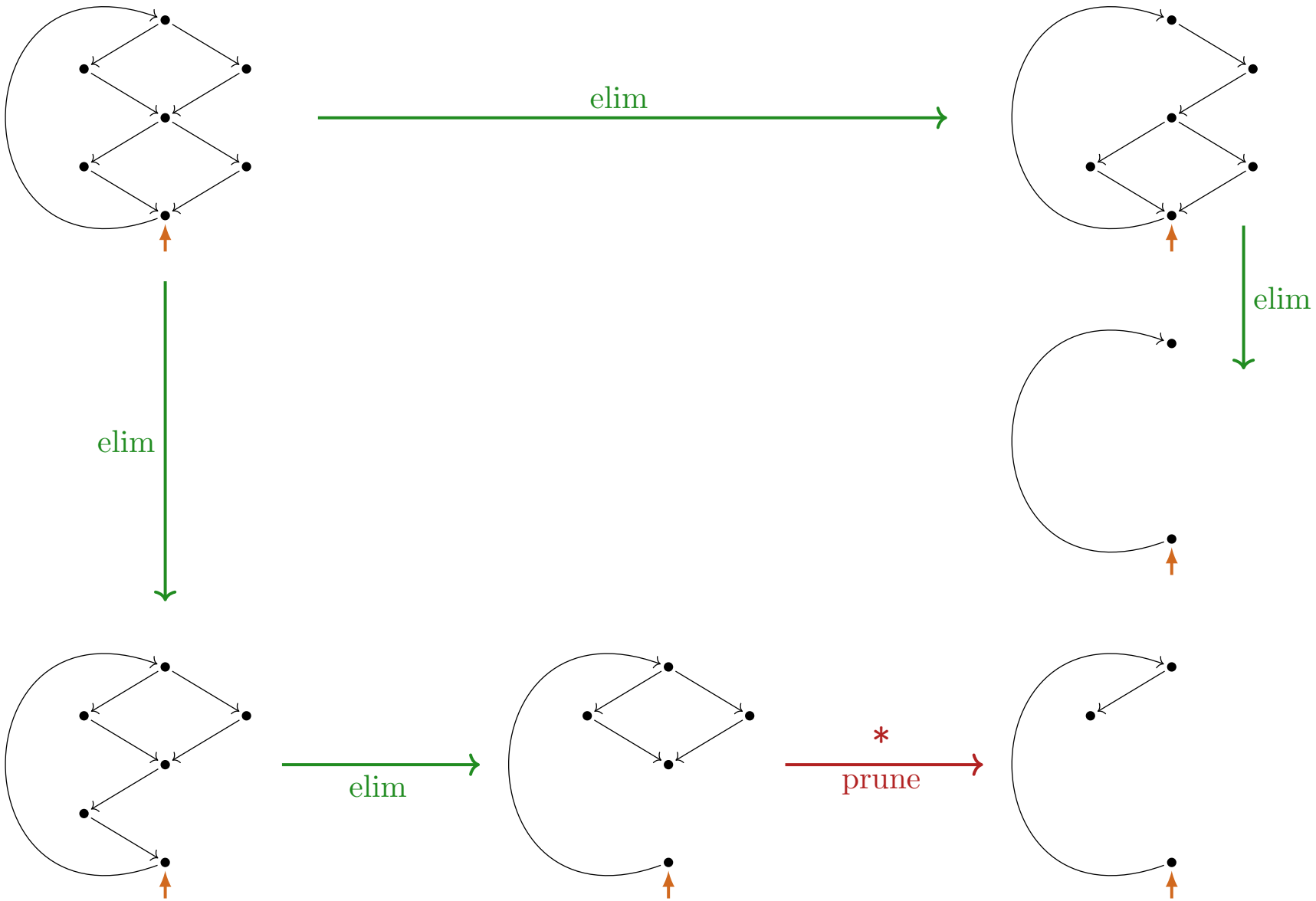

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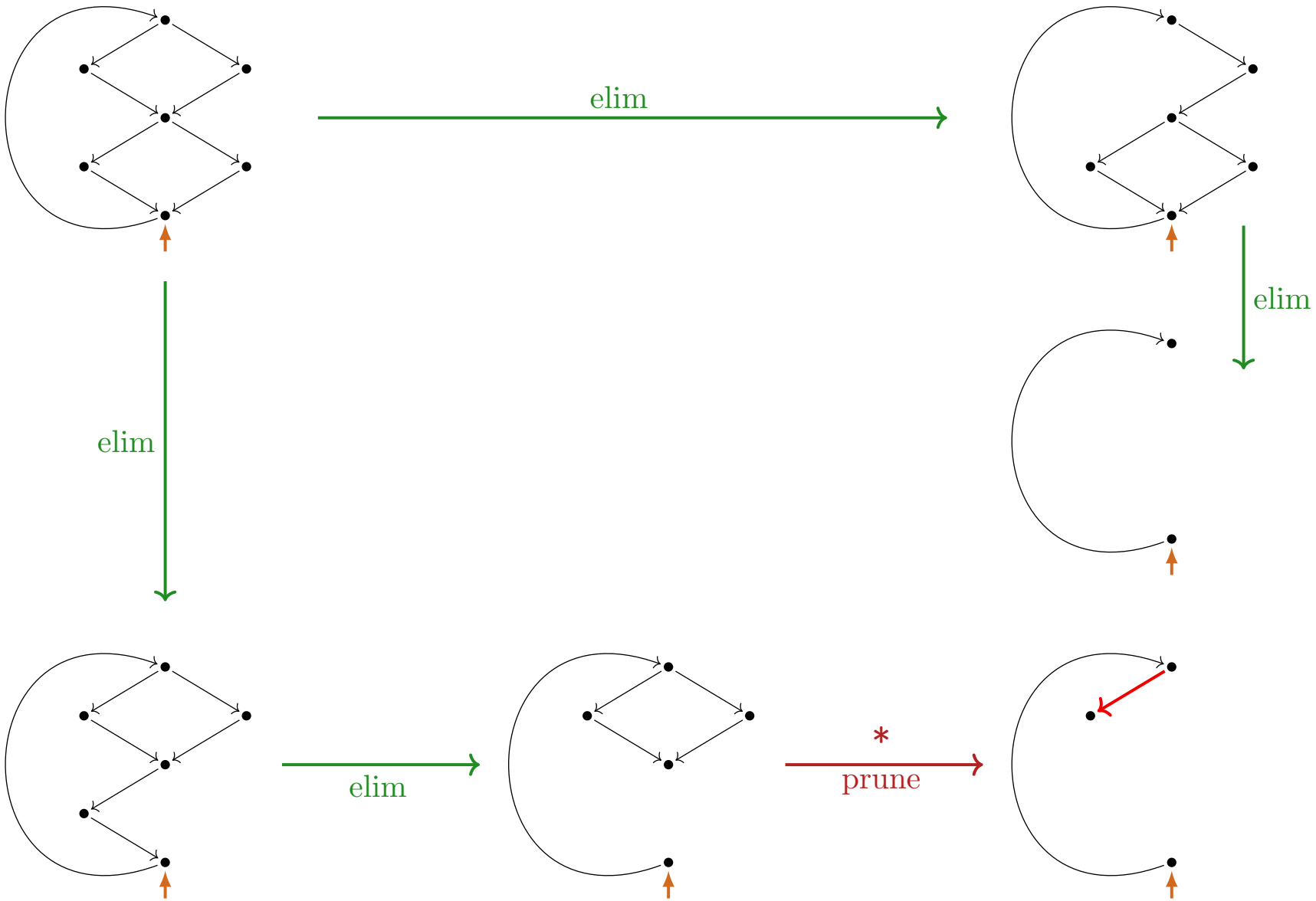

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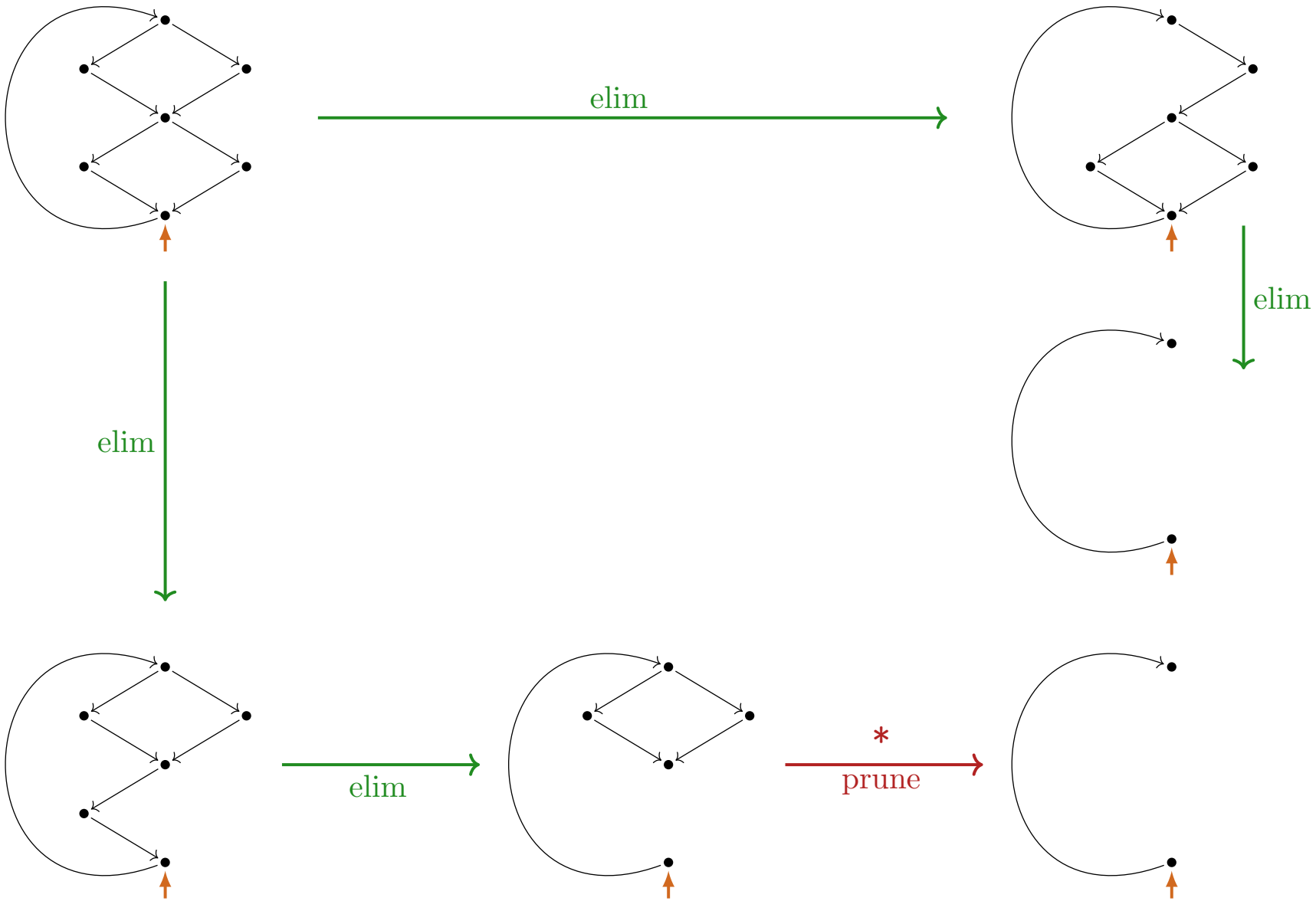

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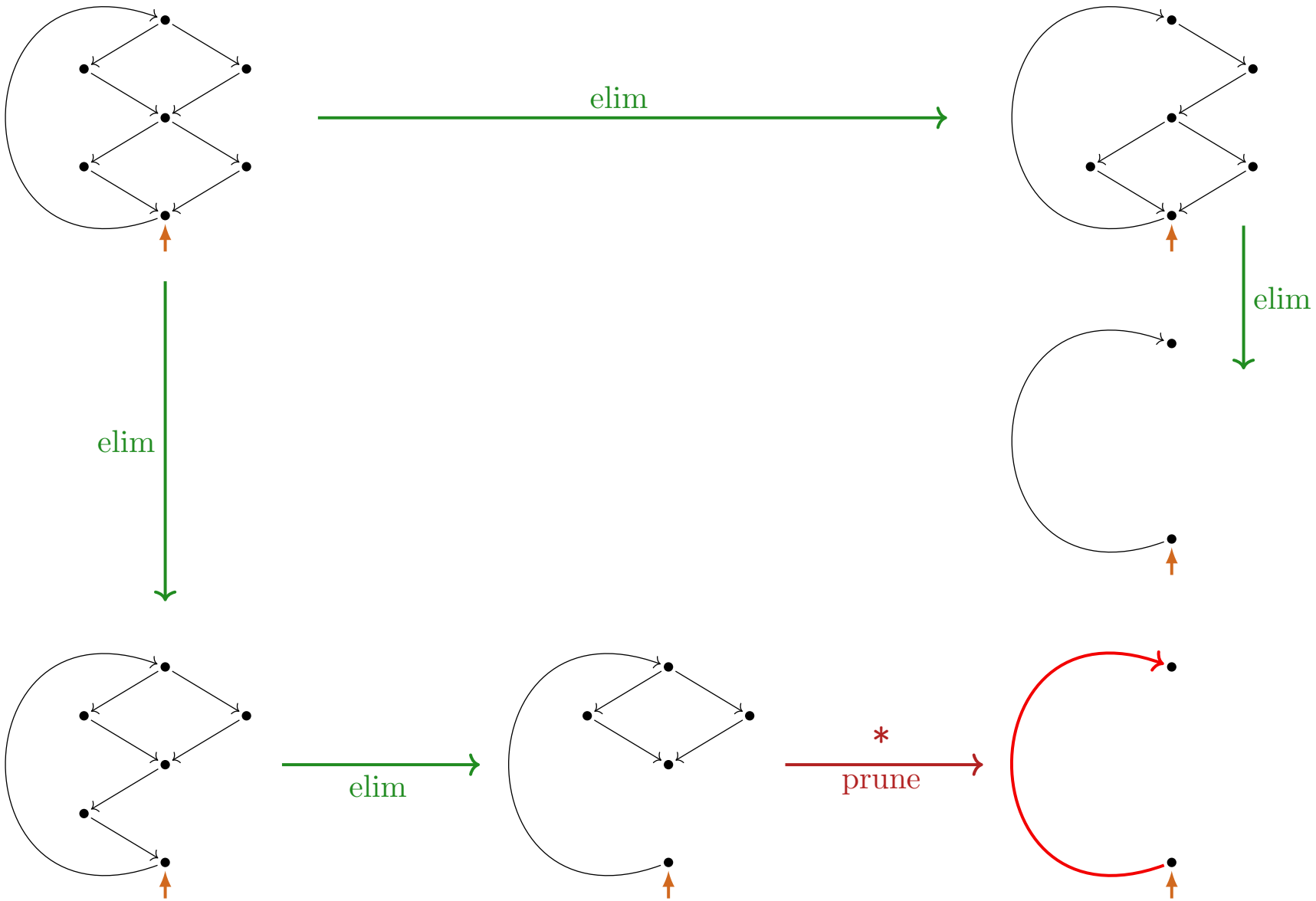

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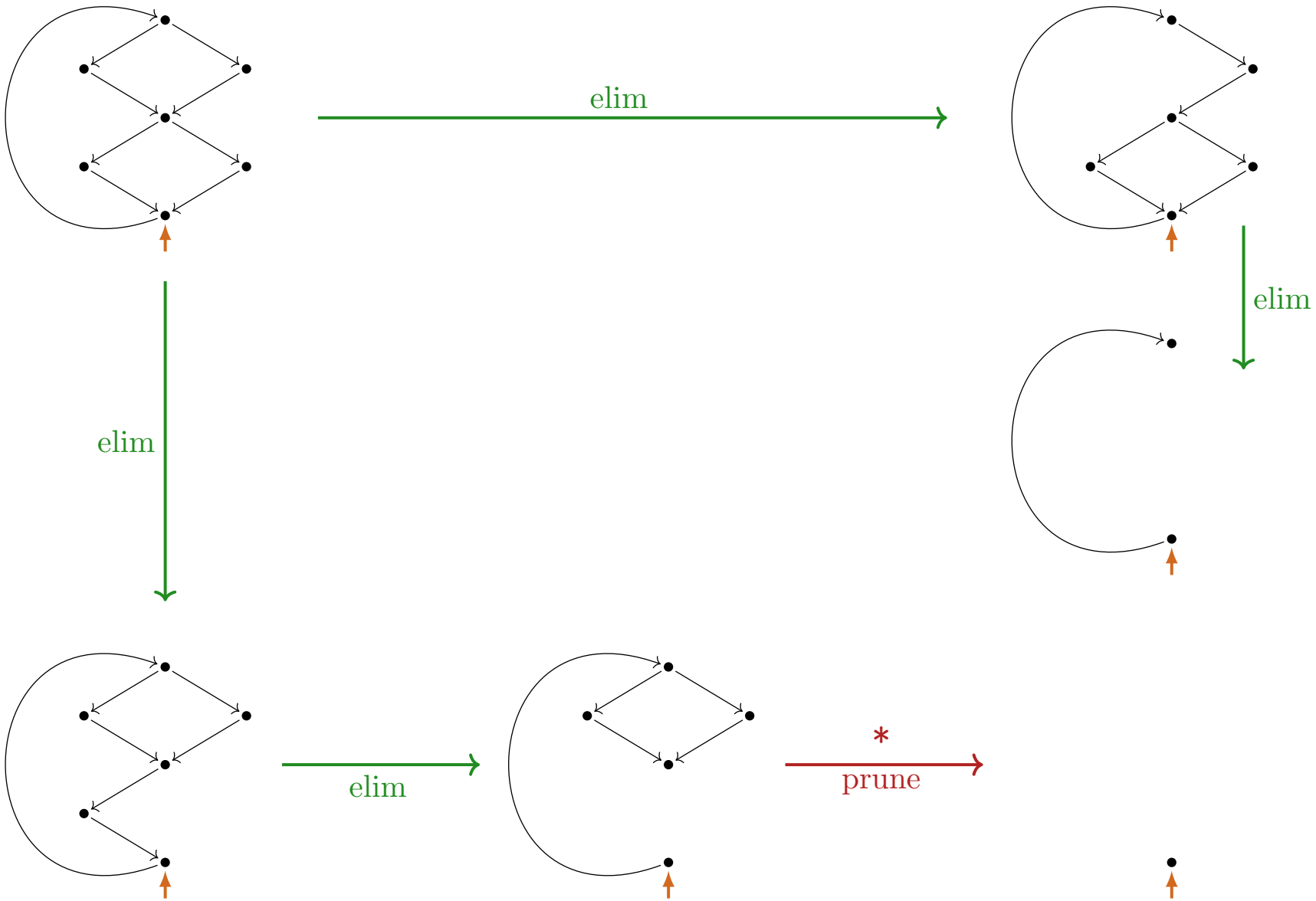

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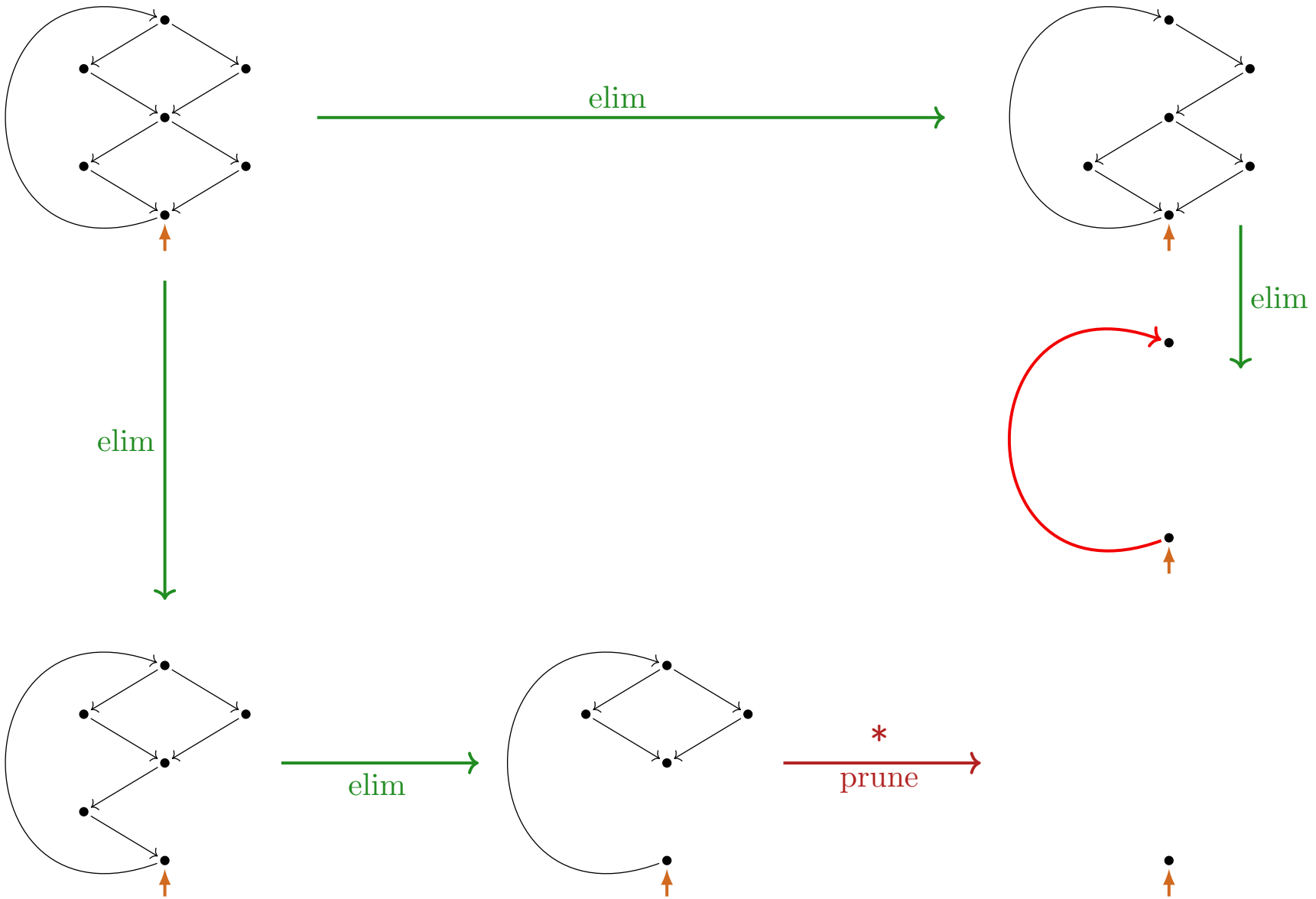

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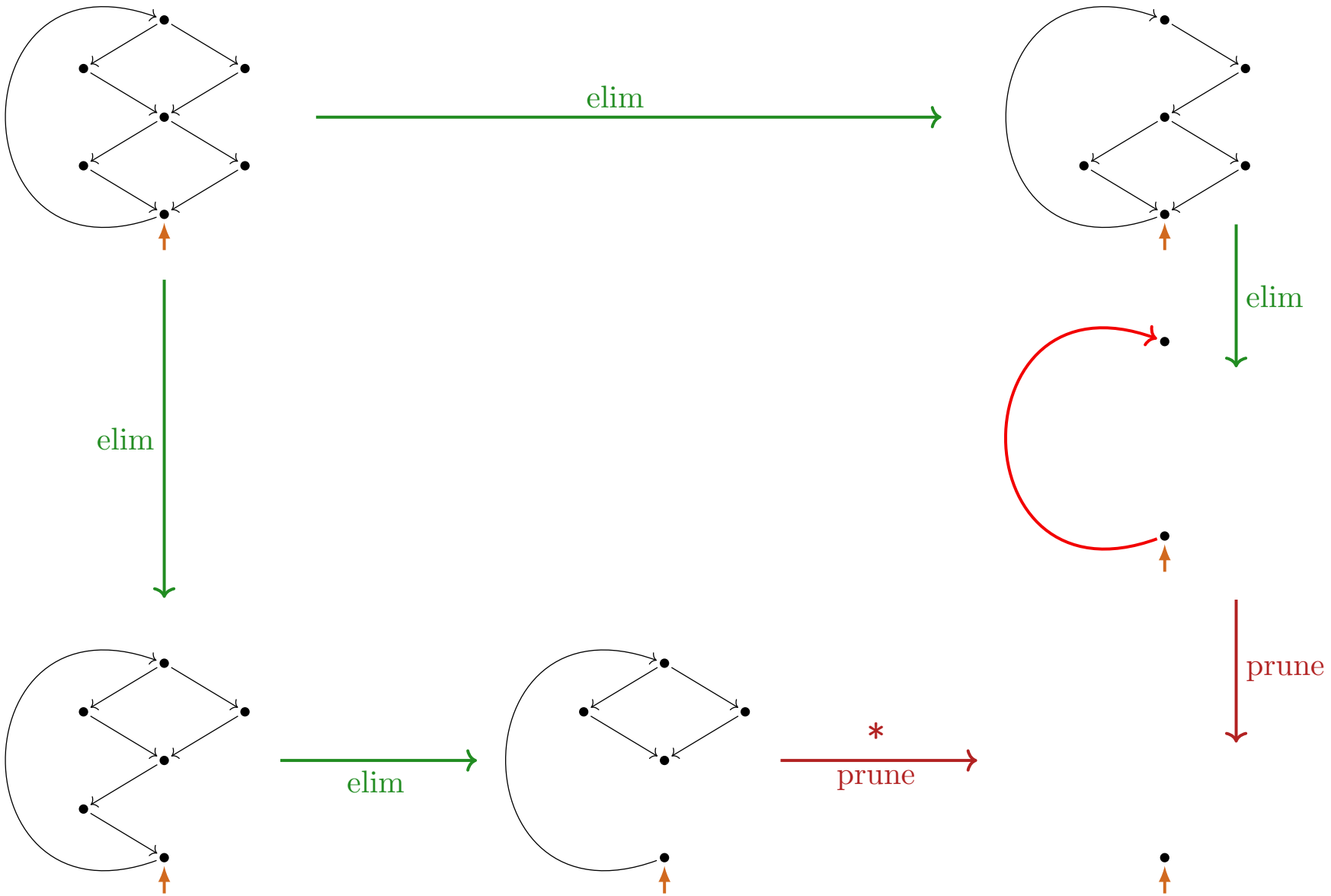
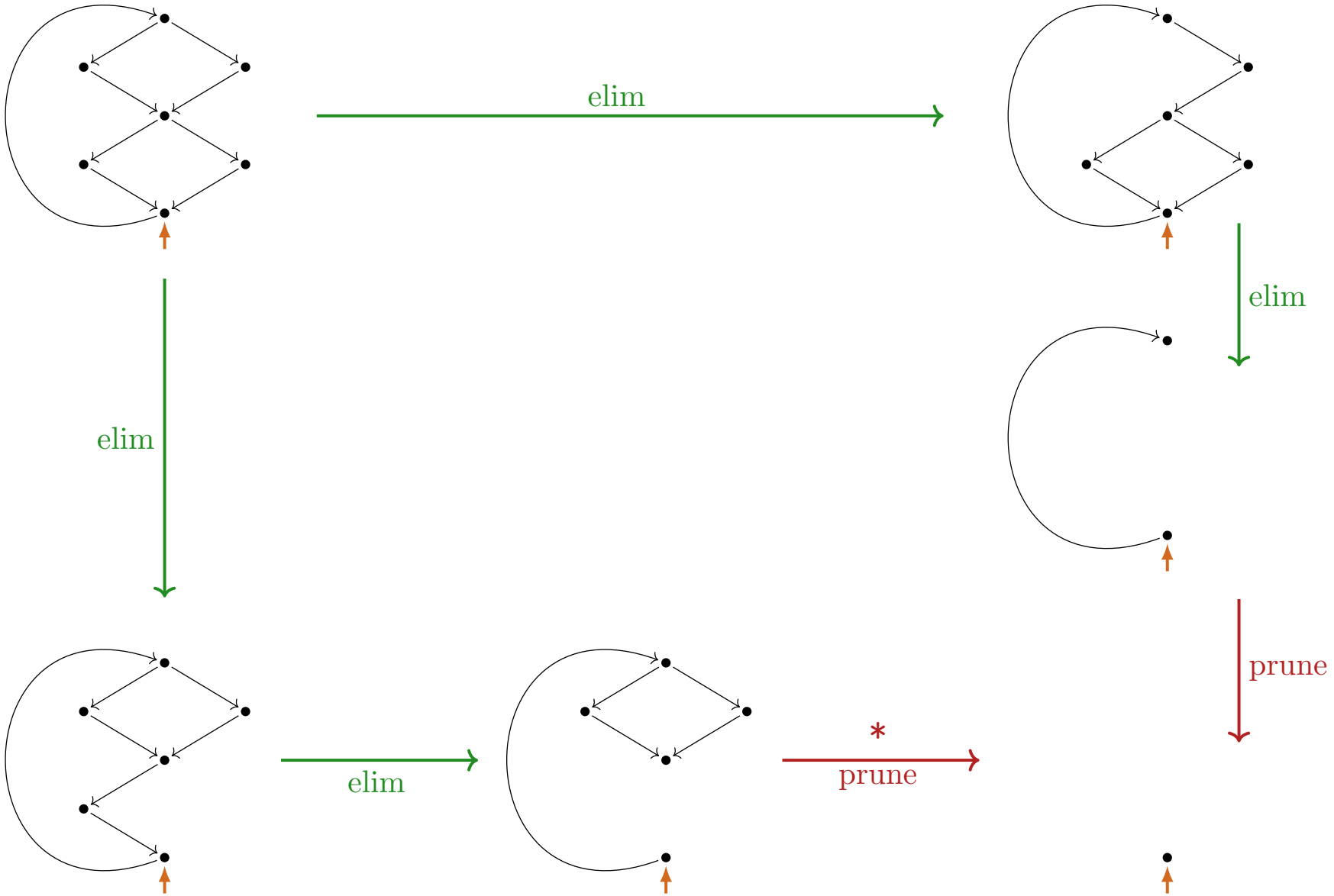


‘Critical pair’: bi-loop elimination



‘Critical pair’: bi-loop elimination