The Graph Structure of Process Interpretations of Regular Expressions

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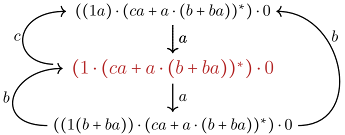

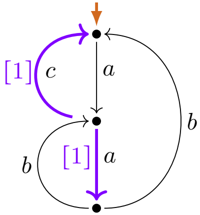

IFIP 1.6 Working Group Meeting

Nancy

July 1, 2024

Overview

- regular expressions (with unary/binary star, under-star-1-free $(*/\!\!\!\!1)$)
- Milner's process interpretation P/semantics $[\![\cdot]\!]_P$
  - P-/$[\![\cdot]\!]_P$-expressible graphs ($\rightsquigarrow$ expressibility question)
  - axioms for $[\![\cdot]\!]_P$-identity ($\rightsquigarrow$ completeness question)
- loop existence and elimination (LEE)
  - defined by loop elimination rewrite system, its completion
  - describes interpretations of $(*/\!\!\!\!1)$ reg. expr.s (extraction possible)
  - LEE-witnesses: labelings of process graphs with LEE
  - LEE is preserved under bisimulation collapse (stepwise collapse)
- 1-LEE = sharing via 1-transitions facilitates LEE
- LEE/1-LEE characterize image of $P^{\bullet}$ (restricted/unrestricted)
  - where $P^{\bullet}$ a compact (sharing-increased) refinement of P
- outlook on work-to-do

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  - LEE is preserved under bisimulation collapse (stepwise collapse)
- 1-LEE = sharing via 1-transitions facilitates LEE
  - describes interpretations of all reg. expr.s (extraction possible)
  - not preserved under bisimulation collapse (approximation possible)
- LEE/1-LEE characterize image of $P^{\bullet}$ (restricted/unrestricted)
  - where $P^{\bullet}$ a compact (sharing-increased) refinement of P
  - via refined extraction using LEE/1-LEE
- outlook on work-to-do

Regular Expressions

Definition (~ *Copi–Elgot–Wright, 1958* **)**

Regular expressions over alphabet A with unary Kleene star:

$$e, e_1, e_2 \;::=\; \mathbf{0} \;\mid\; \boldsymbol{a} \;\mid\; e_1 + e_2 \;\mid\; e_1 \cdot e_2 \;\mid\; e^{*} \qquad (\text{for } \boldsymbol{a} \in A).$$

- symbol $\mathbf{0}$ instead of $\varnothing$, symbol $\mathbf{1}$ instead of $\{\epsilon\}$

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Regular Expressions

Definition (~ *Kleene, 1951*, ~ *Copi–Elgot–Wright, 1958*)

Regular expressions over alphabet A with unary / binary Kleene star:

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$$e, e_1, e_2 ::= \mathbf{0} \mid \mathbf{1} \mid \boldsymbol{a} \mid e_1 + e_2 \mid e_1 \cdot e_2 \mid e_1{}^{\circledast} e_2 \qquad \text{(for } \boldsymbol{a} \in A\text{).}$$

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Regular Expressions (1-free)

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Definition (for process interpretation)

1-free regular expressions over alphabet A with binary Kleene star:

$$f, f_1, f_2 \;::=\; \mathbf{0} \mid \boldsymbol{a} \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1{}^{\circledast} f_2 \qquad (\text{for } \boldsymbol{a} \in A).$$

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Regular Expressions (under-star-/1-free)

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Definition (for process interpretation)

The set $\mathit{RExp}^{(\cancel{1})}(A)$ of 1-free regular expressions over A is defined by:

$$f, f_1, f_2 \;::=\; \mathbf{0} \mid \boldsymbol{a} \mid f_1 + f_2 \mid f_1 \cdot f_2 \mid f_1{}^{\star} \cdot f_2 \qquad \text{(for } \boldsymbol{a} \in A\text{)},$$

the set $\mathit{RExp}^{(*/\cancel{1})}(A)$ of under-star-1-free regular expressions over A by:

$$uf, uf_1, uf_2 \;::=\; \mathbf{0} \mid \mathbf{1} \mid \boldsymbol{a} \mid uf_1 + uf_2 \mid uf_1 \cdot uf_2 \mid f^{\star} \qquad \text{(for } \boldsymbol{a} \in A\text{)}.$$

Process interpretation P of regular expressions *(Milner, 1984)*

$\mathbf{0} \xmapsto{P}$ deadlock $\boldsymbol{\delta}$, no termination

$\mathbf{1} \xmapsto{P}$ empty-step process $\boldsymbol{\epsilon}$, then terminate

$a \xmapsto{P}$ atomic action a, then terminate

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$e_1 + e_2 \xmapsto{P}$ *(choice)* execute $P(e_1)$ or $P(e_2)$

$e_1 \cdot e_2 \xmapsto{P}$ *(sequentialization)* execute $P(e_1)$, then $P(e_2)$

$e^* \xmapsto{P}$ *(iteration)* repeat (terminate or execute $P(e)$)

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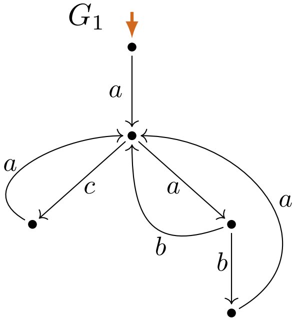
$e^* \xmapsto{P}$ (*iteration*) repeat (terminate or execute $P(e)$)

$[\![e]\!]_P \;:=\; [P(e)]_{\underline{\leftrightarrow}}$ (bisimilarity equivalence class of process $P(e)$)

P-expressibility and $[\![\cdot]\!]_P$-expressibility (example, informally)



$$P\big(\overbrace{(a\cdot(c\cdot a+a\cdot(b+b\cdot a))^*)\cdot 0}^{f}\big)$$

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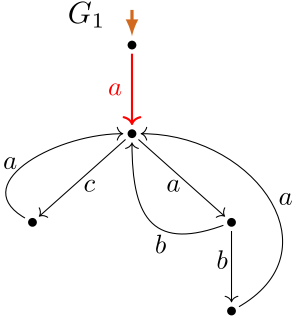

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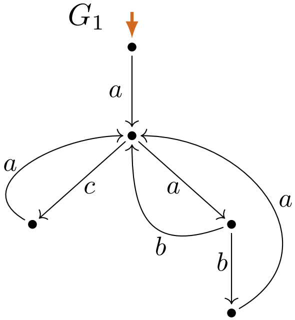

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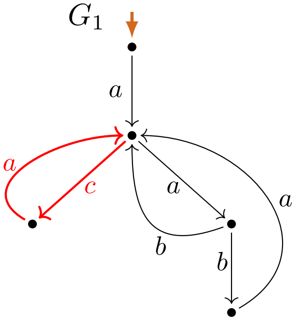

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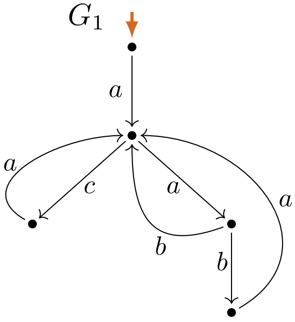

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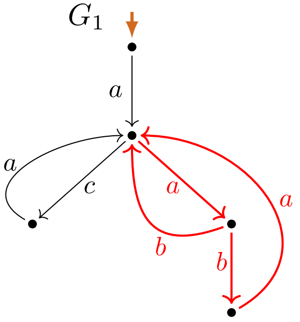

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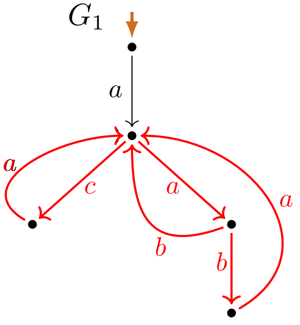

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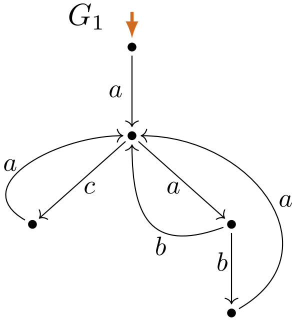

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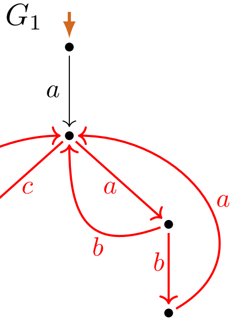

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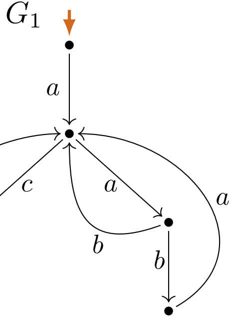

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$$G_1 \in [\![f]\!]_P$$

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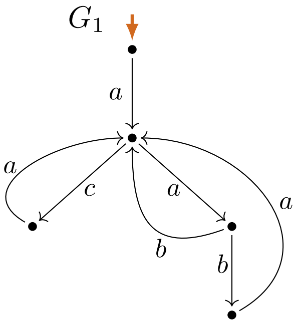

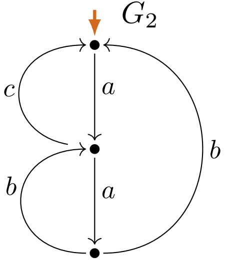

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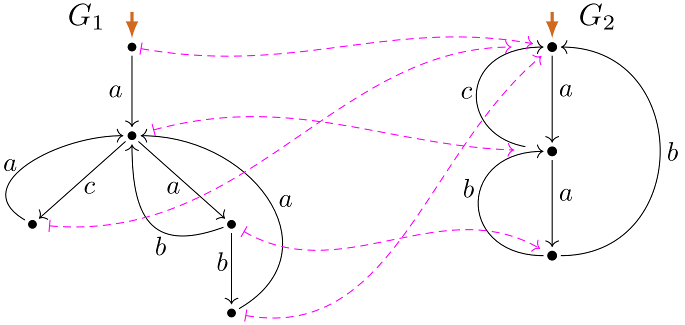

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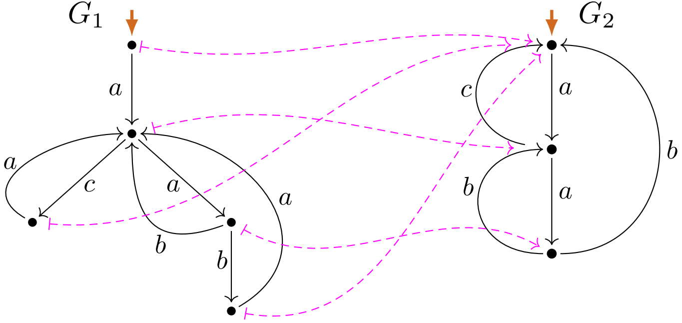

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$$G_2 \in [\![f]\!]_P$$

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Process interpretation P (formally)

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{a \xrightarrow{a} 1} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'}\ (i \in \{1,2\})$$

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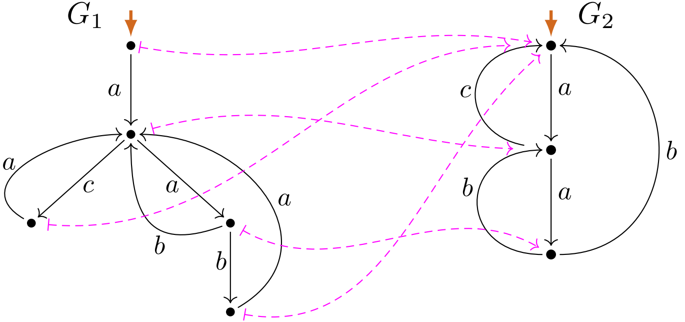
Definition

The process (graph) interpretation $P(e)$ of a regular expression e:

$P(e)$:= labeled transition graph generated by e by derivations in $\mathcal{T}$.

P-expressibility and $[\![\cdot]\!]_P$-expressibility (example, informally)



$$P\Big(\overbrace{a\cdot((c\cdot a+a\cdot(b+b\cdot a)^*)\cdot 0)}^{f}\Big)$$

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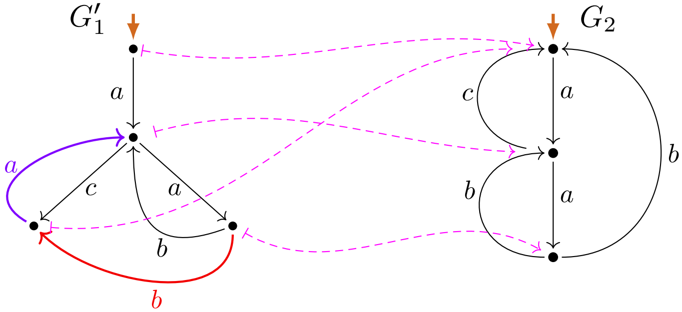
P-expressibility and $[\![\cdot]\!]_P$-expressibility (example, formally)



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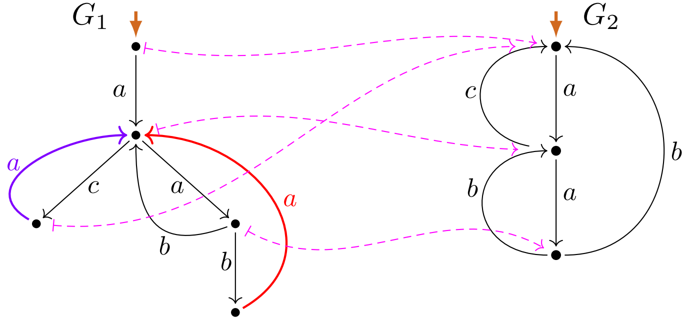

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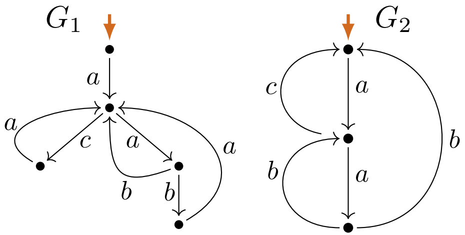
P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



P-expressible

$[\![\cdot]\!]_P$-expressible $[\![\cdot]\!]_P$-expressible

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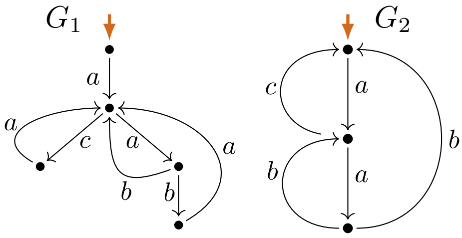
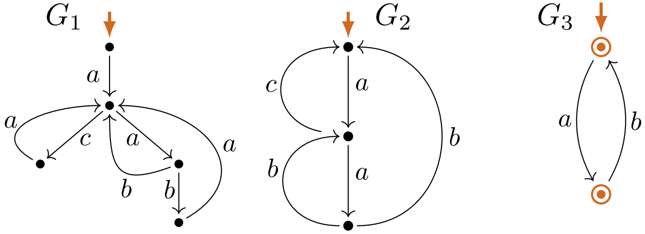


P-expressible ?

$[\![\cdot]\!]_P$-expressible $[\![\cdot]\!]_P$-expressible

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)



G_1	G_2	G_3
P-expressible	**?**	**not** P-expressible
$[\![\cdot]\!]_P$-expressible	$[\![\cdot]\!]_P$-expressible	**not** $[\![\cdot]\!]_P$-expressible

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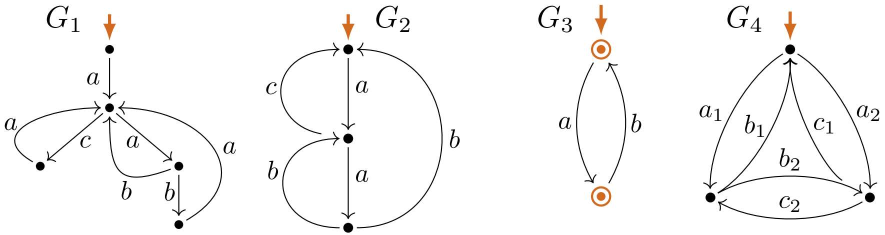
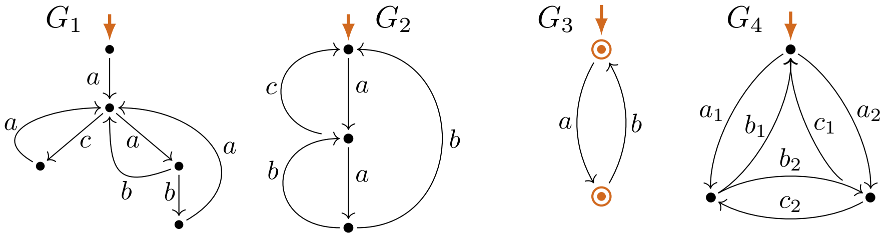


G_1	G_2	G_3, G_4
P-expressible	**?**	**not** P-expressible
$[\![\cdot]\!]_P$-expressible	$[\![\cdot]\!]_P$-expressible	**not** $[\![\cdot]\!]_P$-expressible

P-expressibility and $[\![\cdot]\!]_P$-expressibility (examples)

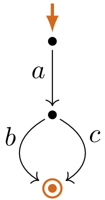


G_1	G_2	G_3, G_4
P-expressible	**?**	**not** P-expressible
$[\![\cdot]\!]_P$-expressible	$[\![\cdot]\!]_P$-expressible	**not** $[\![\cdot]\!]_P$-expressible

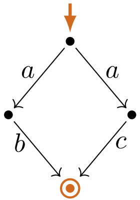
Q2: How can P-expressibility and $[\![\cdot]\!]_P$-expressibility be characterized?

Process semantics equality $=_{[\![\cdot]\!]_P}$

- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: .



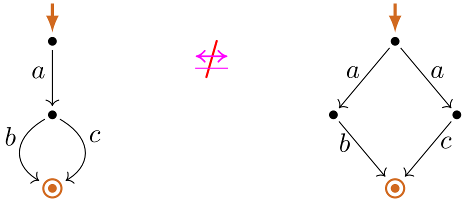
$P(a \cdot (b + c))$



$P(a \cdot b + a \cdot c)$

Process semantics equality $=_{[\![\cdot]\!]_P}$

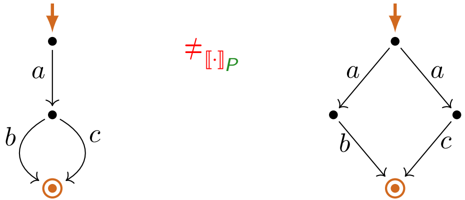
- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$:



$$P(a \cdot (b + c)) \quad \not\underline{\leftrightarrow} \quad P(a \cdot b + a \cdot c)$$

Process semantics equality $=_{[\![\cdot]\!]_P}$

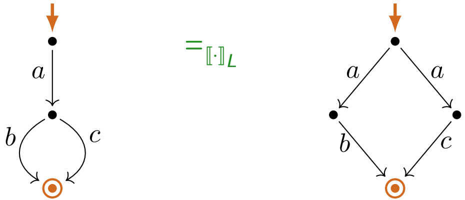
- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: .



$$a \cdot (b + c) \quad \neq_{[\![\cdot]\!]_P} \quad a \cdot b + a \cdot c$$

Process semantics equality $=_{[\![\cdot]\!]_P}$

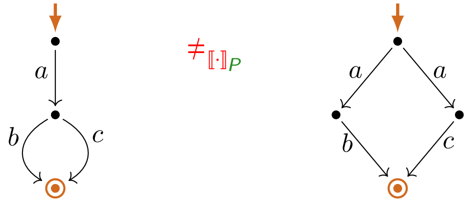
- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: .



$$a \cdot (b + c) \quad =_{[\![\cdot]\!]_L} \quad a \cdot b + a \cdot c$$

Process semantics equality $=_{[\![\cdot]\!]_P}$

- Fewer identities hold for $=_{[\![\cdot]\!]_P}$ than for $=_{[\![\cdot]\!]_L}$: $=_{[\![\cdot]\!]_P} \subsetneqq =_{[\![\cdot]\!]_L}$.



$$a \cdot (b + c) \quad \neq_{[\![\cdot]\!]_P} \quad a \cdot b + a \cdot c$$

Milner's proof system Mil

Axioms:

(A1)	$e + (f + g) = (e + f) + g$	(A7)	$e = 1 \cdot e$
(A2)	$e + 0 = e$	(A8)	$e = e \cdot 1$
(A3)	$e + f = f + e$	(A9)	$0 = 0 \cdot e$
(A4)	$e + e = e$	(A10)	$e^* = 1 + e \cdot e^*$
(A5)	$e \cdot (f \cdot g) = (e \cdot f) \cdot g$	(A11)	$e^* = (1 + e)^*$
(A6)	$(e + f) \cdot g = e \cdot g + f \cdot g$		

But: $e \cdot (f + g) \neq e \cdot f + e \cdot g$ But: $e \cdot 0 \neq 0$

Inference rules: rules of equational logic *plus*

$$\frac{e = f \cdot e + g}{e = f^* \cdot g}\ \mathrm{RSP}^*$$ (if f does not terminate immediately)

Milner's Question (Q1)

Is Mil complete with respect to $=_{[\![\cdot]\!]_P}$? (Does $e =_{[\![\cdot]\!]_P} f \Longrightarrow e =_{\mathsf{Mil}} f$ hold?)

Milner's questions

(Q1) Complete axiomatization:

Is the proof system Mil *complete for* $=_{[\![\cdot]\!]_P}$?

(Q2) $[\![\cdot]\!]_P$-Expressibility:

What structural property characterizes process graphs that are $[\![\cdot]\!]_P$*-expressible* ?

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(Q1) Complete axiomatization:

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- is decidable (Baeten/Corradini/G, 2007)

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(Q1) Complete axiomatization:

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- partial new answer (G/Fokkink, 2020):
  - bisimulation collapse has loop existence & elimination property (LEE) if expressible by under-star-1-free regular expression

Milner's questions

(Q1) Complete axiomatization:

Is the proof system Mil *complete for* $=_{[\![\cdot]\!]_P}$*?*

- series of partial completeness results for:
  - exitless iterations (Fokkink, 1998)
  - with a stronger fixed-point rule (G, 2006)
  - under-star 1-free, and without 0 (Corradini/de Nicola/Labella, 2004)
  - with 0 but under-star-1-free (G/Fokkink, 2020)

(Q2) $[\![\cdot]\!]_P$-Expressibility:

What structural property characterizes
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Milner's questions

(Q1) Complete axiomatization:

Is the proof system Mil *complete for* $=_{[\![\cdot]\!]_P}$*?*

- Yes! (G, 2022, proof summary, employing LEE and crystallization)
- series of partial completeness results for:
  - exitless iterations (Fokkink, 1998)
  - with a stronger fixed-point rule (G, 2006)
  - under-star 1-free, and without 0 (Corradini/de Nicola/Labella, 2004)
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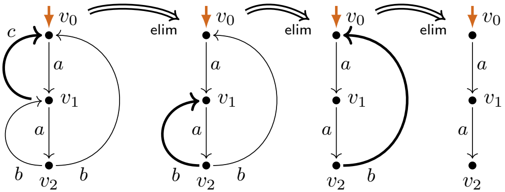
Question (Q2) specialized

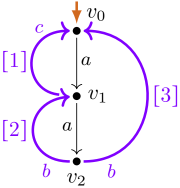
$(Q2)_0$ P-Expressibility and P-$(*/\not{1})$-Expressibility:

What structural property characterizes:

- *process graphs that are P-expressible ?*
  (... in the image of P?)
- *process graphs that are P-expressible by $(*/\not{1})$ regular expressions?*
  (... in the image of $(*/\not{1})$ expressions under P?)

Loop Existence and Elimination (LEE)





Loop graphs (interpretations of innermost iterations without 1)

Definition

A process graph is a loop graph if:

(L1) There is an infinite path from the start vertex.

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(L3) Termination is only possible at the start vertex.

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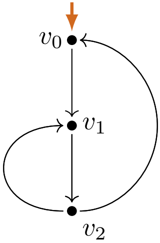

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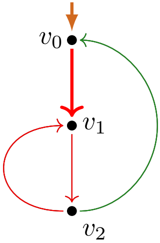

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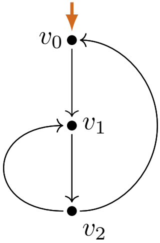

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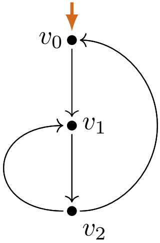

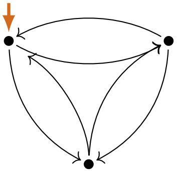

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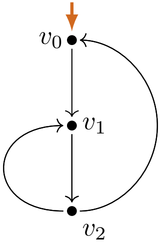

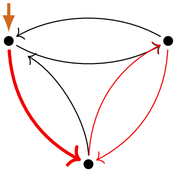

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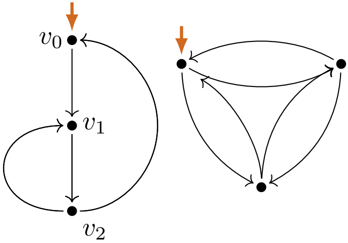

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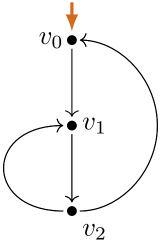

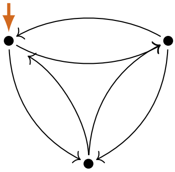

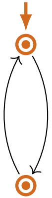

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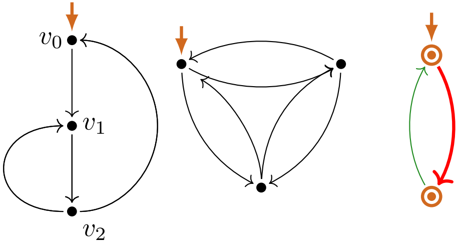

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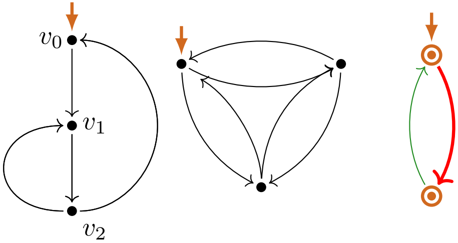

(L1),~~(L2)~~ (L1),(L2),~~(L3)~~

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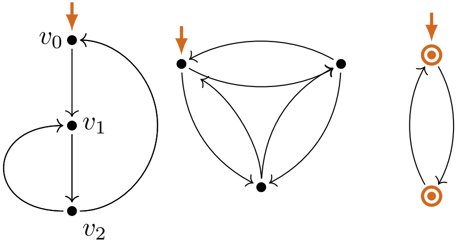

(L1),~~(L2)~~

(L1),(L2),~~(L3)~~

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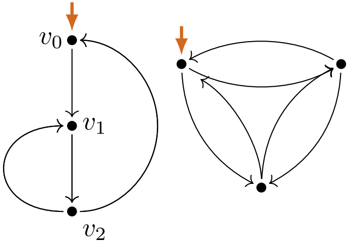
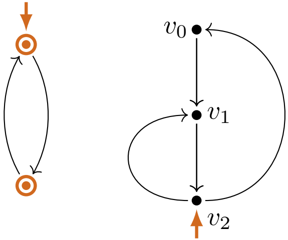


(L1),~~(L2)~~



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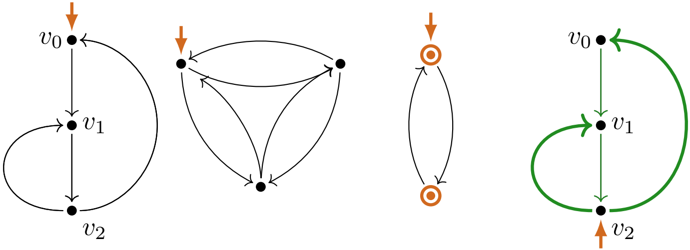

(L1),~~(L2)~~ (L1),(L2),~~(L3)~~

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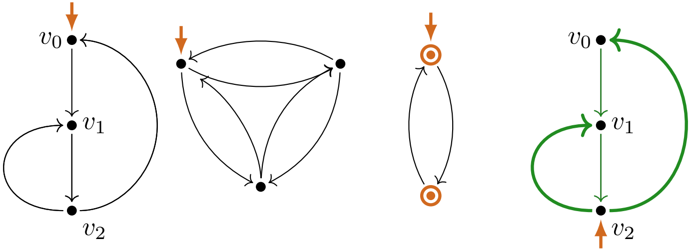

(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart

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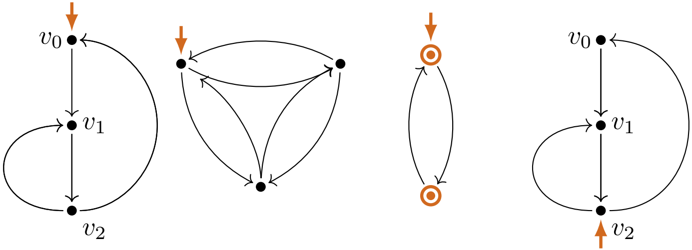

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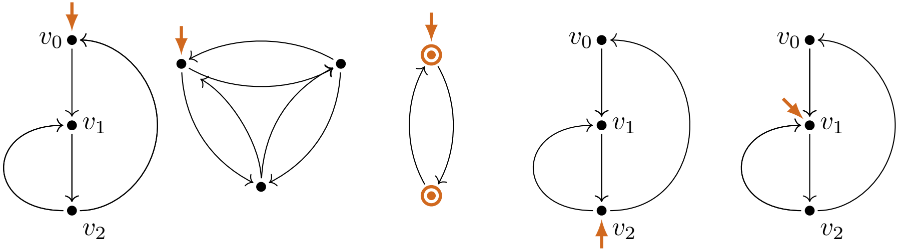

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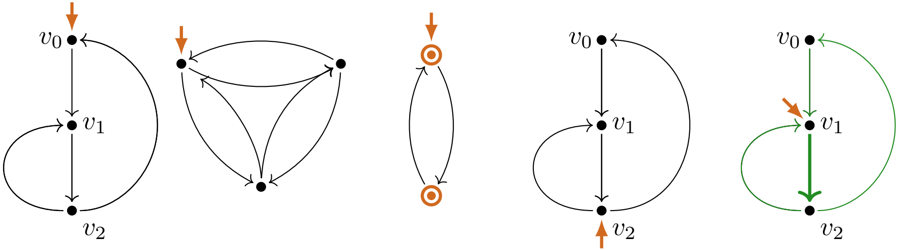

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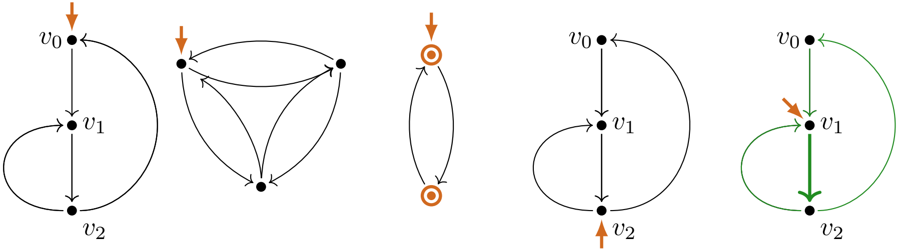

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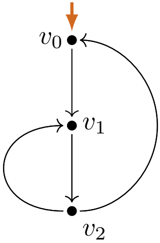
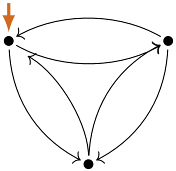
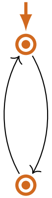
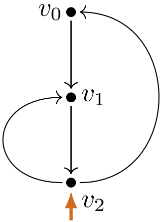




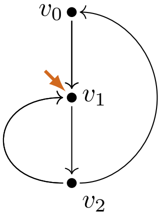
(L1),~~(L2)~~



(L1),(L2),~~(L3)~~



loop chart



loop chart

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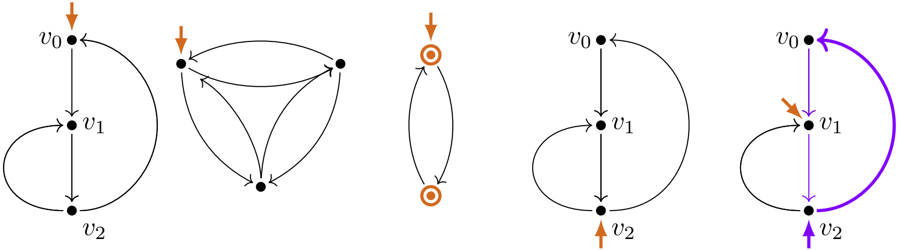
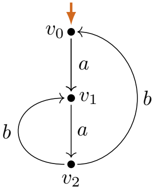


(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart loop subchart

Loop existence and elimination (example 1)



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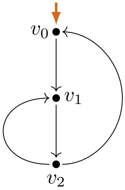

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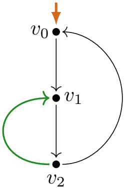

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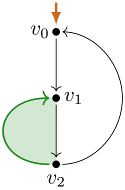

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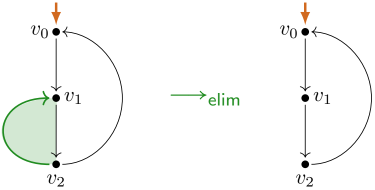

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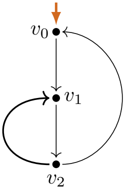
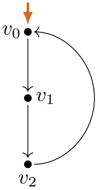


$\longrightarrow_{\text{elim}}$



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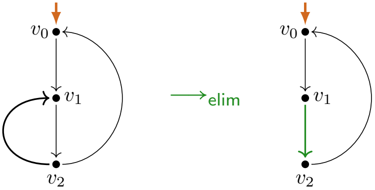

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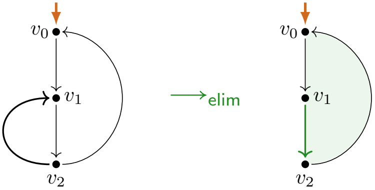

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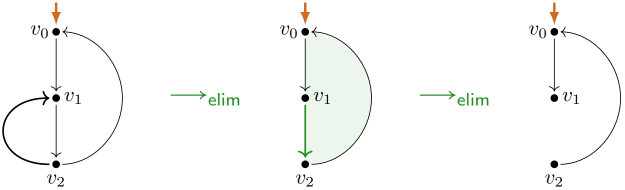

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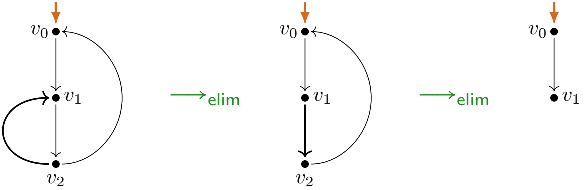

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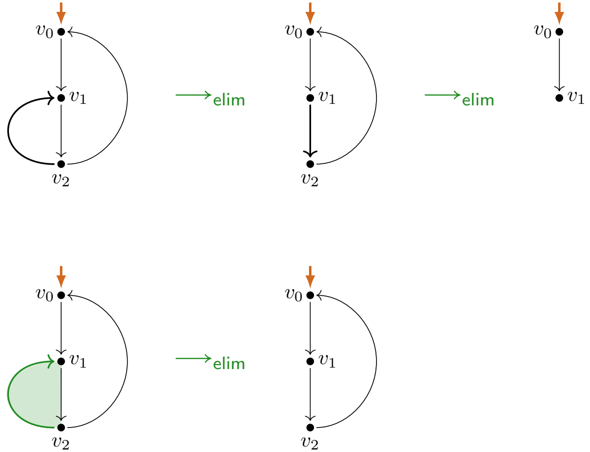

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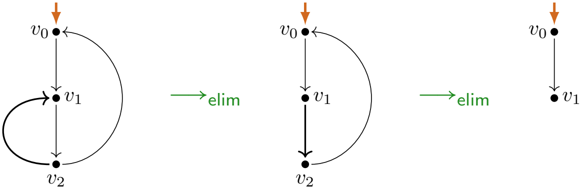

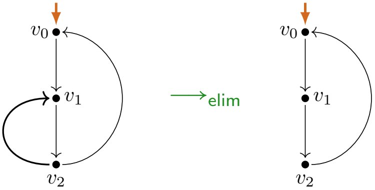

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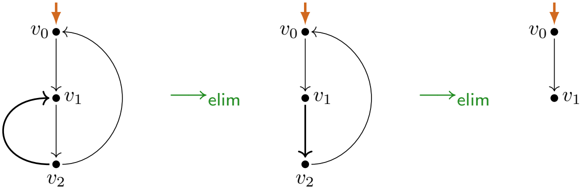

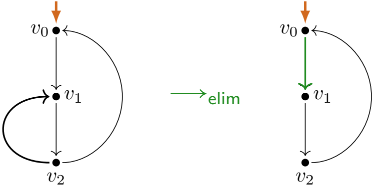

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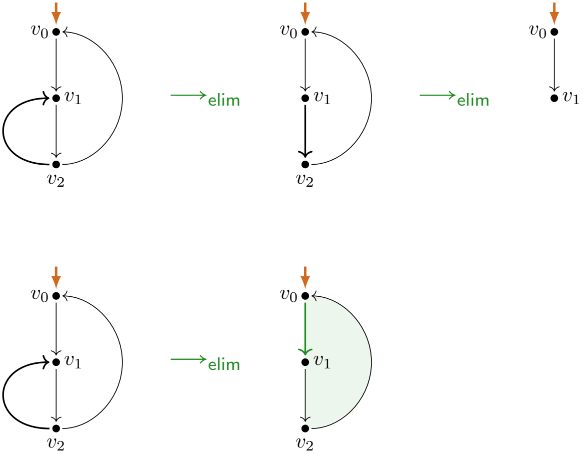

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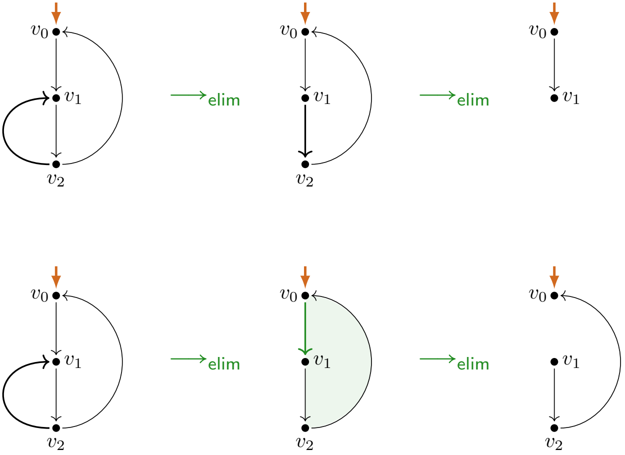

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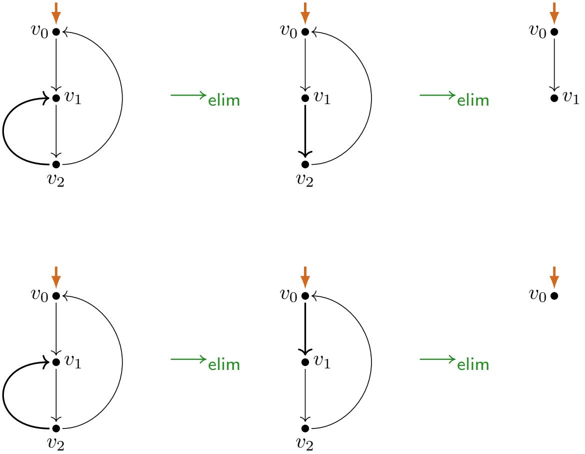

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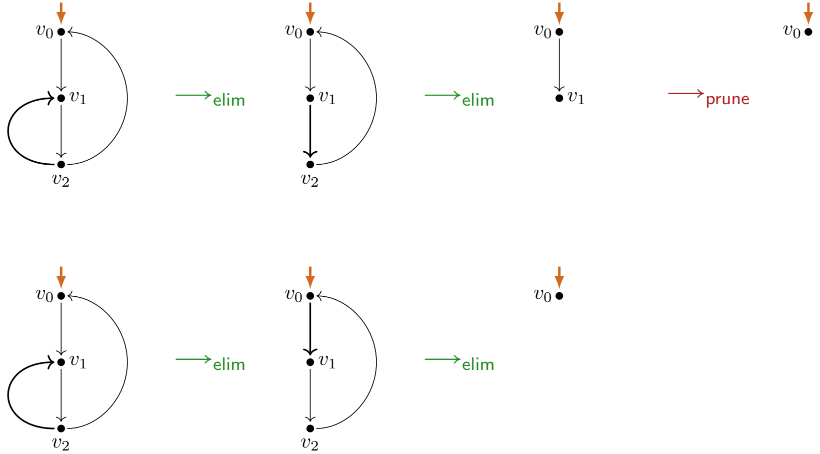
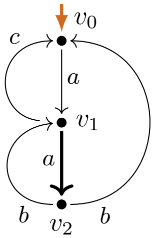


Loop existence and elimination (example 2)



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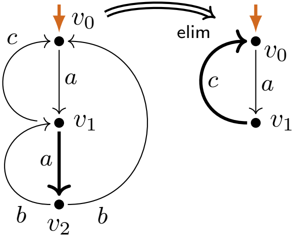

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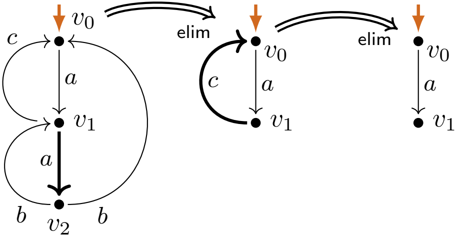

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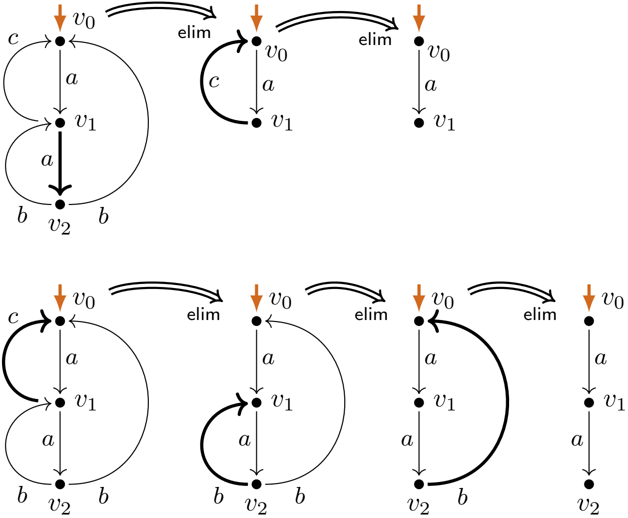

LEE

Definition

A chart $\mathcal{C}$ satisfies LEE (*loop existence and elimination*) if:

$$\exists \mathcal{C}_0 \left(\mathcal{C} \longrightarrow^{*}_{\text{elim}} \mathcal{C}_0 \not\longrightarrow_{\text{elim}} \right.$$
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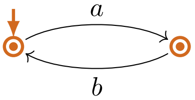
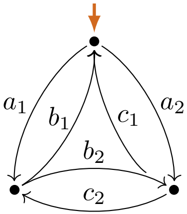
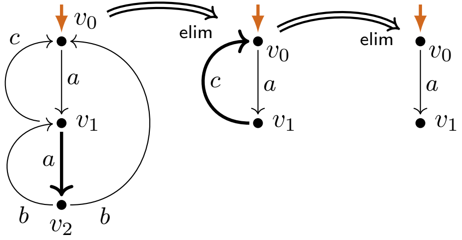
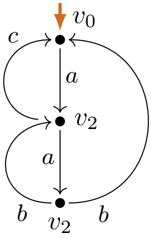




~~LEE~~ ~~LEE~~

LEE





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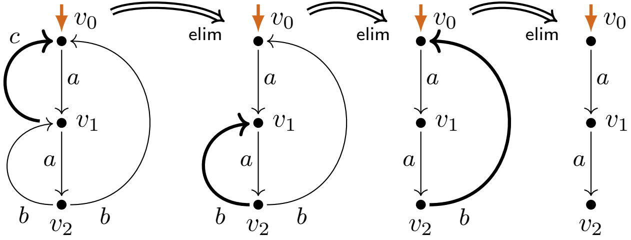

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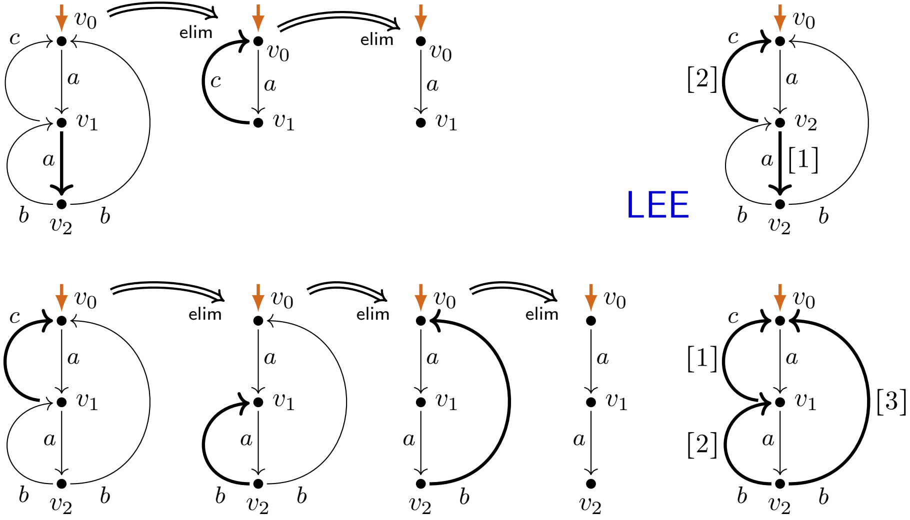

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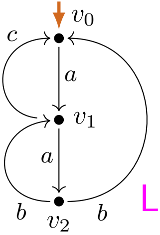
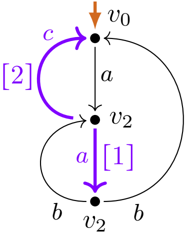


LEE-graph

LEE-witness



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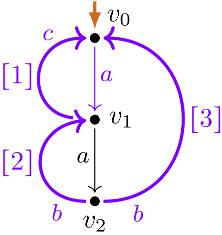
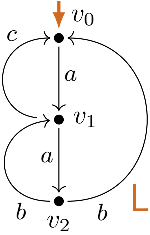
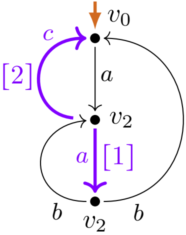


Layered LEE



LLEE-graph

layered
LLEE-witness



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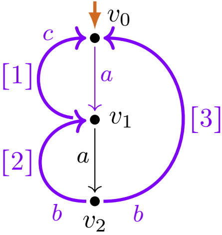
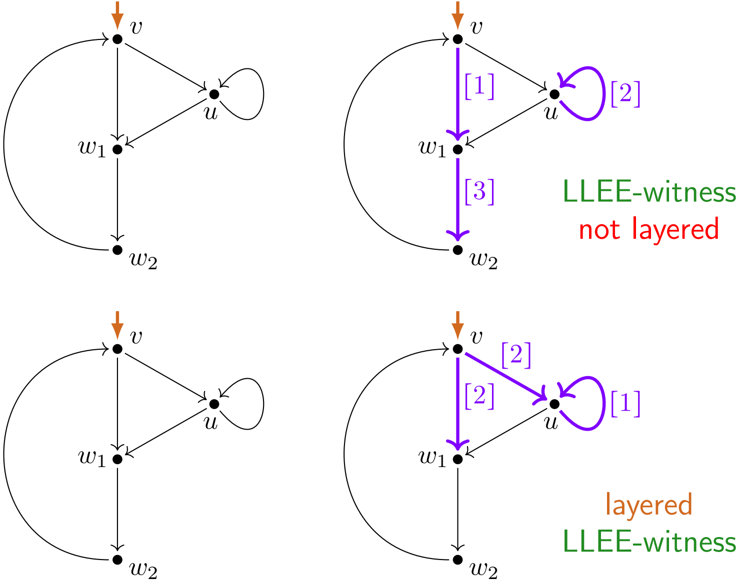
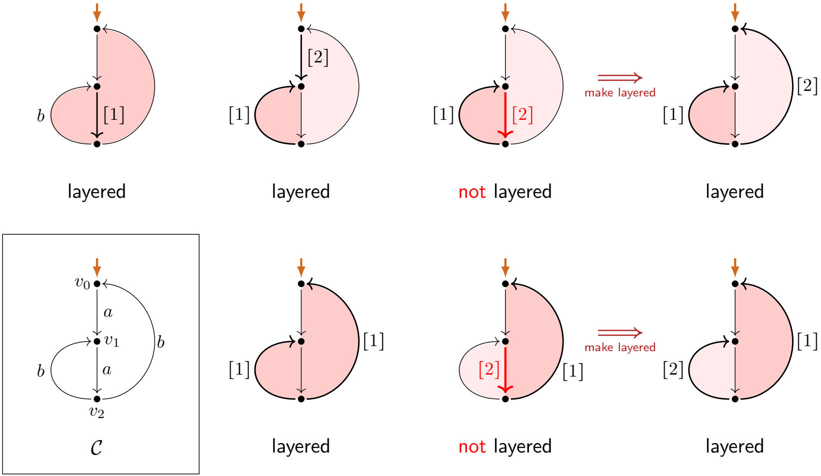


Layered LEE-witness (LLEE-witness)



7 LEE-witnesses



Loop elimination: properties

$\longrightarrow_{\text{elim}}$: eliminate a transition-induced loop by:

- removing the loop-entry transition(s)
- garbage collection

$\longrightarrow_{\text{prune}}$: remove a transition to a deadlocking state

Lemma

(i) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is terminating.*

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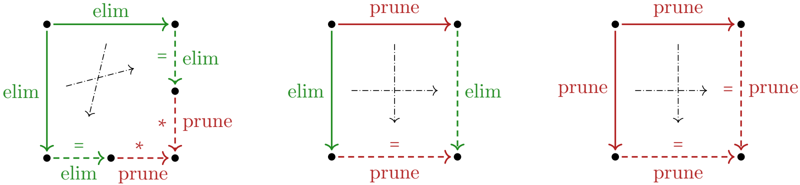
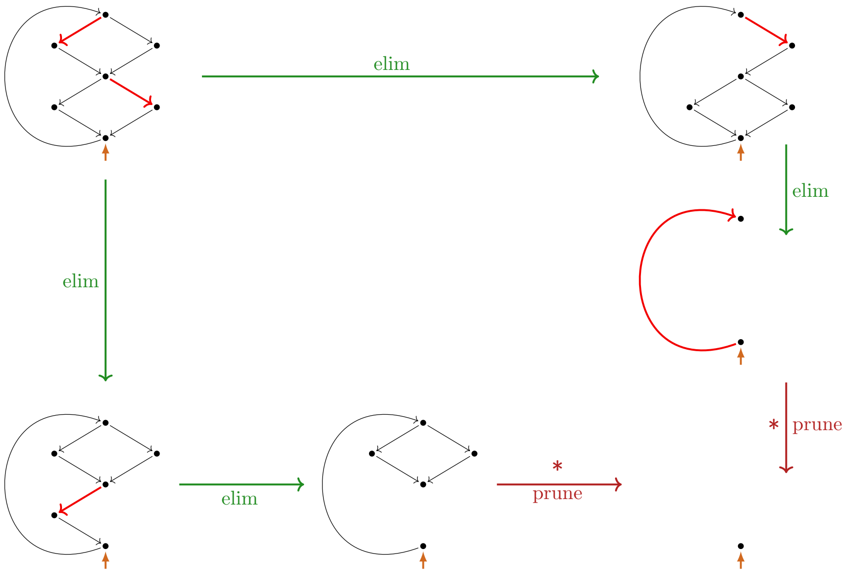
(ii) $\longrightarrow_{\text{elim}} \cup \longrightarrow_{\text{prune}}$ *is decreasing* [Van Oostrom, de Bruijn]



‘Critical pair’: bi-loop elimination



Loop elimination, and properties

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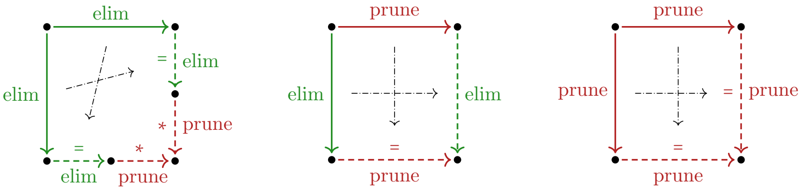

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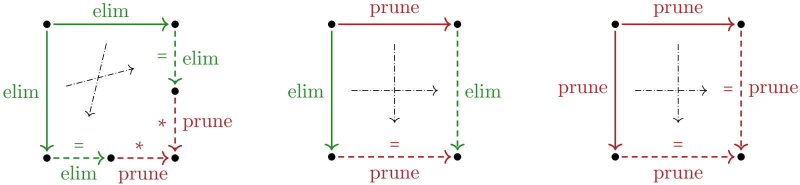

Structure property LEE

$$\mathsf{LEE}(G) \;:\Longleftrightarrow\; \exists G_0 \,\big(\, G \longrightarrow^{*}_{\mathsf{elim}} G_0 \not\longrightarrow_{\mathsf{elim}} \;\wedge\; G_0 \text{ has no infinite trace} \,\big)\,.$$

Lemma (by using termination and confluence)

For every process graph G the following are equivalent:

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Theorem (efficient decidability)

The problem of deciding $\mathsf{LEE}(G)$ for process graphs G is in PTIME.

Interpretation/extraction correspondences with LEE

($\Leftarrow$ G/Fokkink 2020, G 2021)

(Int)$_{P}^{(*/\cancel{1})}$: *P-$(*/\cancel{1})$-expressible graphs have the structural property LEE.*

Process **i**nterpretations $P(e)$ of $(*/\cancel{1})$ regular expressions e are finite process graphs that satisfy LEE.

(Extr)$_P$: LEE *implies* $[\![\cdot]\!]_P$*-expressibility*

From every finite process graph G with LEE a regular expression e can be **e**xtracted such that $G \underline{\leftrightarrow} P(e)$.

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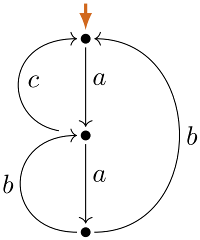
(Coll): LEE *is preserved under collapse*

The class of finite process graphs with LEE is closed under bisimulation **c**ollapse.

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

G_2



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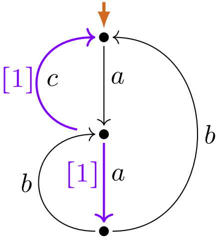
$\widehat{G_2}$



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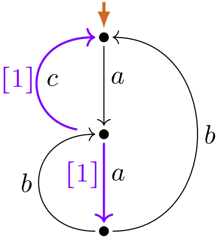
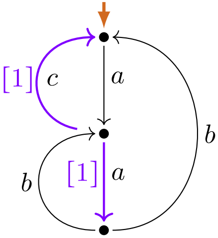


$(\qquad\qquad\qquad\qquad)^* \cdot 0$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

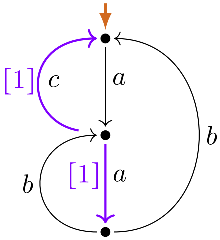
$\widehat{G_2}$



$$\Big(\qquad \overset{\vdots\, a}{\downarrow} \qquad \Big)^* \cdot 0$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



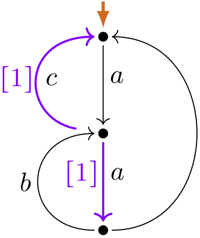
$$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$$

$$\downarrow a$$

$$(\qquad)^* \cdot 0$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

a

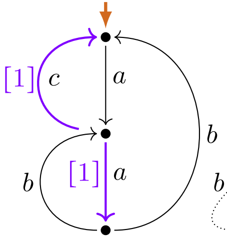
$(\qquad)^* \cdot 0$

b

b

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$(a \cdot 1) \cdot ((\qquad)^{*} \cdot 0)$

$\downarrow a$

$(\qquad)^{*} \cdot 0$

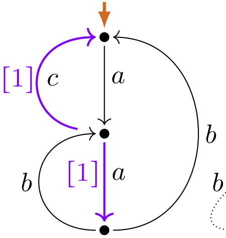
b

$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad)^{*}) \cdot 0)$

b

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



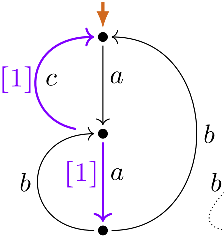
$(a \cdot 1) \cdot ((\qquad)^* \cdot 0)$

$(\qquad)^* \cdot 0$

$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot ((\qquad)^*) \cdot 0)$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$$(a \cdot 1) \cdot \big((\qquad\qquad)^* \cdot 0 \big)$$

$$\downarrow a$$

$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0$$

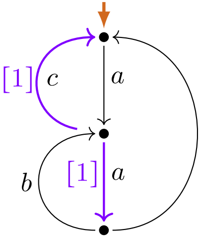
$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \big((\qquad\qquad)^*) \cdot 0 \big)$$

c b b

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$$(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)$$

$$\downarrow a$$

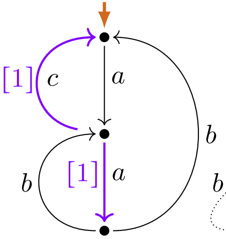
$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \big) \cdot 0 \Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\Big\downarrow a$$

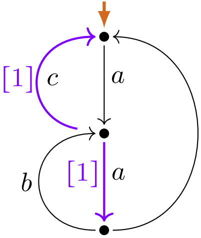
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0 \quad \xrightarrow{c} (a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)$$

$$\Big\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$P(e) \rightleftharpoons G_2$

$$\overbrace{(a \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^* \cdot 0\big)}^{e}$$

$$\Big\downarrow a$$

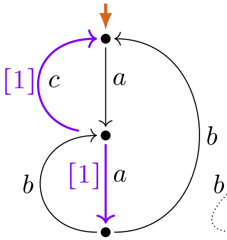
$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0$$

$$\Big\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0\Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$$P(e) \mathrel{\underline{\rightarrow}} G_2$$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0 \Big)}^{e}$$

$$\downarrow a$$

$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \cdot 0$$

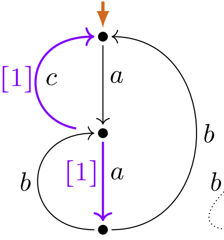
$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^{*} \big) \cdot 0 \Big)$$

c b b

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$P(e) \succeq G_2$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

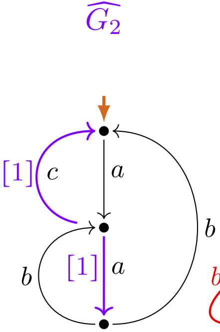
$$\downarrow a$$

$$\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0$$

$$\downarrow a$$

$$(b \cdot 1 + b \cdot (a \cdot 1)) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0\Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)



$P(e) \rightleftharpoons G_2$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

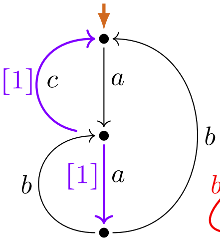
$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

Expression extraction using LLEE (G/Fokkink 2020, G 2021/22)

$\widehat{G_2}$



$$P(e) \mathrel{\underline{\to}} G_2 \not\cong P(e)$$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0$$

$$\downarrow a$$

$$(b\cdot 1+b\cdot(a\cdot 1))\cdot\big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*})\cdot 0\big)$$

Interpretation of extracted expression

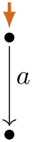
G'_2

$$P(e) = G'_2$$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^{*} \cdot 0\Big)}^{e}$$

Interpretation of extracted expression

G_2'



$$P(e) = G_2'$$

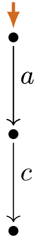
$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)$$

Interpretation of extracted expression

G_2'



$$P(e) = G_2'$$

$$\overbrace{(a \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \cdot 0 \big)}^{e}$$

$$\downarrow a$$

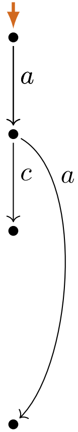
$$(1 \cdot 1) \cdot \big((c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \cdot 0 \big)$$

$$\downarrow c$$

$$\big((1 \cdot (a \cdot 1)) \cdot (c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)))^{*} \big) \cdot 0$$

Interpretation of extracted expression

G'_2



$P(e) = G'_2$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

$$\big\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big) \xrightarrow{a}$$

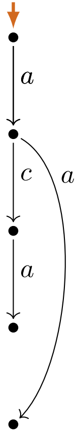
$$\big\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

$$\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

Interpretation of extracted expression

G_2'



$P(e) = G_2'$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\big\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\big\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

$$\big\downarrow a$$

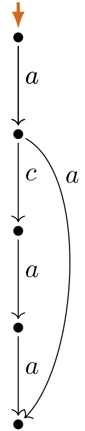
$$\big((1\cdot 1)\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0\big)$$

$$a$$

$$\big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\big)\cdot 0$$

Interpretation of extracted expression

G'_2



$P(e) = G'_2$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

$$\downarrow a$$

$$\Big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0\Big)$$

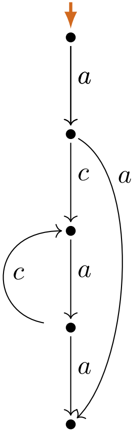
$$\downarrow a$$

$$\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

a

Interpretation of extracted expression

G_2'



$P(e) = G_2'$

$$\overbrace{(a\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\Big(\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\cdot 0\Big)$$

$$\downarrow c$$

$$\Big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

$$\downarrow a$$

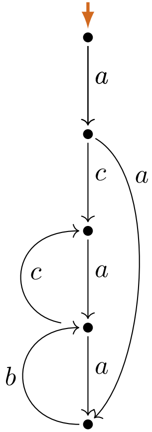
$$\Big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0\Big)$$

$$\downarrow a$$

$$\Big((1\cdot(b\cdot 1+b\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\Big)\cdot 0$$

Interpretation of extracted expression

G_2'



$\textcolor{green}{P}(\textcolor{orange}{e}) = G_2'$

$$\overbrace{(a\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1\cdot 1)\cdot\big((c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1)))^{*}\cdot 0\big)$$

$$\downarrow c$$

$$\big((1\cdot(a\cdot 1))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

$$\downarrow a$$

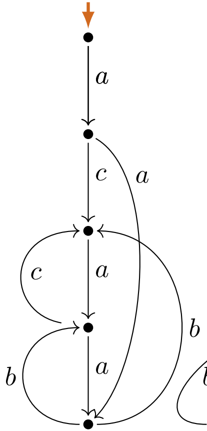
$$\big((1\cdot 1)\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0\big)$$

$$\downarrow a$$

$$\big((1\cdot(b\cdot 1+\textcolor{red}{b}\cdot(a\cdot 1)))\cdot\big(c\cdot(a\cdot 1)+a\cdot(b\cdot 1+b\cdot(a\cdot 1))\big)^{*}\big)\cdot 0$$

Interpretation of extracted expression

G_2'



$P(e) = G_2'$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \cdot 0 \big)$$

$$\downarrow c$$

$$\big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0$$

$$\downarrow a$$

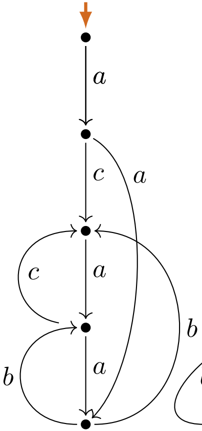
$$\big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0 \big)$$

$$\downarrow a$$

$$\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1)) \big)^* \big) \cdot 0$$

Interpretation of extracted expression

G_2'



$$P(e) = G_2' \rightarrow G_2$$

$$\overbrace{(a \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\big)}^{e}$$

$$\downarrow a$$

$$(1 \cdot 1) \cdot \big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\big)$$

$$\downarrow c$$

$$\big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\big) \cdot 0$$

$$\downarrow a$$

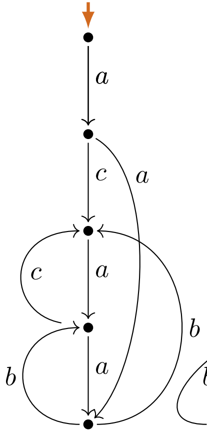
$$\big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\big) \cdot 0\big)$$

$$\downarrow a$$

$$\big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\big) \cdot 0$$

Interpretation of extracted expression

G_2'



$$P(e) = G_2' \rightrightarrows G_2 \not\simeq G_2'$$

$$\overbrace{(a \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)}^{e}$$

$$\Big\downarrow a$$

$$(1 \cdot 1) \cdot \Big(\big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^* \cdot 0\Big)$$

$$\Big\downarrow c$$

$$\Big((1 \cdot (a \cdot 1)) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

$$\Big\downarrow a$$

$$\Big((1 \cdot 1) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0\Big)$$

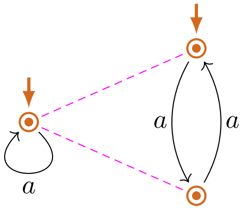
$$\Big\downarrow a$$

$$\Big((1 \cdot (b \cdot 1 + b \cdot (a \cdot 1))) \cdot \big(c \cdot (a \cdot 1) + a \cdot (b \cdot 1 + b \cdot (a \cdot 1))\big)^*\Big) \cdot 0$$

LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.

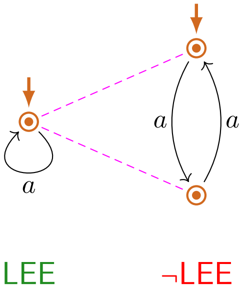


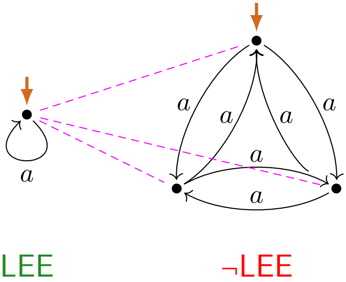
LEE ¬LEE

LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.

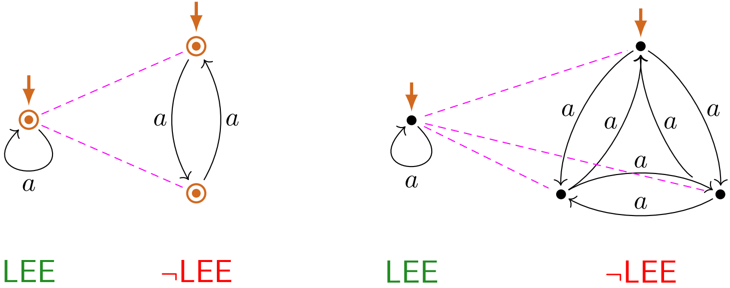




LEE under bisimulation

Observation

- LEE is not invariant under bisimulation.
- LEE is not preserved by converse functional bisimulation.



LEE under functional bisimulation

Lemma

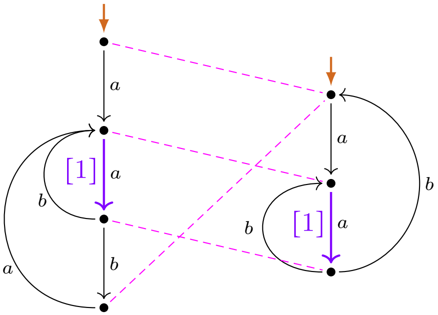
(i) LEE is preserved by functional bisimulations:

$$\mathsf{LEE}(G_1) \wedge G_1 \mathrel{\underline{\to}} G_2 \implies \mathsf{LEE}(G_2)\,.$$

Proof (Idea).

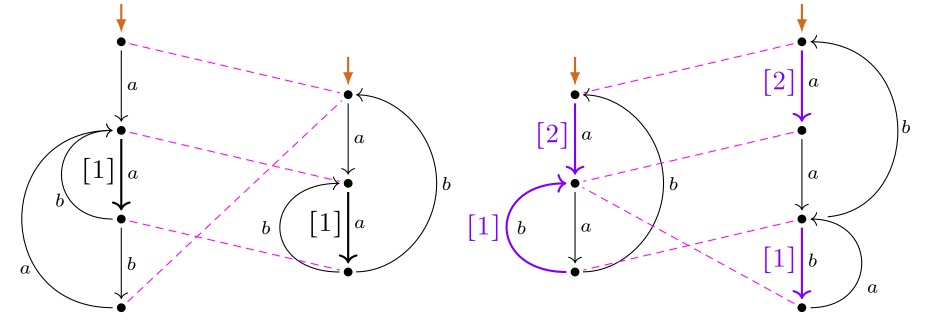
Use loop elimination in G_1 to carry out loop elimination in G_2.

Collapsing LEE-witnesses



$P(a(a(b+ba))^* \cdot 0)$

Collapsing LEE-witnesses



$P(a(a(b+ba))^* \cdot 0)$ $\qquad$ $P((aa(ba)^* \cdot b)^* \cdot 0)$

LEE under functional bisimulation / bisimulation collapse

Lemma

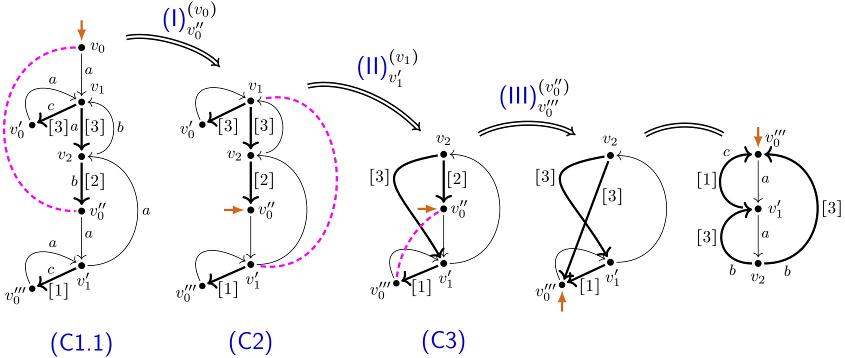
(i) LEE *is preserved by functional bisimulations:*

$$\mathsf{LEE}(G_1) \wedge G_1 \mathrel{\underline{\rightarrow}} G_2 \implies \mathsf{LEE}(G_2)\,.$$

(ii) LEE *is preserved from a process graph to its bisimulation collapse:*

$$\mathsf{LEE}(G) \wedge G \textit{ has bisimulation collapse } C \implies \mathsf{LEE}(C)\,.$$

LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)
(no 1-transitions!)



Lemma

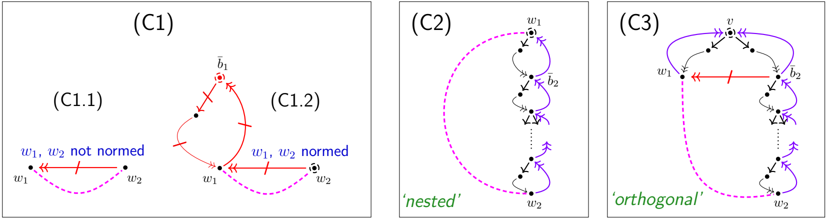
The bisimulation collapse of a LLEE-chart is again a LLEE-chart.

Reduced bisimilarity redundancies in LLEE-graphs (no 1-trans.!)

(G/Fokkink, LICS'20)

w_1, w_2 in different scc's

w_1, w_2 in the same scc



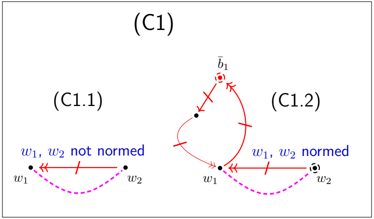
Lemma

Every not collapsed LLEE-graph contains bisimilar vertices $w_1 \neq w_2$ of kind (C1), (C2), or (C3) (a reduced bisimilarity redundancy $\langle w_1, w_2 \rangle$):

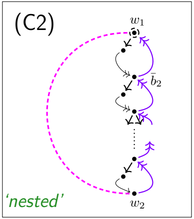
Reduced bisimilarity redundancies in LLEE-graphs (no 1-trans.!)

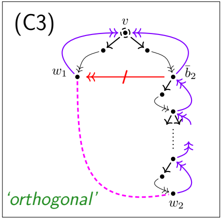
(G/Fokkink, LICS'20)

w_1, w_2 in different scc's



w_1, w_2 in the same scc





Lemma

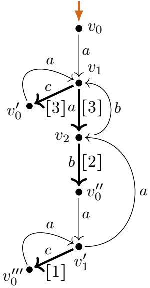
Every not collapsed LLEE-graph contains bisimilar vertices $w_1 \neq w_2$ of kind (C1), (C2), or (C3) (a reduced bisimilarity redundancy $\langle w_1, w_2 \rangle$):

Lemma

Every reduced bisimilarity redundancy in a LLEE-graph can be eliminated LLEE-preservingly.

LLEE-preserving collapse of LLEE-charts (G/Fokkink, LICS'20)

(no 1-transitions!)



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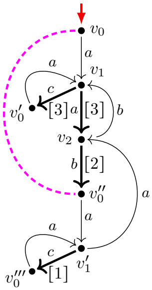

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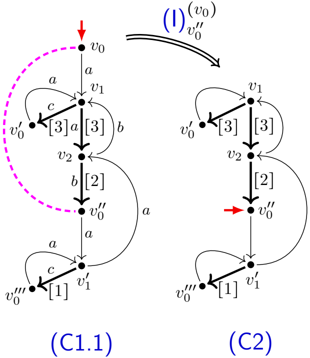

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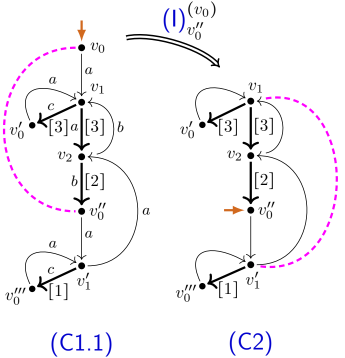

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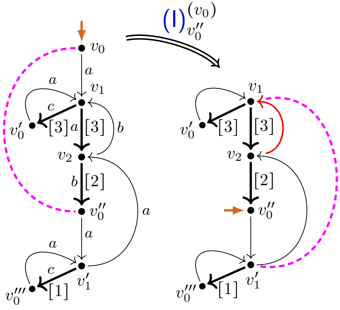

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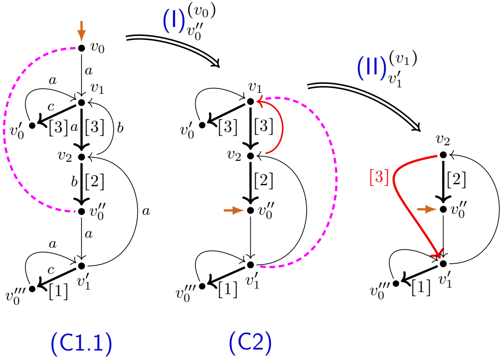

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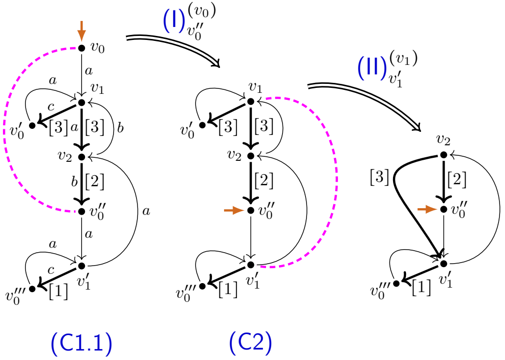

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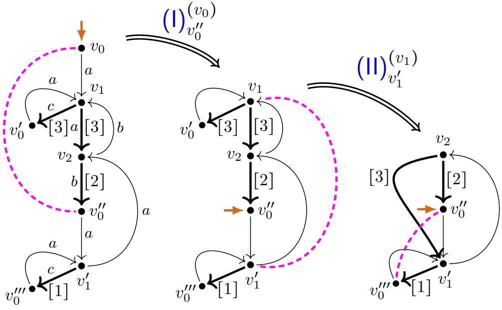

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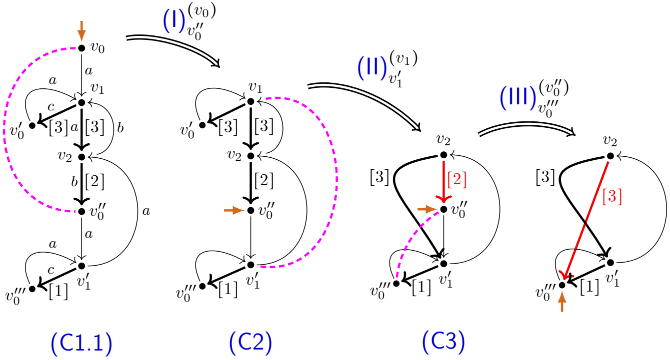

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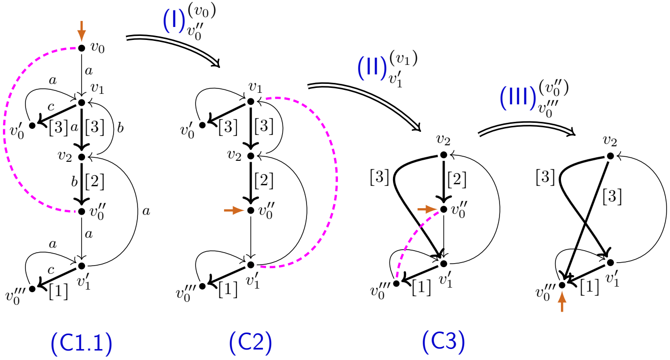

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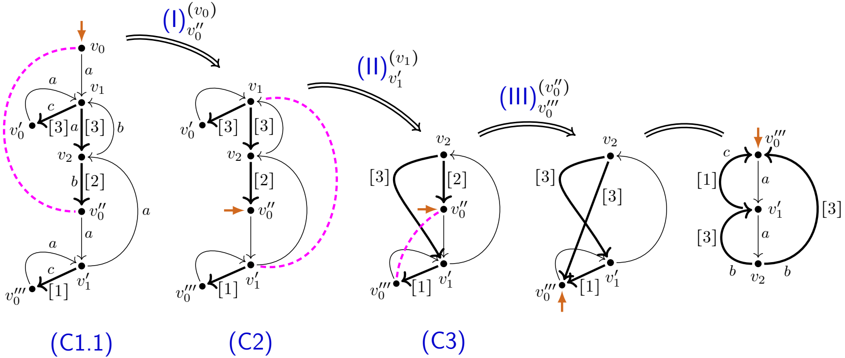

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Properties of LEE-charts

Theorem (⇐ G/Fokkink, 2020)

A process graph G

is $[\![\cdot]\!]_P$-expressible by an under-star-1-free regular expression

(i.e. P-expressible modulo bisimilarity by an $(\cancel{1}\backslash *)$ reg. expr.)

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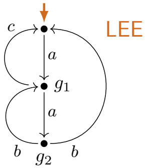
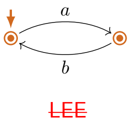
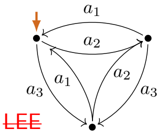
Hence $[\![\cdot]\!]_P$-expressible | **not** $[\![\cdot]\!]_P$-expressible by 1-free regular expressions:







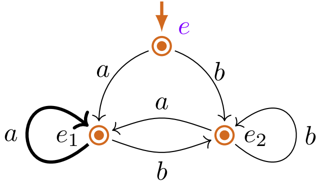
1-LEE

$\hat{=}$ sharing via 1-transitions facilitates LEE

Failure of LEE in general (example)

$(a^* \cdot b^*)^*$ not $(*/\cancel{1})$!

$$P((a^* \cdot b^*)^*)$$



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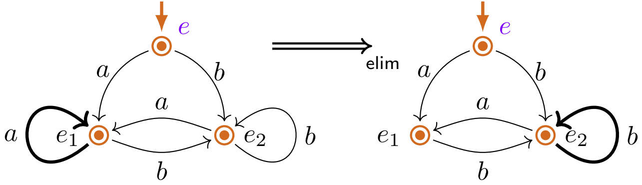

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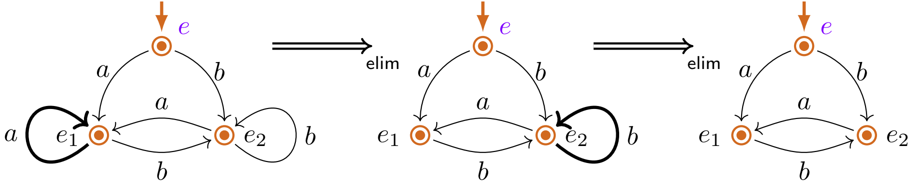

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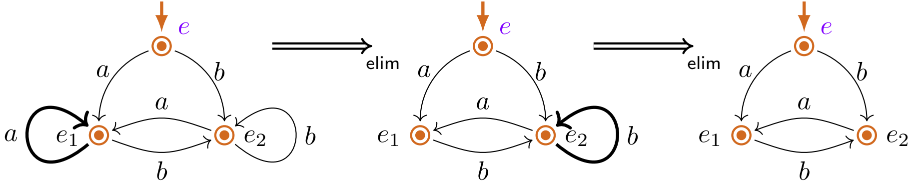

no loop subchart,
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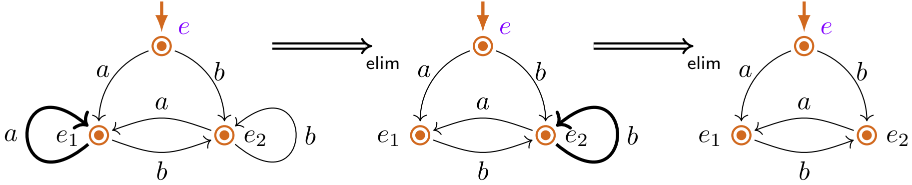

~~LEE~~

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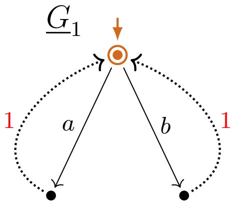
1-Graphs and induced graphs

Definition

$\xrightarrow{1} \cdot \cdots \cdot \xrightarrow{1} \cdot \xrightarrow{a}$ $\widehat{=}$ $\xrightarrow{(a]}$ induced a-transitions, for $a \in A$

$\xrightarrow{1} \cdot \cdots \cdot \xrightarrow{1} \cdot \Downarrow$ $\widehat{=}$ $\Downarrow^{(1)}$ induced termination.



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$$v_1 \xrightarrow{1} \cdot \;\cdots\; \cdot \xrightarrow{1} \cdot \xrightarrow{a} v_2 \quad \widehat{=} \quad v_1 \xrightarrow{(a]} v_2$$ induced a-transitions, for $a \in A$

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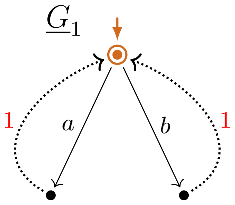

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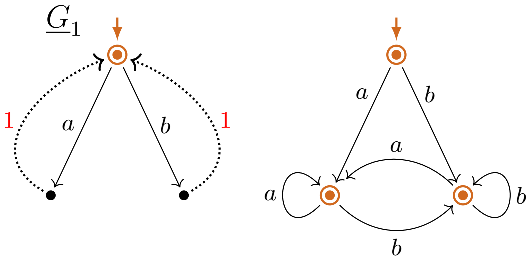

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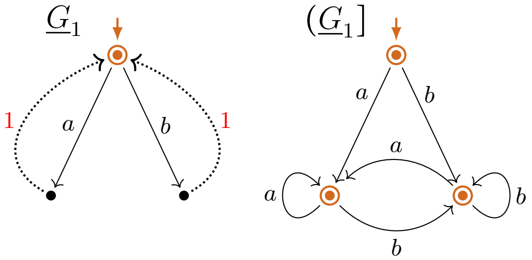

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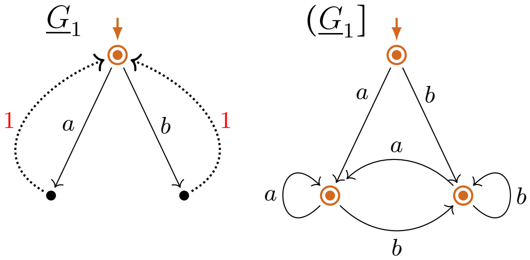

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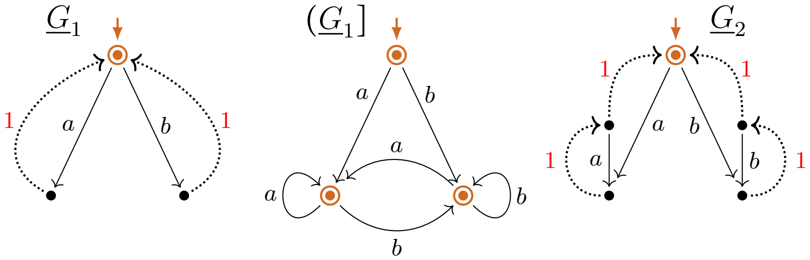

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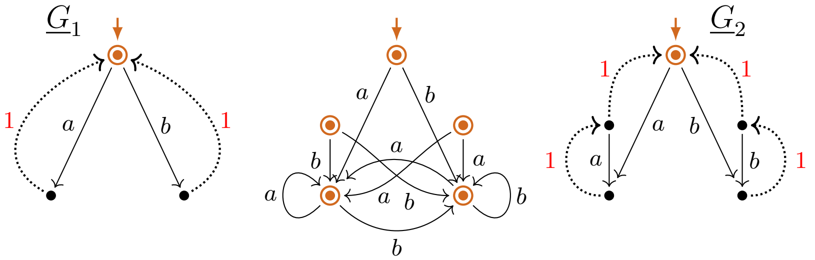

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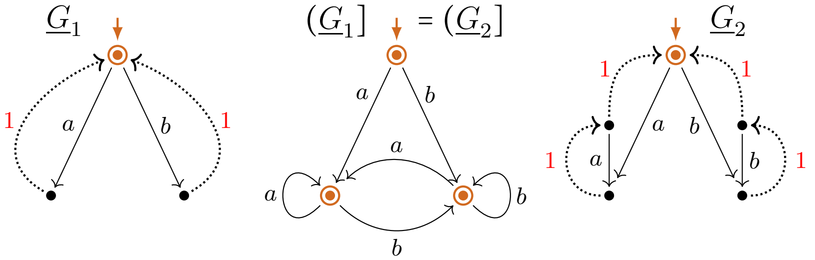

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1-LEE(G) holds for a graph G,
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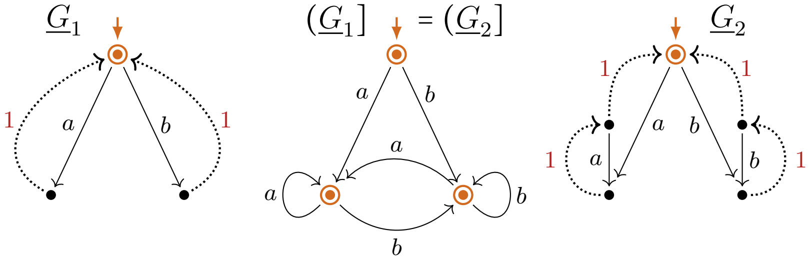

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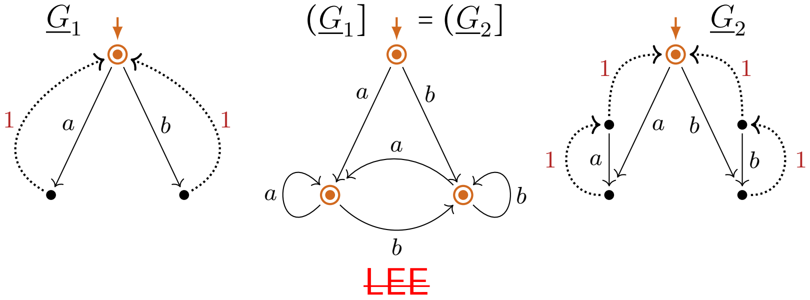

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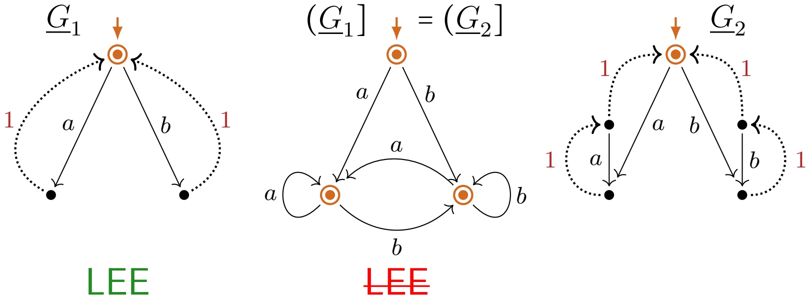

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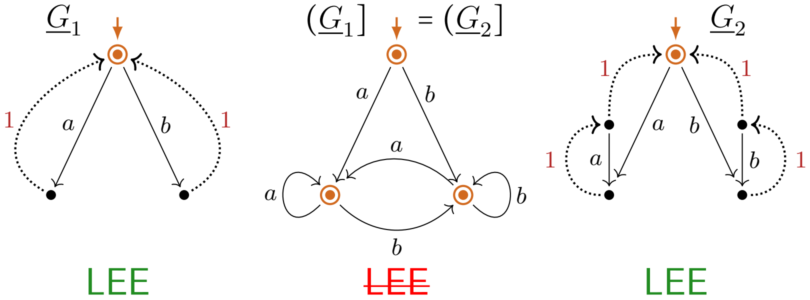

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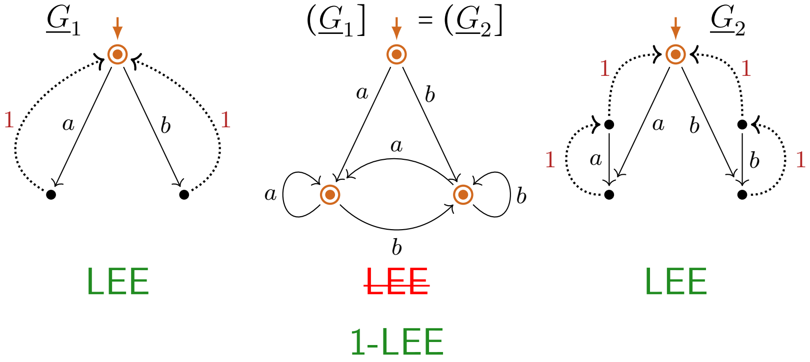
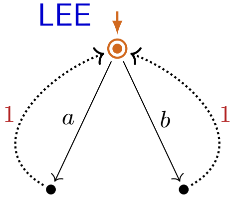
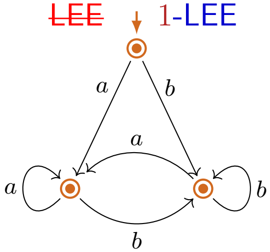


1-LEE holds for process interpretations





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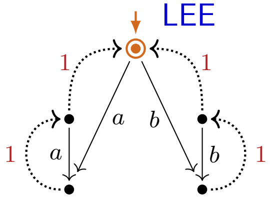

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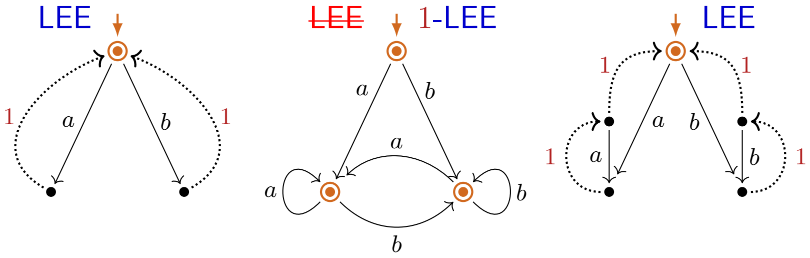

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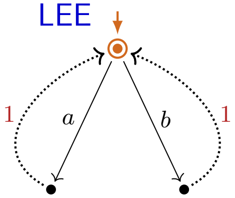

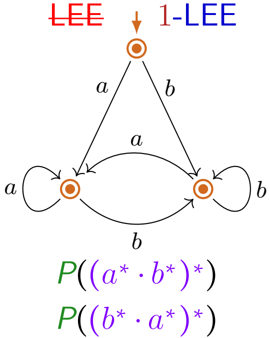

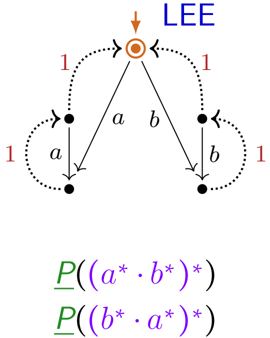

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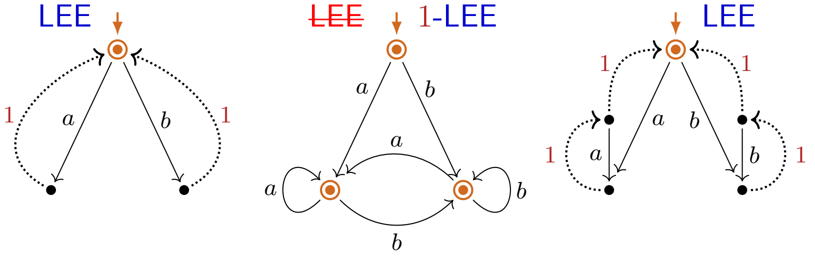

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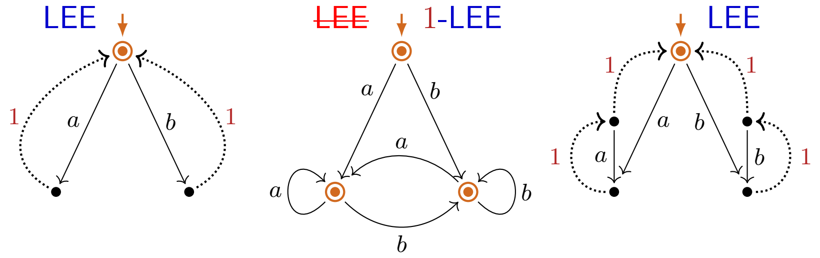

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Interpretation/extraction correspondences with 1-LEE

($\Leftarrow$ G 2021/22/23)

(Int)$_P$: *P-expressible graphs have the structural property* 1-LEE

Process **int**erpretations $P(e)$ of regular expressions e
are finite process graphs that satisfy 1-LEE.

(Extr)$_P$: 1-LEE *implies* $[\![\cdot]\!]_P$*-expressibility*

From every finite 1-process-graph $\underline{G}$ with 1-LEE
a regular expression e can be **extr**acted such that $\underline{G} \mathrel{\underline{\leftrightarrow}} P(e)$.

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Process **int**erpretations $P(e)$ of regular expressions e
are finite process graphs that satisfy 1-LEE.

(Extr)$_P$: 1-LEE *implies* $[\![\cdot]\!]_P$*-expressibility*

From every finite 1-process-graph $\underline{G}$ with 1-LEE
a regular expression e can be **extr**acted such that $\underline{G} \; \underline{\leftrightarrow} \; P(e)$.

~~**(Coll)**~~: 1-LEE *is not preserved under collapse*

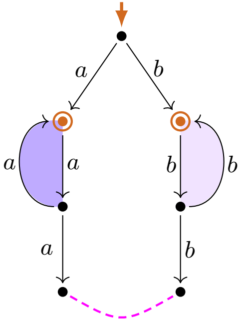
The class of finite process graphs with 1-LEE
is not closed under bisimulation **coll**apse.

1-LEE/ LEE characterize

the un-/restricted image of compact version $P^{\bullet}$ of P

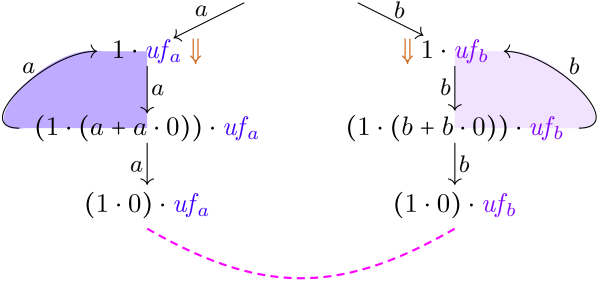
Image of P is **not closed** under bisimulation collapse
not even for $(*/\cancel{1})$ regular expressions (example)

$P(uf)$



$P(uf)$

$$uf := a \cdot \overbrace{(a \cdot (a + a \cdot 0))^*}^{uf_a} + b \cdot \overbrace{(b \cdot (b + b \cdot 0))^*}^{uf_b}$$



Compact process interpretation $P^\bullet$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{}{\mathbf{1}\Downarrow} \qquad \frac{e_i\Downarrow}{(e_1 + e_2)\Downarrow}\;(i \in \{1,2\}) \qquad \frac{e_1\Downarrow \quad e_2\Downarrow}{(e_1 \cdot e_2)\Downarrow} \qquad \frac{}{(e^*)\Downarrow}$$

$$\frac{}{a \xrightarrow{a} \mathbf{1}} \qquad \frac{e_i \xrightarrow{a} e_i'}{e_1 + e_2 \xrightarrow{a} e_i'}\;(i \in \{1,2\})$$

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \qquad \frac{e_1\Downarrow \quad e_2 \xrightarrow{a} e_2'}{e_1 \cdot e_2 \xrightarrow{a} e_2'} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

$$\frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e' \cdot e^*} \text{ (if } e' \text{ is normed)} \qquad \frac{e \xrightarrow{a} e'}{e^* \xrightarrow{a} e'} \text{ (if } e' \text{ is not normed)}$$

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

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Definition

The compact process (graph) interpretation $P^{\bullet}(e)$ of a reg. expr's e:

$P^{\bullet}(e) :=$ labeled transition graph generated by e by derivations in $\mathcal{T}^{\bullet}$.

Compact process interpretation $P^{\bullet}$

Definition (Transition system specification $\mathcal{T}^{\bullet}$, changed rules w.r.t. $\mathcal{T}$)

$$\frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1' \cdot e_2} \text{ (if } e_1' \text{ is normed)} \qquad \frac{e_1 \xrightarrow{a} e_1'}{e_1 \cdot e_2 \xrightarrow{a} e_1'} \text{ (if } e_1' \text{ is not normed)}$$

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Definition

The compact process (graph) interpretation $P^{\bullet}(e)$ of a reg. expr's e:

$P^{\bullet}(e) :=$ labeled transition graph generated by e by derivations in $\mathcal{T}^{\bullet}$.

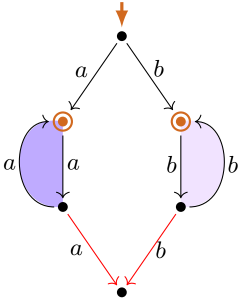
Lemma ($P^{\bullet}$ increases sharing; $P^{\bullet}$, P have same bisimulation semantics)

(i) $P(e) \Rightarrow P^{\bullet}(e)$ *for all regular expressions* e.

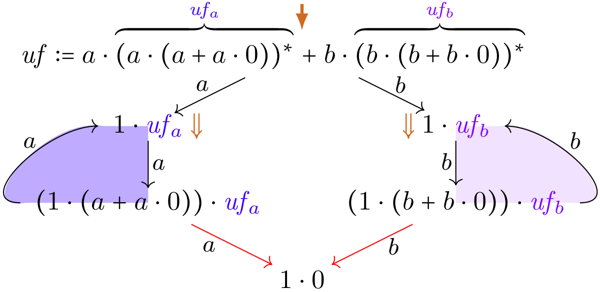
(ii) *(*G *is* $[\![\cdot]\!]_{P^{\bullet}}$*-expressible* $\Longleftrightarrow$ G *is* $[\![\cdot]\!]_{P}$*-expressible) for all graphs* G.

Image of P restricted to $(*/\cancel{1})$ regular expressions ... contains all of its bisimulation collapses (example)

$P^{\bullet}(uf)$



$P^{\bullet}(uf)$



Interpretation correspondence of $P^{\bullet}$ with LEE

(Int)$^{(*/\cancel{1})}_{P^{\bullet}}$: *By under-star-1-free expressions $P^{\bullet}$-expressible graphs satisfy* LEE:

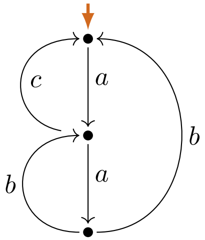
Compact process interpretations $P^{\bullet}(uf)$
of under-star-1-free regular expressions uf
are finite process graphs that satisfy LEE.

(Extr)$^{(*/\cancel{1})}_{P^{\bullet}}$: LEE *implies $[\![\cdot]\!]_P$-expressibility by under-star-1-free reg. expr's*:

From every finite process graph G with LEE
an under-star-1-free regular expression uf can be extracted
such that $G \trianglerighteq P(uf)$.

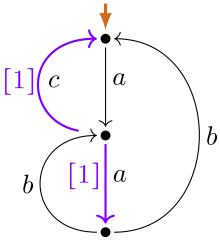
Refined extraction expression (example)

G_2

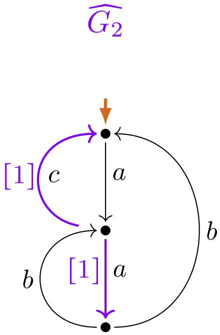


Refined extraction expression (example)

$\widehat{G_2}$



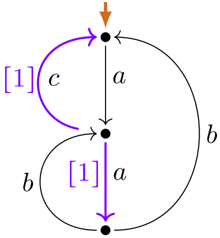
Refined extraction expression (example)



$$(1 \cdot (\qquad\qquad\qquad)^{*}) \cdot 0$$

Refined extraction expression (example)

$\widehat{G_2}$

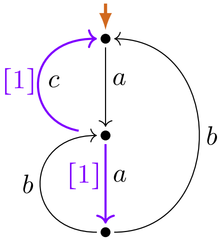


$$a$$

$$(1 \cdot (\qquad\qquad)^*) \cdot 0$$

Refined extraction expression (example)

$\widehat{G_2}$



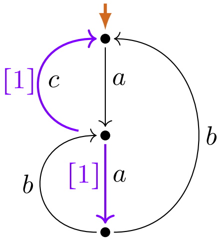
$$((1 \cdot a) \cdot (\qquad\qquad)^*) \cdot 0$$

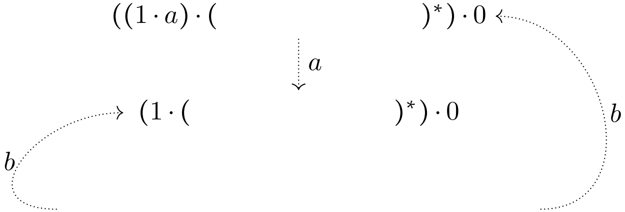
$$\Big\downarrow a$$

$$(1 \cdot (\qquad\qquad)^*) \cdot 0$$

Refined extraction expression (example)

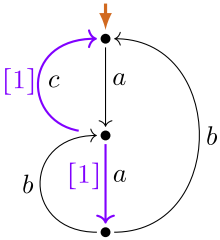
$\widehat{G_2}$





Refined extraction expression (example)

$\widehat{G_2}$



$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$

a

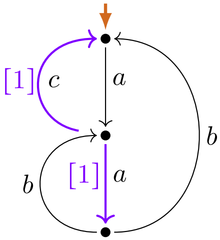
$(1 \cdot (\qquad)^*) \cdot 0$

b $\qquad$ b

$((1 \cdot (b + b \cdot a)) \cdot (\qquad)^*) \cdot 0$

Refined extraction expression (example)

$\widehat{G_2}$



$((1 \cdot a) \cdot (\qquad)^*) \cdot 0$

a

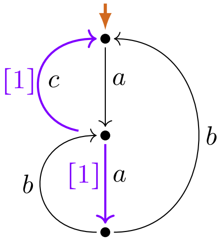
$(1 \cdot (\qquad)^*) \cdot 0$

a

$((1 \cdot (b + b \cdot a)) \cdot (\qquad)^*) \cdot 0$

Refined extraction expression (example)

$\widehat{G_2}$



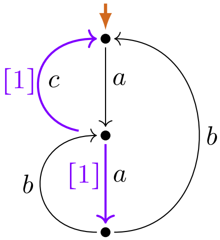
$$((1 \cdot a) \cdot (\qquad\qquad)^{*}) \cdot 0$$

$$(1 \cdot (c \cdot a + a \cdot (b + b \cdot a))^{*}) \cdot 0$$

$$((1 \cdot (b + b \cdot a)) \cdot (\qquad\qquad)^{*}) \cdot 0$$

Refined extraction expression (example)

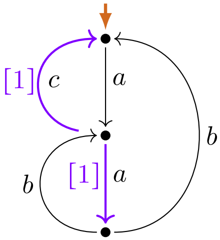
$\widehat{G_2}$



$$\begin{tikzcd}
((1\cdot a)\cdot(c\cdot a + a\cdot(b+b\cdot a))^*)\cdot 0 \arrow[d, dotted, "a"] \arrow[d, dotted, bend right=60, "c"'] \\
(1\cdot(c\cdot a + a\cdot(b+b\cdot a))^*)\cdot 0 \arrow[d, dotted, "a"] \\
((1\cdot(b+b\cdot a))\cdot(c\cdot a + a\cdot(b+b\cdot a))^*)\cdot 0 \arrow[u, dotted, bend left=60, "b"] \arrow[uu, dotted, bend right=60, "b"']
\end{tikzcd}$$

Refined extraction expression (example)

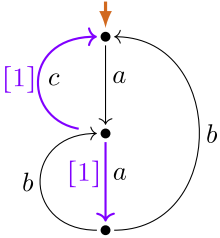
$\widehat{G_2}$



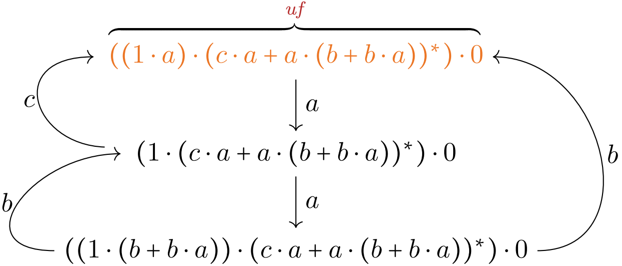
$$
\begin{tikzcd}
((1 \cdot a) \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0 \arrow[d, "a"] \\
(1 \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0 \arrow[d, "a"] \arrow[u, bend left, "c"] \\
((1 \cdot (b + b \cdot a)) \cdot (c \cdot a + a \cdot (b + b \cdot a))^*) \cdot 0 \arrow[u, bend left, "b"] \arrow[uu, bend right, "b"']
\end{tikzcd}
$$

Refined extraction expression (example)

$\widehat{G_2}$



$P^{\bullet}(uf) = P(uf) \simeq G_2$



Interpretation/extraction correspondences of $P^{\bullet}$ with LEE

(Int)$_{P^{\bullet}}^{(*/\cancel{1})}$: *By under-star-1-free expressions $P^{\bullet}$-expressible graphs satisfy* LEE:

Compact process **i**nterpretations $P^{\bullet}(uf)$
of under-star-1-free regular expressions uf
are finite process graphs that satisfy LEE.

(Extr)$_{P^{\bullet}}^{(*/\cancel{1})}$: LEE *implies $[\![\cdot]\!]_P$-expressibility by under-star-1-free reg. expr's*:

From every finite process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \mathrel{\underline{\rightarrow}} P^{\bullet}(uf)$.

From every finite collapsed process graph G with LEE
an under-star-1-free regular expression uf can be **e**xtracted
such that $G \simeq P^{\bullet}(uf)$.

Interpretation/extraction correspondences of $P^{\bullet}$ with LEE

(Int)$_{P^{\bullet}}^{(*/\cancel{1})}$: *By under-star-1-free expressions $P^{\bullet}$-expressible graphs satisfy LEE:*
Compact process interpretations $P^{\bullet}(uf)$
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(Extr)$_{P^{\bullet}}^{(*/\cancel{1})}$: LEE *implies $[\![\cdot]\!]_P$-expressibility by under-star-1-free reg. expr's:*
From every finite process graph G with LEE
an under-star-1-free regular expression uf can be extracted
such that $G \rightarrow\!\!\!\!\!\!\!\!\:\underline{\ \ } P^{\bullet}(uf)$.
From every finite collapsed process graph G with LEE
an under-star-1-free regular expression uf can be extracted
such that $G \simeq P^{\bullet}(uf)$.

(ImColl)$_{P^{\bullet}}^{(*/\cancel{1})}$: The image of $P^{\bullet}$,
restricted to under-star-1-free regular expressions,
is closed under bisimulation collapse.

Interpretation/extraction correspondences of $P^{\bullet}$ with 1-LEE

(Int)$_{P^{\bullet}}$: *$P^{\bullet}$-expressible graphs satisfy 1-LEE*:

Compact process **i**nterpretations $P^{\bullet}(e)$ of regular expressions e are finite process graphs that satisfy 1-LEE.

(Extr)$_{P^{\bullet}}$: LEE *implies $[\![\cdot]\!]_P$-expressibility:*

From every finite process graph G with 1-LEE an regular expression e can be **e**xtracted such that $G \mathrel{\underline{\to}} P^{\bullet}(e)$.

From every finite collapsed process graph G with 1-LEE a regular expression e can be **e**xtracted such that $G \simeq P^{\bullet}(e)$.

Interpretation/extraction correspondences of $P^{\bullet}$ with 1-LEE

(Int)$_{P^{\bullet}}$: *$P^{\bullet}$-expressible graphs satisfy 1-LEE*:

Compact process interpretations $P^{\bullet}(e)$ of regular expressions e are finite process graphs that satisfy 1-LEE.

(Extr)$_{P^{\bullet}}$: LEE *implies* *$[\![\cdot]\!]_P$-expressibility:*

From every finite process graph G with 1-LEE an regular expression e can be extracted such that $G \rightarrow\!\!\!\!\!\!\!\;\_ \; P^{\bullet}(e)$.

From every finite collapsed process graph G with 1-LEE a regular expression e can be extracted such that $G \simeq P^{\bullet}(e)$.

~~**(ImColl)**$_{P^{\bullet}}$~~: The image of $P^{\bullet}$ is not closed under bisimulation collapse .

LEE $\triangleq$ image of $P^\bullet|_{RExp^{(*/\cancel{1})}}$

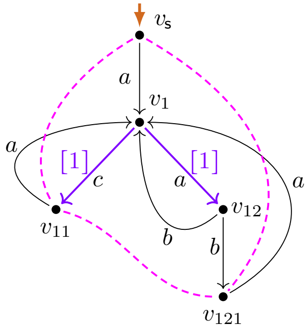
Theorem

For every process graph G TFAE:

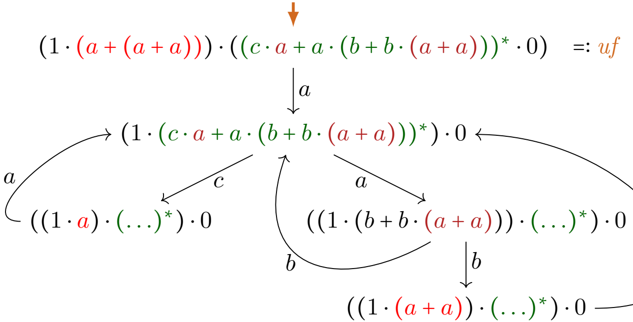
- *(i)* $\mathrm{LEE}(G)$.
- *(ii)* G *is* $P^\bullet$*-expressible by an* $(*/\cancel{1})$ *regular expression (i.e.* $G \simeq P^\bullet(e)$ *for some* $e \in RExp^{(*/\cancel{1})}$*).*
- *(iii)* G *is isomorphic to a graph in the image of* $P^\bullet$ *on* $(*/\cancel{1})$ *reg. expr's (i.e.* $G \simeq G'$ *for some* $G' \in im(P^\bullet|_{RExp^{(*/\cancel{1})}})$*).*

Adapted (refined) extraction from LLEE-graph

$G_1 \,/\, \widehat{G_1}$



$P^{\bullet}(uf) = P(uf) \simeq G_1$



1-LEE $\triangleq$ image of $P^{\bullet}$

Theorem

For every process graph G TFAE:

(i) 1-LEE(G)
(i.e. $G = (\underline{G}]$ for some 1-transition-process-graph $\underline{G}$ with LEE($\underline{G}$)*).*

(ii) G *is $P^{\bullet}$-expressible by a regular expression*
(i.e. $G \simeq P^{\bullet}(e)$ for some $e \in \mathit{RExp}$).

(iii) G *is isomorphic to a graph in the image of $P^{\bullet}$*
(i.e. $G \simeq G'$ for some $G' \in \mathit{im}(P^{\bullet})$).

Summary

- Characterizations of the image of $P^{\bullet}$ (refinement of P):
  - LEE $\triangleq$ image of $P^{\bullet}|_{RExp^{(*/\cancel{1})}}$ $\supsetneqq$ image of $P|_{RExp^{(*/\cancel{1})}}$
  - 1-LEE $\triangleq$ image of $P^{\bullet}$ $\supsetneqq$ image of P

Summary

- process interpretation P/semantics $[\![\cdot]\!]_P$ of regular expressions
  - expressibility and completeness questions
- loop existence and elimination (LEE)
  - loop elimination rewrite system can be completed
  - interpretation/extraction correspondences with $(*/\cancel{1})$ reg. expr.s
  - LEE-witnesses: labelings of graphs with LEE
  - stepwise LEE-preserving bisimulation collapse
- 1-LEE = sharing via 1-transitions facilitates LEE
  - interpretation/extraction correspondences with all regular expressions
  - not preserved under bisim. collapse (approximation possible)
- Characterizations of the image of $P^{\bullet}$ (refinement of P):
  - LEE $\triangleq$ image of $P^{\bullet}|_{RExp^{(*/\cancel{1})}}$ $\supsetneqq$ image of $P|_{RExp^{(*/\cancel{1})}}$
  - 1-LEE $\triangleq$ image of $P^{\bullet}$ $\supsetneqq$ image of P
- outlook on work-to-do

My next aims

Completeness problem, solution:

A1: graph structure of regular expression processes (LEE/1-LEE)

A2: motivation of crystallization

A4: details of crystallization procedure,
and completeness of Milner's proof system

Expressibility problem

A3: LEE is decidable in polynomial time.

Q: Is 1-LEE decidable in polynomial time**?**

P: Is expressibility by a regular expression, for a finite process graph,
decidable in polynomial time/fixed-parameter tractable time**?**

Resources

- Slides/abstract on `clegra.github.io`
  - slides: `.../lf/IFIP-1_6-2024.pdf`
  - abstract: `.../lf/abstract-IFIP-1_6-2024.pdf`
- CG: Closing the Image of the Process Interpretation of 1-Free Regular Expressions Under Bisimulation Collapse
  - TERMGRAPH 2024, extended abstract.
- CG: The Image of the Process Interpretation of Regular Expressions is Not Closed under Bisimulation Collapse,
  - arXiv:2303.08553, 2021/2023.
- CG: Milner's Proof System for Regular Expressions Modulo Bisimilarity is Complete,
  - LICS 2022, arXiv:2209.12188, poster.
- CG, Wan Fokkink: A Complete Proof System for 1-Free Regular Expressions Modulo Bisimilarity,
  - LICS 2020, arXiv:2004.12740, video on youtube.
- CG: Modeling Terms by Graphs with Structure Constraints,
  - TERMGRAPH 2018, EPTCS 288, arXiv:1902.02010.

Language semantics $[\![\cdot]\!]_L$ of reg. expr's *(Copi–Elgot–Wright, 1958)*

$\mathbf{0} \xmapsto{L}$ empty language $\varnothing$

$\mathbf{1} \xmapsto{L} \{\epsilon\}$ (ϵ the empty word)

$a \xmapsto{L} \{a\}$

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$\mathbf{1} \xmapsto{L} \{\epsilon\}$ (ϵ the empty word)

$a \xmapsto{L} \{a\}$

$e_1 + e_2 \xmapsto{L}$ union of $L(e_1)$ and $L(e_2)$

$e_1 \cdot e_2 \xmapsto{L}$ element-wise concatenation of $L(e_1)$ and $L(e_2)$

$e^* \xmapsto{L}$ set of words formed by concatenating words in $L(e)$, and adding the empty word ϵ

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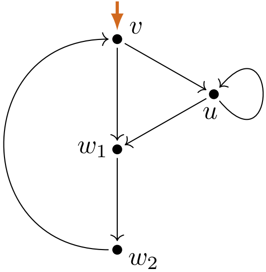
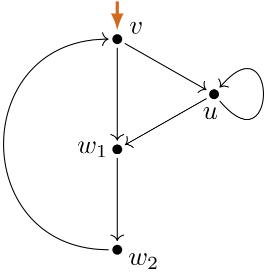
$e^* \xmapsto{L}$ set of words formed by concatenating words in $L(e)$, and adding the empty word ϵ

$[\![e]\!]_L := L(e)$ (language defined by e)

Layered LEE-witness (LLEE-witness)





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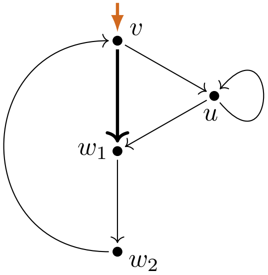
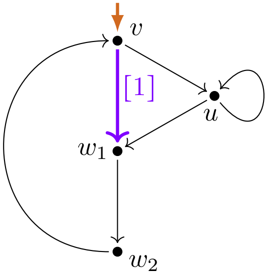
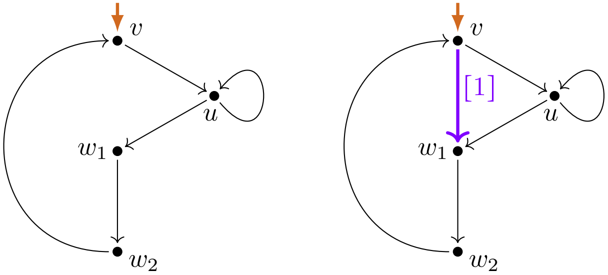
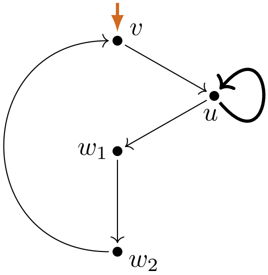


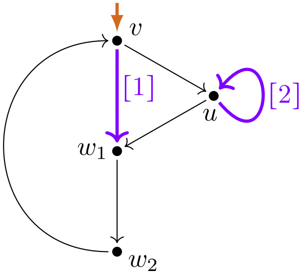


Layered LEE-witness (LLEE-witness)



Layered LEE-witness (LLEE-witness)





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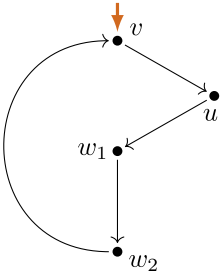

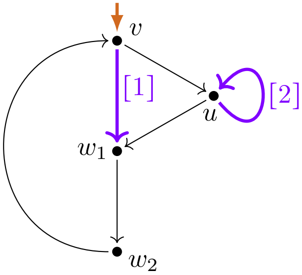

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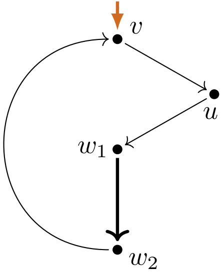

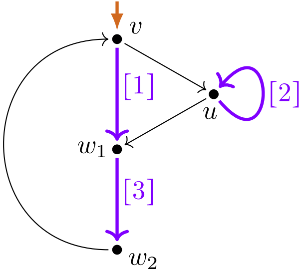

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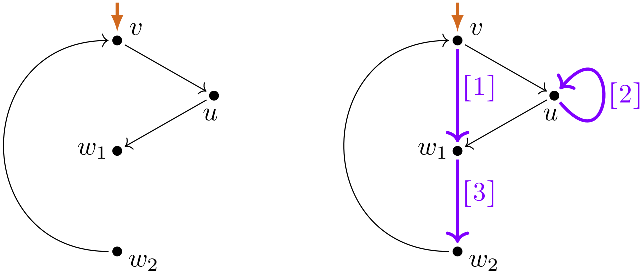

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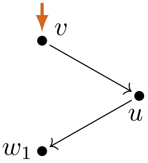

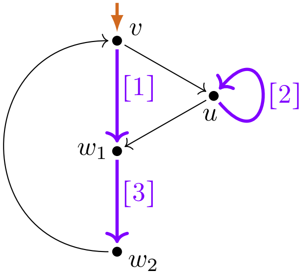

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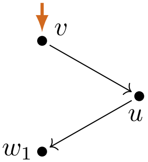
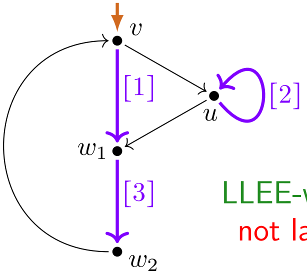
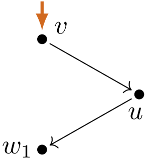
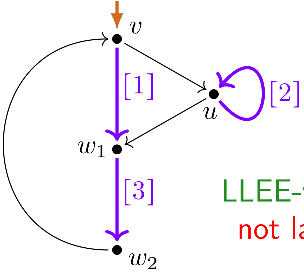




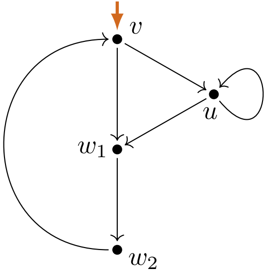
LLEE-witness
not layered

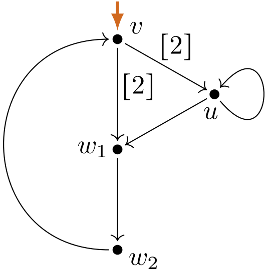
Layered LEE-witness (LLEE-witness)





LLEE-witness
not layered





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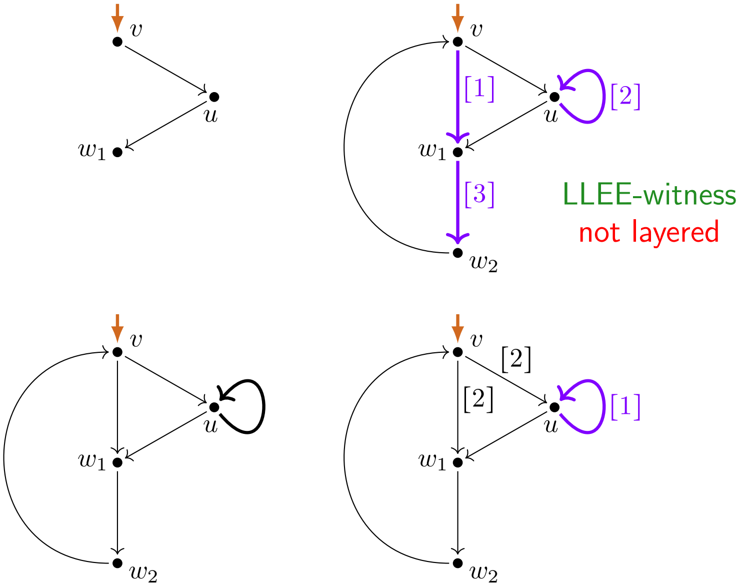

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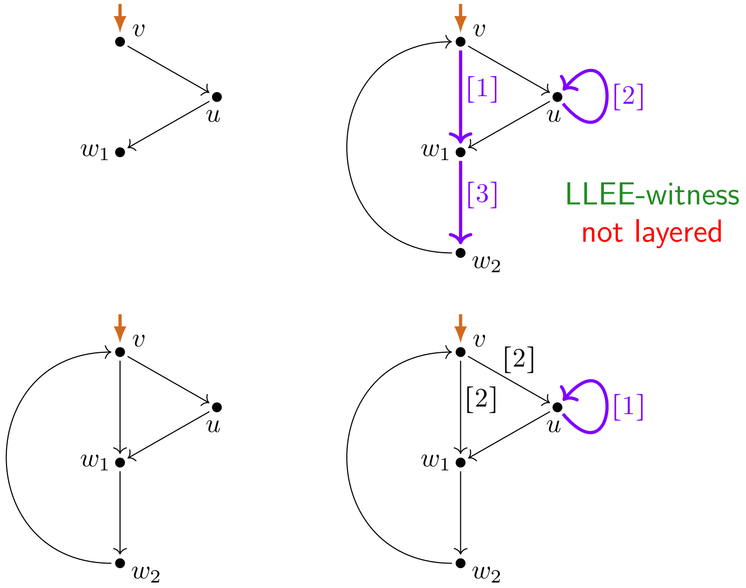

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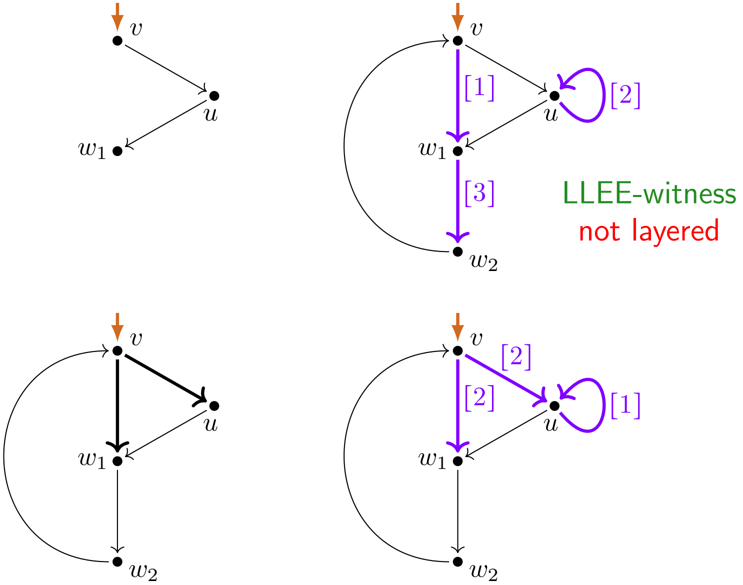

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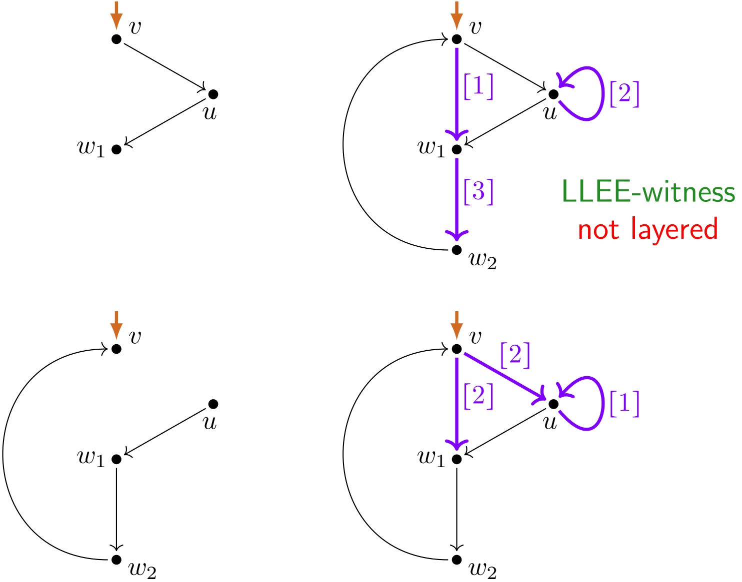

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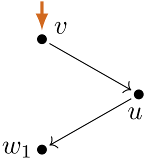
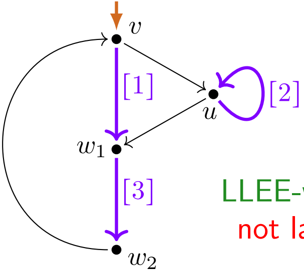
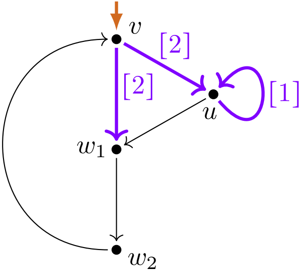




LLEE-witness
not layered



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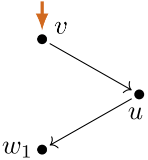
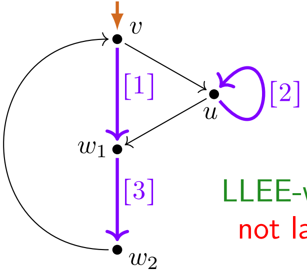
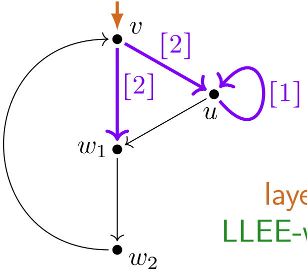


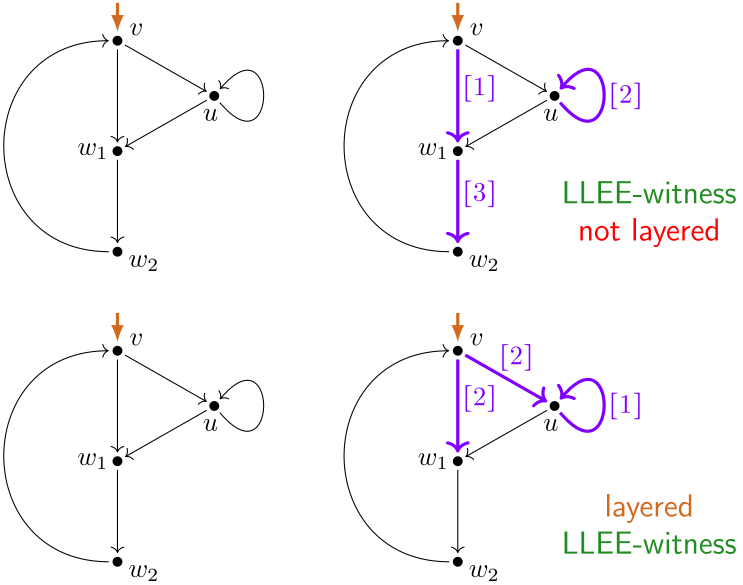


LLEE-witness
not layered

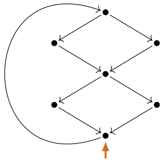


layered
LLEE-witness

Layered LEE-witness (LLEE-witness)



'Critical pair': bi-loop elimination



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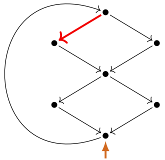

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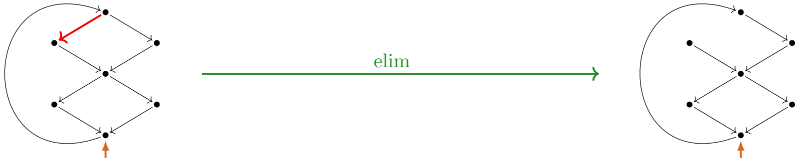

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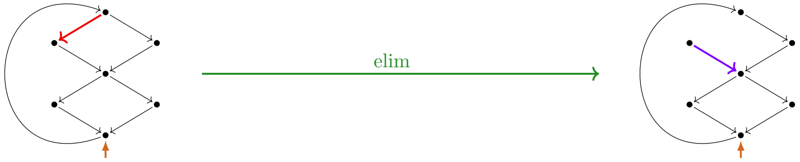

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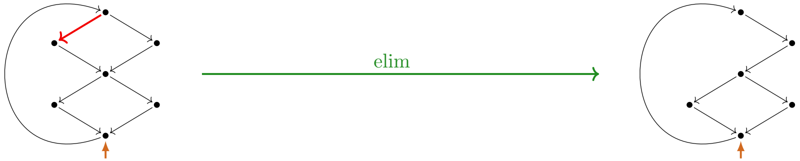

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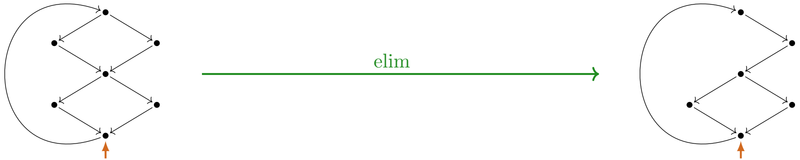

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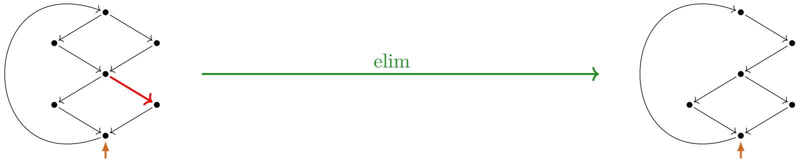

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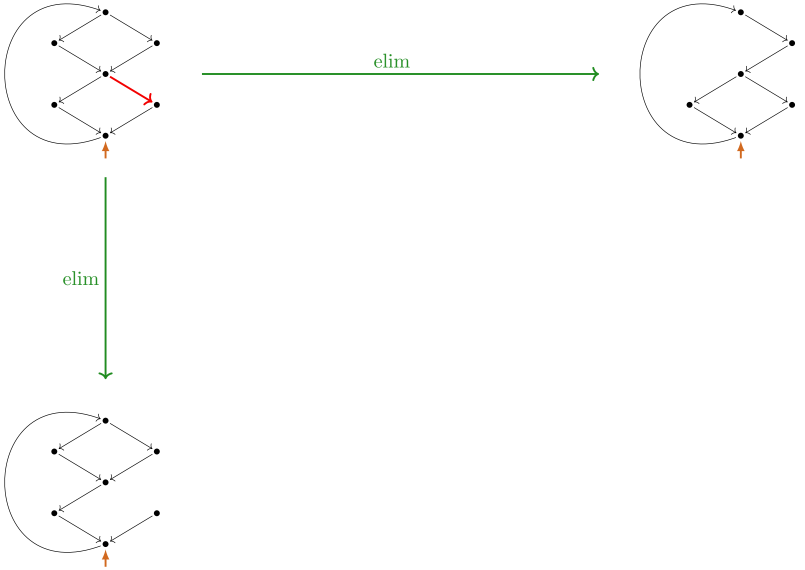

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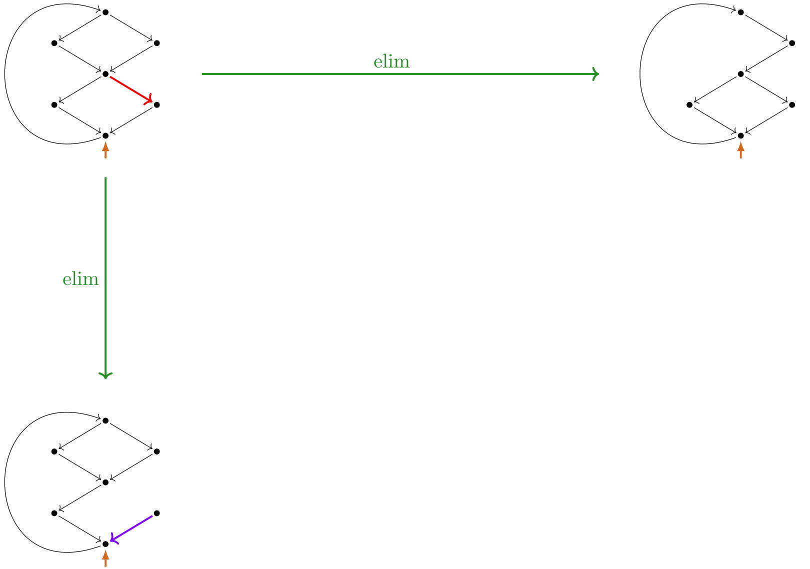

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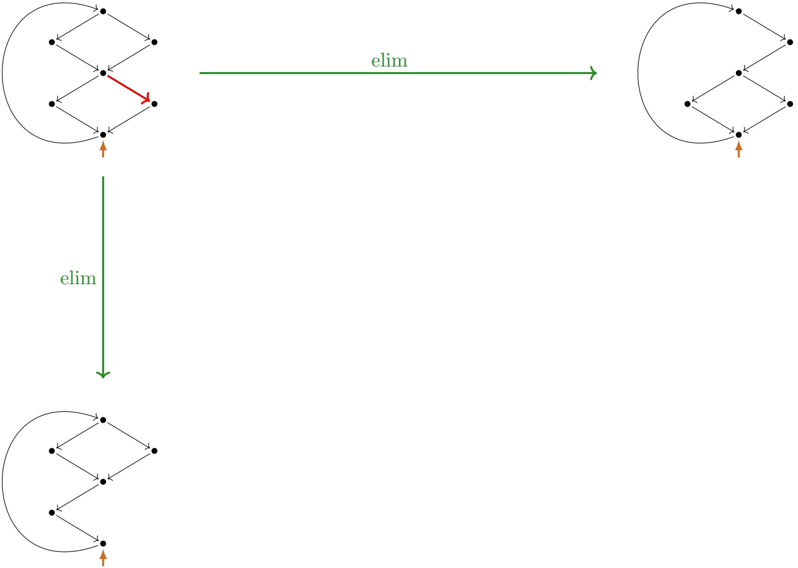

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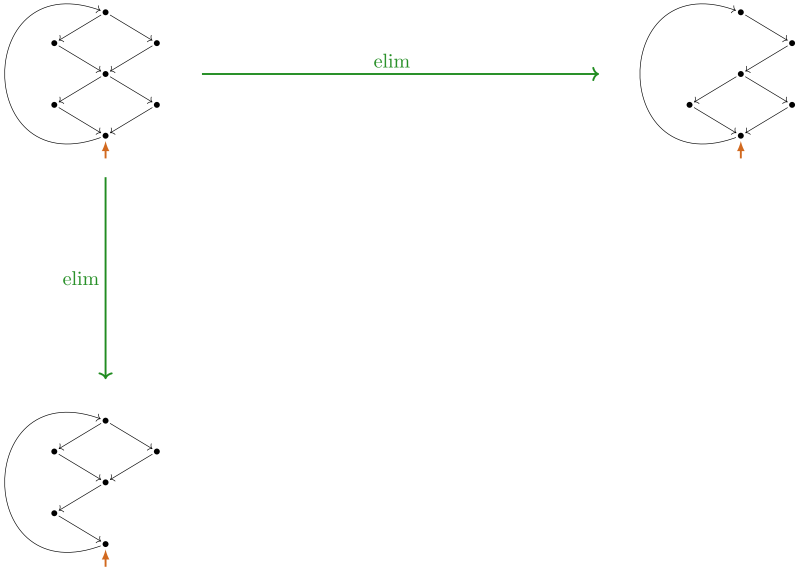

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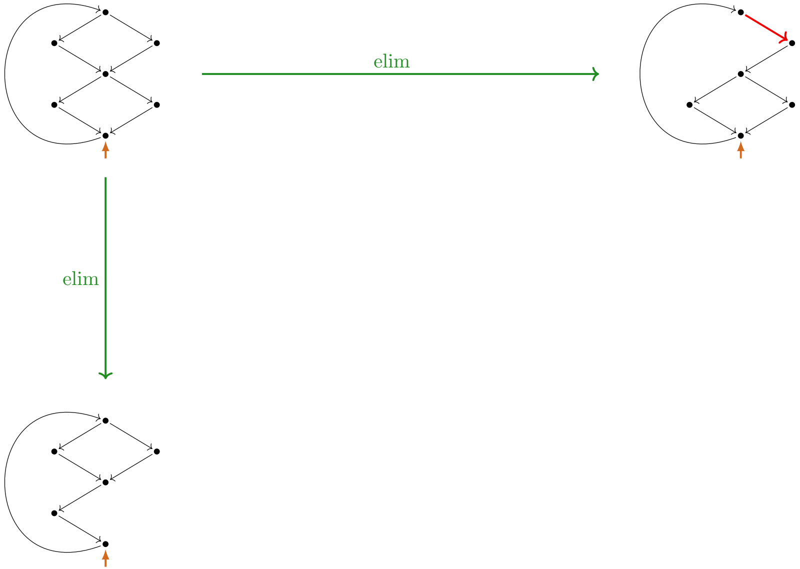

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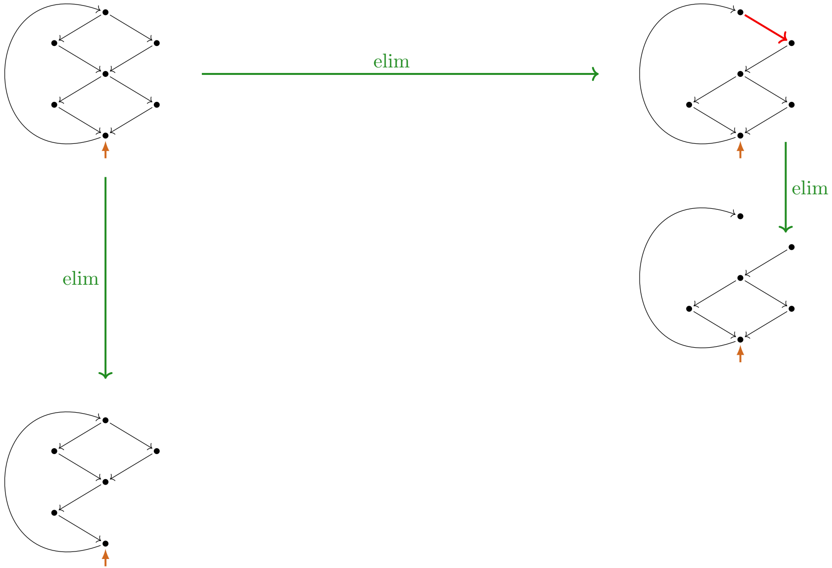

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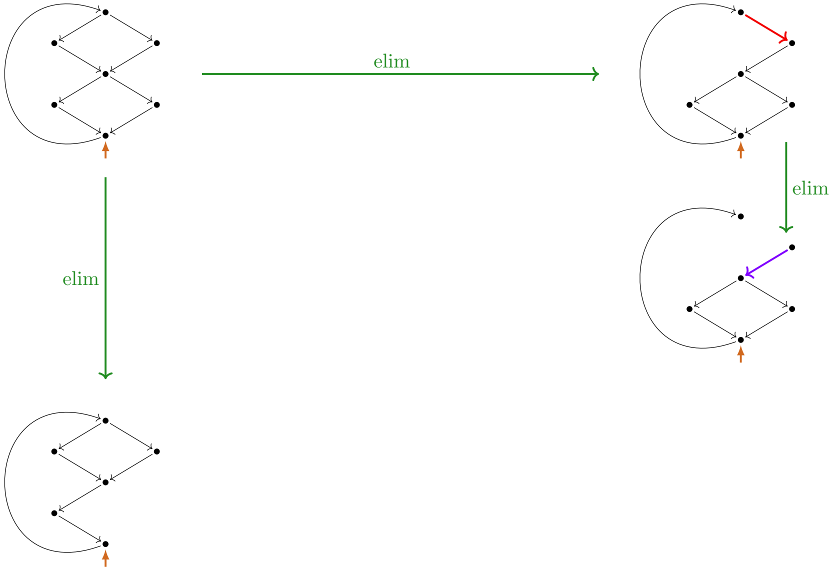

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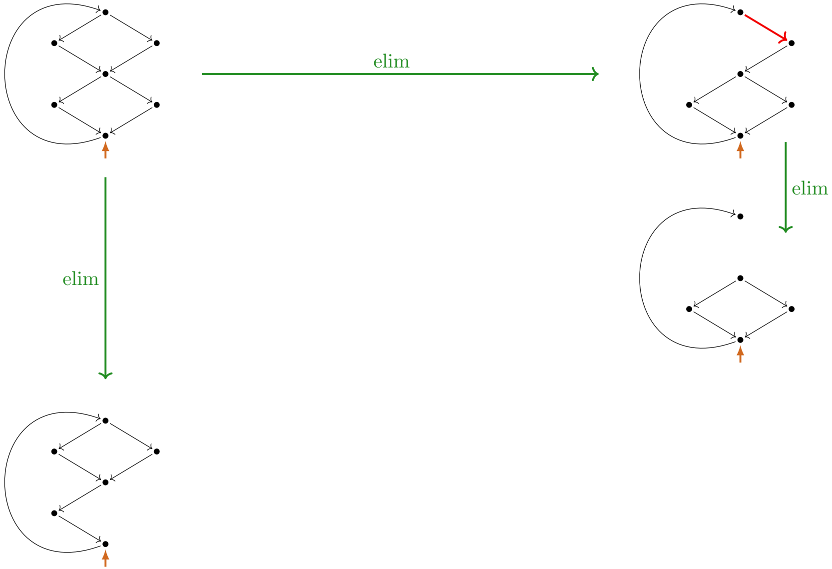

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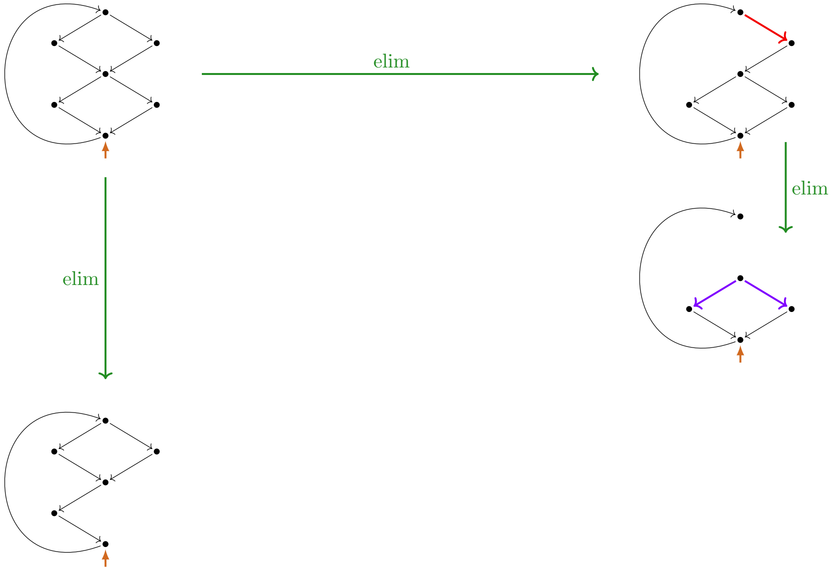

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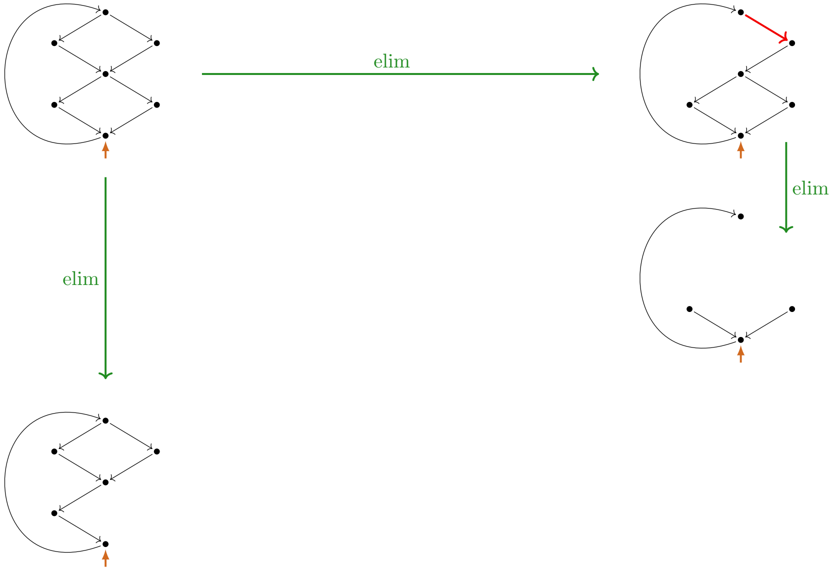

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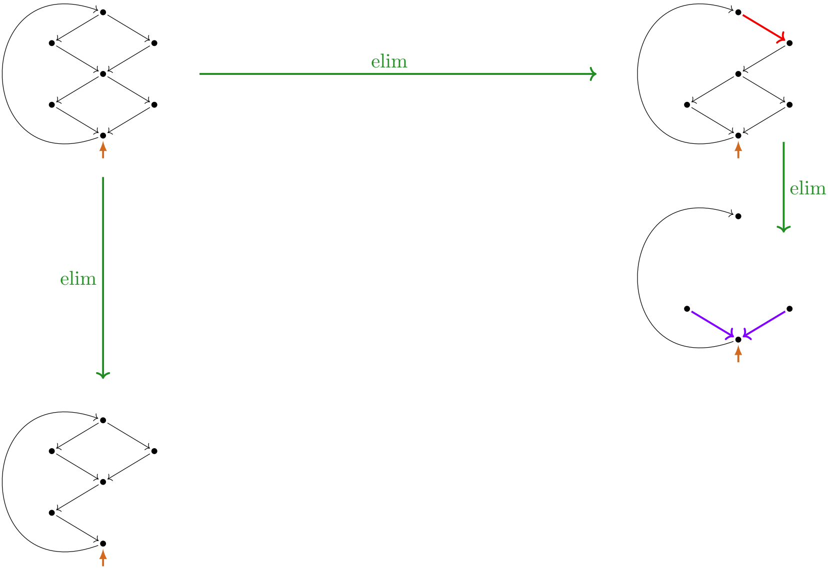

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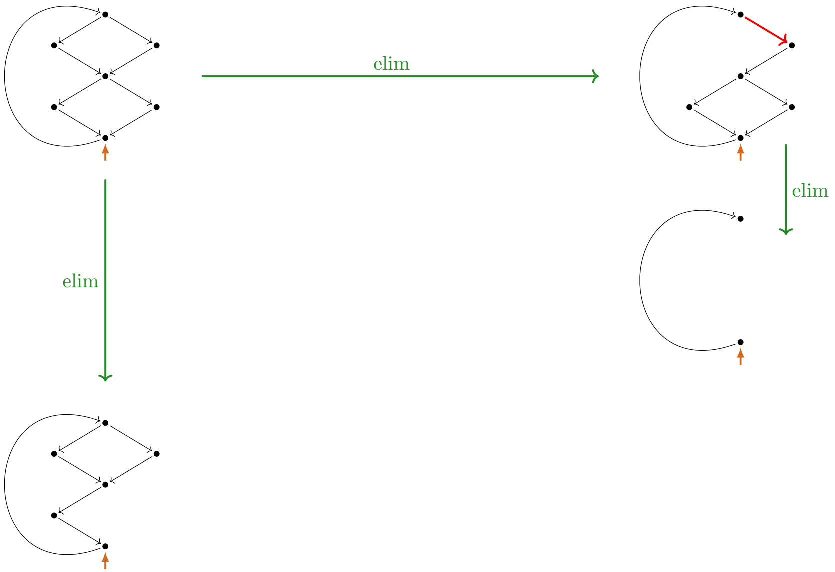

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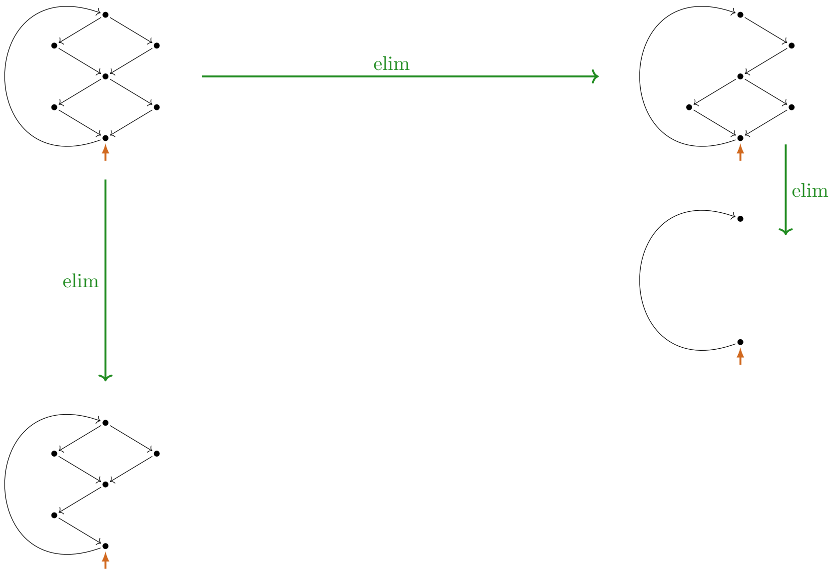

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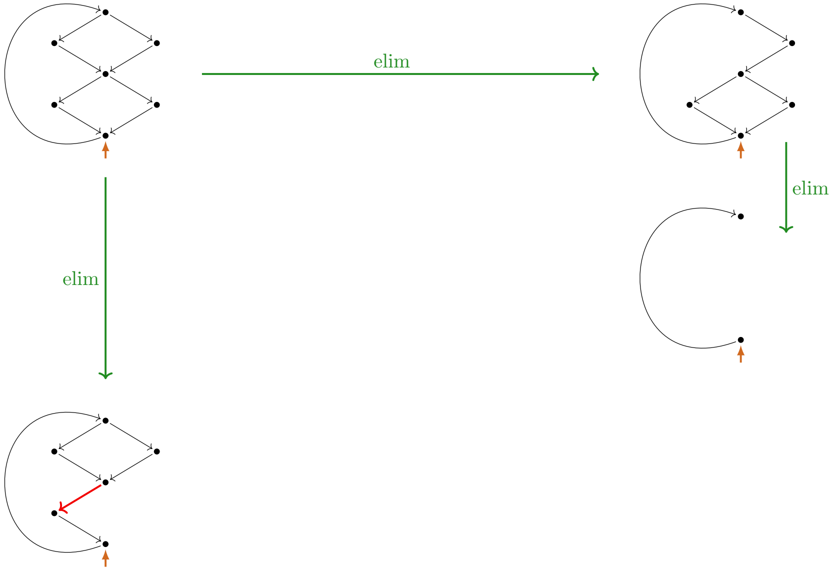

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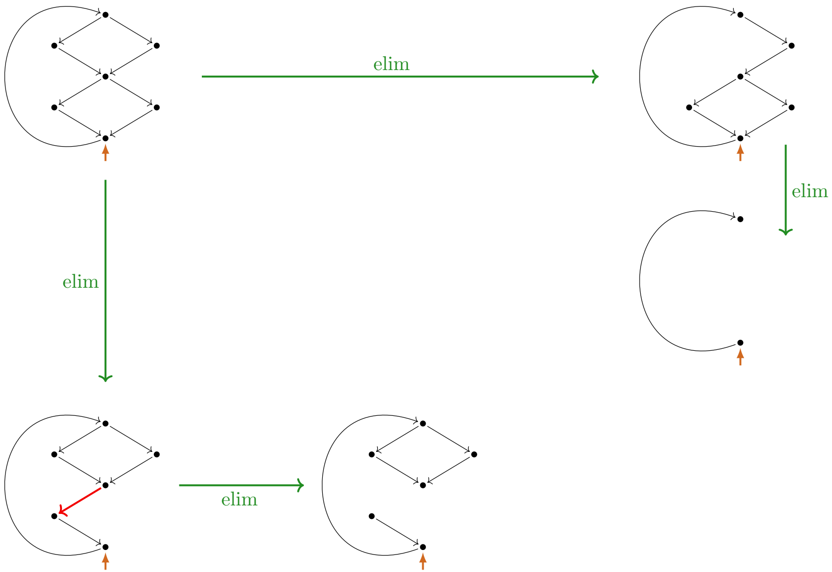

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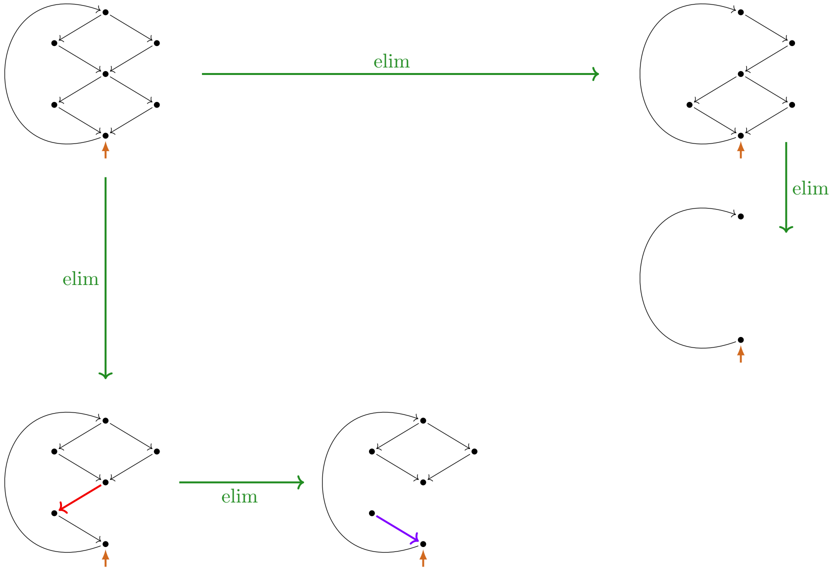

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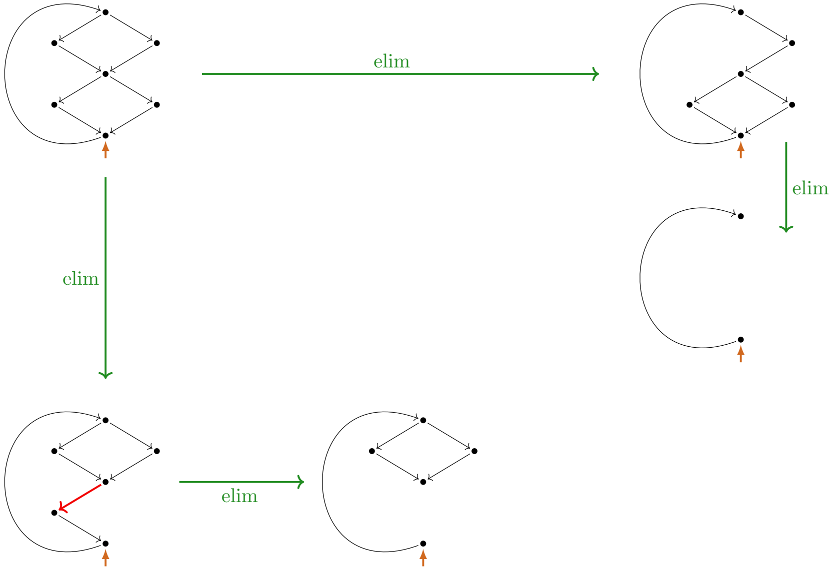

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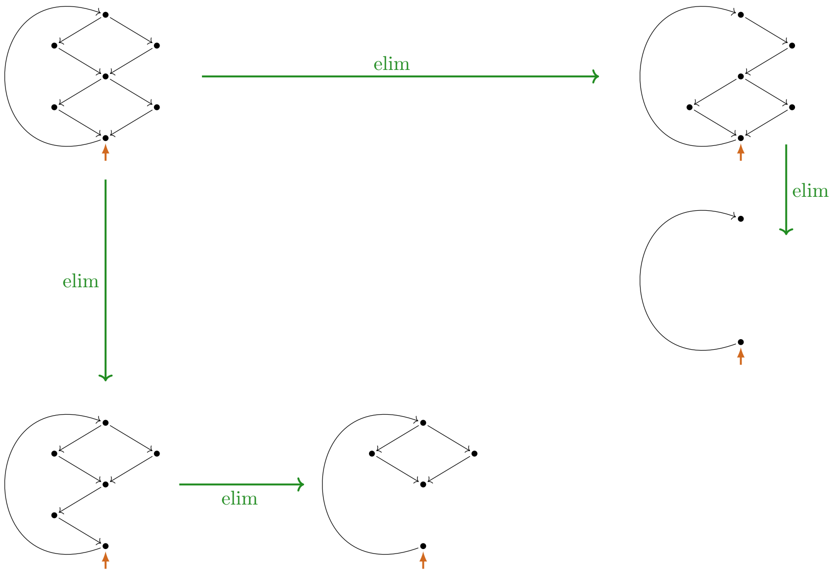

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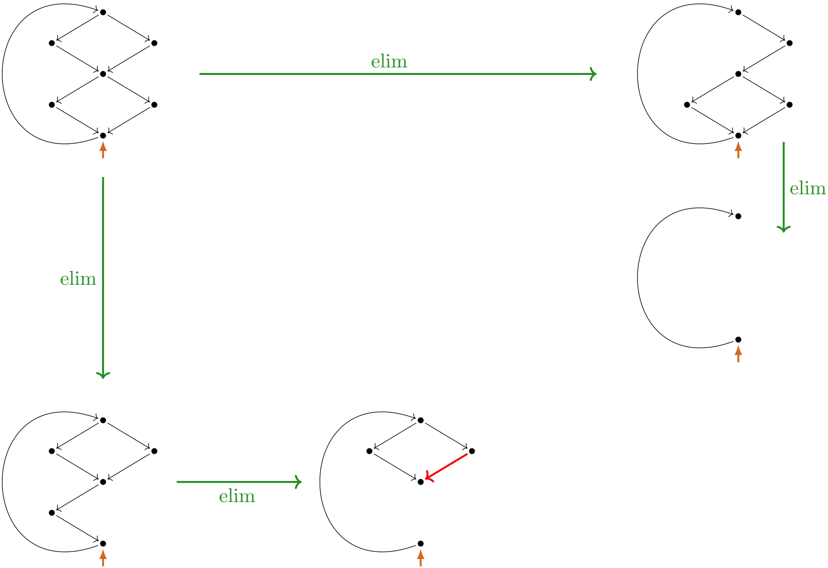

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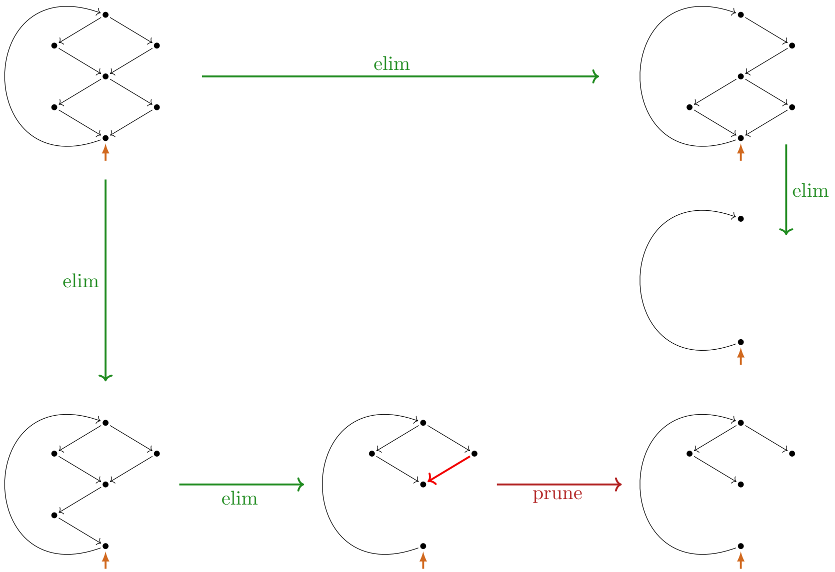

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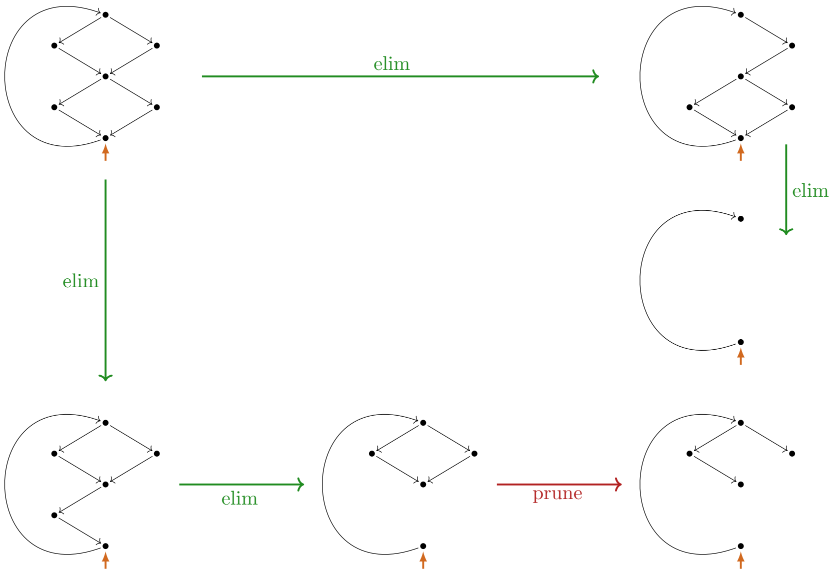

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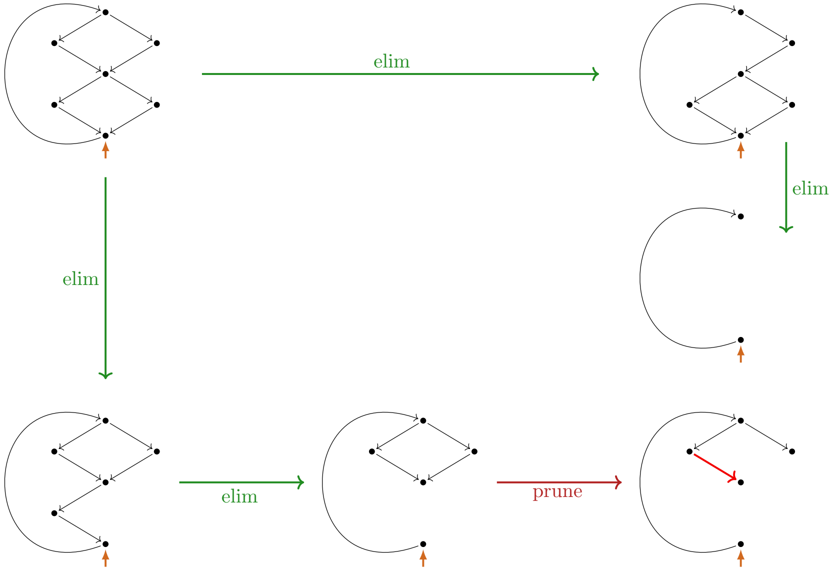

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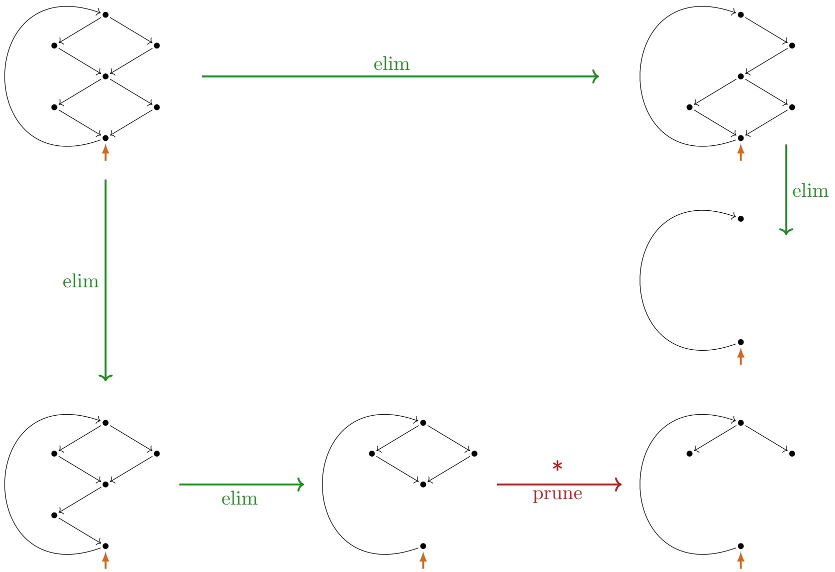

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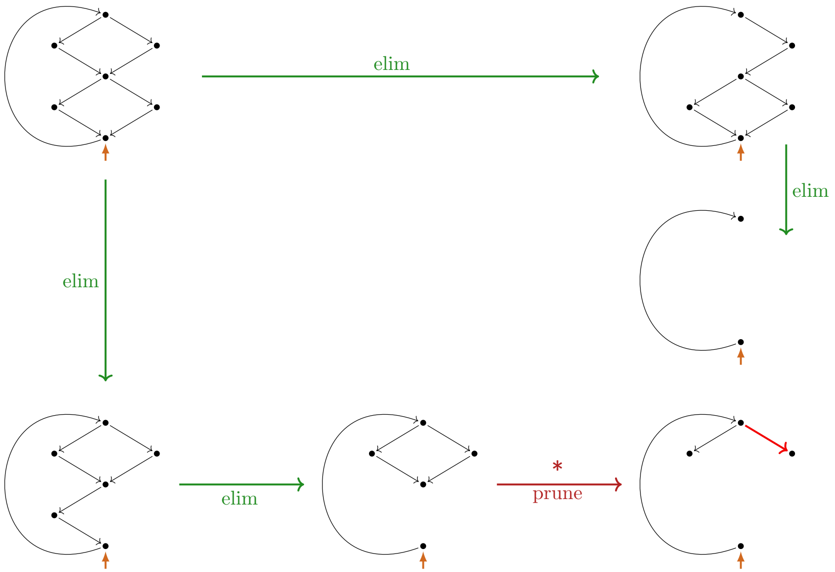

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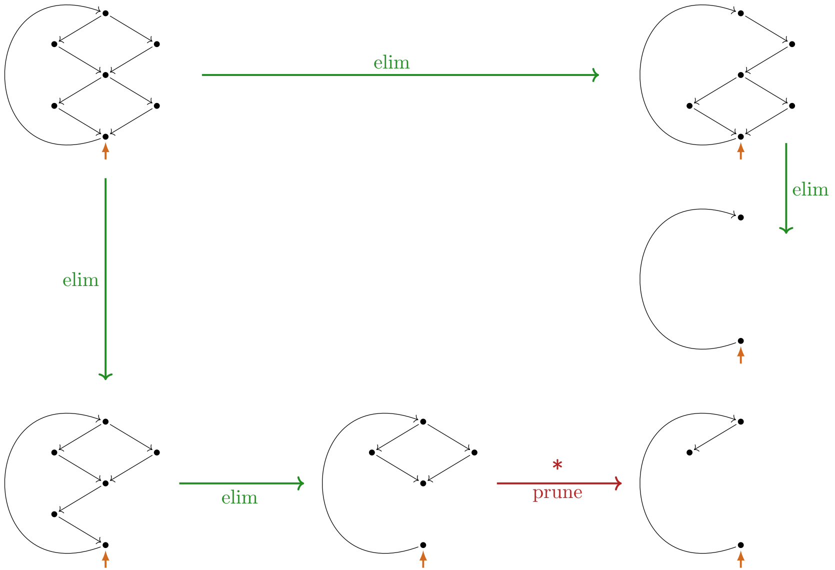

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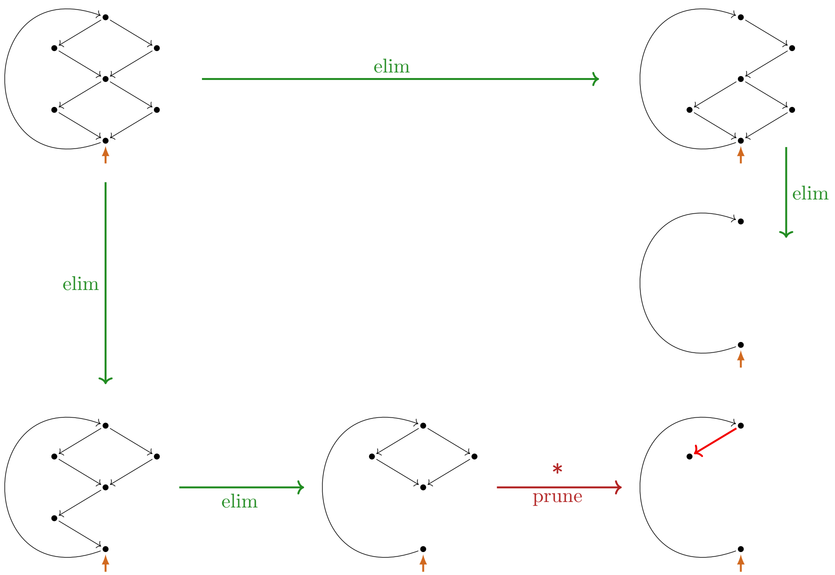

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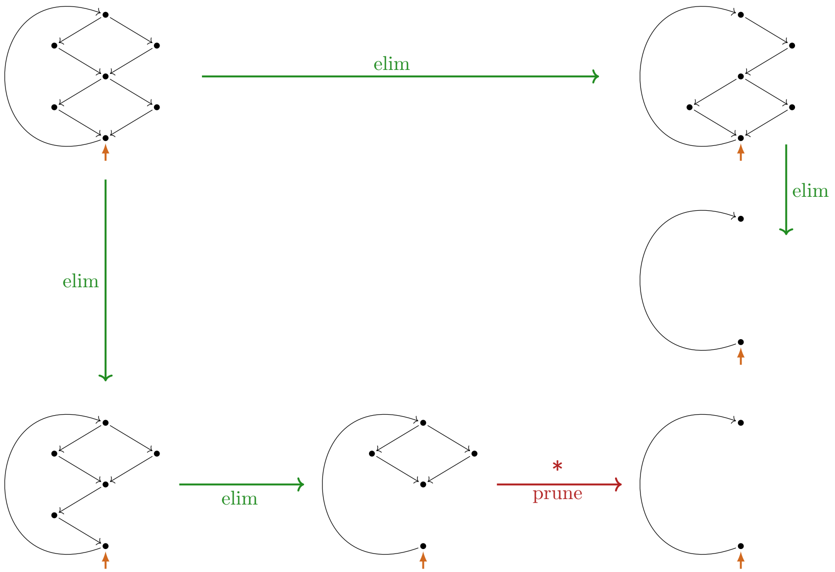

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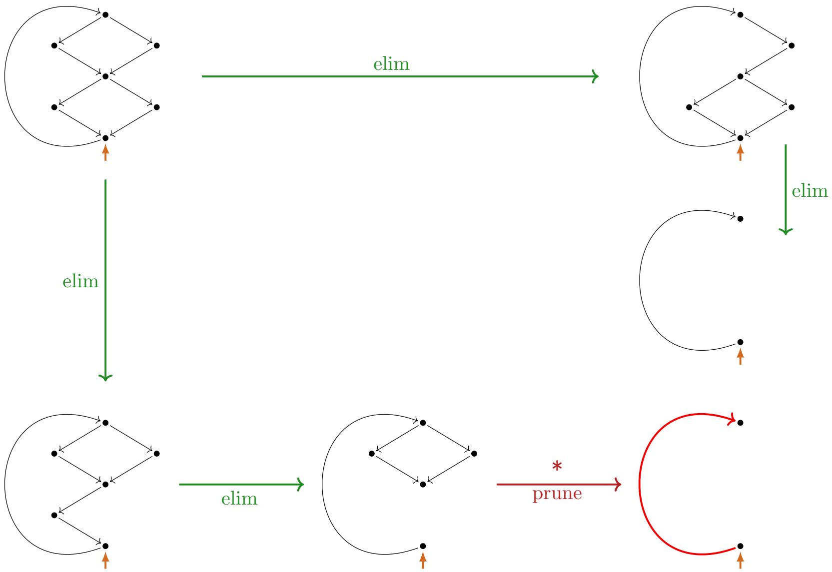

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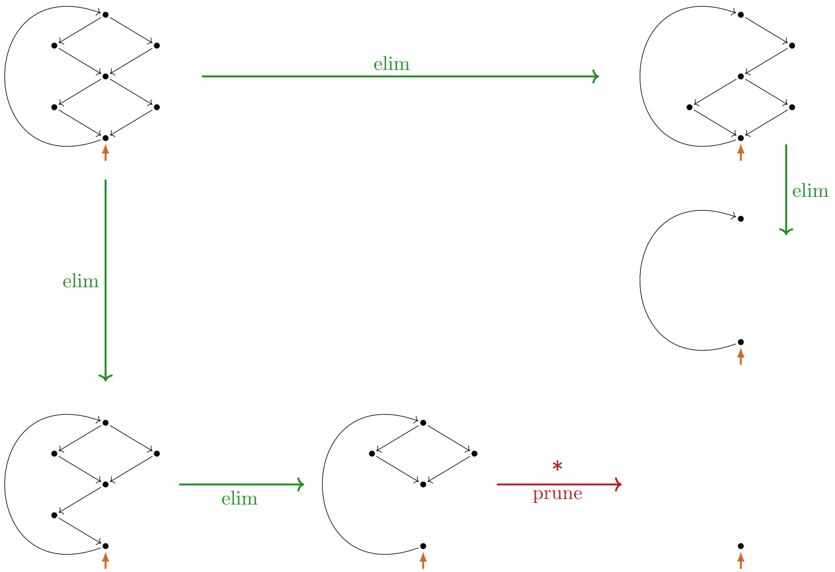

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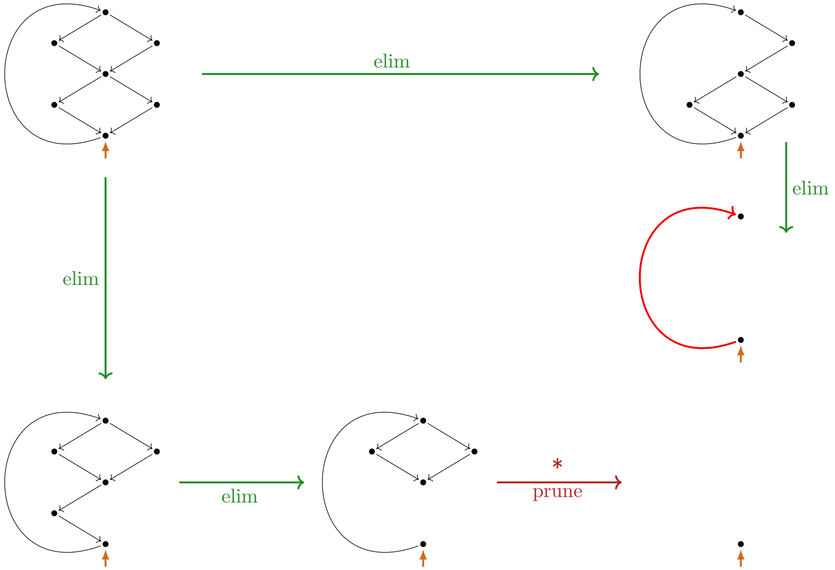

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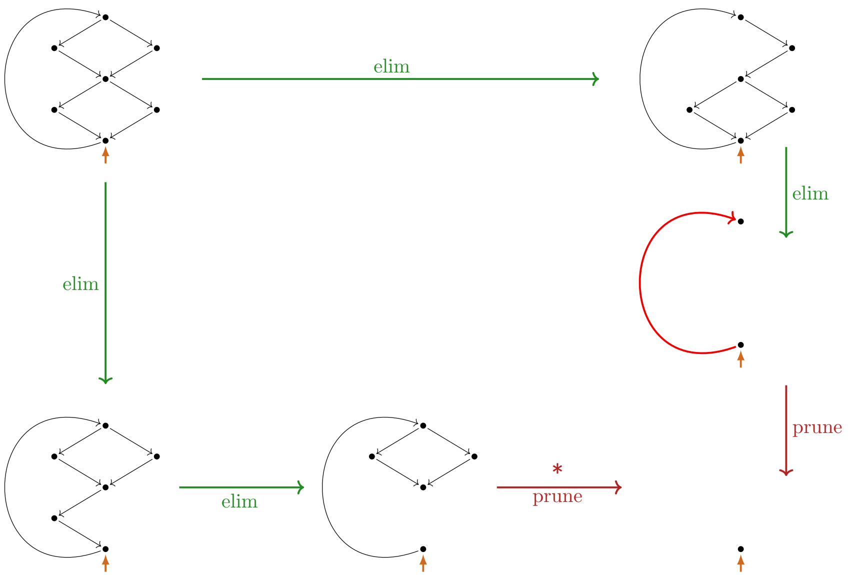

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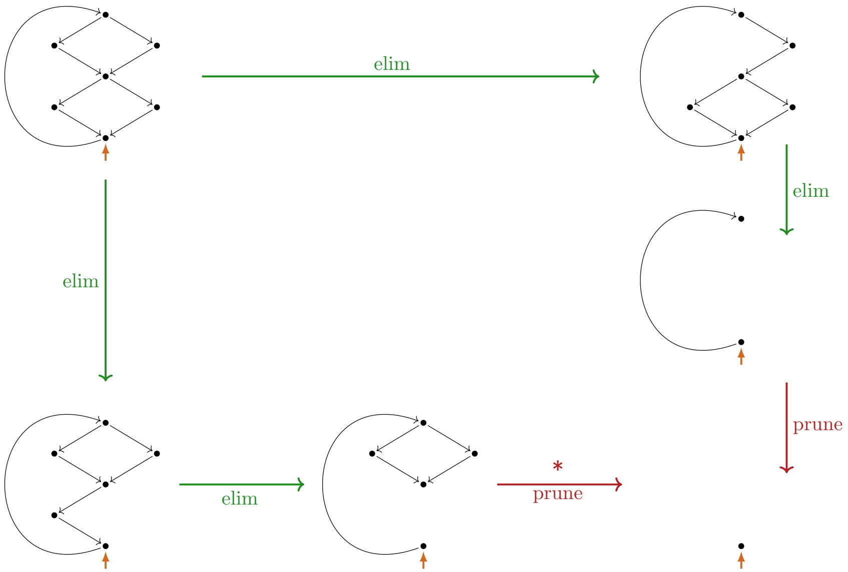

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