Forms of Graph Sharing, and Expressibility of Process Graphs by Regular Expressions

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Overview

Forms of sharing (in term graphs / rooted directed graphs)

- horizontal, vertical, and twisted sharing
- twisted $\Rightarrow$ only vertical sharing: exponential size increase possible

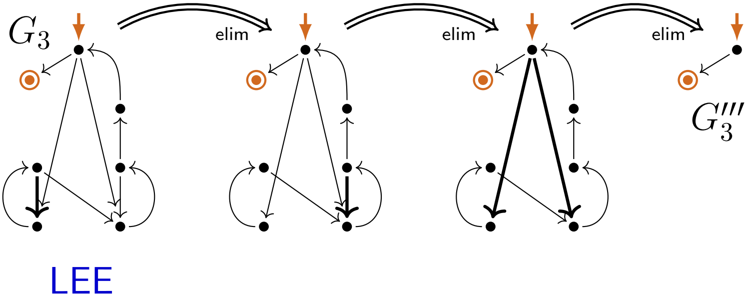
Expressibility of process graphs by regular expressions

- loop-elimination property LEE
  - guarantees expressibility by a regular expression
- process interpretations of reg. expressions do not always satisfy LEE
  - but can be 1-transition refined to guarantee LEE
- expressibility problem of process graphs by regular expressions
- steps to solve the expressibility problem (perhaps) efficiently

Forms of Sharing

Expressibility of process graphs by regular expressions

Loop Existence and Elimination (LEE)



Loop charts (interpretations of innermost iterations)

Definition

A chart is a loop chart if:

(L1) There is an infinite path from the start vertex.

(L2) Every infinite path from the start vertex returns to it.

(L3) Termination is only possible at the start vertex.

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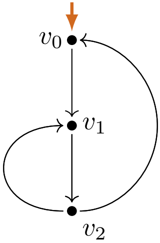

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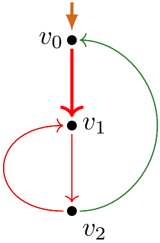

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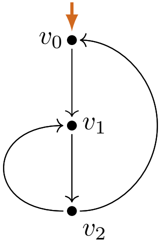

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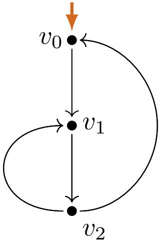

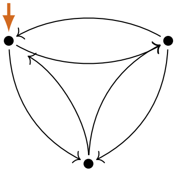

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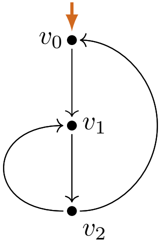

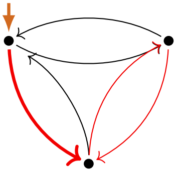

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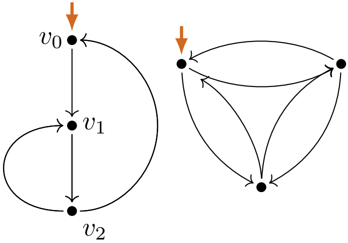

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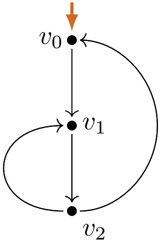

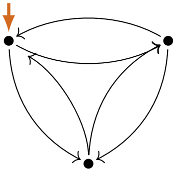

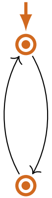

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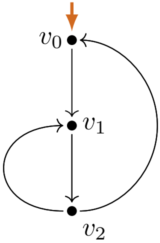

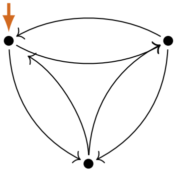

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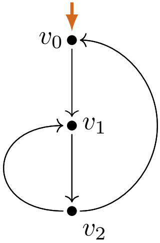

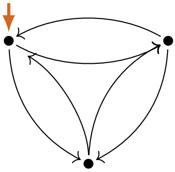

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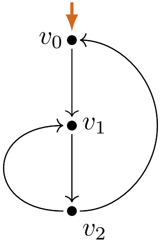

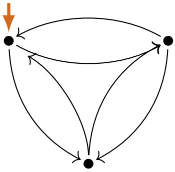

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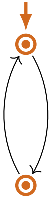

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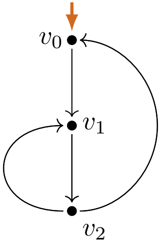

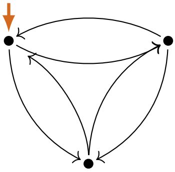

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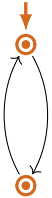

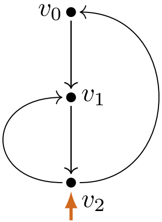

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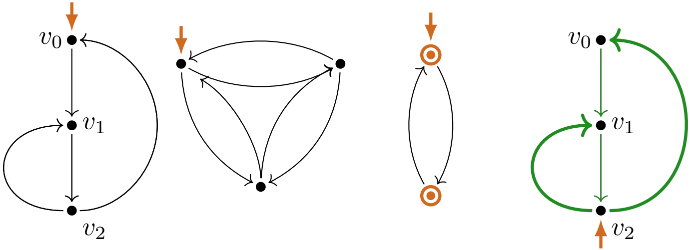

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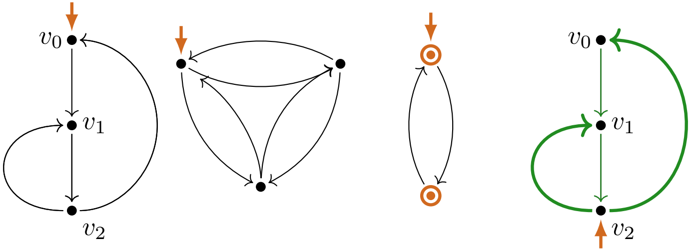

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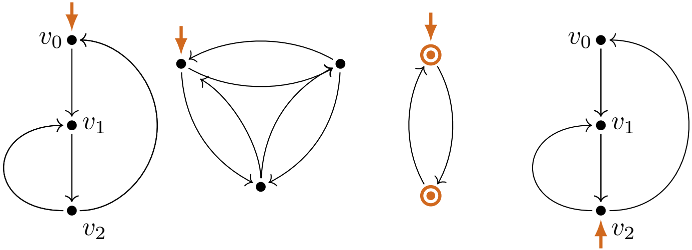

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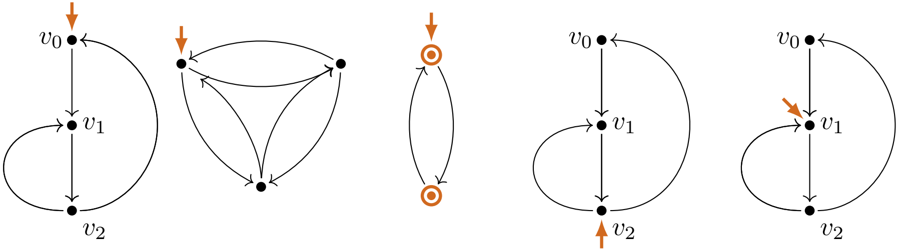

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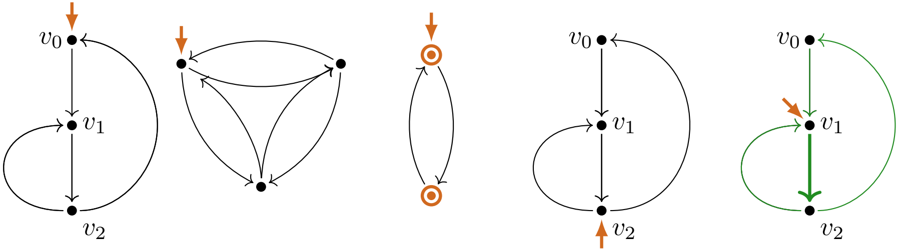

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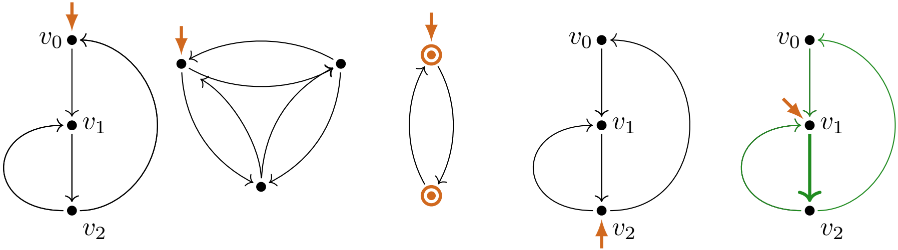

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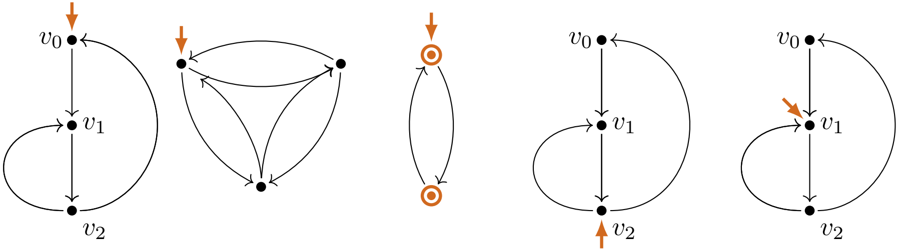

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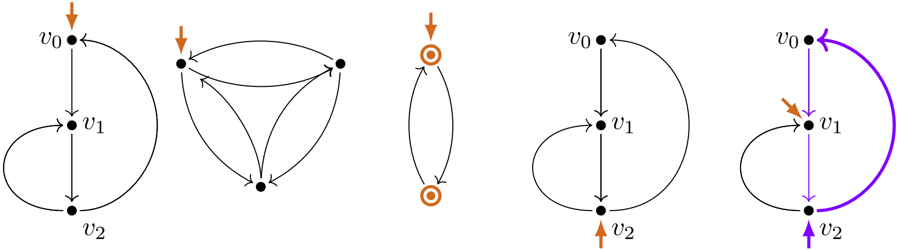
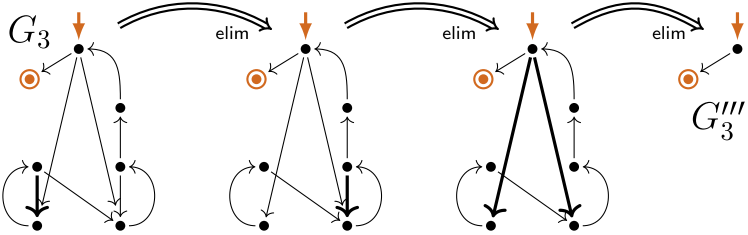


(L1),~~(L2)~~ (L1),(L2),~~(L3)~~ loop chart loop subchart

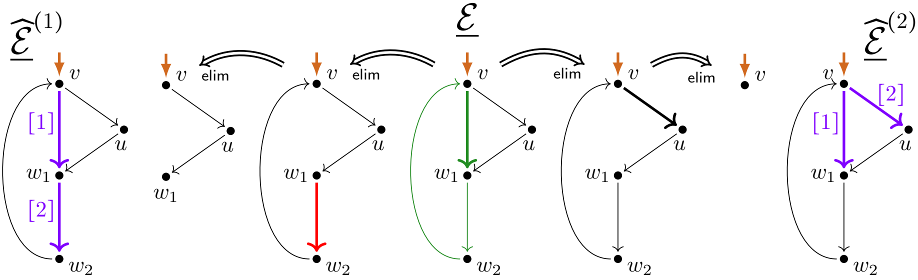
Layered Loop Existence and Elimination (LLEE)



LLEE-chart

LLEE: loop subcharts not eliminated
from bodies of previously eliminated loop subcharts

LEE-witness / layered LEE-witness



LEE-witness

LLEE-witness
layered LEE-witness

Deciding (L)LEE

Proposition

A 1-chart $\underline{\mathcal{C}}$ satisfies LEE if and only if it satisfies LLEE.

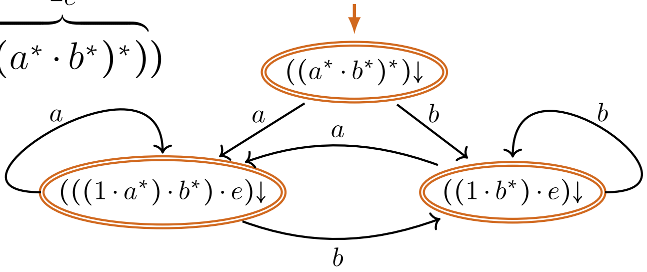
DECIDING-(L)LEE
  Instance: A 1-chart $\underline{\mathcal{C}}$.
  Question: Does $\underline{\mathcal{C}}$ satisfy LLEE?

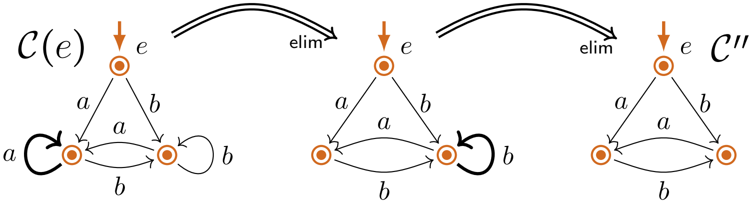
Proposition

DECIDING-(L)LEE $\in$ P.

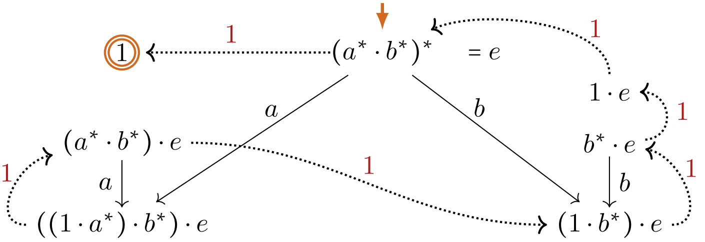
Process interpretations do not always satisfy LEE

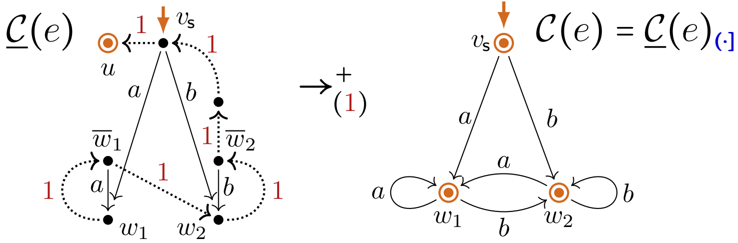
$\mathcal{C}(\underbrace{((a^* \cdot b^*)^*)}_{=e})$



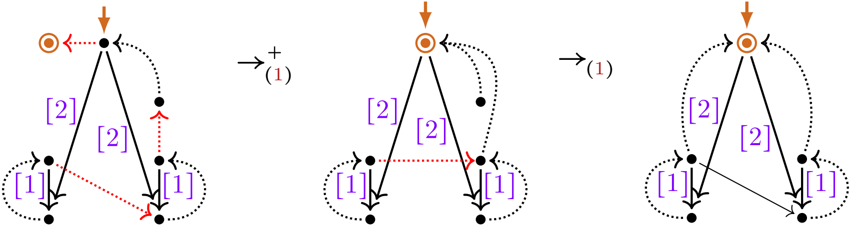


Process interpretations can be refined into LLEE-1-charts





1-Transition reduced LLEE-witnesses

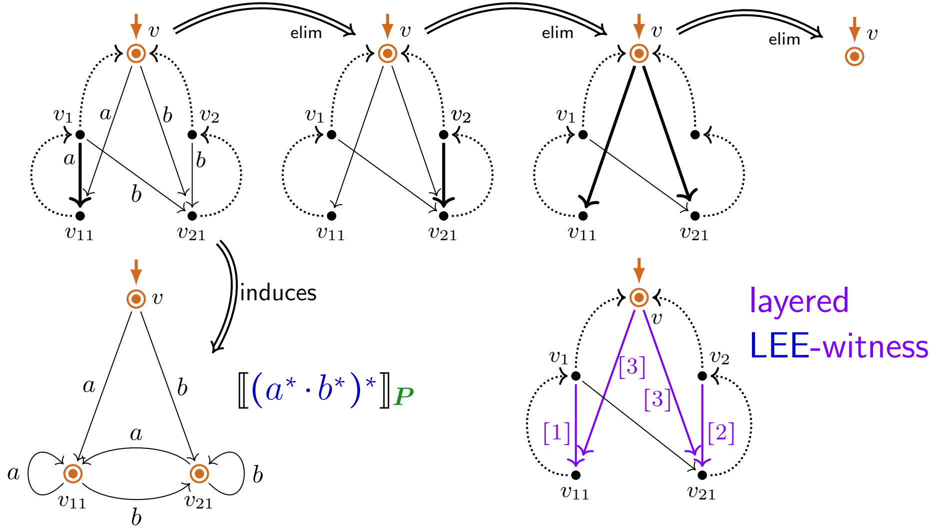


this LLEE-witness
is 1-transition reduced:
only backlinks
are 1-transitions

Lemma

Every LLEE-1-chart $\underline{\mathcal{C}}$ 1-transition refines a LLEE-1-chart $\underline{\mathcal{C}}_r$ that is 1-transition reduced, and it holds $\underline{\mathcal{C}} \to^{*}_{(1)} \underline{\mathcal{C}}_r$.

LEE, and LLEE-witness, induced process graph



$[\![(a^* \cdot b^*)^*]\!]_{P}$

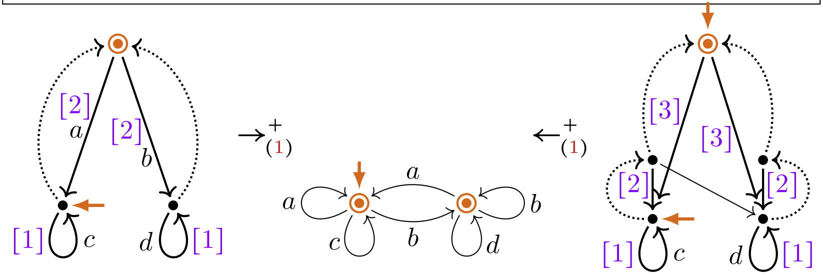
Deciding refinability into a LLEE-1-chart

A 1-chart $\underline{\mathcal{C}}$ is 1-transition refinable into a 1-chart $\underline{\mathcal{C}}'$
if $\underline{\mathcal{C}}' \to^{*}_{(1)} \underline{\mathcal{C}}$ (that is, $\underline{\mathcal{C}}$ arises by 1-transition elimination steps from $\underline{\mathcal{C}}'$).

REFINABILITY-INTO-LLEE-1-CHART

Instance: A 1-chart $\underline{\mathcal{C}}$.

Question: Can $\underline{\mathcal{C}}$ be 1-transition refined into a 1-chart with LLEE?



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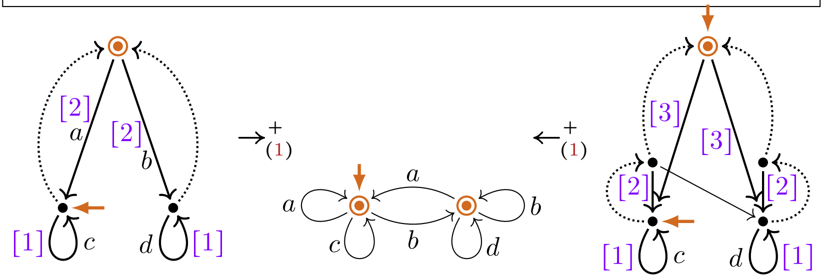

Proposition

REFINABILITY-INTO-LLEE-1-CHART $\in$ P.

Expressibility problem

A chart $\mathcal{C}$ is called expressible by a regular expression modulo bisimilarity
if $\mathcal{C}$ is bisimilar to the process interpretation of a regular expression.

EXPRESSIBILITY-MODULO-BISIMILARITY
Instance: A chart $\mathcal{C}$ (finite process graph).
Question: Is $\mathcal{C}$ expressible by a regular expression modulo bisimilarity?

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Lemma

If a chart $\mathcal{C}$ is refinable into a LLEE-1-chart,

$\Longrightarrow$

$\mathcal{C}$ is expressible by a regular expression modulo bisimilarity.

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$\Longrightarrow$

$\mathcal{C}$ is expressible by a regular expression modulo bisimilarity.

Theorem (Baeten–Corradini–G, 2007)

EXPRESSIBILITY-MODULO-BISIMILARITY *is decidable*
(yet by a (highly) super-exponential decision procedure).

Expandability into crystallized LLEE-1-chart

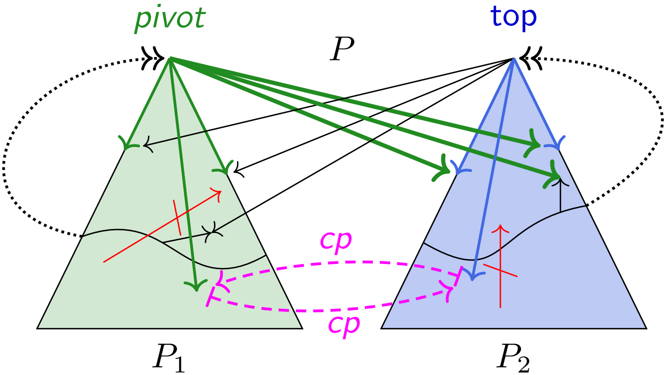
Theorem (follows from LICS'22 article)

A chart $\mathcal{C}$ is expressible by a regular expression modulo bisimilarity

$\Longleftrightarrow$

the bisimulation collapse $\underline{\mathcal{C}}_0$ of $\underline{\mathcal{C}}$
can be expanded into a crystallized LLEE-1-chart $\mathcal{C}_{0,\text{ref}}$
($\mathcal{C}_0$ results from $\mathcal{C}_{0,\text{ref}}$ by 'connect-through' and 1-transition elim. steps).

Twin-Crystal



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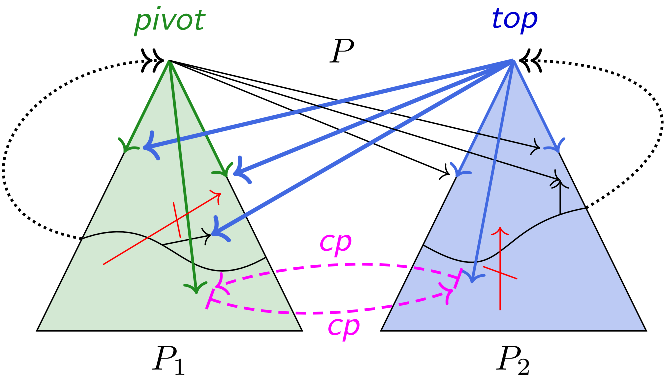

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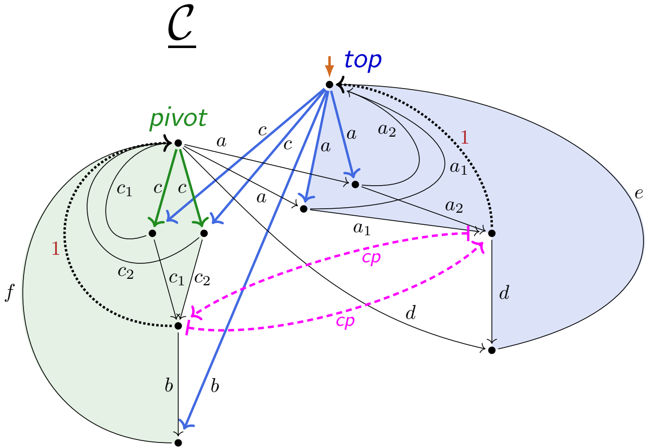

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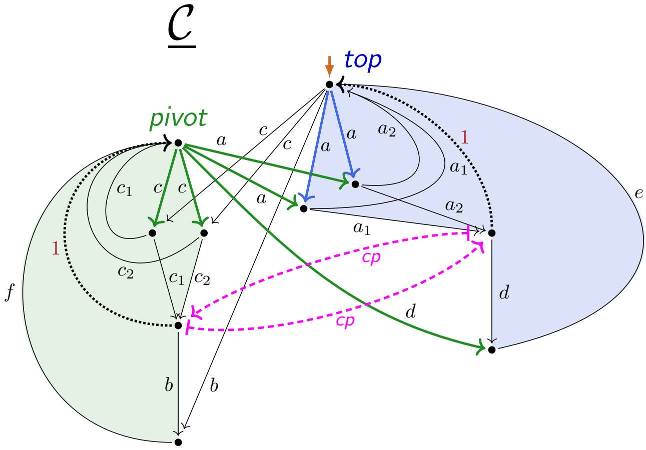
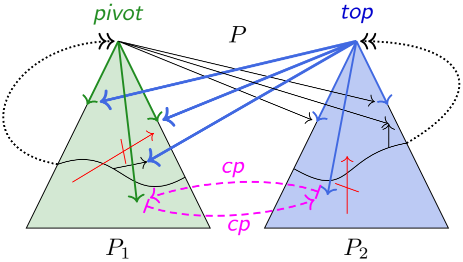


Crystallization

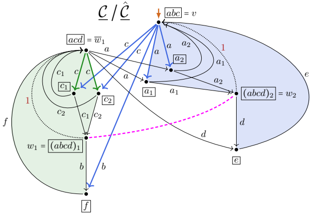


twin-crystal

Crystallized 1-charts = LLEE-1-charts that are collapsed apart from some scc's that are twin-crystals.

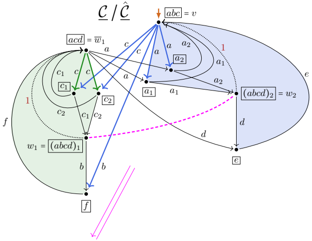
(CR) Crystallization: Every LLEE-1-chart can be reduced under bisimilarity to a bisimilar crystallized 1-chart.

1-Collapses and Bisimulation Collapse of Twin-Crystal

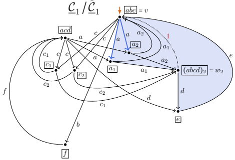


Twin-Crystal

1-Collapses and Bisimulation Collapse of Twin-Crystal

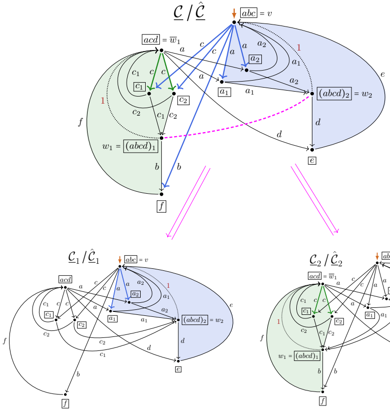


Twin-Crystal



1-Collapses

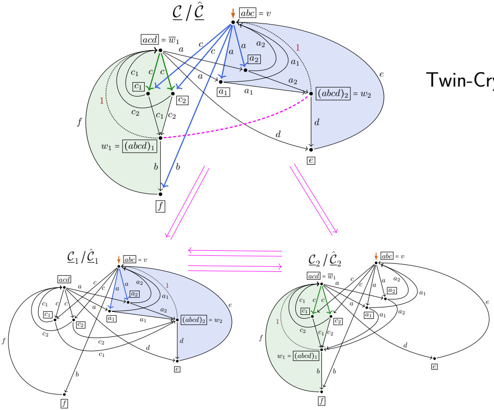
1-Collapses and Bisimulation Collapse of Twin-Crystal



Twin-Crystal

1-Collapses

1-Collapses and Bisimulation Collapse of Twin-Crystal



Twin-Crystal

1-Collapses

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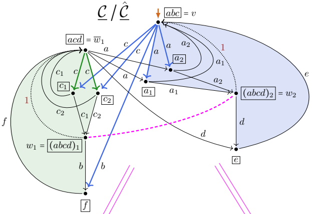
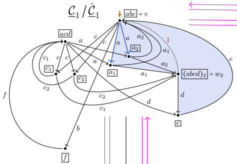
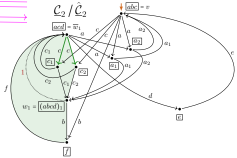
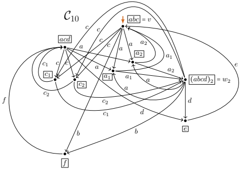


Twin-Crystal



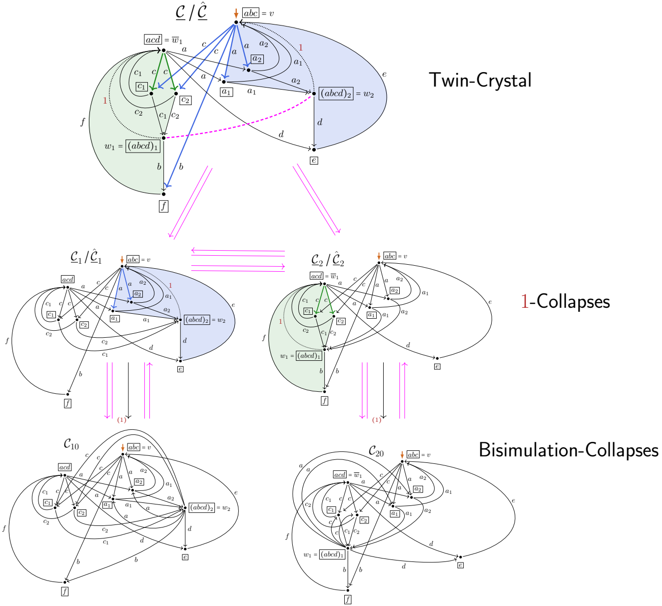


1-Collapses

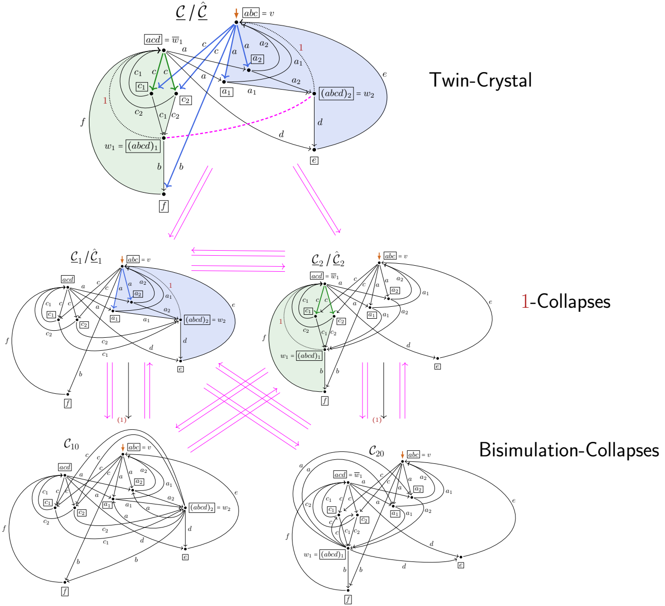


Bisimulation-Collapses

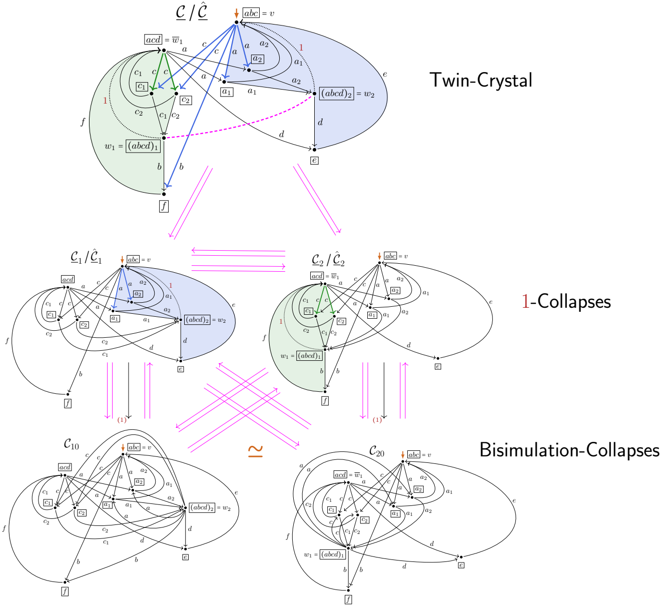
1-Collapses and Bisimulation Collapse of Twin-Crystal



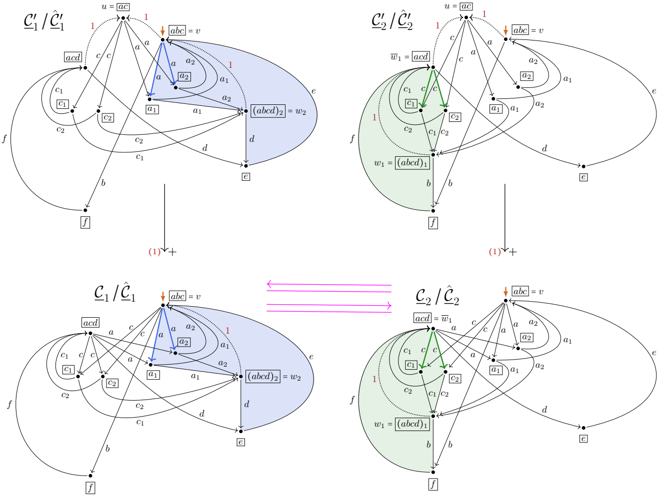
1-Collapses and Bisimulation Collapse of Twin-Crystal



1-Collapses and Bisimulation Collapse of Twin-Crystal



Not 1-transition refinable into LLEE-1-chart



Expandability into crystallized LLEE-1-chart

Theorem (follows from LICS'22 article)

A chart $\mathcal{C}$ is expressible by a regular expression modulo bisimilarity

$\Longleftrightarrow$

the bisimulation collapse $\underline{\mathcal{C}}_0$ of $\underline{\mathcal{C}}$

can be expanded into a crystallized LLEE-1-chart $\mathcal{C}_{0,\mathsf{ref}}$

($\mathcal{C}_0$ results from $\mathcal{C}_{0,\mathsf{ref}}$ by 'connect-through' and 1-transition elim. steps).

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EXPANDABILITY-INTO-CRYSTALLIZED-LLEE-1-CHART

Instance: A bisimulation-collapsed chart $\mathcal{C}$.

Question: Can $\mathcal{C}$ be expanded into a crystallized LLEE-1-chart?

Expandability into crystallized LLEE-1-chart

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Expandability-into-crystallized-LLEE-1-chart **Instance:** A bisimulation-collapsed chart $\mathcal{C}$. **Question:** Can $\mathcal{C}$ be expanded into a crystallized LLEE-1-chart?

Question

Expandability-into-crystallized-LLEE-1-chart $\in$ P ?

Expandability into crystallized LLEE-1-chart

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EXPANDABILITY-INTO-CRYSTALLIZED-LLEE-1-CHART
Instance: A bisimulation-collapsed chart $\mathcal{C}$.
Question: Can $\mathcal{C}$ be expanded into a crystallized LLEE-1-chart?

Question

EXPANDABILITY-INTO-CRYSTALLIZED-LLEE-1-CHART $\in$ P ?

Conjecture

p-EXPANDABILITY-INTO-CRYSTALLIZED-LLEE-1-CHART $\in$ FPT,
with the maximum outdegree of vertices of $\underline{\mathcal{C}}$ as parameter.

Aims and questions

Articles

- motivation of crystallization
- crystallization procedure

Tool implementation

- first step: efficiently deciding refinability into a LLEE-1-chart
- second step (envisaged):
  - deciding expandability
    of a given collapsed process graph into a crystallized LLEE-1-chart

Questions

- relation with attribute grammars?
- examples, where efficient local manipulation or evaluation of process graphs with twisted sharing is used/would be advantageous?

Summary

Forms of sharing (in term graphs / rooted directed graphs)

- horizontal, vertical, and twisted sharing
- twisted ⇒ only vertical sharing: exponential size increase possible

Expressibility of process graphs by regular expressions

- loop-elimination properties LEE and LLEE
  - cover *many* graphs with twisted sharing
- process interpretations of reg. expressions do not always satisfy LEE, but can be 1-transition refined to guarantee LEE
- expressibility problem of process graphs by regular expressions
- steps to solve the expressibility problem (perhaps) efficiently:
  - deciding refinability by 1-transitions to a graph with LEE
  - using crystallization to solve the expressibility problem